NINETEENTH EUROPEAN ROTORCRAFT FORUM

Paper n° *H* 8

ROBUST ANALYSIS OF FLIGHT CONTROL SYSTEM TO MEET ROTORCRAFT HANDLING QUALITIES SPECIFICATIONS AGAINST SENSOR FAILURES

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September 14-16, 1993

CERNOBBIO (Como)

ITALY

ASSOCIAZIONE INDUSTRIE AEROSPAZIALI

ASSOCIAZIONE ITALIANA DI AERONAUTICA ED ASTRONAUTICA

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1 Abstract

Aim of this work is the design of an active control system with fault tolerance properties against scale factor sensor failure for the stabilization and the dynamic decoupling of an high performance helicopter at hover, in order to decrease the pilot workload. In this flight condition, in fact, most of the high performance helicopter are unstable and exibit a strong coupling between longitudinal and lateral motion so that its manual control is very difficult. Furthemore, in the paper is also proposed a new robust analysis procedure in order to evaluate the robustness property of the controller with respect to significant helicopter parameter variations. Simulation results showing the performances of the proposed control system design technique are also presented with reference to the Agusta $A - 109$ helicopter.

2 Introduction

The increased demand on rotorcraft performances asks for the use of more sophisticated control systems: e.g. the extreme coupling between the longitudinal and lateral modes and the open-loop instabilities that exists in most high performance helicopters at hover, make extremely difficult the manual control of the helicopter at this flight condition, and thus leads to increase the pilot workload. A feedback control system for modal decoupling and stability augmentation is then required for better performances and manoeuvrability. Moreover, mere stability is not enough, and the added stability augmentation system must be designed to meet handling quality requirements.

By using decoupled linearized models of longitudinal, lateral, vertical velocity and yaw dynamics, with reference to a specified flight condition and to fixed values of aerodynamic, propulsive, inertial, structural, and controller parameters, the handling qualities can be expressed in terms of desired pole location with an assigned tolerance in the complex plane, [1], [2]. Since the values of these parameters are uncertain and the helicopter is highly vulnerable to incidents like failure of components (actuators, sensors, and flight computers), the control system must be also designed to achieve, for each flight condition within the flight envelope, robustness to off-nominal flight conditions and parameter uncertainties, and to provide fault tolerance against failure of components. To meet these requirements for the closed-loop system, generally a hierarchical concept is used in the control system design. The basic level consists of a controller (static or dynamic compensator) that, for a given flight condition, assigns closed-loop poles in specified regions of the complex plane according to handling quality specifications and shapes closed-loop eigenvectors according to decoupling requirements. Moreover, the controller can be designed to meet robustness criteria with respect to off-nominal flight conditions, parameter uncertainties, unmodelled dynamics and some kinds of sensor-actuator failures like changes in sensor scale factor and actuator gains. All of the more sophisticated tasks like failure detection and redundancy management, plant parameter identification, and controller scheduling are assigned to higher levels, [3], [4].

Thus, from a control system point of view the helicopter basic flight control system design can be formulated as a robust eigenstructure assignment problem. Recently a number of papers appeared which discuss the application of various modern feedback control design techniques to helicopter flight control system synthesis based on eigenstructure assignment concept, [5] [6]. Nevertheless, the proposed control schemes does not guarantee the required robustness properties with respect to both unmodelled dynamics and parameter uncertainties. As far as robustness with respect to unmodelled dynamics is concerned, many multivariable robustness analysis and design techniques have been proposed, [5], [7], [8]. The handling quality robustness problem under parameter uncertainty, i.e. the ability of the control system to maintain the desired handling quality level despite the presence of uncertainties in the mathematical model used for the design, have not been sufficiently investigated, mostly due to the absence of appropriate methodologies.

By using a new computationally tractable procedure for robust stability analysis of dynamic system with highly structured parameter uncertainties [9], successfully experienced in fixed wing flight control system applications, [9], [10], the authors propose a new control system design scheme for an helicopter at hover. In particular we design a full state feedback controller which assures the helicopter handling quality specifications in the assigned flight condition and guarantee the required dynamic and command decoupling properties. By using the results in [10], the controller degrees of freedom which result from the handling quality requirements, will be utilized in order to achieve robustness against scale factor sensor failures. Moreover, in order to prove the robustness properties of the controller with respect to off-nominal flight conditions, the true region of Level 1 handling qualities will be calculated in the weight and balance envelope plane, with reference to a fault sensor configuration.

Simulation results showing the performances of the proposed approach will also be presented with reference to the AGUSTA A-109 helicopter detailed mathematical model.

