EIGENSTRUCTURE ASSIGNMENT FOR HANDLING QUALITIES IN HELICOPTER FLIGHT CONTROL LAW DESIGN

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Abstract

The requirements of active control technology, together with increased operational demands, present a range of new problems for the designers of helicopter flight control laws. Multivariable control system design techniques, such as eigenstructure assignment, provide a potentially powerful set of tools for control law development which take account of the inherently multi-input multi-output nature of the helicopter. This paper examines the use of eigenstructure assignment for the design of control laws based on state variable feedback. The merits of this approach are examined through the use of an illustrative design example with particular emphasis being given to decoupling and robustness properties.

1. Introduction

Multivariable control law design techniques are of potential interest for helicopter applications in that they provide an integrated approach to the design of multi-input multi-output control systems and have proved to be of value in other fields of application. A review of multivariable design methods and details of a strategy for assessment of such techniques for helicopter applications can be found in a companion paper presented at this Forum [Ref. 1].

Eigenstructure assignment is a method of multivariable control law design in which eigenvalue placement is used to ensure stability while the selection of eigenvectors gives decoupling. This approach has been applied previously in a number of published studies involving fixed-wing aircraft and rotorcraft. Indeed reviews of the literature suggest that many of the theoretical developments in eigenstructure-based design methods have been stimulated by aerospace applications. Examples of the use of eigenstructure assignment techniques to problems of helicopter flight control are, however, small in number compared with the fixed-wing case. Parry and Murray-Smith [Ref. 2], Garrard and Liebst [Ref. 3], Innocenti and Stanziola [Ref. 4] and Ekblad [Ref. 5] have all described design methods based on eigenstructure assignment concepts which they have applied to helicopter flight control problems. Apkarian [Ref. 6] has also applied eigenstructure assignment as part of a hybrid approach to design which also involved the use of nonlinear programming.

One reason for applying eigenstructure assignment to the problems of helicopter flight control law design is that by using a technique based on eigenvalues and eigenvectors one provides a natural link between quantities commonly used to describe the dynamics of the vehicle itself and the processes involved in the development of the controller. This allows the design process to be related directly to the overall dynamic characteristics of the vehicle and provides a level of visibility in the design calculations which is not present in certain other methods.

2. Eigenstructure Assignment

The techniques of eigenstructure assignment have their origins in the more restricted topic of eigenvalue assignment and can be traced to comments by Rosenbrock in 1962 [Ref. 7] concerning the control of linear systems. This early work involving eigenvalue assignment was concerned with pole placement for system stabilisation using state feedback rather than with the satisfaction of transient response specifications. The resulting controllers were not unique for a multi-input system.

Theoretical work carried out by Moore [Ref. 8] provided a basis for extending the use of full state feedback to allow consideration of eigenvectors as well as eigenvalues and Srinathkumar [Ref. 9] subsequently showed that for controllable systems with n state variables and m inputs the following general results apply:-

i) n eigenvalues and a maximum of n x m eigenvector elements can be chosen arbitrarily

and

ii) no more than m entries in one eigenvector can be chosen arbitrarily.

Some insight concerning these design constraints may be obtained by considering the standard state-space linear time-invariant formulation

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad (1)$$

$$\underline{y} = C\underline{x} \qquad (2)$$

where $\underline{x}(t)$ is the state vector, $\underline{u}(t)$ is the input vector, $\underline{y}(t)$ is the output vector and A, B, C are matrices with constant coefficients. If one applies a linear feedback control law

$$\underline{u}(t) = F\underline{x}(t) \qquad (3)$$

the corresponding closed-loop system is described by the equation

$$\dot{\underline{x}} = (A + BF)\underline{x} \qquad (4)$$

Assume now that a set of given eigenvalues λ_i (i = 1,2, ... n) are the desired closed-loop eigenvalues with corresponding eigenvectors $\underline{\nu}_i$. Then for each eigenvalue-eigenvector pair we have

$$(A + BF)\underline{\nu}_i = \lambda_i\underline{\nu}_i \qquad (5)$$

or

$$\underline{\nu}_i = (\lambda_i I - A)^{-1} BF\nu_i \qquad (6)$$

With a full state feedback matrix of dimension m x n one does not have complete freedom in the eigenstructure assignment and the assignable eigenvectors $\underline{\nu}_i$ must lie in a subspace defined by

$$\Gamma = (\lambda_i I - A)^{-1} B \qquad (6)$$

where λ_i is the desired eigenvalue for the mode. As the rank of B is limited to m it is clear that for each choice of eigenvalue λ_i the assignable eigenvectors must be restricted to an m-dimensional assignable subspace. Since m, the number of inputs, is generally smaller than the system order, n, it follows that arbitrary assignment of all n elements of the eigenvector $\underline{\nu}_i$ is impossible.

The problem of eigenstructure assignment is essentially one of choosing $(\lambda_i,\underline{\nu}_i)$ pairs which are assignable. In terms of eigenvalue positions in the complex plane it is relatively straightforward to define desired regions which ensure stability and allow constraints in terms of bandwidth, rise time or damping factor to be satisfied. For example a stable dominant eigenvalue close to the origin will produce a smaller system bandwidth than one located further into the left half plane. The distance of the eigenvalue from the negative real axis will affect the damping of the mode.

Although there are several different procedures available for the assignment of eigenvectors which involve specification of desired eigenvector subspaces [Ref. 2] or desired eigenvectors [Ref. 10], or desired eigenvector elements [Ref. 10], most eigenstructure assignment algorithms have a common basis. Once one has a desired eigenvalue a desired eigenvector is chosen for that mode and this eigenvector is then projected on to the assignable subspace Γ to obtain the assignable eigenvector in a least squares sense. One approach commonly used to find the assignable eigenvectors involves the use of singular value decomposition.

It should be noted that the elements of the eigenvector show how strongly each mode will appear in each state variable response. In aircraft applications it is generally desirable to decouple longitudinal responses from lateral directional responses and this can be attempted by specifying desired eigenvectors for longitudinal modes with zero elements on the lateral-directional states and vice-versa. In general the better the understanding which the designer has of the dynamics of the system and the performance requirements the easier is the task of selecting desired eigenvectors.

