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ABSTRACT

At present, most of the structural dynamic design modifications are being done primarily by the use of analytical finite element modelling techniques. Design verifications and testing are accomplished by tests on the full scale rotorcraft with and without modifications. This limits the options for such design modifications. The responsibility primarily lies on the analytical modelling capability. In this paper, a different approach that complements the analytical modelling has been suggested. This approach is by the use of structural dynamic physical scale models, optimization techniques with frequency constraints and general structural dynamic system identification techniques.

INTRODUCTION

Structural dynamic design modifications of rotorcraft become necessary for a variety of different reasons (1-7). For example, Prouty and Amer (1) have discussed several design modifications that were incorporated into the original design of YAH-64. These design modifications were necessary to correct various technical problems that were not recognized until the rotorcraft was flight tested. The particular modifications discussed in reference (1) were to the empenage and the tail rotor. During these modifications, it was necessary to address some structural dynamic design problems. A propoosed 'T' tail design resulted in mode shapes and frequencies that were very close to some excitation frequencies. The design objective was then to obtain satisfactory frequency separations. The procedure used was as follows. Flight tests were performed with different proposed design modifications and a configuration that yielded the most satisfactory result was selected. It is very difficult and expensive to achieve an optimum design modification by following such a procedure.

In a different application, King (2) has discussed a modal approach to structural modifications. His objective is to reduce the undesirable vibration characteristics. He has developed a simple algorithm that is capable of estimating changes to normal modes and natural frequencies when the modes and frequencies of the original structure and the structural changes are specified. Effects of damping and deviations of experimental results from analytical results have not been considered. The method provides a tool for preliminary analysis. It is also to be noted that this method depends primarily on the analytical modelling capabilities. In reference (3), Wang, Kitis, Pilkey, and Palazzolo have discussed a numerical method for the calculation of the frequency response of a vibrating structure as the design changes are incorporated into the structure. This technique and other predecessors (6-11) depend on numerical techniques like finite element methods and the resulting analytical models.

Viswanathan and Myers (4) have discussed design modifications to 206 L Bell Helicopter. A particular Pylon support system called SAVITAD has been designed and and incorporated as a design modification. The design modification has been accomplished by using a mathematical model and flight test verifications. In connection with "wing mounted store" designs, Smith and Wei `5 have presented a technique to predict the structural dynamic behavior of a given system by combining the experimental and the analytical models. Further needed developments and validations of the technique for conducting structural design modifications have not been discussed in this paper. In particular, the procedure discussed in the paper is to combine analytically developed helicopter finite element models with experimentally generated frequencies and mode shapes of "wing stores". The required experimental data on wing stores have been obtained by supporting the wing stores at one end and testing the system by using modal testing techniques. The modal data for the store has been converted to mass and stiffness matrices by using Berman and Flannely's theory developed in their 1971 paper (11-18). These mass and stiffness matrices are then combined with the helicopter finite element model by using modal synthesis techniques. In this approach, possible deviations of the observed helicopter mode shapes and frequencies from the finite element model have been assumed to be negligible. Even though the wing stores are tested to obtain the structural dynamic parameters, the design modifications still rely on the finite element model for the helicopter and analytical predictions by modal synthesis techniques.

In this paper, a slightly different approach has been proposed for structural dynamic design modifications. The proposed approach is to use recent developments in the field of structural dynamic system identification techniques, optimization techniques with frequency constraints and structural dynamic physical scale models along with the analytical finite element models and flight tests. The proposed approach has the potential of providing near optimum designs before flight tests.

THE PROCEDURE

As a first step, it is assumed that the required design modifications, applicable design constraints and the design objectives are specified. As an example, a very simple design modification problem of a specific helicopter is stated as follows. This helicopter is assumed to have a wing of a specified span and a specified cantilevered length. The current design is also assumed to contain stores on each wing. The required design modification is (a) to increase the weight of the store at a location or (b) to consider adding wing mounted stores at other locations. A constraint of concern is the dynamic behavior of the complete helicopter system. In particular, it is required that the new natural frequencies of the system resulting from the design modifications are not in the proximity of the exciting frequencies. Other constraints are to assure that the stores are supported by members with sufficient strength and desired safety margins. The design objective is to achieve the required design modification with a minimum increase of weight.