3 The model

The mathematical model used in this paper is a nonlinear 10 *DOF* model typical of a modern high-performance attack helicopter [11]. The model consists of the rigid body dynamics augmented with main rotor and actuator dynamics. The controls for the helicopter consist of the main rotor collective pitch, longitudinal cyclic pitch, lateral cyclic pitch and tail rotor collective pitch.

The $8th$-order linearized model used for the control system design and for the robustness analysis of the closed loop system has been obtained by linearization and model order reduction of the nonlinear model around the hovering flight condition. The linear model is automatically generated by the *ARMCOP* code for different values of the vector parameters $\pi = (x_{cg}, w)^T$, and its state space form is as follows:

$$\dot{x} = A(\pi)x + B(\pi)\delta \tag{1}$$

where $x = (u, w, q, \theta, v, p, \phi, r)^T$ is the nondimensional rigid-body state vector, $\delta = (\delta_1, \delta_2, \delta_3, \delta_4)^T$ is the nondimensional input control vector and $A(\pi)$, $B(\pi)$ are the dynamic and input matrices parametrized with respect to the weight and balance vector parameter π. The rigid-body states were nondimensionalized to allow for better identification of the modes and their corresponding eigenvectors.

The open-loop matrices and the corresponding eigenstructure of the model (1), with reference to *AGUSTA A-109* helicopter at hover flight condition characterized by the nominal value of the parametric vector $\bar{\pi} = (132.4, 5401)^T$ and by $\bar{\theta} = 3^o, \bar{\phi} = -3.4^o$, are given in table 1. Examining the eigenvectors and eigenvalues in this table, the cross coupling that occurs between modes and the open loop instability are evident. Moreover, from the input distribution matrix is also evident the considerable coupling and the influence the control inputs have on forward, side, and heave acceleration.

4 Handling quality requirements

For the control system design we refer to the handling quality specification criterias proposed in [1]. Those criterias are based on a correlation between the pilot opinion and some characteristic

$\bar{A}=$	-0.0218	0.0128	0.0132	-0.4490	-0.0348	-0.0165	0	-0.0029
	-0.0008	-0.2477	-0.0002	-0.0237	-0.0085	-0.0013	0.0268	0.0245
	1.3659	-0.1291	-0.4038	0	0.7592	0.2422	0	0.0409
	0	0	0.9982	0	0	0	0	0.0597
	0.0318	0.0029	-0.0174	0.0014	-0.1002	-0.0137	0.4482	0.0063
	3.2534	0.3057	-1.0319	0	-2.4930	-1.3518	0	-0.0060
	0	0	-0.0032	0	0	1.0000	0	0.0528
	0.3475	0.0880	0.1155	0	0.5462	-0.1015	0	-0.2510

$\bar{B}=$	-0.0562	0.0211	-0.0043	0.0002
	-0.0058	-0.3853	-0.0004	0
	0.8755	-0.1423	0.0719	0.0364
	0	0	0	0
	-0.0064	-0.0009	0.0380	-0.0465
	-0.4693	-0.1617	2.5798	-0.3849
	0	0	0	0
	-0.0372	0.6383	0.2023	1.3933

	mode 1	mode 2	mode 3	mode 4	mode 5
u	$-0.0481-0.0630i$	$-0.1697+0.1234i$	$-0.0298+0.1143i$	0.0233	-0.0301
w	$-0.0139-0.0077i$	$-0.0032+0.0150i$	$-0.0122-0.0029i$	-0.3833	-0.3411
q	$0.2898+0.1574i$	$0.0158+0.3794i$	$-0.0120+0.1710i$	0.0534	-0.0458
θ	$-0.1593-0.1566i$	$0.3953+0.1750i$	$0.2201+0.0431i$	0.0081	-0.0318
v	$-0.1635-0.0299i$	$0.0657+0.2395i$	$-0.3040-0.1583i$	-0.0565	-0.0427
p	$-0.6981+0.2533i$	$-0.4686-0.1200i$	$0.5323+0.1051i$	0.0432	-0.0524
ϕ	$0.5028-0.0385i$	$-0.3525+0.3932i$	$0.1946-0.6207i$	0.0285	0.0088
r	$-0.0098+0.0363i$	$0.1863+0.1247i$	$-0.1356+0.2298i$	-0.9186	0.9354
λ	$-1.4216+0.3978i$	$0.4184+0.7917i$	$0.0703+0.8239i$	-0.1915	-0.3192

Table 1: Nondimensional state space open-loop model

	roll rate	pitch rate	forward vel.	lateral vel.	vertical vel.	yaw rate
u	0	α_2	1	0	0	0
w	0	0	0	0	1	0
q	0	$\lambda_{3/4}$	α_3	0	0	0
θ	0	1	α_3/λ_5	0	0	0
v	α_1	0	0	1	0	0
p	$\lambda_{1/2}$	0	0	α_4	0	0
ϕ	1	0	0	α_4/λ_6	0	0
r	0	0	0	0	0	1

