SIXTH EUROPEAN ROTORCRAFT AND POWERED LIFT AIRCRAFT FORUM

Paper No. 72

GUST RESPONSE OF ROTARY WING AND ITS ALLEVIATION

by

Shigeru SAITO, Akira AZUMA

University of Tokyo

and

Masaki NAGAO

Olympus Optical Co., Ltd.

September 16-19, 1980

Bristol, England

THE UNIVERSITY, BRISTOL, BS8 1HR, ENGLAND

ABSTRUCT

A simple feedback system to alleviate the gust response of the helicopter rotor is analyzed. The system is comprised of sensors each of which can detect the flapping motion of the respective blade and of actuators which can control the pitch angle of the blade following to the signal shaped adequately to the gust alleviation. The validity of this system is demonstrated by applying the theoretical calculation based on the local momentum theory to the gust responses of the complete dynamic system of a helicopter penetrating into a sinusoidal gust and a step gust.

1. INTRODUCTION

The helicopter has the potential to fly close to the ground over which the motion of atmosphere may be thought of turbulent flow. In order to avoid unfavorable dynamic characteristics in flying and riding qualities and structural vibrations caused by such gusty wind, it is required to analyze the gust response of the rotary wing and to study how to alleviate them.

The unsteady characteristics of the gust response of helicopter rotor were made clear in Ref. 1 by applying the local momentum theory (LMT)[2]. The theory may also be applicable to the study of maneuver response of the helicopter without essential change of the computor program.

There have been proposed several ways to reduce the vibration of the helicopter by introducing the active pitch control corresponding to the change of load at the hub and blades[3-7]. Modern technology in the aircraft control engineering enables us to utilize advanced actuators which can operate the pitch control link of individual rotor blade in response to the control or command signal with high frequency.

Thus more free consideration on the design of feedback control system is possible for the gust alleviation of the helicopter rotor. By referring to the past results[1,8,9] the present paper is to present a simple proposal for the control guidance of the gust alleviation at the initial stage in this field.

2. BLADE FLAPPING MOTION

The irregular flapping motion of the rotor blade must be a good indication for the existence of the gust along the flight course. Let us consider first the most simple blade-flapping motion of the rotor under the following assumptions: (i) a set of blades is made rigid and uniform, (ii) the gust is, by referring to Fig. 1, sinusoidal or

$$\begin{aligned} w_G(X,t) &= w_{G_0}\sin\{\kappa(X_0-X+V_G t)\} \\ &= w_{G_0}\sin\{\kappa(X_0-X)+\omega_G t\}, \end{aligned} \qquad (1)$$

in which (iii) the angular frequency ω_G is smaller than that of the rotor speed or $\omega_G/\Omega \ll 1$, (iv) the blade pitch is given by the first harmonics such as

$$\theta = \theta_0+\theta_t(x-3/4)+\theta_{1c}\cos\psi+\theta_{1s}\sin\psi, \qquad (2)$$

and (v) the induced velocity is a kind of funnel shape given by

$$v = \bar{v}\{1-(2/3)K_0+x(K_0+K_{1c}\cos\psi+K_{1s}\sin\psi)\} \quad (3)$$

where the meaning of all symbols have been listed in the NOMENCLATURE.

Then the equation of flapping motion of the blade is given by

$$\ddot{\beta}/\Omega^2+A(\psi)\dot{\beta}/\Omega+B(\psi)\beta$$

$$= C(\psi)+\int_{x_\beta}^{B}\frac{1}{2}\rho acR^3\Omega^2(x+\mu\sin\psi)(w_G/R\Omega)R(x-x_\beta)dx \quad (4)$$

where coefficients $A(\psi)$, $B(\psi)$, and $C(\psi)$ are explained in APPENDIX A and where the final forced term in the right hand side of the above equation is given in APPENDIX B[1].

By expanding the flapping angle β and the feathering angle θ in the series of first harmonics of the azimuth angle and by dividing them into non-perturbed and gust-perturbed terms,

$$\begin{aligned}\beta(t) &= \beta_n(t)+\Delta\beta(t)\\ &=\beta_{0n}(t)+\Delta\beta_0(t)+\{\beta_{n,1c}(t)+\Delta\beta_{1c}(t)\}\cos\psi+\{\beta_{n,1s}(t)+\Delta\beta_{1s}(t)\}\sin\psi\\ \theta(t) &= \theta_n(t)+\Delta\theta(t)\\ &=\theta_{0n}(t)+\Delta\theta_0(t)+\{\theta_{n,1c}(t)+\Delta\beta_{1c}(t)\}\cos\psi+\{\beta_{n,1s}(t)+\Delta\beta_{1s}(t)\}\sin\psi\end{aligned} \qquad (5)$$

equation (4) yields the following two sets of equations of perturbed quantities which are mutually independent:

$$\Delta\ddot{\beta}_0/\Omega^2+K_\beta\Delta\dot{\beta}_0/\Omega+(1+K_\beta)\Delta\beta_0$$

$$=(K_1+\tfrac{1}{2}K_3\mu^2)\Delta\theta_0+(w_{G_0}/R\Omega)(K_2-\tfrac{1}{4}k^2K_4)\sin(\omega_G t) \quad (6)$$

and

$$\Delta\ddot{\boldsymbol{\beta}}/\Omega^2+(K_{\dot{\beta}}-2i)\Delta\dot{\boldsymbol{\beta}}/\Omega+(K_\beta-iK_{\dot{\beta}})\Delta\boldsymbol{\beta}$$

$$= (K_1+\tfrac{1}{2}K_3\mu^2)\Delta\boldsymbol{\theta}+(w_{G_0}/R\Omega)\{\boldsymbol{i}(\tfrac{1}{8}k^3K_5-kK_1)\cos(\omega_G t)$$

$$+(\tfrac{1}{8}k^2K_1\mu-K_3\mu)\sin(\omega_G t)\}, \qquad (7)$$

where

$$\boldsymbol{\beta} = -\beta_{1s} + i\beta_{1c}$$
$$\boldsymbol{\theta} = -\theta_{1s} + i\theta_{1c}. \tag{8}$$

It is, then, obvious that the first and the second equations, (6) and (7), are related to the coning angle and the complex tilt angle of a rigid cone which is assumed to be configured by the first harmonic motion of the one blade[10].