3. Invariant Zeros and Invariant Zero Directions

The invariant zeros of a multivariable system correspond to the complex frequencies at which transmission through the system is blocked [Ref. 11]. In terms of a state space representation, the invariant zeros are the values of s for which the matrix

$$P(s) = \begin{bmatrix} sI - A & -B \\ C & D \end{bmatrix} \qquad (7)$$

is singular. The matrices A and B in eqn. (7) are the system state matrix and control input matrix of eqn. (1) while C and D in eqn. (7) are (m x n) and (m x m) matrices relating the output vector y to the state vector $\underline{x}$ and input vector $\underline{u}$ through an equation of the form

$$\underline{y} = C\underline{x} + D\underline{u} \qquad (8)$$

Thus $\det(P(s)) = 0$ when s is an invariant zero. The number of invariant zeros is given by

$$\ell = n-m-d \tag{9}$$

where d is the rank deficiency of the matrix product CB.

The difference between the invariant zeros and the transmission zeros of the system is that the invariant zeros are defined, as above, in terms of a state space description of the system while transmission zeros relate to a transfer function form of description [Ref. 11]. For a system which is both controllable and observable the invariant zeros and the transmission zeros are the same.

The concept of an invariant zero direction is, for an invariant zero, similar to the concept of an eigenvector being associated with an eigenvalue. The invariant zero directions, x_o, for an invariant zero at s = z are given by

$$P(z) \begin{bmatrix} x_o \\ \\ g \end{bmatrix} = 0 \tag{10}$$

where g is an m vector. The poles and zeros in a classical single-input single-output system are thus replaced in the case of a multivariable system by the eigenstructure and the invariant zero structure. Knowledge of the invariant zero structure can add considerably to the information available about a multivariable system and additional physical insight is gained which can greatly facilitate the choice of desirable eigenvalues and eigenvectors.

4. Eigenstructure Assignment for Decoupled Tracking

In the helicopter flight control problem, as with many other control applications, the objective is to design a decoupled tracking system. That is, each input should be tracked by a single output, thus giving a set of decoupled control channels. Hence the design emphasis shifts from the state space $\underline{x}$ to the output space $\underline{y}$. In addition, in order to have a system which has predictable response characteristics for the pilot, efforts should be made to simplify the transfer function for each channel as far as possible. The following design algorithm uses eigenstructure assignment to satisfy these requirements.

Simple control channel transfer functions imply a need to reduce the order of the system from the pilot's point of view. In classical single-input single-output control the order of the system can be reduced by using pole-zero cancellation. Extending this concept for a multivariable system it can be seen that if eigenvalues and eigenvectors are selected to cancel invariant zeros and invariant zero directions then the order of the system is effectively reduced. As in the single input case, only invariant zeros which are in the left half plane can be cancelled if stability is to be guaranteed.

If one has available a number of possible output spaces $\underline{y}$ these should all be checked to find the output space which has the most benign invariant zero locations. Given the A and B matrices for a system, the choice of C and D for different output spaces $\underline{y}$ will determine the invariant zero structure and hence the nature of the dynamics to be controlled. For example, if there are ℓ invariant zeros for the output space $\underline{y}_1$ and these are all minimum phase, then it is clear that the output space $\underline{y}_1$ will allow a greater number of cancellations than some other output space $\underline{y}_2$ for which some invariant zeros are nonminimum phase. The use of output space $\underline{y}_1$ will therefore give a greater reduction in the effective order. If the matrix product CB has full rank such that in eqn. (9) ℓ = n−m, and if all ℓ of the invariant zeros are minimum phase, then the effective order of the system can be reduced to m by assigning ℓ modes to cancel the invariant zero structure. In this case the task then becomes one of assigning the remaining m modes in such a way that the m control channels are decoupled. This may be achieved from consideration of the null space of the matrix C. If, for example, a mode $(\lambda_i, \underline{\nu}_i)$ is to be present on the first control channel then the m-dimensional output space description of the mode

$$\underline{\mu}_i = C\underline{\nu}_i \tag{11}$$

should have the form $\underline{\mu}_i = [1 \quad 0 \ldots 0]^T$. The eigenvector must therefore lie in the null space of the matrix C_{N1} where

$$C_{N1} = [C_2 \ldots C_m]^T \tag{12}$$

and where the elements C_i are the ith rows of the output matrix C. Hence by determining the null spaces

$$\Sigma_i = \mathbb{N}(C_{Ni}) \qquad i = 1, \ldots m \tag{13}$$

one is defining desired subspaces for the remaining modes. If the effective order is m then one mode can be assigned to each control channel and the responses will be first order and decoupled as far as the pilot is concerned, provided the modes are excited by a feedforward controller [Ref. 12]. If the effective system order is greater than m more than one mode must be assigned to some channels.

5. System Dynamics and Design Objectives

One of the main problems to be overcome in helicopter flight control law design is the high level of coupling which exists between high frequency modes of the rotor and actuators and the lower frequency rigid body modes. Because of this coupling the high order dynamics tend to restrict the performance which can be achieved by controlling the rigid body variables.

The companion paper [Ref. 1], referred to in Section 1, concerned more generally with the problems of multivariable methods for flight control law design, provides details of ways in which rotorcraft dynamics influence the design process. The approach most generally adopted involves tailoring the response of the low-order rigid-body dynamics to satisfy appropriate design objectives in the presence of higher-order dynamics involving the engines, the actuators and the main rotor.

In addition to the constraints which are imposed by the dynamics of the system the use of eigenstructure assignment methods must be matched carefully to the design objectives. An extensive discussion of requirements can be found within the handling qualities documentation [Ref. 13] but in addition to specifications dealing with system bandwidth, damping and levels of coupling, the handling quality requirements also call for particular response-types which, depending on the operational requirements of the vehicle, may implicitly involve different levels of stabilisation. Further discussion of this aspect of the handling quality requirements is provided by Hoh [Ref. 14].