It is assumed that an analytical finite element model for the helicopter and some test results on the complete helicopter are available. In particular mode shapes of natural frequencies of interest are assumed to have been measured. In general, these natural frequencies and mode shapes are in the form of complex numbers. The measured frequencies and mode shapes need not agree with those predicted from the finite element model. It is also assumed that the exciting frequencies have been specified.

First step is to improve the finite element model to agree with the experimental results by using structural dynamic system identification techniques (12-18). Recent developments in structural dynamic system identification techniques are discussed in the next section. The next step is to obtain a simpler finite element model that addresses the particular design modification problem. For the example problem in this paper the first step has been eliminated and a simple finite element model has been directly obtained. At this stage, one option is to proceed to study the required design modifications and optimizations by using the simple analytical model that accurately reproduces the observed experimental results. The specific optimization procedure and the selection of a specific design modification are discussed in a subsequent section with reference to the selected example problem.

A second design option is to use structural dynamic physical scale models. In the paper, they are simply called as the scale models. A simple scale model is designed and fabricated from a knowledge of the existing geometry, analytical finite element models and experimental results (18). The optimum design changes are then incorporated as changes into the scale model. The scale model is tested and the results interpreted for the full scale structure to evaluate if the design constraints are achieved. If the identified analytical model and the interpreted scale model test results do not agree, an iterative design modification is necessary before flight tests are initiated. The use of the scale model in the structural dynamic design modification is also illustrated with reference to an example problem.

STRUCTURAL DYNAMIC SYSTEM IDENTIFICATION

In scientific developments, observations have resulted in the development of mathematical models. Experiments have been later systematically designed and conducted to validate and improve the mathematical models. However, in many engineering applications including the field of structural dynamics, analytical models are usually first developed and solved for specific cases. Selected experiments are later designed and conducted to verify the models. The structural dynamic system identification techniques, however, provide tools to use the experimental data and the preliminary analytical model to obtain an improved or identified model that reproduces the experimental data while retaining the required assumptions and relevant physics. Even though the subject of structural dynamic system identification is relatively new there has been a considerable amount of research activity in this field (14, 15). The references (14 and 15) provide a good survey and limitations of the state-of-the-art of structural dynamic system identification. One of the problems that has attracted the attention of research workers in this field is to obtain improved mass, stiffness and damping matrices of a linear model from measured natural frequencies and mode shapes which may be complex in general. Recently two articles have appeared in this field. The reference (18) is used in this paper.

WING STORE DESIGN MODIFICATION

It is assumed that the analytical finite element model for the helicopter is represented in the following form.

$$M_H \ddot{q} + K_H \ddot{q} = f \qquad (1)$$

This model usually contains a large number of degrees of freedom. It is also assumed that the modal test information on the helicopter is available in the form of natural frequencies ω_i and mode shapes ϕ_j only. Some of the frequencies correspond to wing deformations. Instead of a complicated model with large number of degrees of freedom, it is possible to obtain a simple model for the helicopter, wing and store combination. Of course, this simple model is expected to yield the significant measured natural frequencies and mode shapes that correspond to using deformations. As a first step, a model similar to that shown in figure 1 is considered. For simplicity the mass of the helicopter is assumed to be a nonstructured mass at the center of the wing. This simple model is capable of yielding frequencies and mode shapes that are approximately equal to the measured values. A further simplification is considered by considering the wing in the form shown in Figure 2 with spring supports instead of a large nonstructured mass. Then, it is possible to obtain an analytical model with very few degrees of freedom instead of a full scale model with large number of degrees of freedom.

$$M_R \ddot{q} + K_R \ddot{q} = f \tag{2}$$

In many cases, the natural frequencies and mode shapes that are calculated from this model are only approximately equal to the measured values. A comparison of natural frequencies are discussed in the section on numerical results.