3. FEEDBACK SYSTEM

Now let us consider the following simple feedback system:

$$\Delta\theta_0 = \{b_\beta \Delta\beta_0 + b_{\dot\beta}\Delta\dot\beta_0/\Omega + b_{\ddot\beta}\Delta\ddot\beta_0/\Omega^2\}/(K_1 + \tfrac{1}{2}K_3\mu^2)$$
$$\Delta\boldsymbol{\theta} = \{a_\beta \Delta\boldsymbol{\beta} + a_{\dot\beta}\Delta\dot{\boldsymbol{\beta}}/\Omega + a_{\ddot\beta}\Delta\ddot{\boldsymbol{\beta}}/\Omega^2\}/(K_1 + \tfrac{1}{2}K_3\mu^2). \tag{9}$$

where any kind of time lag of the actuator has been out of consideration.. Then, the characteristic equations for equations (6) and (7) are respectively given by

$$(1-b_{\ddot\beta})(S/\Omega)^2 + (K_{\dot\beta} - b_{\dot\beta})(S/\Omega) + (1 + K_\beta - b_\beta) = 0 \tag{10}$$

$$(1-a_{\ddot\beta})(S/\Omega)^2 + (K_{\dot\beta} - 2i - a_{\dot\beta})(S/\Omega) + (K_\beta - iK_{\dot\beta} - a_\beta) = 0 \tag{11}$$

Fig. 2 a, b, and c show the root loci of equation (10) for systems in which individual gain is feedback independently. The stable region of this feedback systems are respectively given by

$$\begin{aligned} &\text{(a)} \quad \text{for } b_\beta \; : \; b_\beta < K_\beta + 1 \\ &\text{(b)} \quad \text{for } b_{\dot\beta} \; : \; b_{\dot\beta} < K_{\dot\beta} \\ &\text{(c)} \quad \text{for } b_{\ddot\beta} \; : \; b_{\ddot\beta} \leq 1 . \end{aligned} \tag{12}$$

The undamped natural frequency and the damping ratio of equation (10) can be given by

$$\omega_n = \sqrt{(1 + K_\beta - b_\beta)/(1 - b_{\ddot\beta})}$$
$$\xi = \{(K_{\dot\beta} - b_{\dot\beta})/2\}/\sqrt{(1 + K_\beta - b_\beta)(1 - b_{\ddot\beta})} . \tag{13}$$

Similarly, Fig. 3 a, b, and c show the root loci of individual feedback system represented by equation(11). It is interesting to say that the root loci are not symmetrical about the real axis because the coefficients in equation (11) are complex number. The stable regions of such second-order dynamic system with complex variable were, as precisely discussed in Ref. 10 and 11, given respectively by

(a) for $\alpha_\beta = a_{\beta,R} + i a_{\beta,I}$:

$$1+K_\beta - a^2_{\beta,I}/K^2_{\dot\beta} > a^2_{\beta,R} \tag{14}$$

(b) for $\alpha_{\dot\beta} = a_{\dot\beta,R} + i a_{\dot\beta,I}$:

$$k_{\dot\beta} > a_{\dot\beta,R} \tag{15a}$$

and

$$K_\beta a^2_{\dot\beta,R} - 2(K_\beta+1)K_{\dot\beta} a_{\dot\beta,R} - K_{\dot\beta} a_{\dot\beta,R} a_{\dot\beta,I} + K^2_{\dot\beta}(a_{\dot\beta,I}+K_\beta+1) > 0 \tag{15b}$$

(c) for $\alpha_{\ddot\beta} = a_{\ddot\beta,R} + i a_{\ddot\beta,I}$:

$$K_{\dot\beta}(1-a_{\ddot\beta,R}) + 2a_{\ddot\beta,I} > 0 \tag{16a}$$

and

$$K^2_\beta a^2_{\ddot\beta,R} + 2K_{\dot\beta}\{K_\beta(a_{\ddot\beta,R}-1)-(K_\beta+2)\}a_{\ddot\beta,I} + K^2_{\dot\beta}(a_{\ddot\beta,R}-1)(a_{\ddot\beta,R}+K_\beta+1) < 0. \tag{16b}$$

From either Fig. 3 or equations (14-16) the stable regions of individual gain of the feedback system are those shown in Fig. 4 a, b, and c. Actually, for the selection of a set of gain combination the following items must be considered: gain margin, natural frequency, response time, allowable overshoot, terminal amplitude, and so on.

The actual pitch link motion must be related to the above feedback gains. This is performed by combining the direct feedback relation,

$$\Delta\theta(t) = c_\beta \Delta\beta + c_{\dot\beta}\Delta\dot\beta/\Omega + c_{\ddot\beta}\Delta\ddot\beta/\Omega^2 , \tag{17}$$

with the perturbed relations given by equations (5) and the feedback relations given by equations (9) as follows:

$$
\begin{aligned}
c_{\ddot\beta} &= b_{\ddot\beta} = \alpha_{\ddot\beta} \\
c_{\dot\beta} &= b_{\dot\beta} = \alpha_{\dot\beta}+2i\alpha_{\ddot\beta} \\
c_{\beta} &= b_{\beta} = \alpha_{\beta}-\alpha_{\ddot\beta}+i\alpha_{\ddot\beta}
\end{aligned} \tag{18}
$$

By referring to Fig. 3 and Fig. 4 the above equations suggest that the following combination may be a solution for this feedback system:

$$
\begin{aligned}
\alpha_{\ddot\beta} &= 0 \\
\alpha_{\dot\beta} &= a_{\dot\beta,R} = b_{\dot\beta} = K_{\dot\beta}-2\sqrt{K_\beta+1} \\
\alpha_{\beta} &= a_{\beta,R}-i(K_{\dot\beta}-2\sqrt{K_\beta+1})
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
c_{\ddot\beta} &= b_{\ddot\beta} = 0 \\
c_{\dot\beta} &= b_{\dot\beta} = \alpha_{\dot\beta} = K_{\dot\beta}-2\sqrt{K_\beta+1} \\
c_{\beta} &= b_{\beta} = \alpha_{\beta}+i\alpha_{\dot\beta} = a_{\beta,R}
\end{aligned} \tag{20}
$$

where the real part of the α_β, $a_{\beta,R} = b_\beta$, may be selected to satisfy the restriction (a) in the stability conditions (12) and (14) and to get the quick response of the system.

4. GUST RESPONSE OF A ROTOR AND A HELICOPTER

Numerical examples of the present calculation have been performed for a rotor fixed in space and a helicopter flying in cruising condition, the detailed dimensions of which are respectively given in Table 1 and 2.

Fig. 5 a,b,c, and d show the time responses of the thrust coefficient, the tilt angles of tip-path plane, and the pitch input for the exemplified rotor penetrating into a sinusoidal vertical gust. The feedback gains were selected as follows:

$$c_{\ddot\beta} = 0 \ , \ c_{\dot\beta} = -1.1, \text{ and } c_\beta = -2,0 \ .$$

It can be seen from these figures that the deviations of (a) the thrust coefficient and (d) the tilt angles represented by the first harmonic flapping angles from the mean value are appreciably reduced by adopting the above feedback gain if the required pitch angle shown in Fig. 5 c is perfectly followed.