6. Design Example

Consider an eighth order representation of the Lynx helicopter in straight and level flight at 80 knots forward speed as given by the HELISTAB theoretical flight mechanics model [Refs. 15,16]. Equations (1) and (2) are the linearised equations of motion in state space form with the elements of the A and B matrices given in Appendix 1. The elements of the output vector depend upon the chosen response-type of the controlled system. Eigenvalues of the A matrix are also given in Appendix 1.

Figure 1 shows a possible form of multivariable control system structure involving a full state feedback matrix F and a precompensator matrix P. The feedback matrix F can be designed using the eigenstructure assignment approach outlined in Section 2 and the precompensator P may be obtained using a special form of the Broussard Command Generator approach as described by Sobel and Shapiro [Ref. 12].

The required response-type chosen for this example corresponds to an attitude command attitude hold (ACAH) controller. Outputs to be controlled were chosen as $\underline{y} = [\dot{h}\ \theta\ \varphi\ r]^T$. Invariant zeros and invariant zero directions for this system were found using methods described by MacFarlane and Karcanias [Ref. 11]. Two minimum phase invariant zeros exist in this example problem and therefore two of the modes of the system were designed to cancel these zeros. The remaining six modes could then be assigned to give decoupled height rate, pitch attitude, roll attitude and yaw rate responses. Desired eigenvalues for these responses were selected in an iterative fashion involving assessment of the bandwidth of the closed-loop system resulting from each trial set.

The eigenstructure of the system resulting from the complete design process is given in Appendix 2 together with the corresponding feedback and precompensator matrices. The responses of the controlled system (with aactuator dynamics included) for step inputs on the four pilot inceptors are shown in Figures 2, 3, 4 and 5. The results show that the outputs in each case track the pilot inputs well and little coupling appears between channels in the steady state. The actuator dynamics involved a first order lag for each control channel with time constants of 80 ms for the three actuators associated with the main rotor and 40 ms for actuator on the rail rotor.

The pitch, roll and yaw rate control responses were also evaluated in the frequency domain. The bandwidth figures were found to be 5.5, 7.3 and 4.17 rad/s respectively in pitch, roll and yaw with corresponding phase delays of 0.041, 0.028 and 0.025 seconds. This gives Level 1 performance in terms of bandwidth and phase delay characteristics for all three cases for combat/target tracking tasks.

Evaluation of the performance of the system with six additional rotor states incorporated in the helicopter model to allow for second order rotor flapping dynamics shows that there is a deterioration in the bandwidth for pitch, roll and yaw channels. The bandwidth figures in this case are approximately 3 rad/s in terms of pitch and 2 rad/s in both the roll and yaw channels with corresponding phase delay figures of 0.11, 0.09 and 0.17 seconds respectively. This no longer provides a Level 1 performance for combat/target tracking tasks but keeps the bandwidth/phase delay performance within the Level 1 region or at the Level 1 boundary for all other mission task elements.

The results illustrate clearly the effect of additional high frequency dynamics on the basic design. The inclusion of rotor modes which were not modelled at the original design stage has altered the eigenvalues and eigenvectors of the system, thus reducing the bandwidth and increasing the phase delay. Lack of robustness in terms of unmodelled dynamics is generally regarded as a problem with design techniques based on eigenstructure assignment. The results found in this example study are not, however, too discouraging in this respect. The coupling levels remain small when unmodelled dynamics are taken into account and stability is maintained, as can be seen from Figure 6 which shows responses to a unit step command in pitch angle. This set of responses is also typical of the responses for step command inputs for the other channels.

A modified form of controller involving an outer loop with proportional plus integral control has also been evaluated. The block diagram of this system is shown in Figure 7. Elements of the proportional and integral path gain matrices KP and KI were found using classical design methods and the values used are given in Appendix 3. The resulting responses, as might be expected with this form of control, are considerably more sluggish than in the previous design although the coupled response transients are of smaller magnitude, Figure 8, which is typical, shows responses to a unit step command in pitch.

Robustness to changes of flight conditions have also been investigated. Figures 9 and 10 show responses to a unit step command in pitch angle for flight conditions at 50 knots and 120 knots respectively for the controller with additional proportional plus integral terms designed using the helicopter model valid for 80 knots. The controlled system is still stable for these two conditions and coupled transients are still small, although in comparison with those of Figure 8 their character has changed. The responses in Figures 8, 9 and 10 are all for the helicopter model structure which incorporates actuator dynamics and second-order rotor flapping dynamics.

The magnitude of the coupled transients shown in Figures 9 and 10 can be reduced significantly by appropriately scheduling a number of linear controllers developed for spot points in the flight envelope. For two spot points V_1 and V_2 which straddle the flight condition V controller parameters were calculated from the expression

$$g_i(V) = \frac{V - V_1}{V_2 - V_1} \left[g_i(V_2) - g_i(V_1) \right] \tag{14}$$

Figures 11 and 12 show pitch command responses for the controller described above scheduled using this form of linear interpolation with sixteen spot points in the range 50 to 120 knots. The transients are clearly much smaller than in the unscheduled case.

Figure 13 shows responses obtained from a nonlinear simulation based upon the HELISIM nonlinear model [Ref. 15,16]. In this case a pitch attitude command was applied to take the helicopter from an initial trimmed forward flight condition of 80 knots to a maximum forward speed of 120 knots, then to a minimum of 50 knots and back eventually to 80 knots. It can be seen that the system remains stable throughout this manoeuvre and that the coupled transients are relatively small. The control law used in this case was the proportional plus integral controller from 80 knots case and the scheduled controller matrices F and P. Similar results, but with increased transient coupling levels, have been obtained in tests involving nonlinear simulation without control law scheduling.

7. Discussion and Conclusions

It is believed that a thorough understanding of the dynamics of the system to be controlled is essential before any control law design technique can be used to full effect. The application of multivariable design methods to helicopter flight control law design can present considerable difficulties even when the dynamics of the vehicle itself are well understood due to the complexity of the system and of the design objectives.