(*) $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathcal{D}_1;\ \lambda_5, \lambda_6, \lambda_7 \in \mathcal{D}_2;\ \lambda_8 \in \mathcal{D}_3$
$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ arbitrary values

Table 2: Desired eigenvalues and eigenvectors

parameters of the linearized model around the specified flight condition. With reference to the hovering flight condition the authors in [2] translated the imposed specifications for such parameters in desired eigenstructure of the corresponding linearized model and in desired decoupling properties of the input matrix.

Table 2 and figure 1 illustrate the desired eigenstructure corresponding to Level 1 handling quality requirements and to the hovering flight condition. As we can see the desired eigenstructure has been expressed in terms of eigenvalues location with an assigned tolerance in the complex plane as shown in figure 1 ($\mathcal{D}$-stability requirements) and corresponding desired eigenvectors (see table 2). In table 3 the desired input distrubution matrix showing the required decoupling control inputs properties is also presented. The regions $\mathcal{D}_i, i = 1, 2, 3$ are described by:

$$\zeta_i^- \leq \zeta_i \leq \zeta_i^+ ,\ \ \omega_{n_i}^- \leq \omega_{n_i} \leq \omega_{n_i}^+ ,\ \ i = 1, 2, 3.$$

where:

	ζ_i^-	ζ_i^+	$\omega_{n_i}^-$	$\omega_{n_i}^+$
$i = 1$	0.44	0.9	1.04	1.78
$i = 2$	1	1	0.19	0.4
$i = 3$	1	1	2	4

As far as handling quality requirements is concerned, throughout this paper we say the complex vector $\lambda = (\lambda_1, \ldots, \lambda_8)^T$ is an admissible eigenvalue configuration for the model (1) if it belongs to the set $\Lambda_{\mathcal{D}}$ defined as:

$$\Lambda_{\mathcal{D}} \triangleq \{\lambda \in \mathcal{C}^8 : \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathcal{D}_1;\ \lambda_5, \lambda_6, \lambda_7 \in \mathcal{D}_2; \lambda_8 \in \mathcal{D}_3\}$$

5 Control system design

To meet handling quality requirements analized in the previous section, the conceptual proposed control scheme is shown in figure 2. In this scheme P is the model to be controlled, M is the diagonal scale factor sensor gain matrix, K is a full state feedback gain matrix and F is a feedforward gain matrix which allows the pilot reference input signals interface.

The matrix $M = diag(m)$, with $m = (m_u, m_w, m_q, m_\theta, m_v, m_p, m_\phi, m_r)^T \leq 1$ the scale factor sensor gain vector, is an identity matrix in the nominal unfailed configuration.

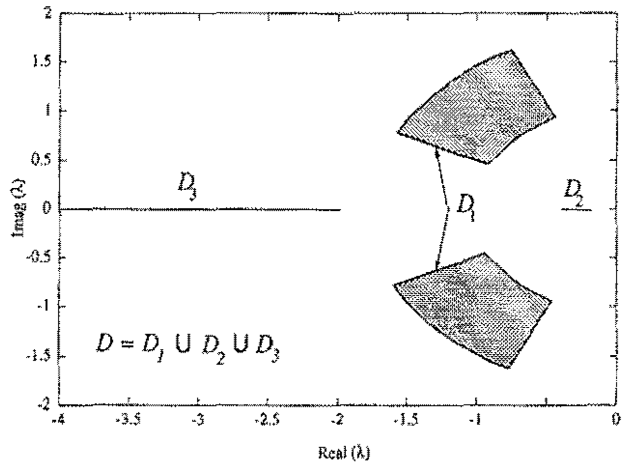

Figure 1: $\mathcal{D}$-stability region

$B_d =$	1	0	0	0
	0	1	0	0
	0	0	0	0
	0	0	0	0
	0	0	1	0
	0	0	0	0
	0	0	0	0
	0	0	0	1

Table 3: Desired input distribution matrix

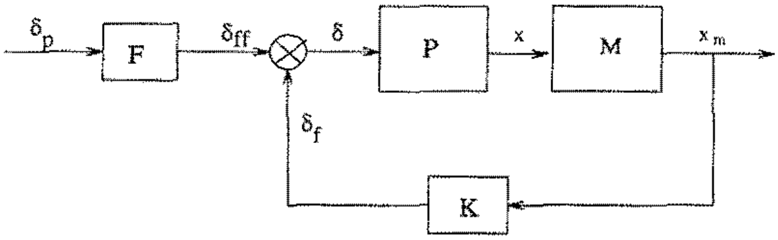

Figure 2: Proposed control scheme

Feedback control action.