Fig. 6 a, b, c, and d are step responses for the same rotor penetrating into a step gust. The gust alleviation is obvious for such adequate feedback system.

Fig. 7 a and b, and Fig. 8 a and b show the gust responses of the helicopter penetrated into a sinusoidal gust and a step gust respectively. The alleviation effects due to the feedback system are obvious for all quantities related not only to the longitudinal motion but also to the lateral motion.

5. CONCLUSION

A simple analytic method has been proposed to construct a combination of feedback gains in active control system which was designed to alleviate the gust response of the helicopter rotor by sensing the respective blade motion and by making feedback it to the pitch control link of individual blade. The validity of the method was demonstrated in the theoretical calculation based on the local momentum theory for the gust responses of a rotor which is fixed in space and of a helicopter which is flying in cruising condition while they are penetrating into a sinusoidal gust and a step gust.

The deviation of every quantity from a trimmed value is appreciably reduced by adopting such active control system.

REFERENCES

1. A. Azuma, S. Saito, Application of the Local Momentum Theory to the Gust Response of Helicopter. 5th European Rotorcraft and Powered Lift Aircraft Forum, No. 27, Amsterdam, The Netherlands, Sept., 4-7, (1979)
2. A. Azuma, K. Kawachi, Local Momentum Theory and its Application to the Rotary Wing. J. of Aircraft, Vol. 16, No. 1, Jan., (1979)
3. J. Shaw, N. Albin, Active Control of Rotary Blade Pitch for Vibration Reduction: A Wind Tunnel Demonstration. Vertica, Vol. 4, No. 1, (1980)
4. M. Kretz, M. Larché, Future of Helicopter Rotor Control. Vertica, Vol. 4, No. 1, (1980)
5. N.D. Ham, A Simple System for Helicopter Individual-Blade-Control using Modal Decomposition. Vertica, Vol. 4, No. 1, (1980)
6. J.L. McCloud III, The Promise of Multicyclic Control. Vertica, Vol. 4, No. 1, (1980)
7. E.R. Wood, R.W. Powers, C.E. Hammond, On Method for Application of Harmonic Control. Vertica, Vol. 4, No. 1, (1980)
8. M. Yasue, C.A. Vohlow, N.D. Ham, Gust Response and its Alleviation for a Hingless Helicopter Rotor in Cruising Flight. 4th European Rotorcraft and Powered Lift Aircraft Forum, Stresa, Italy, Sept., 13-15, (1978)
9. H.J. Dahl, A.J. Faulkner, Helicopter Simulation in Atmospheric Turbulence. 4th European Rotorcraft and Powered Lift Aircraft and Powered Lift Aircraft Forum, Stresa, Italy, Sept., 13-15, (1980)
10. A. Azuma, Dynamic Analysis of the Rigid Rotor System. J. of Aircraft, Vol. 4, No. 3, May-June (1967), pp. 203-209
11. A. Azuma, Effect of Slow Spin on the Motion of Axisymmetrical Missiles. Transaction of the Japan Society for Aeronautical and Space Sciences, Vol. 2, No. 3, (1959)

APPENDIX A Coefficients of Equation (4)

$$A(\psi) = K_{\dot\beta}+K_{2,a}\mu\sin\psi$$
$$B(\psi) = K_\beta+1+K_2\mu\cos\psi+K_3\mu^2\sin\psi\cos\psi$$
$$\begin{aligned}C(\psi) = {} & K_4\theta_t+K_1(\theta_0+\theta_{1c}\cos\psi+\theta_{1s}\sin\psi-\tfrac{3}{4}\theta_t+2\mu\theta_t\sin\psi)+K_2\{(\theta_0+\theta_{1c}\cos\psi \\ & +\theta_{1s}\sin\psi-\tfrac{3}{4}\theta_t)2\mu\sin\psi+\mu^2\theta_t\sin^2\psi\}+K_3(\theta_0+\theta_{1c}\cos\psi+\theta_{1s}\sin\psi \\ & -\tfrac{3}{4}\theta_t)\mu^2\sin^2\psi-\mu\tan i_s(K_2+K_3\mu\sin\psi)-\frac{ng}{R\Omega^2}Kg+\frac{k_f}{x_f^2}\beta_0 \\ & -(\frac{\bar{v}}{R\Omega})\{(K_1+K_2\mu\sin\psi)(K_0+K_{1c}\cos\psi+K_{1s}\sin\psi)+(1-\tfrac{2}{3}K_0)(K_2+K_3\mu\sin\psi)\},\end{aligned}$$

where,

$$K_\beta = \frac{1}{x_f^2}(k_f+x_{f,a}^2)-1$$
$$K_{\dot\beta} = \frac{k_{\dot f}}{x_f^2}+K_{1,a}$$
$$k_f = k_\beta/m_\beta R^2\Omega^2$$
$$k_{\dot f} = k_{\dot\beta}/m_\beta R^2\Omega.$$

APPENDIX B Expression of a Sinusoidal Gust in the Rotor Coordinate System

By referring to Ref.1, the expression of a sinusoidal gust in the rotor coordinate system is given by

$$\begin{aligned}w_G(r,t)/w_{G_0} = {} & \sin(\omega_G t+\kappa X_0)\{J_0(kx)+2\sum_{n=1}^{\infty}(-1)^n J_{2n}(kx)\cos 2n\alpha_\tau\} \\ & -\cos(\omega_G t+\kappa X_0)\{2J_1(kx)\cos\alpha_\tau \\ & +2\sum_{n=1}^{\infty}(-1)^n J_{2n+1}(kx)\cos(2n+1)\alpha_\tau\}\ ,\end{aligned} \quad \text{(B-1)}$$

where,

$$\begin{aligned}k &= \kappa R = \omega_G/\mu\Omega \\ \omega_G &= \kappa(V\cos\Psi+V_G) \\ \alpha_\tau &= \psi-\Psi \\ r &= Rx.\end{aligned} \quad \text{(B-2)}$$

By assuming that the wave length of the sinusoidal gust is extremely larger than the rotor radius, the following approximation will be established:

$$\begin{aligned}J_0(kx) &= 1-\frac{k^2x^2}{4}+\frac{k^4x^4}{64}-\ \dots \simeq 1-\frac{k^2x^2}{4} \\ J_1(kx) &= \frac{kx}{2}(1-\frac{k^2x^2}{8}+\frac{k^4x^4}{32}-\dots) \simeq \frac{kx}{2}(1-\frac{k^2x^2}{8}) \\ J_2(kx) &= \frac{k^2x^2}{4}(\frac{1}{2}-\frac{k^2x^2}{24}+\dots) \simeq \frac{k^2x^2}{8} \\ J_n(kx) &\simeq 0 \qquad (n \geqq 3).\end{aligned} \quad \text{(B-3)}$$