Eigenstructure assignment provides a degree of visbility in the design process in the sense that the control technique can be tailored naturally to the dynamics of the helicopter and to the objectives to be satisfied. In the attitude-command attitude-hold example which has been considered the eigenstructure assignment made use of knowledge concerning the invariant zero structure of the system to reduce the order of the response in each control channel. The design freedoms available were then used to meet bandwidth criteria and to produce a decoupled tracking system.

In order to meet handling qualities requirements for combat/target tracking tasks the controller bandwidth must extend to frequencies at which unmodelled and unmeasurable elements of the system are active. Imprecise information concerning these high order modes inevitably restricts the ability of the controller to improve performance at high frequencies. Robustness of the system to unmodelled dynamics has been investigated both in terms of actuator and rotor flapping modes. Although the attitude-command attitude-hold system designed by this method could not provide Level 1 performance for combat/tracking tasks in terms of bandwidth and phase delay requirements in the presence of second-order rotor flapping dynamics, Level 1 performance was just achievable for other mission tasks.

A second form of control law involving additional output feedback with dynamic precompensation in the form of proportional plus integral control has also been considered. This provides an illustration of the way in which eigenstructure assignment principles can be blended with classical control design concepts. Robustness of this system to unmodelled dynamics and to variation of flight condition has been assessed and the tracking performance for scheduled and unscheduled controllers has been investigated with both linear and nonlinear simulations of the Lynx helicopter.

It is intended that the design example considered in this paper should provide an illustration of the strategy for assessment of multivariable control law design techniques which is described in more detail in the companion paper [Ref. 1]. Factors which have not been discussed and which must be addressed in a complete evaluation of a design include the response of the controlled vehicle to atmospheric disturbances and the effects of sensor noise on system performance.

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Appendices

Appendix 1: Linearised description of dynamics of Lynx helicopter for condition involving straight and level trimmed flight at 80 knots forward speed as obtained from the HELISTAB program (Refs. 15 and 16)

The matrices of the state space model are:

$$A = \begin{bmatrix}
-0.0322 & 0.0403 & -0.2261 & -9.8080 & -0.0021 & -0.1086 & 0.0000 & 0.0000 \\
-0.0096 & -0.8018 & 41.0911 & -0.2113 & -0.0194 & -0.4512 & 0.3223 & 0.0000 \\
0.0271 & 0.0288 & -2.3408 & 0.0000 & 0.0104 & 0.4102 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.9995 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0329 \\
0.0043 & 0.0143 & -0.1284 & 0.0069 & -0.1665 & 0.1986 & 9.8028 & -40.6861 \\
-0.0373 & 0.2344 & -1.9960 & 0.0000 & -0.1633 & -10.5358 & 0.0000 & -0.2865 \\
0.0000 & 0.0000 & -0.0007 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0215 \\
-0.0258 & 0.0024 & -0.0885 & 0.0000 & 0.1014 & -1.7934 & 0.0000 & -1.3488
\end{bmatrix}$$

$$B = \begin{bmatrix}
4.3447 & -7.6327 & 2.0578 & 0.0000 \\
-117.7857 & -30.3913 & 0.0000 & 0.0000 \\
14.0778 & 28.5401 & -5.8552 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 \\
1.4985 & -1.5282 & -9.3201 & 6.7038 \\
32.0714 & -25.0312 & -153.2298 & -1.3416 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 \\
13.9472 & -5.9564 & -26.8072 & -18.0693
\end{bmatrix}$$

The elements of the state vector are:

u – longitudinal velocity (m/s)
w – z–axis velocity (m/s)
q – pitch rate (rad/s)
θ – pitch angle (rad)
v – lateral velocity (m/s)
p – roll rate (rad/s)
φ – roll angle (rad)
r – yaw rate (rad/s)

The eigenstructure is as follows:

Eigenvalue	-10.5525	-3.1994	-0.6531 ± 2.2539i	+0.1339 ± 0.3766i	-0.4053	-0.0305
Eigenvector						
u	-0.0093	0.0381	0.0013 ± 0.0044i	-0.5559 ± 0.5834i	-0.5831	-0.3621
w	-0.2039	0.9567	0.0145 ± 0.0136i	0.1788 ± 0.5311i	0.8076	-0.0053
q	0.0392	-0.0551	-0.0010 ± 0.0009i	-0.0008 ± 0.0141i	0.0081	-0.0030
θ	-0.0033	0.0170	0.0012 ± 0.0001i	0.0325 ± 0.0132i	-0.0193	0.0001
v	-0.6015	0.2793	-0.9707 ± 0.2323i	0.0360 ± 0.1829i	0.0708	0.8420
p	-0.7551	0.0384	0.0146 ± 0.0086i	0.0055 ± 0.0041i	0.0184	-0.0138
φ	0.0718	-0.0121	-0.0048 ± 0.0051i	0.0141 ± 0.0098i	-0.0448	0.3891
r	-0.1401	0.0186	0.0002 ± 0.0553i	0.0049 ± 0.0038i	-0.0101	0.0908

Appendix 2: Feedback matrix and precompensator matrix for attitude–command attitude–hold flight control law together with resulting eigenstructure

$$F = \begin{bmatrix} 0.0006 & -0.0307 & -0.0453 & 1.4631 & 0.0012 & 0.0006 & 0.0120 & -0.0002 \\ 0.0007 & -0.0141 & 0.1804 & -0.2572 & 0.0000 & 0.0109 & -0.0558 & 0.0028 \\ 0.0003 & -0.0101 & -0.0255 & 0.3420 & 0.0014 & -0.0155 & -0.2430 & 0.0021 \\ 0.0013 & -0.0134 & -0.0518 & 0.7067 & -0.0067 & 0.1191 & 0.3882 & -0.2063 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.0382 & -0.1100 & 0.0131 & 0.0020 \\ -0.0167 & 0.4297 & -0.0563 & -0.0058 \\ 0.0106 & -0.0925 & -0.2427 & 0.0045 \\ 0.0193 & -0.0893 & 0.3888 & -0.2799 \end{bmatrix}$$

Eigenstructure:

Eigenvalue	-0.0237	-0.1080	-4.0000	-7.0000	-5.0000	-2.5000	-5.5000	-4.5000
Eigenvector								
u	0.9998	-0.0017	0.0759	0.0003	0.0007	0.0959	0.0003	0.0681
w	0.0214	-0.0329	-0.9969	-0.0106	0.0326	0.9931	-0.0143	0.9914
q	0.0000	0.0000	0.0000	0.0000	-0.0041	-0.0601	0.0000	-0.1081
θ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0240	0.0000	0.0240
v	-0.0043	-0.9995	0.0199	-0.1997	0.9981	0.0176	-0.2813	0.0131
p	0.0000	0.0000	0.0000	-0.9700	-0.0027	0.0000	-0.9440	-0.0001
φ	0.0000	0.0000	0.0000	0.1386	0.0000	0.0000	0.1716	0.0000
r	0.0000	0.0000	0.0000	0.0000	0.1235	0.0000	0.0000	0.0000

Appendix 3: Proportional (KP) and integral (KI) gain matrices for outer loop controller.

$$KP = \begin{bmatrix} 0.20 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.20 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.10 & 0.00 \\ 0.00 & 0.00 & 0.13 & 0.30 \end{bmatrix}$$

$$KI = \begin{bmatrix} 1.80 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.80 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.50 & 0.00 \\ 0.00 & 0.00 & 0.00 & 2.80 \end{bmatrix}$$

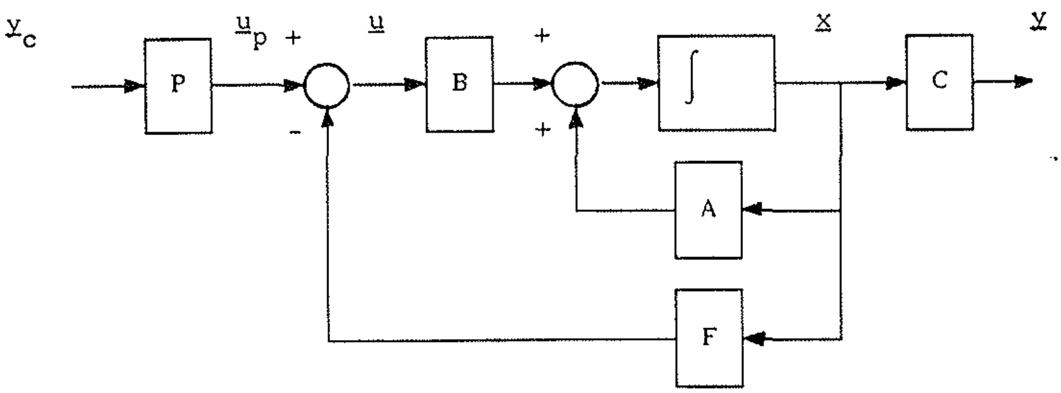

Figure 1 : A linear multivariable controller using full state feedback and precompensation.

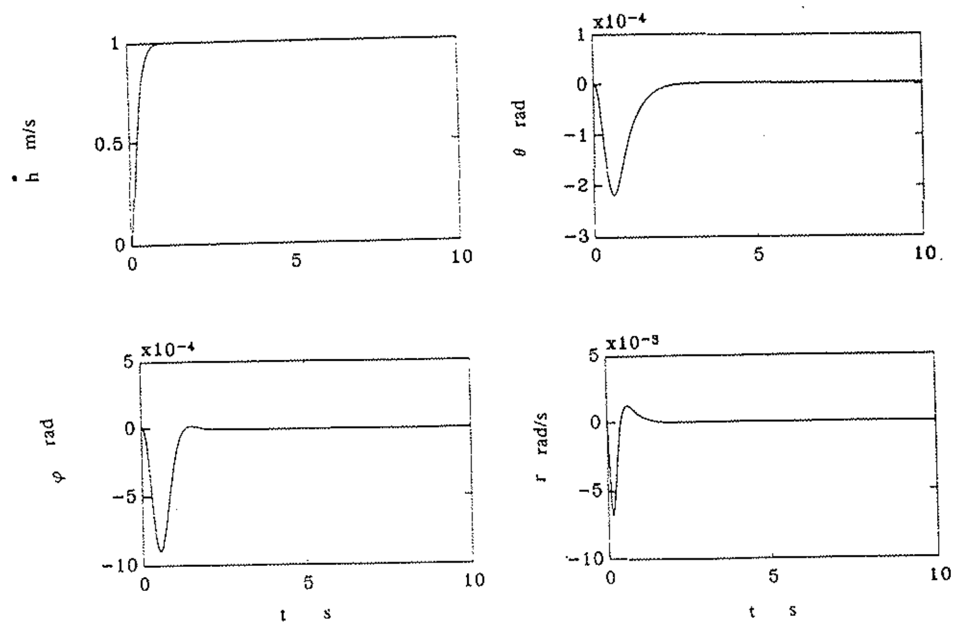

Figure 2: ACAH system response to a height rate unit step command. Linear 8th order system model with additional first order actuator dynamics for each control channel.

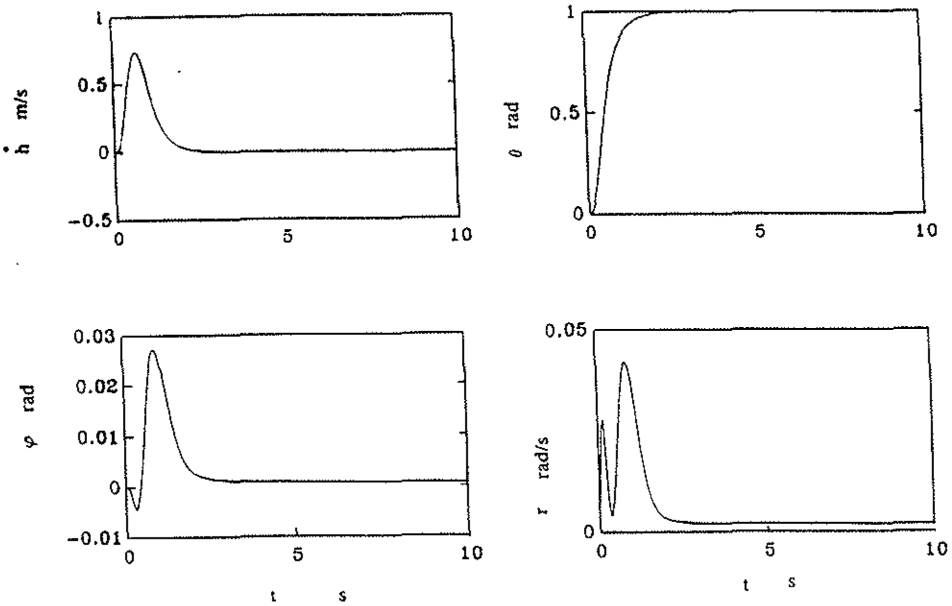

Figure 3: ACAH system response to a unit step command in pitch angle. Linear 8th order system model with additional first order actuator dynamics for each control channel.