The purpose of this control action is to meet the desired eigenstructure handling quality requirements. Formally, this means that the feedback gain matrix K will be designed in order to impose the eigenvalues of the nominal feedback plant model $A(\overline{\pi}) + B(\overline{\pi})KM$ to belong to the handling qualities admissible region $\Lambda_{\mathcal{D}}$ and the corresponding eigenvectors to be as close as possible to the desired eigenvectors $u_{d_i}, i = 1, \ldots, 8$. Since it is possible to verify that, in the case under investigation, the closeness property of the actual eigenvectors to the desired ones is quite insensitive with respect to the selection of the desired eigenvalues $\lambda_d \in \Lambda_{\mathcal{D}}$ (see [12]), it is immediate to define the set $\mathcal{K}_{\mathcal{D}}$ of all $\mathcal{D}$-stabilizing feedback matrices K which ensures the desired eigenstructure handling quality requirements:

$$\mathcal{K}_{\mathcal{D}} = \left\{ K = \mathcal{A}(\lambda_d) \in \mathcal{R}^{8\times4} | \lambda_d \in \Lambda_{\mathcal{D}} \right\}$$

where the operator $\mathcal{A} : \mathcal{C}^8 \rightarrow \mathcal{R}^{8\times4}$ can be defined through the following classical eigenstructure assignment algorithm [9].

Algorithm 1

Given the desired eigenvalues vector λ_d and the desired eigenvectors $u_{di}, i = 1, \ldots, 8$ the algorithm is defined by the following steps:

step 1 for each mode associated with the dynamic system (1), compute the vectors

$$w_i = Q_{ip_i}^{+} u_{di}, \; u_i = Q_i w_i, \; i = 1, \ldots 8,$$

where :

$Q_{ip_i}^{+}$ is the weighted pseudoinverse matrix of Q_i with weight matrix P_i and $Q_i = (\lambda_{di} I - \overline{A})^{-1} \overline{B}$.

step 2 compute $K = W \cdot U^{-1}$ where:

$$W = (w_1, \ldots, w_8) \text{ and } U = (u_1, \ldots, u_8).$$

With reference to the pevious algorithm it is possible to prove (see [9] for details) that the closed loop matrix $\overline{A}_c = \overline{A} + \overline{B} \cdot K \cdot M$ has eigenvalues vector λ_d and eigenvectors $u_i, \; i = 1, \ldots 8$ such that:

$$u_i : \min_{u_i} ||u_i - u_{di}||_{P_i} \quad , i = 1, \ldots, 8$$

where $|| \cdot ||_{P_i}$ is the euclidean weighted norm with weight matrix P_i.

The weight matrices $P_i, \; i = 1, \ldots, 8$ have been chosen diagonal with zero diagonal elements in correspondence with arbitrary component of the desired eigenvectors (see table 3) and 1 in the other position of the diagonal, in order to meet non arbitrary desired eigenvector components requirements.

Feedforward control action

The control action exerted by the feedforward gain matrix F in the control scheme of figure 2 has the objective to meet command decoupling handling quality requirements. Formally this means that the gain matrix $F = (f_1, \ldots, f_n)$ will be designed in order to impose the nominal controlled input distribution matrix $\overline{B}_c = \overline{B} \cdot F$ to be as close as possible to the desired input matrix B_d of table (3). This can be accomplished by solving for each column of the matrix $B_d = (b_{d_1}, \ldots, b_{d_4})$ the following problem :

$$f_i : \min_{f_i} ||\bar{B} f_i - b_{di}||, \; i = 1, \ldots, 4 \tag{2}$$

by means of pseudoinversion techniques.

Robustness against scale factor sensor failure

Now we refer to the sensor scale factor failure problem, which can be viewed as a robust stability problem of a linear dynamical sistem with structured perturbations.

According to previous terminology, $\bar{\pi}$ denotes the vector of nominal flight condition parameters and $\bar{m}$ denotes the vector of nominal sensor gains. With reference to the nominal value of the flight condition parameters $\bar{\pi}$, the closed loop characteristic polynomial associated to the closed-loop model of figure 2 can be written for each $\lambda_d \in \Lambda_{\mathcal{D}}$ and for each value of the sensor gain vector m as:

$$p(s, \lambda_d, m) = det(sI - A(\bar{\pi}) - B(\bar{\pi})K(\lambda_d)\text{diag}(m))$$

where $K(\lambda_d) \in \mathcal{K}$ is designed according to algorithm 1 in order to meet handling quality requirements.