By using the above approximation, equation (B-1) yields

$$\begin{aligned}w_G(r,t)/w_{G_0} = {} & \sin(\omega_G t+\kappa X_0)\{J_0(kx)-2J_2(kx)\cos 2\alpha_\tau\} \\ & -2J_1(kx)\cos(w_G t+\kappa X_0)\cos\alpha_\tau\ .\end{aligned}$$

NOMENCLATURE

a	lift curve slope
$\mathfrak{a}_\beta$	feedback gain of $\Delta\boldsymbol{\beta}$, $= a_{\beta,R} + ia_{\beta,I}$
$\mathfrak{a}_{\dot\beta}$	feedback gain of $\Delta\dot{\boldsymbol{\beta}}$, $= a_{\dot\beta,R} + ia_{\dot\beta,I}$
$\mathfrak{a}_{\ddot\beta}$	feedback gain of $\Delta\ddot{\boldsymbol{\beta}}$, $= a_{\ddot\beta,R} + ia_{\ddot\beta,I}$
B	tip loss factor
b_β	feedback gain of $\Delta\beta_0$
$b_{\dot\beta}$	feedback gain of $\Delta\dot\beta_0$
$b_{\ddot\beta}$	feedback gain of $\Delta\ddot\beta_0$
C_T	thrust coefficient $= T/\rho(\pi R^2)(R\Omega)^2$
c_β	feedback gain of $\Delta\beta$, eq. (18)
$c_{\dot\beta}$	feedback gain of $\Delta\dot\beta$, eq. (18)
$c_{\ddot\beta}$	feedback gain of $\Delta\ddot\beta$, eq. (18)
g	gravity acceleration
i_s	inclination of rotor shaft
i	imaginary $= \sqrt{-1}$
J_n	Bessel function of n-th order
$K_{1,a}$	$= (\frac{\gamma}{2})(\frac{B^4}{4} - \frac{2}{3}B^3x_\beta + \frac{1}{2}B^2x_\beta^2)$
$K_{2,a}$	$= (\frac{\gamma}{2})(\frac{B^3}{3} - B^2x_\beta + Bx_\beta^2)$
K_1	$= (\frac{\gamma}{2})(\frac{B^4}{4} - \frac{1}{3}B^3x_\beta)$
K_2	$= (\frac{\gamma}{2})(\frac{B^3}{3} - \frac{B^2}{2}x_\beta)$
K_3	$= (\frac{\gamma}{2})(\frac{B^2}{2} - Bx_\beta + \frac{x_\beta^2}{2})$
K_4	$= (\frac{\gamma}{2})(\frac{B^5}{5} - \frac{B^4}{4}x_\beta)$
K_g	$= (\bar{x} - x_\beta)/x_f^2$
K_0, K_{1c}, K_{1s}	coefficients in eq. (3)
K_β	equivalent spring stiffness
$K_{\dot\beta}$	equivalent damping coefficent
k	$= \omega_G/\mu\Omega$
k_β	mechanical spring stiffness
$k_{\dot\beta}$	mechanical damping coefficient
n	load factor
r	radius
r_β	radius at flapping hinge
s	Laplace operator
T	thrust
t	time
V	forward velocity
V_g	gust forward velocity
v	induced velocity
$\bar{v}$	mean induced velocity
w_G	gust velocity, positive upward
X	point of X coordinate in stationary space
X_0	initial point of gust in stationary space
x	$= r/R$
x_β	$= r_\beta/R$
x_f	$= \sqrt{I_\beta/m_\beta R^2}$
$x_{f,a}$	$= \sqrt{(\bar{x}^2 - x_\beta\bar{x})_\beta}$
β	flapping angle $= \beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi + \cdots$
$\boldsymbol{\beta}$	$= -\beta_{1s} + i\beta_{1c}$
γ	Lock number $= \rho acR^4/I_\beta$
ζ	= damping ratio
θ	blade pitch angle $= \theta_0 + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi + \cdots$
$\boldsymbol{\theta}$	$= -\theta_{1s} + i\theta_{1c}$
θ_t	blade twist
κ	wave number
ρ	air density
ψ	azimuth angle
ω_n	undamped natural frequency
ω_G	gust angular frequency
Δ	deviation from equilibrium point

Subscripts

R	real part
I	imaginary part
n	non-perturbed part
0	constant or initial value

Superscript

$(\dot{\ })$	$= d(\ )/dt$

Table 1. Dimensions for a Rotor

R	Rotor radius	, 8.53 m	r_{CG}	C. G. position of blade	,	2.74 m
b	Number of blades	, 4	m_β	Blade mass	,	10.86 $kg \cdot sec^2 \cdot m^{-1}$
c	Blade chord	, 0.417 m	I	Moment of inertia of blade	,	162.6 $kg \cdot m \cdot sec^2$.
Ω	Rotor rotational speed	, 23.67 rad./sec.	M_β	Mass moment of blade	,	169.3 $kg \cdot m \cdot sec^2$.
θ_t	Blade twist angle	, -8 deg.	γ	Lock number	,	8.84
θ_0	Collective pitch angle	, 8 deg.		Wing section	,	NACA 0012
θ_{1c}	Longitudinal cyclic pitch angle	, 0 deg.	μ	Advance ratio	,	0.18
θ_{1s}	Lateral cyclic pitch angle	, 0 deg.	W	Gross weight	,	6,353 kg
r_β	Position of flapping hinge	, 0.3 m	ω_G	Gust angular velocity	,	3.14 rad./sec.
r_C	Blade cut off	, 0.594 m	V_G	Gust forward velocity	,	0 m/sec.
			v_{G_0}	Gust amplitude	,	1.80 m/sec.