An identification procedure (18) is now used to improve the model to reproduce the significant natural frequencies and mode shapes of the full scale structure that have been obtained experimentally and that concern wing deformations. The details of the procedure are discussed in reference (18) by the authors of this paper. For purposes of this application, the procedure is discussed as follows. From the complex eigenvalues and mode shapes the following equations are formulated

$$MZF^2 + CZF + KZ = 0 \tag{3}$$

where

$$Z = \begin{matrix} Re(\phi_1)\, Re(\phi_2) \ldots Re(\phi_n) \\ Im(\phi_1)\, Im(\phi_2) \ldots Im(\phi_n) \end{matrix} \tag{4}$$

$$F = \begin{bmatrix} \Lambda_R & \Lambda_I \\ -\Lambda_I & \Lambda_R \end{bmatrix} \tag{5}$$

$$\Lambda_R = \text{diag} \quad Re(\lambda_1)\, Re(\lambda_2) \ldots Re(\lambda_n) \tag{6}$$

$$\Lambda_I = \text{diag} \quad Im(\lambda_1)\, Im(\lambda_2) \ldots Im(\lambda_n) \tag{7}$$

$$L_1 = ZF^2(F^T)^2Z^T, L_2 = ZF(F^T)^2Z^T \quad (8)$$

$$L_3 = Z(F^T)^2Z^T, L_4 = ZFF^TZ^T \quad (9)$$

$$L_5 = ZF^TZ^T, L_6 = ZZ^T \quad (10)$$

By assuming the diagonal elements of the mass matrix to be the same as that of the apriori the mass matrix, matrix, equations (3) are solved for the remaining coefficients of the mass matrix, the stiffness matrix and the damping matrix. In these equations the quantities ϕ_i represent the measured mode shapes. λ_i are used to denote the complex measured natural frequencies. As discussed in the Reference 18 , the suggested identification technique satisfies the appropriate symmetry and orthogonality conditions. Symbolically the identified model is written as follows

$$M_{RI}\ \ddot{q} + C_{RI}\ \dot{q} + K_{RI}\ q = f \quad (11)$$

If the damping is considered to be not important, the identification procedure of reference 18 can still be used without the damping terms. The required computations are reduced.

The model represented by equation (11) is used in structural modifications.

The design modification has been achieved by modifying methods developed by Khot , Hanagud and Chattopadhyay ' Hanagud, Meyyappa and Craig . The problem is that of finding the changes in cross sectional areas of members, changes in the nonstructured masses and their locations that satisfy the frequency constraints and minimize the weight of the structure. A summary of the procedure is discussed here. The objective is to minimize the weight

$$Ws = \sum_{i=1}^{N} m_i A_i l_i \quad (12)$$

subject to the following contraints

$$\omega_i^2 = \bar{\omega}_i^2 + \alpha_i^2 \qquad i = 1 \dots q \quad (13)$$

In these equations m_i is the weight density of the material, A_i is the cross sectional area of the i^{th} member, l_i is the length of the finite element, ω_i are the natural frequencies, $\bar{\omega}_i$ are the exciting frequencies and α_i are the tolerances allowed in the design. Following a finite element technique, the natural frequency of the modified structure in terms of the discretized variables are as follows

$$\omega_i^2 = \frac{\phi_i^T\ (K + \Delta K)\ \phi_i}{\phi_i^T\ (M + \bar{M} + M_1 + \Delta M_1)\ \phi_i} \quad (14)$$

In these equations ϕ_i are the mode shapes and K is the original structural stiffness matrix from equations (11), ΔK is the stiffness change matrix, ΔM_I is the change in structural mass matrix, M_I includes the changes in nonstructured mass, $\bar{M}$ is the original nonstructured mass matrix. An option has been introduced to consider the changes in the dynamic stiffnesses from static stiffnesses.

By differentiating equation (12) with respect to the design variables subject to the constraint equations (13) and (14) the optimality criteria are derived.

$$\sum_{s=1}^{q} \frac{A_r(\phi_{s1r}^T k_r \phi_{r} - \omega_s^2 \psi_{s1r}^T m_r \psi_{s1r})}{A_r \phi_s^T (M + \bar{M}) \phi_s} = 1; \quad \phi_s^T = [\phi_{s11}, \cdots \phi_{s1n}] \tag{15}$$

k_r and m_r are the element components of the considered stiffness and mass of the element r. The combination includes the original quantities and changes.

The optimization procedure needs initial successes of design variables which are usually the current or unmodified design variables. These design variables are iteratively changed by following a recursion relationship. Constraint equations and the frequency equation. The recursion relationship is derived from the optimality criteria. Specific steps used in the optimization procedure are as follows.

- o Initial design variables are selected from studying the existing design
- o Equations (14) are solved to obtain natural frequencies and modes.
- o The design variables are modified by using the recursion relationship after calculating the unknown Lagrangian multipliers.
- o The new weights are computed. If the weight between two successive iterations are small the iterations are stopped.