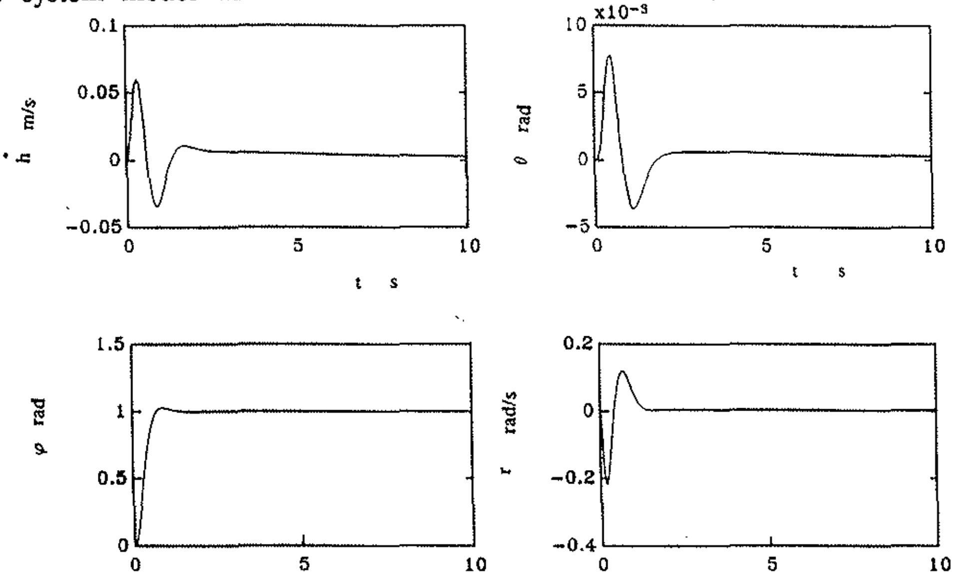

Figure 4: ACAH system response to unit step command in roll angle. Linear 8th order system model with additional first order actuator dynamics for each control channel.

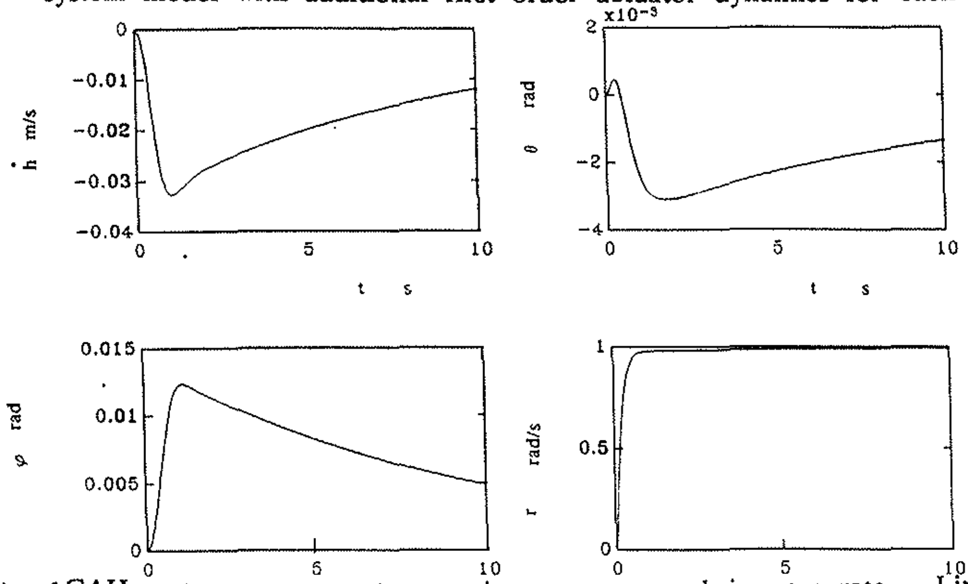

Figure 5: ACAH system response to a unit step command in yaw rate. Linear 8th order system model with additional actuator dynamics for each control channel.

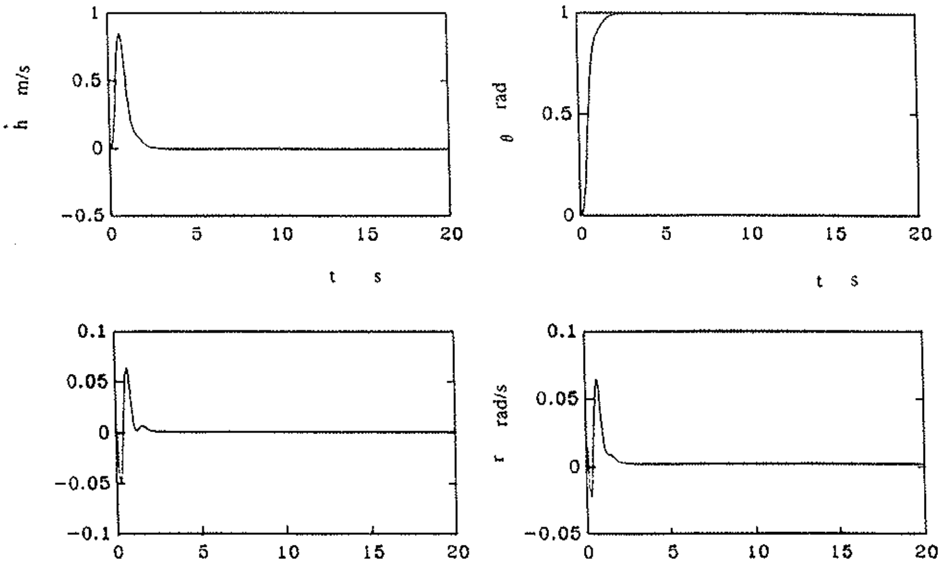

Figure 6: ACAH system response to unit step command in pitch angle for system model incorporating actuator and rotor dynamics.