Table 2. Dimensions for a Exemplified Helicopter

W	Gross weight	, 2994 kg
I_{x_B}	Moment of inertia of body	, 339.96 $kg \cdot m \cdot s^2$
I_{Y_B}	Moment of inertia of body	, 1170.1 $kg \cdot m \cdot s^2$
I_{Z_B}	Moment of inertia of body	, 989.0 $kg \cdot m \cdot s^2$
For Main Rotor		
R	Rotor radius	, 6.706 m
b	Number of blades	, 2
c	Blade chord	, 0.533 m
θ_t	Blade twist	, -10 deg.
Ω	Rotor rotational speed	, 33.9 rad./sec.
m_β	Blade mass	, 8.837 $kg \cdot s^2 \cdot m^{-1}$
I_β	Moment of inertia of blade	, 118.4 $kg \cdot m \cdot s^2$
i_s	Inclination of rotor shaft	, 3 deg.
γ	Lock number	, 6.526
σ	Solidity	, 0.0506
For Tail Rotor		
R_T	Rotor radius	, 1.3 m
b_T	Number of blades	, 2
c_T	Blade chord	, 0.214 m
θ_{tT}	Blade twist	, 0 deg.
Ω_T	Rotor rotational speed	, 173 rad./sec.
$m_{\beta T}$	Blade mass	, 0.23 $kg \cdot s^2 \cdot m^{-1}$
$I_{\beta T}$	Moment of inertia of blade	, 0.128 $kg \cdot m \cdot s^2$
δ_{3T}	δ_3 - angle	, 45 deg.
γ_T	Lock number	, 3.35
σ_T	Solidity	, 0.105
For Horizontal Wing		
S_H	Wing area	, 1.586 m^2
b_H	Span	, 2.844 m
c_H	Chord	, 0.558 m
AR_H	Aspect ratio	, 5.1
η_H	Efficiency	, 0.7
For Vertical Wing		
S_V	Wing area	, 0.883 m^2
b_V	Span	, 1.036 m
c_V	Chord	, 0.884 m
AR_V	Aspect ratio	, 1.22
η_V	Efficiency	, 0.9
Inputs		
θ_0	Collective pitch angle	, 6.4 deg.
θ_{1c}	Longitudinal cyclic pitch angle	, -0.2 deg.
θ_{1s}	Lateral cyclic pitch angle	, -1.45 deg.
Φ	Roll angle of body	, 0 deg.
Θ	Pitch angle of body	, -3.5 deg.
Ψ	Yaw angle of body	, 0 deg.
γ	Flight path angle	, 0 deg.
μ	Advance ratio	, 0.2
ω_G	Gust frequency	, 3.14 rad./sec.
v_{G_0}	Gust amplitude	, 1.8 m/sec.
c_β	Feedback gain of $\Delta\beta$	, -1.0
$c_{\dot\beta}$	Feedback gain of $\Delta\dot\beta$	, -1.1
$c_{\ddot\beta}$	Feedback gain of $\Delta\ddot\beta$	, 0

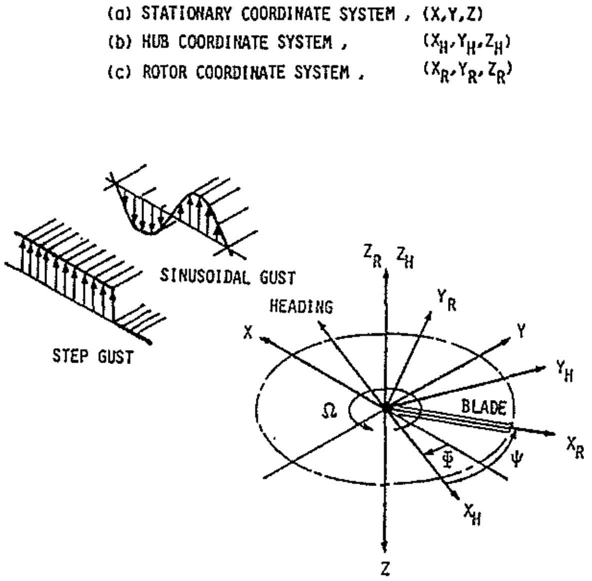


Figure 1 Gust shapes and the related coordinate systems.

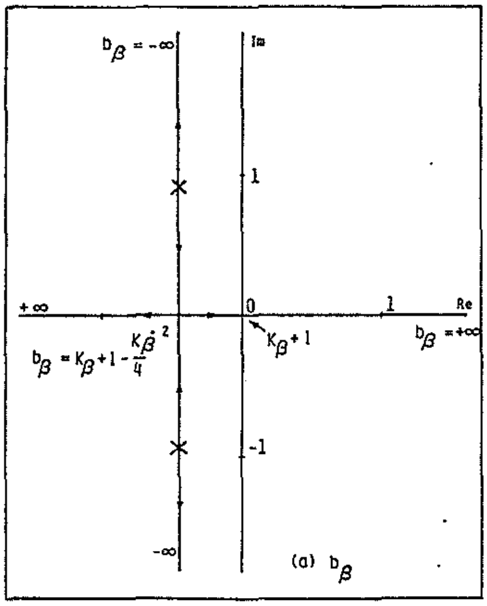
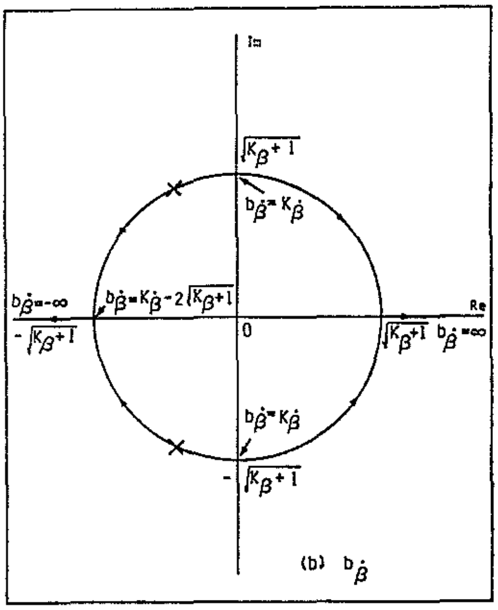
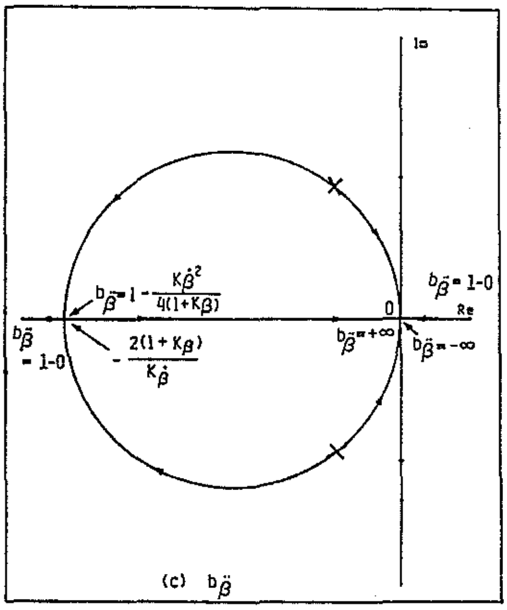
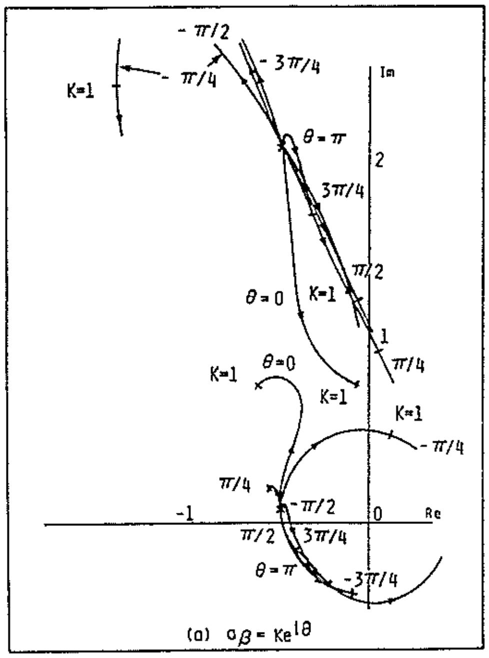