NUMERICAL EXAMPLE

A wing of specified geometry and cantilever length of 90 inches has been considered. The wing has two flexible pylons carrying seven hundred pounds at each of these two locations. Starting with six degrees of freedom at each nodes and eight nodes a finite element model has been constructed. A finite element nodal representation of the system is shown in figure 3. It is also assumed that first seven experimental modes and natural frequencies are available. For purposes of illustration pseudo-experimental data have been created 14 . For use in identification procedures these seven modes and three additional higher modes, that have been calculated from the finite element model, have been used. The input to the identification procedure are as follows.

- o Ten natural frequencies that are complex numbers (seven experimental and three analytically generated values.
- o Ten modes (seven experimental and three analytically generated values)

- o Diagonal elements of the ten degree of freedom mass matrix obtained by condensing the finite element model.

The identification procedure yield, new mass, stiffness and damping matrices. These resulting matrices can be used to calculate the modes and natural frequencies. These are called as the identified modes and frequencies. A comparison of the identified and the original and the experimental natural frequencies are shown in table I. As a first step the following problem of modification is considered.

It is necessary to add an additional mass at any of the nodal locations corresponding to the eight nodes, and it is required to keep the first mode frequency at or above 5 Hz, the amount of mass that can be added to any of the nodes (so that the first frequency equals 5 Hz) is shown in table 2. The values have been obtained by using the optimization technique.

If the mass to be added is fixed, and if the first frequency must be kept at or above 5 Hz, the preceding table can be used to determine the location or the node number at which the mass can be lumped on the other hand it is desired to add a given mass at a specific location for which the maximum allowable mass value (as given in table 2) is less than the given mass, the wing needs to be stiffened so that the frequency constraint is not violated. For example, let it be required to add a weight of 700 lbs. From the table it is seen that this weight cannot be added at locations 5, 6, or 8 without reducing the first frequency below 5 Hz. The wing must therefore be stiffened to raise the first frequency to 5 Hz. This can be accomplished by increasing the stiffness of all the members including the pylons by a constant factor so that the stiffness matrix is increased by a constant factor. This factor is determined as the ratio of the eigenvalues corresponding to 5 Hz and the frequency that would be obtained without any stiffening of the wing. Given below are the factors by which the stiffness matrix must be multiplied when a 700 lb. weight is added at location 5, 6, or 8.

It is to be noted that in this type of modification where the entire wing is stiffened by a constant factor, the higher mode frequencies are also increased correspondingly. Also the modification does not provide an optimum design. However by using the optimization procedure the same result of maintaining the first frequency above SHz is obtained with a minimum increase of the weight. The results are shown in table 3.

SCALE MODELS IN DESIGN MODIFICATION

In order to discuss the potential benefits of using a scale model in design modifications, an idealized model of a wing is considered. The details of the geometry involving varying cross sections have been ignored to preserve the simplicity of presentation. It is assumed the wing-store combination can be approximated by a uniform cantilever beam with a non structured mass. A one-fifth scale model of the beam with masses has been built and tested. Because the model is not a structural dynamically similar model 22 , the model frequencies are multiplied by a factor to interpret the results of interms of the full scale. The model has a cantilevered length of 24 inches and a cross section of 0.5" x 1.75" One nonstructured mass M_1 of weight 7.1 pounds is located at the tip. A finite element model has been constructed for the full scale structure with twelve nodes. Tests have been conducted on the one-fifth physical scale model. Measured frequencies, modes for the model are then interpreted interum full scale frequencies are shown in table 3. The corresponding calculated values are shown in table 4. The experimental data have been used to improve the analytical finite element model by using structural dynamic identification technique of reference 18 . A simple eight node model is shown in table 4.

The identified finite element model is now used to analyze the results of a structural modification. As an assumed modification, a nonstructured mass of M_2 of weight 6.8776 pounds is to be added at a distance of inches from the fixed end. The prediction of the identified model, actual test results and the full scale values for the structure are as shown in table 5. The example displays the fact that even a very simple model with limited number of degrees of freedom yields reasonable predictions when an identified model is available. A scale model provides a tool for validating and improving the results as found necessary in an inexpensive way.