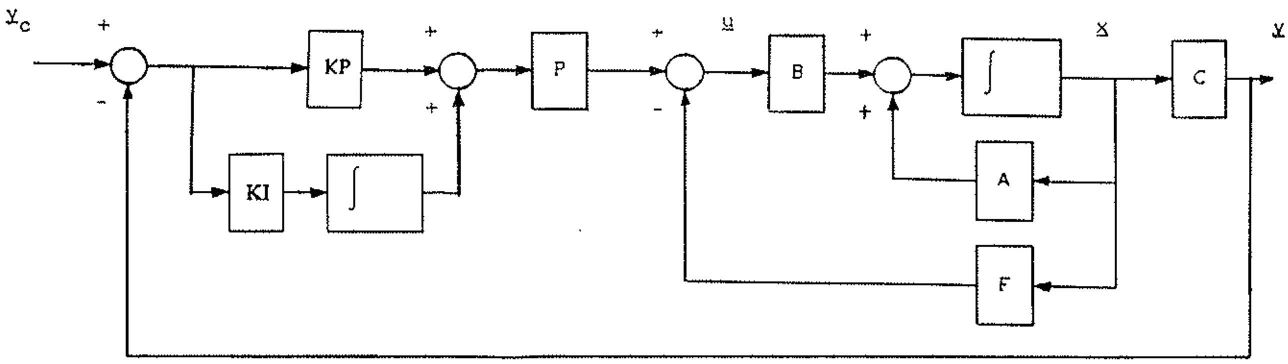

Figure 7: Linear multivariable controller with full state feedback, precompensation and output feedback incorporating proportional plus integral control.

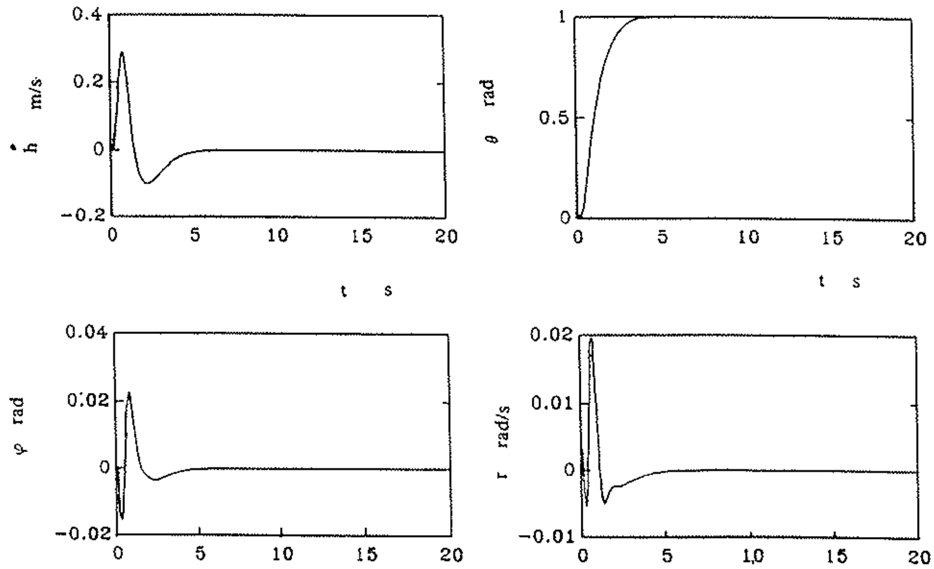

Figure 8: ACAH system with additional proportional plus integral control. Response to unit step command in pitch angle for linear system model incorporating actuator and rotor dynamics.

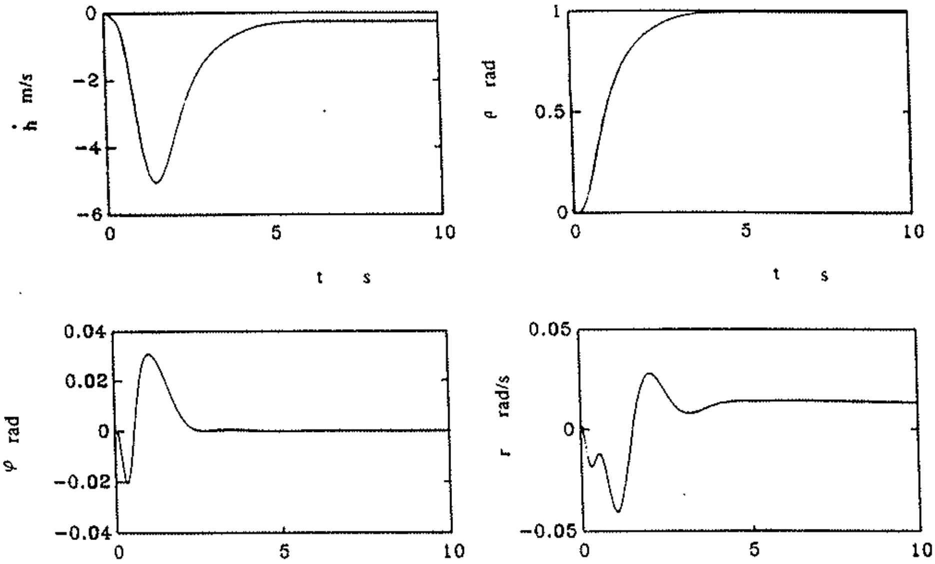

Figure 9: ACAH system with additional proportional plus integral control. Performance of system for 50 knot flight condition.