Denote with $\mathcal{M}_{\mathcal{D}}^{\lambda_d}$ the $\mathcal{D}$-stability region in the sensor scale factor space $\mathcal{M}$ corresponding to a given feedback matrix $K(\lambda_d)$:

$$\mathcal{M}_{\mathcal{D}}^{\lambda_d} \triangleq \left\{ m \in \mathcal{R}^8 \mid p(\hat{s}_i, \lambda_d, m) = 0, \hat{s}_i \in \Lambda_{\mathcal{D}}, i = 1, \ldots, 8 \right\},$$

and with

$$\mathcal{H}(\bar{m}, \rho) \triangleq \left\{ m \in \mathcal{R}^8 \mid \|m - \bar{m}\|_{\infty}^{w} < \rho \right\}$$

an hyperrectangular neighborhood of $\bar{m}$.

Now, a failure of the i-th sensor is equivalent to a reduction of the respective gain m_i from nominal values to zero or some values in between. We say that the controller $K(\lambda_d)$ achieves robustness against a full failure of the i-th sensor if the projection of $\bar{m}$ on the axis $m_i = 0$ is contained in $\mathcal{M}_{\mathcal{D}}^{\lambda_d}$. Analogously, we say that $K(\lambda_d)$ has a $100/n\%$ gain reduction margin if the projection of $\bar{m}$ on the line $m_i = \bar{m}_i/n$ is contained in $\mathcal{M}_{\mathcal{D}}^{\lambda_d}$.

Note that the robustness against full scale sensor failure can be acheived by a compensator with $100/n\%$ gain reduction margin by using an n-redundant sensor system (the parallel of n-sensors, each with nominal gain equals to the full nominal gain divided by n).

Now, the goal of this section can be stated as follows: select a feedback control matrix in the set $\mathcal{K}$ which allows us to enlarge the neighborood $\mathcal{H}(\bar{m}, \rho)$ to the maximum extent with the constarint that the poles vector of the closed-loop system belongs to $\Lambda_{\mathcal{D}}$ for any $m \in \mathcal{H}(\bar{m}, \rho)$.

This can be accomplished by solving the following problems.

Problem A:

$$\bar{\rho}(\lambda_d, \bar{m}) = \sup \rho \quad (3a)$$

$$\text{s.t. } \mathcal{H}(\bar{m}, \rho) \subseteq \mathcal{M}_{\mathcal{D}}^{\lambda_d} \quad (3b)$$

Problem B:

$$\rho^o(\bar{m}) = \max_{\lambda_d} \bar{\rho}(\lambda_d, \bar{m}) \quad (4a)$$

$$\text{s.t. } \lambda_d \in \Lambda_{\mathcal{D}} \quad (4b)$$

The crucial point in solving Problem A is to test condition (3b), which can be viewed as a robust $\mathcal{D}$-stability test of a family of polynomials generated by structured coefficient perturbations $m \in \mathcal{H}(\bar{m}, \rho)$. In [13] and [10] the authors proposed a new computationally tractable procedure to solve the test condition problem (3b), so, by using simultaneously this proposed procedure and an univariate minimization algorithm, it is possible to completely solve Problem A.

As concerns Problem B, note that, since nothing is known about the convexity of the function $\bar{\rho}(\lambda_d, \bar{m})$, there is no guarantee that a global maximum will be found. A good suboptimal solution can be sufficient for our goal.

By using this procedure, it is also possible to give an estimate of the real $\mathcal{D}$-stability region in the sensor gain plane. To this end, denote with $\bar{\mathcal{H}}(\bar{m}) = \mathcal{H}(\bar{m}, \bar{\rho}(\bar{m}))$ the largest neighborhood of $\bar{m}$ contained in $\bar{\mathcal{M}}_\mathcal{D} \triangleq \mathcal{M}_\mathcal{D}^{\lambda_d^o}$. It is easy to show that by varying $\bar{m}$ it is possible to build a sequence of sets that converges to $\bar{\mathcal{M}}_\mathcal{D}$:

$$\mathcal{M}_\mathcal{D}(n) = \bigcup_{m^* \in \mathcal{M}_\mathcal{D}(n-1)} \bar{\mathcal{H}}(m^*), \quad n \in N, \quad \mathcal{M}_\mathcal{D}(1) = \bar{\mathcal{H}}(\bar{m}). \tag{5}$$

Then

$$\lim_n \mathcal{M}_\mathcal{D}(n) = \bar{\mathcal{M}}_\mathcal{D} \tag{6}$$

Obviously, from a practical point of view, it is impossible to obtain the exact region via eq. (6), but generally a good estimate suffices. Moreover, when $dim(m) = 2$ the neighborhood $\bar{\mathcal{H}}(m^*)$ is a rectangle and the solution can be easily represented pictorially.