CONCLUSIONS

It has been demonstrated in this paper that the use of structural dynamic system identification procedures, optimum design techniques and scale model testing provide a different approach to structural dynamic design modifications. This approach complements the approach that uses only the analytical models and flight tests. Additional research work in the fields of identification, optimum design and scale model testing are beneficial in obtaining an improved structural dynamic design modification technology.

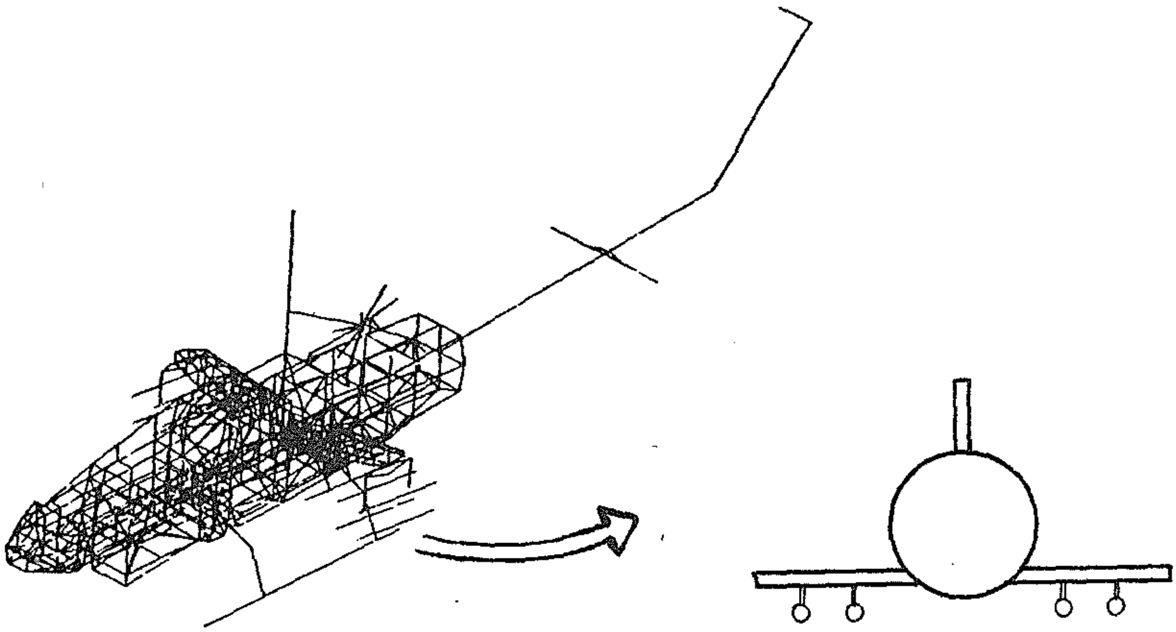


Fig 1 A simple model for the helicopter,

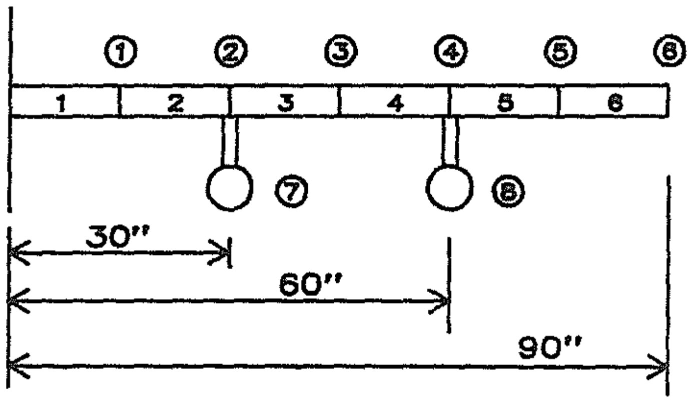


Fig 2 A further simplification

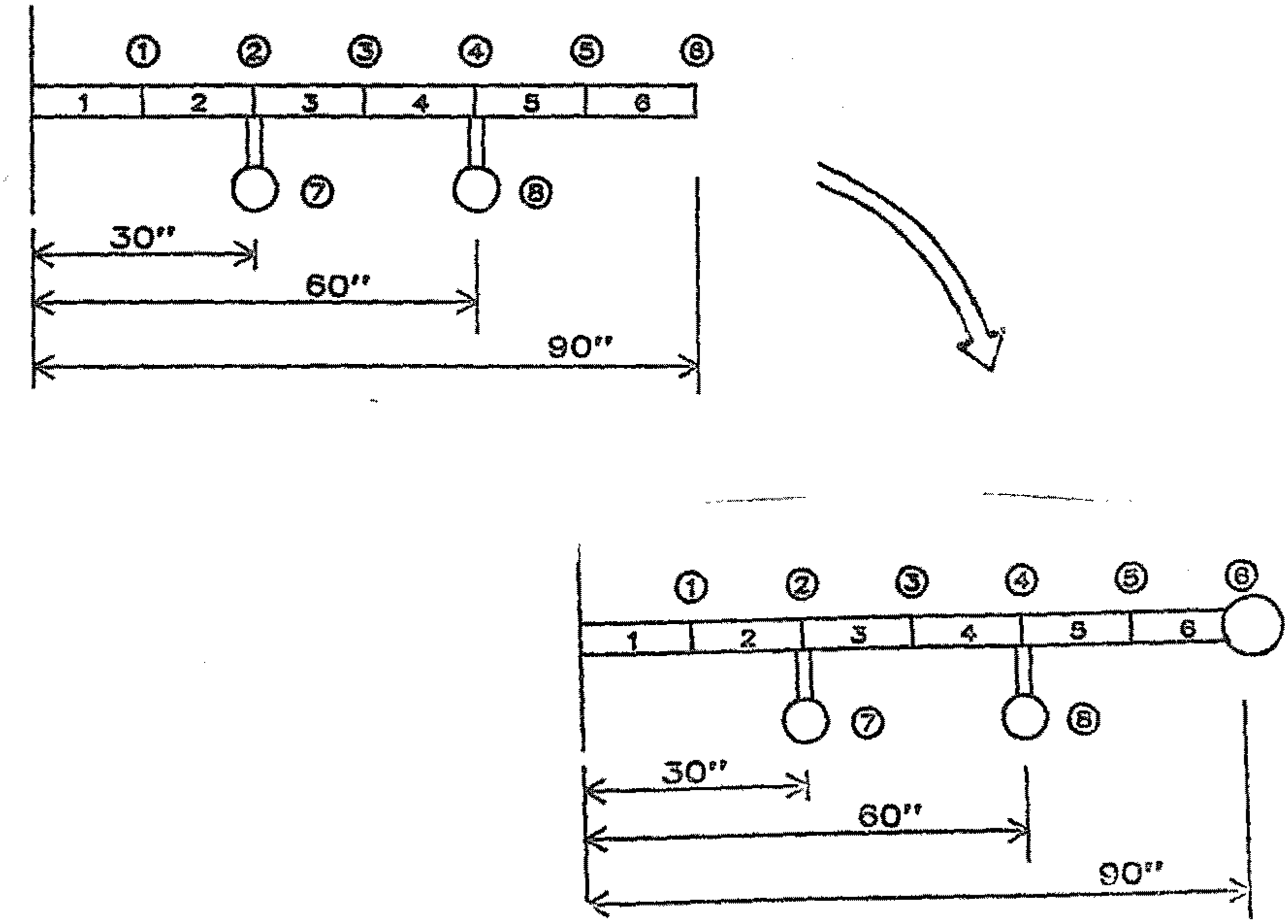
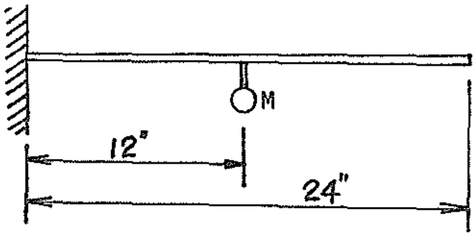


Fig 3 WING STORE DESIGN MODIFICATION

Assumed Exp. (Hz)	Identified (Hz)
6.5	6.5
16.0	16.0
25.8	25.8
53.9	53.9
152.0	152.0
765.1	765.2
879.7	879.8