Figure 2 Root loci for the individual feedback system of the coning angle.







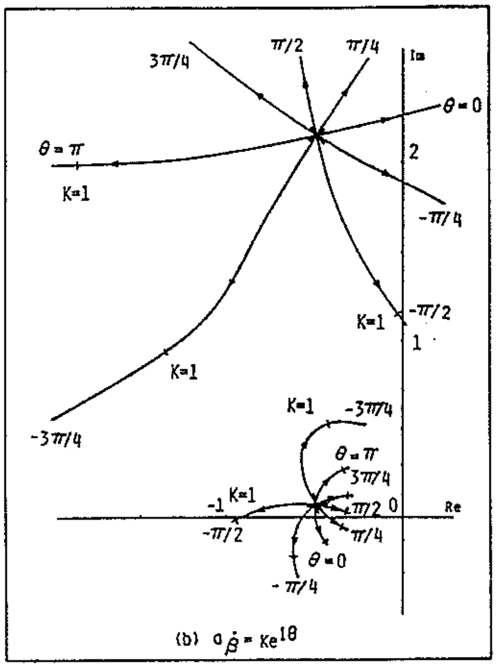
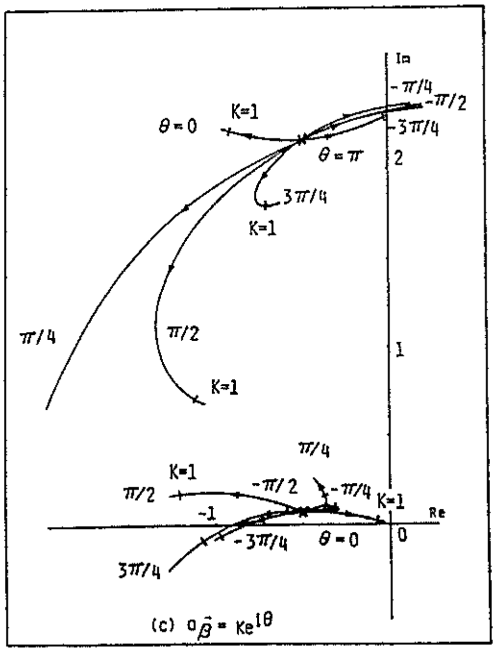


Figure 3 Root loci for the individual feedback system of the complex tilt angle.



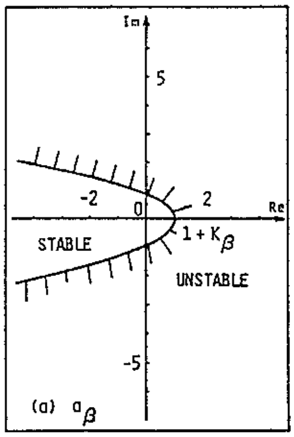
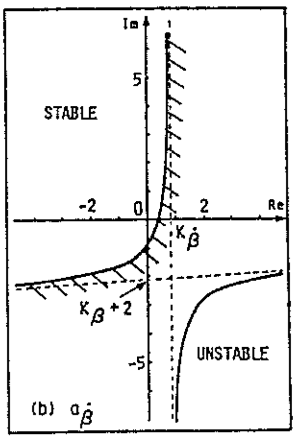
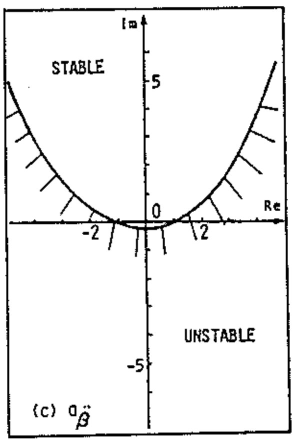


Figure 4 Stable regions of the individual feedback gain for the complex tilt angle.