6 Flight condition robustness analysis

The control design procedure illustrated in the previous section guarantees handling quality specifications only for the nominal vector value $\bar{\pi}$ of flight condition parameters and for all failed sensor gains configurations $m \in \bar{\mathcal{M}}_\mathcal{D}$. For a given fault sensor configuration $\bar{m} \in \bar{\mathcal{M}}_\mathcal{D}$ it would be very interesting to evaluate the $\mathcal{D}$-stability region $\Pi_\mathcal{D}$ in the flight condition parameter plane x_{cg}-w. By using the same terminolgy of the previous section this can be accomplished by solving the following problem having the same properties of problem A:

$$\begin{aligned} \bar{\rho}(\bar{\pi}) &= \sup \rho \qquad &(7a)\\ \text{s.t. } \mathcal{H}(\bar{\pi}, \rho) &\subseteq \Pi_\mathcal{D} \qquad &(7b) \end{aligned}$$

where

$$\mathcal{H}(\bar{\pi}, \rho) \triangleq \left\{ \pi \in \mathcal{R}^2 \mid \|\pi - \bar{\pi}\|_\infty^w < \rho \right\}$$

and, by using a $\Pi_\mathcal{D}$ estimation procedure similar to that given by (5) and (6):

$$\Pi_\mathcal{D}(n) = \bigcup_{\pi^* \in \Pi_\mathcal{D}(n-1)} \bar{\mathcal{H}}(\pi^*), \quad n \in N, \quad \Pi_\mathcal{D}(1) = \bar{\mathcal{H}}(\bar{\pi}). \tag{8}$$

with

$$\lim_n \Pi_\mathcal{D}(n) = \bar{\Pi}_\mathcal{D} \tag{9}$$

7 Simulation results

By using the robust control system design and analysis tools illustrated in §5 and §6 respectively, the handling quality and robustness properties of the control scheme of fig. 2 have been verified with reference to *AGUSTA A-109* helicopter at the hover flight condition detailed in §3.

The nominal rigid-body open-loop linearized model is given in table 1, while the assumed handling quality requirements are those described in §4.

The feedforward and feedback control gain matrices designed according to the procedure illustrated in §5 are given in table 4. In particular, the feedback control gain matrix K has been obtained by solving Problem B with a pattern search optimization procedure starting from $\lambda_{do} = (-1.28 \pm 0.846i, -1.18 \pm 0.75i, -0.29, -0.25, -0.3, -3)^T$ and $\bar{\rho}(\lambda_{do}, \bar{m}) = 0.02$, and by solving Problem A at each optimization step. A suboptimal solution has been obtained in correspondence of $\lambda_d^o = (-1.31 \pm 0.91i, -1.2 \pm 0.83i, -0.26, -0.3, -0.38, -3)^T$ and $\rho^o(\bar{m}) = 0.28$.

$K =$	0.0162	0.0102	-7.1268	-9.2675	-0.0316	-0.6620	0.3566	0.0492
	-0.0004	0.0125	0.1049	-0.0204	-0.0004	0.0024	0.1981	0.1570
	-0.0464	-0.0036	-0.2193	-1.7314	0.0124	-1.3378	-2.8348	-0.8905
	-0.0023	-0.0076	-0.4433	0.0070	-0.0182	0.3829	0.3277	-5.5939
$F =$	-0.0547	-0.0139	-0.0016	-0.0050				
	0.0008	-0.0382	-0.0000	0.0006				
	-0.0099	-0.0003	0.0210	0.0155				
	-0.0006	0.0050	-0.0031	0.1050				

Table 4: Feedback matrix K and feedforward matrix F

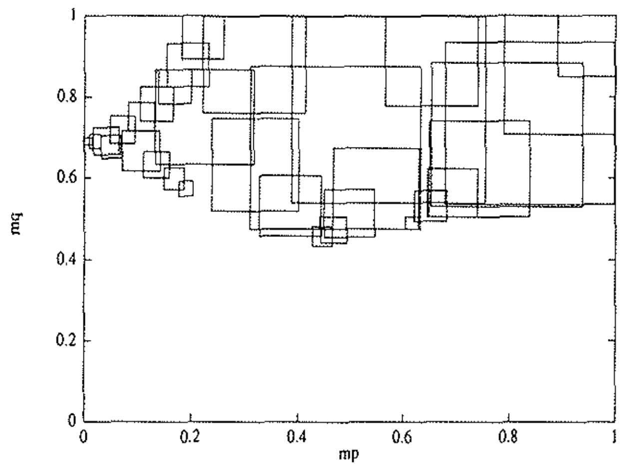

Figure 3: $\mathcal{D}$-stability region in sensor gain plane (Level 1 h.q.)