TABLE I: COMPARISON OF FREQUENCIES

Original Configuration



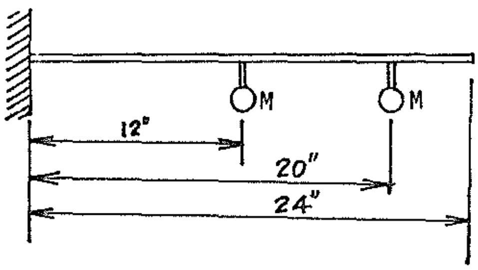
M = 7 lbs

Comparison of Frequencies

Experimental	Analytical		Full-Scale
	A Priori	Identified	
20.4	21.8	20.4	5.1
108.7	105.4	108.7	27.2
406.0	406.9	406.0	101.5

table 4   SCALE MODELING EXAMPLE

Modified Configuration



M = 7 lbs

Comparison of Frequencies

Experimental	Analytical		Full-Scale
	A Priori	Identified	
12.7	13.2	12.8	3.2
85.1	87.6	85.1	21.3
355.1	341.6	348.1	88.8

table 5.   SCALE MODELING EXAMPLE (Continued)

REFERENCES

1. Prouty, R.W., and Amer, K. B., "The YAH-64 Empenage and Tail Rotor - A Technical History," Proceedings of 38th Annual Forum of the American Helicopter Society, 1982, p. 247-261.

2. King, S.P., "The Modal Approach to Structural Modification," J. of the American Helicopter Society, 1983, p. 30-36.

3. Wang, B.P., Kitis, L., Pilkey, W.D., and Palazzolo, A.B., "Helicopter Vibration Reduction by Local Structural Modification," J. of the American Helicopter Society, 1982, p. 43-67.

4. Viswanathan, S.P., and Myers, A.W., "Reduction of Helicopter Vibration Through Control of Hub-Impedance"," J. of the American Helicopter Society, p. 3-12, 1980.

5. Smith, M.R., and Wei, F.S., "Predicting Structural Dynamic Behavior Using a Combined Experimental/Analytical Model," Proc. of 39th Annual Forum of the American Helicopter Society, 1983.

6. Done, G.T.S., and Hughes, A.D., "Reducing Vibration by Structural Modification," Vertica, 1, (1), 1976, pp. 31-38.

7. Done, G.T.S., and Hughes, A.D., "The Response of a Vibrating Structure as a Function of Structural Parameters," Journal of Sound and Vibration, 38, (2), 1975, pp. 255-266.

8. Done, G.T.S., Hughes, A.D., and Webby, J., "The Response of a Vibrating Structure as a Function of Structural Parameters--Application and Experiment," Journal of Sound and Vibration, 49, (2), 1976, pp. 149-159.

9. Hanson, H.W., and Calapodas, N.J., "Evaluation of the Practical Aspects of Vibration Reduction Using Structural Optimization Techniques," Journal of the American Helicopter Society, 25, (3), July 1980, pp. 37-45.

10. Kavanagh, K.T., "An Approximate Algorithm for the Reanalysis of Structures by the Finite Element Method," Computers and Structures, 2, (5/6), 1972, pp. 713-722.

11. Wang, B.P., Palazzolo, A.B., Pilkey, W.D., "Reanalysis, Modal Synthesis and Dynamic Design," State-of-the-Art Survey of Finite Element Methods, Ch. 8, ASME, 1982.

12. Ibanez, P., "Review of Analytical and Experimental Techniques for Improving Structural Dynamic Models," W.R.C. Bulletin 249, 1979.

13. Berman, A., "Limitations of the Identification of Discrete Structural Dynamic Models," paper presented at the 2nd International Conference on Recent Advances in Structural Dynamics, Southampton, April 1984.

14. Ibrahim, S.R., and Mikulcik, E.C., "A Method for the Direct Identification of Vibration Parameters from Free Response," Shock and Vibration Bulletin 46, Part 5, August 1978.

15. Baruch, M., "Optimization Procedure to Correct Stiffness and Flexibility Matrices Using Vibration Tests," AIAA Journal 1978, 120.

16. Baruch, M., "Optimal Correction of Mass and Stiffness Matrices Using Measured Modes," AIAA Journal, 1982, p. 1623.

17. Baruch, H., and Meirovitch, L., "Identification of the Eigen Solutions of Distributed Parameter Systems," Proc. of 23rd Structures, Structural Dynamics and Materials Conference, Paper No. 82-0771, 1982.

18. Hanagud, S., Meyyappa, M., Cheng, Y.P., and Craig, J.I., "Identification of Structural Dynamic Systems with Nonproportional Damping," Paper No. 84 - ..., [illegible]ted at the 25th AIAA, AHS, ASME, and ASLE Structures, Structural [illegible]d Materials Conference, 1984.

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