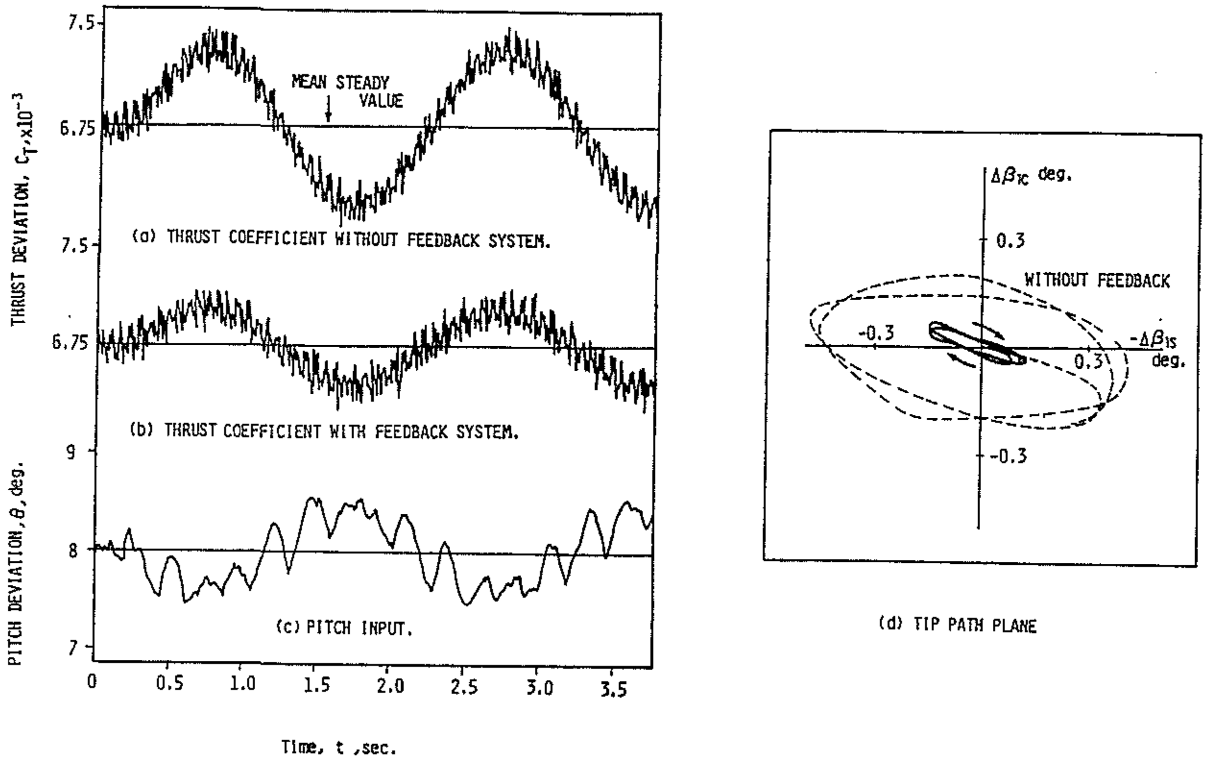


Figure 5 Time responses for a sinusoidal gust.
(w_{GO} = 1.8 m/sec., c_{β} = -2.0, $c_{\dot{\beta}}$ = -1.1, $c_{\ddot{\beta}}$ = 0, μ = 0.18)

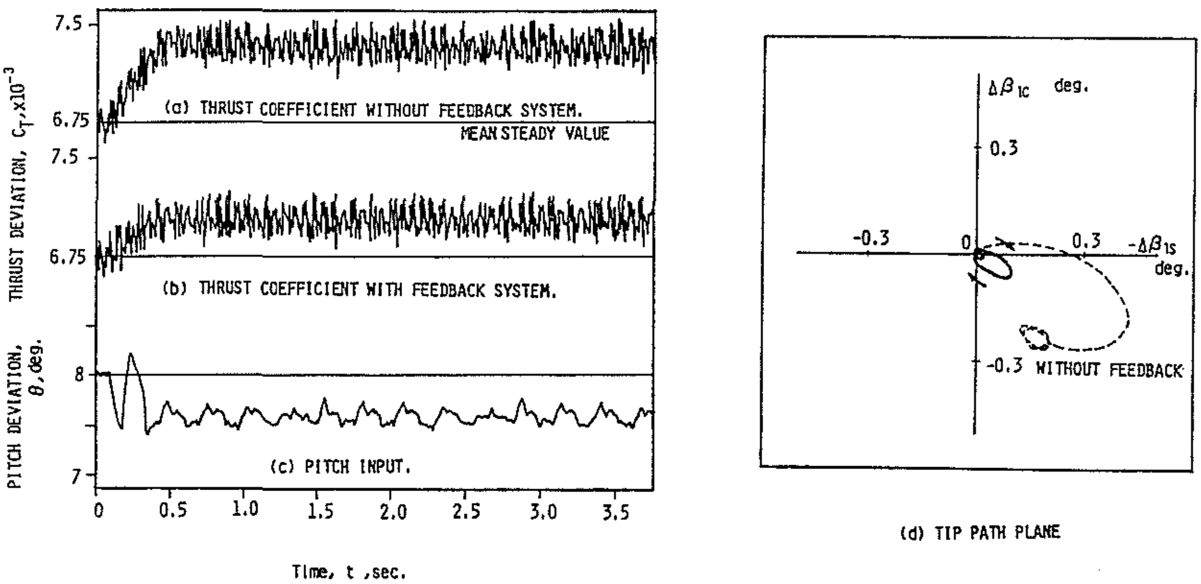
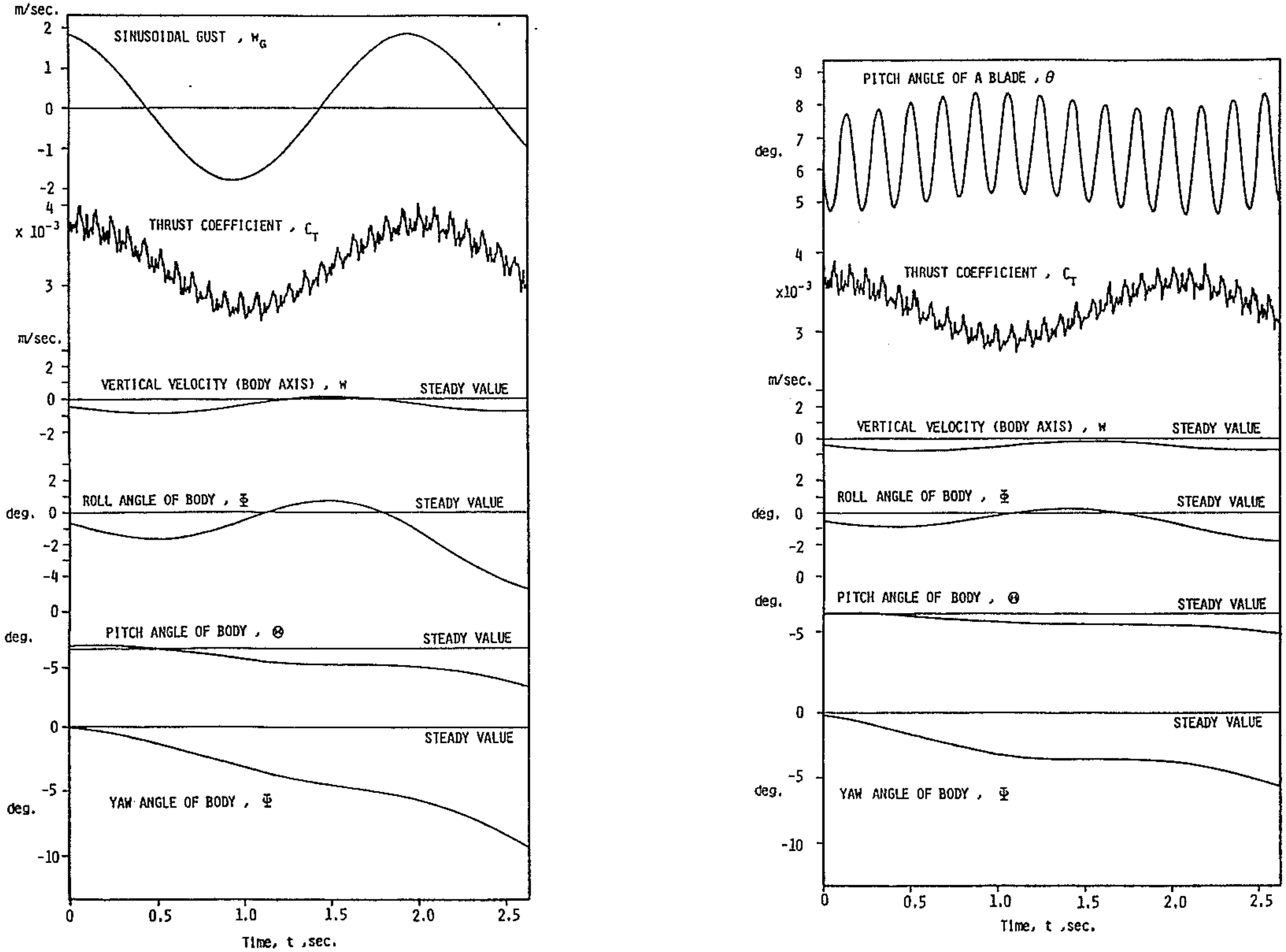