For the so obtained closed-loop system an estimate of the real $\mathcal{D}$-stability region in the sensor gain plane m_q-m_p is given in fig.3. As one can see, the system is tolerant to an independent reduction of 50% in sensor gains, then a two redundant sensor system is sufficient to achieve fault-tolerance.

As far as off-nominal flight condition robustness is conserned, since the $\mathcal{D}$-stability region in x_{cg}-w plane contains the full weight and balance envelope of the vehicle in the nominal unfault condition, an estimate of the real $\mathcal{D}$-stability region in x_{cg}-w plane has been also calculated with reference to the fault condition with $\hat{m} = (0.5, 0.5)^T$ (see fig. 4).

In fig 5 the $\mathcal{D}$-stability region in x_{cg}-w plane in comparison with weight and balance envelope is depicted in order to show the critical points inside the flight envelope.

Moreover, in fig. 6 some step responses of the fault closed-loop system are illustrated in order to show the exibited handling quality performances.

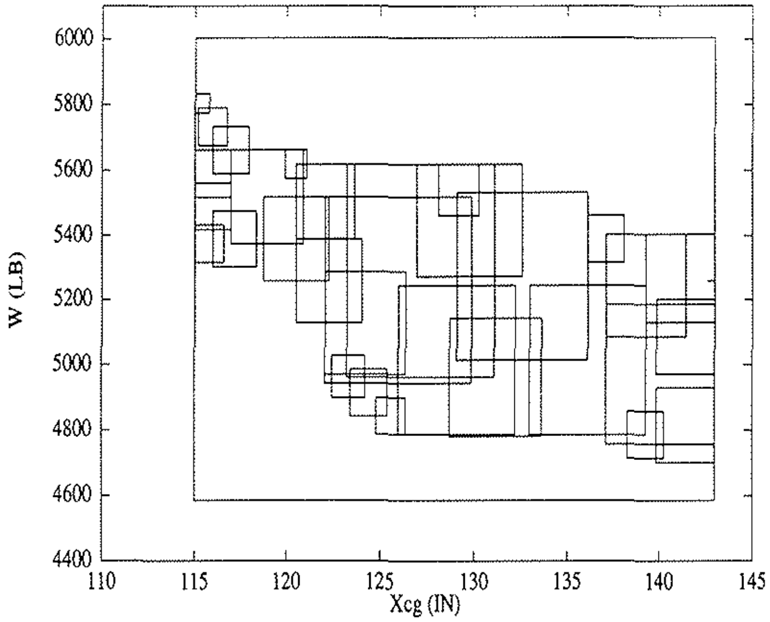

Figure 4: $\mathcal{D}$-stability region in x_{cg}-w plane (Level 1 h.q.)