Figure 6 Time responses for a step gust.
(w_{GO} = 1.8 m/sec., c_{β} = -2.0, $c_{\dot{\beta}}$ = -1.1, $c_{\ddot{\beta}}$ = 0, μ = 0.18)

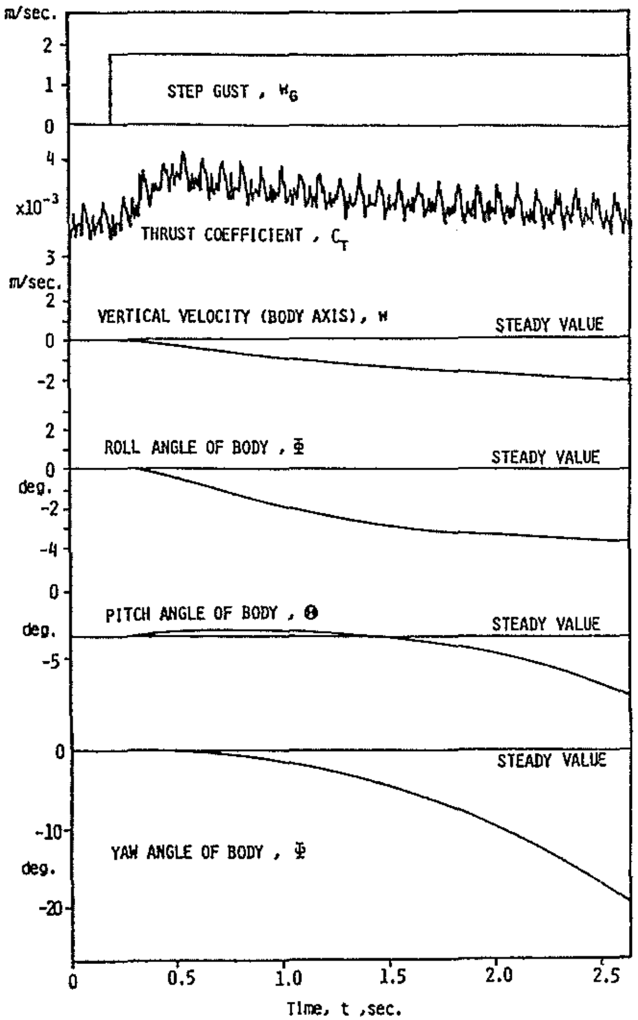


(a) WITHOUT FEEDBACK CONTROL SYSTEM.

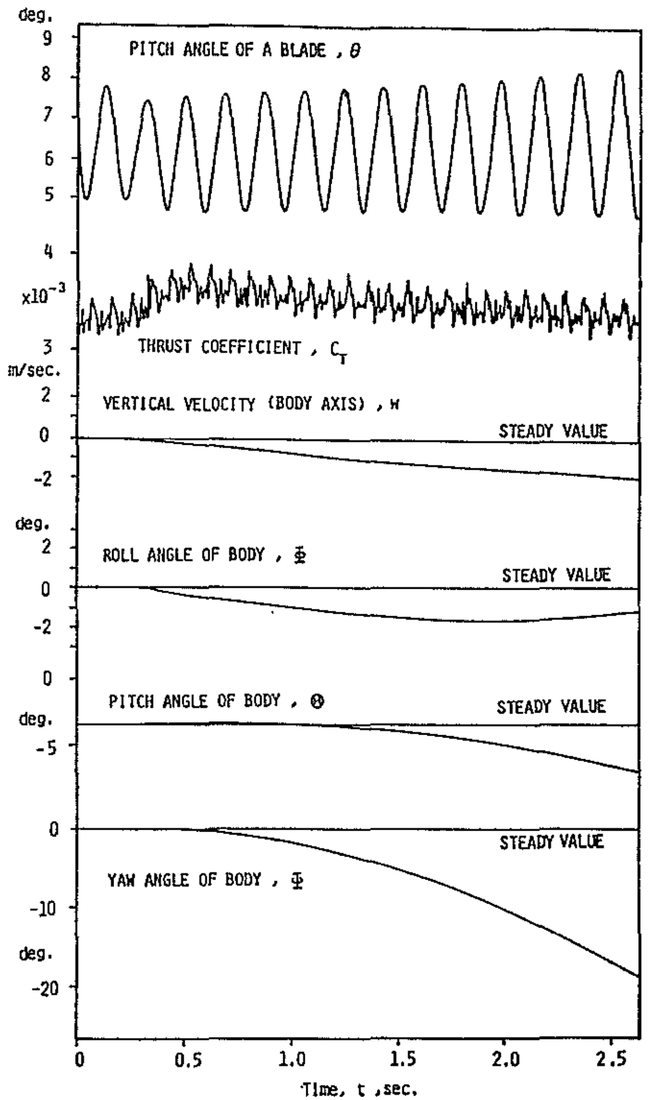
(b) WITH FEEDBACK CONTROL SYSTEM.

Figure 7 Examples of the reduction due to feedback control system.

(w_{G0}= 1.8 m/sec., ω_G= 3.14 rad/sec., c_β=-1.0, $c_{\dot\beta}$=-1.1, $c_{\ddot\beta}$= 0)



(a) WITHOUT FEEDBACK CONTROL SYSTEM.



(b) WITH FEEDBACK CONTROL SYSTEM.

Figure 8 Examples of the reduction due to feedback control system.
(w_{G0}= 1.8 m/sec., c_{β}=-1.0, $c_{\dot{\beta}}$=-1.1, $c_{\ddot{\beta}}$= 0)