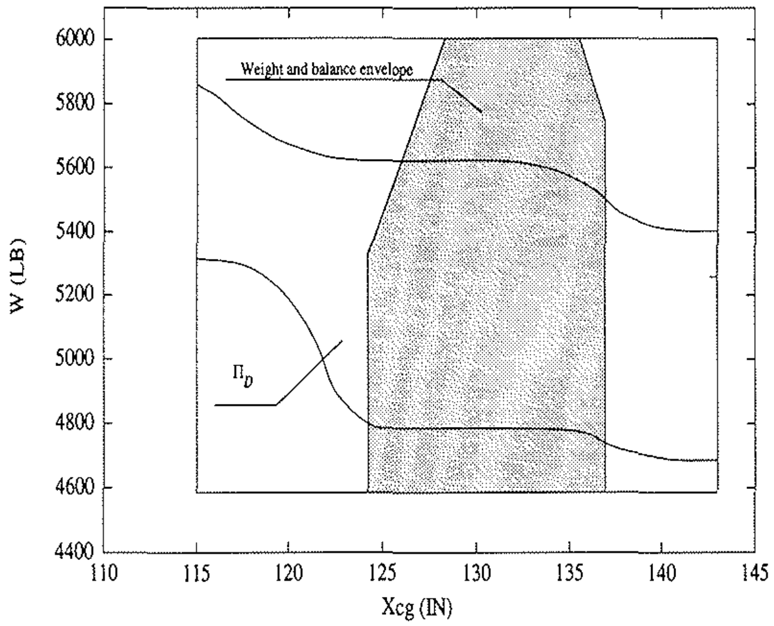

Figure 5: $\mathcal{D}$-stability region in the wieght and balance envelope

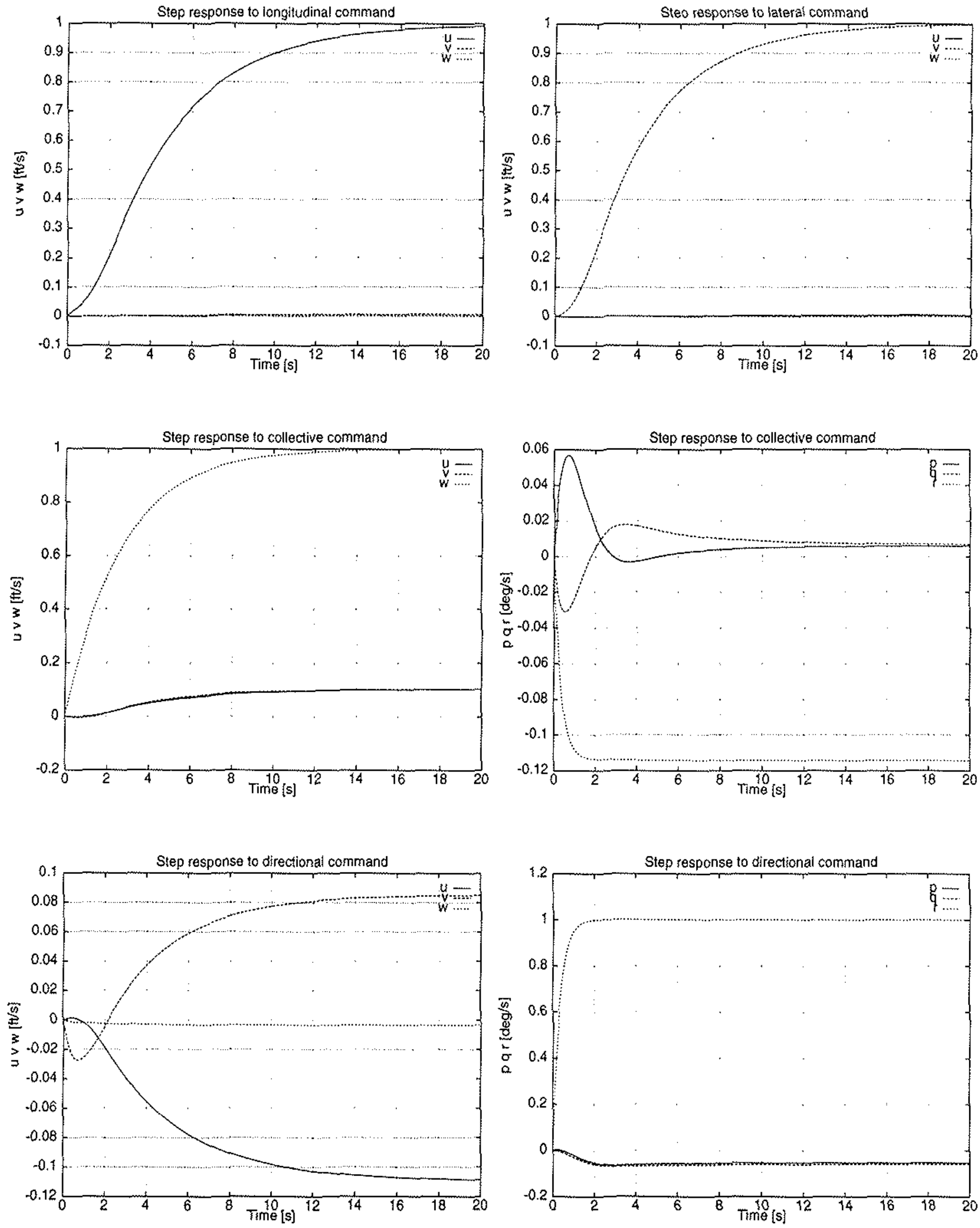

Figure 6: Controlled system step responses

List of simbols

u	= longitudinal velocity (ft/s)	δ_1	= main rotor collective pitch command (in)
v	= lateral velocity (ft/s)	δ_2	= main rotor longitudinal cyclic command (in)
w	= vertical velocity (ft/s)	δ_3	= lateral cyclic pitch command (in)
p	= roll rate (rad/s)	δ_4	= tail rotor collective pitch (in)
q	= pitch rate (rad/s)	x_{cg}	= center of mass longitudinal position (in)
r	= yaw rate (rad/s)	w	= mass of the helicopter (lb)
θ	= pitch angle (rad)	$\|\|x\|\|_\infty^w$	= weighted l_∞ norm of the vector x, i.e. $\|\|x\|\|_\infty^w = \max_i w_i \|x_i\|$, $w_i > 0$
ϕ	= roll angle (rad)		

Inequalities between vectors will be intended component-wise

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