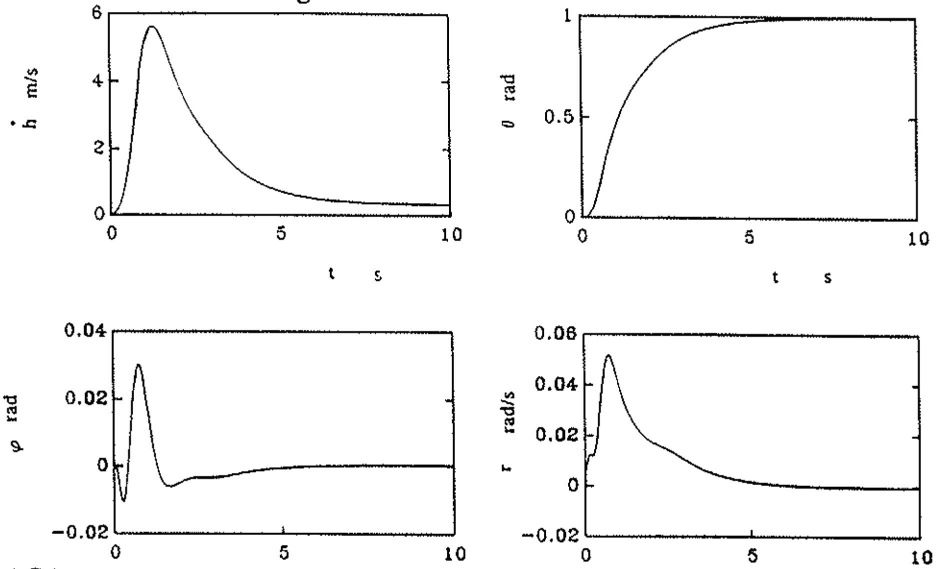

Figure 10: ACAH system with additional proportional plus integral control. Performance of system for 120 knot flight condition.

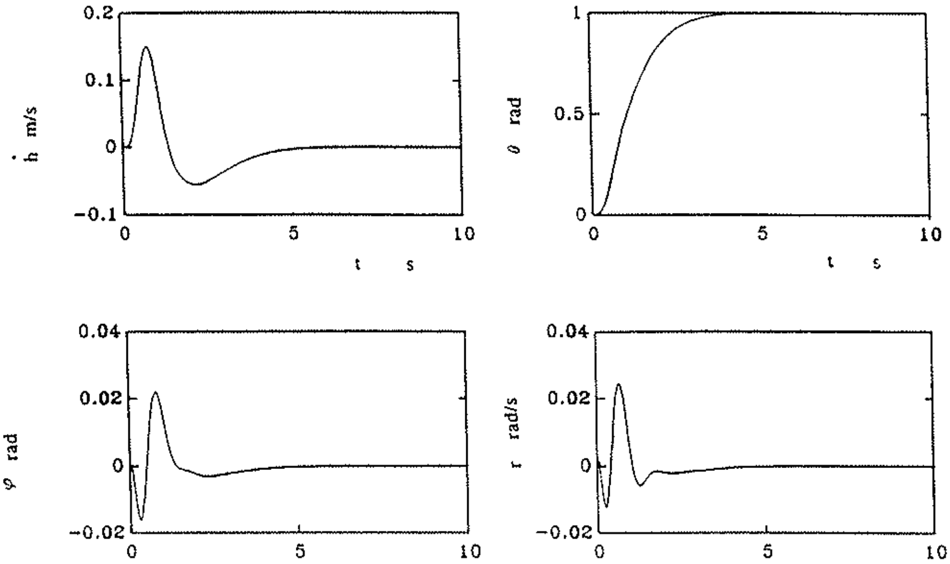

Figure 11: Gain-scheduled controller responses for unit-step command of pitch angle at 50 knots flight condition.

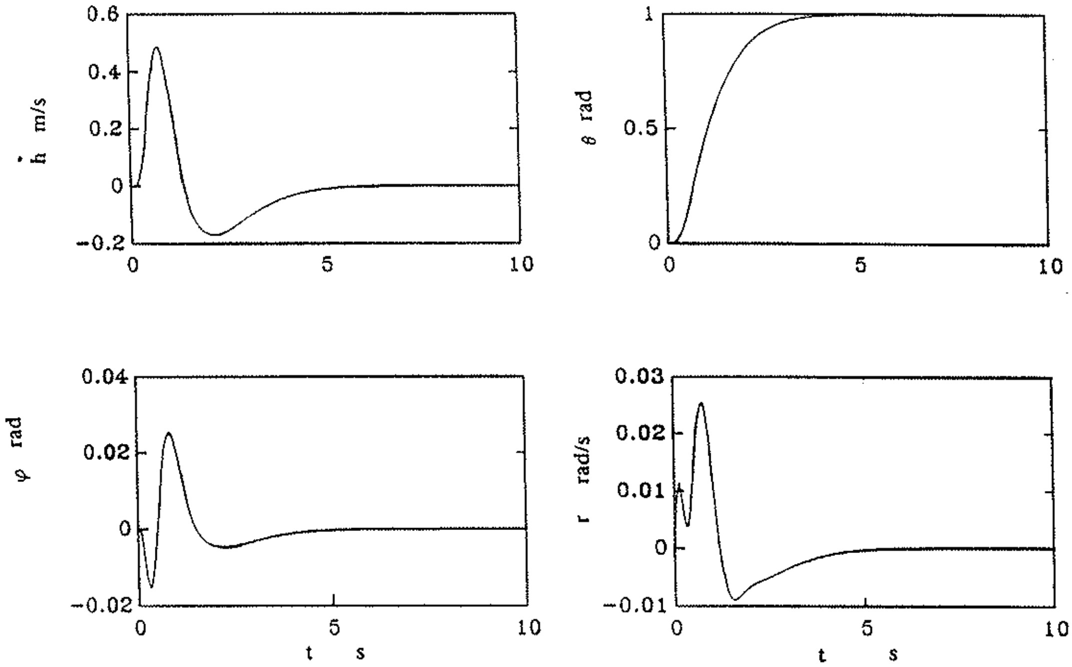

Figure 12: Gain-scheduled controller responses for unit-step command of pitch angle at 120 knots flight condition.

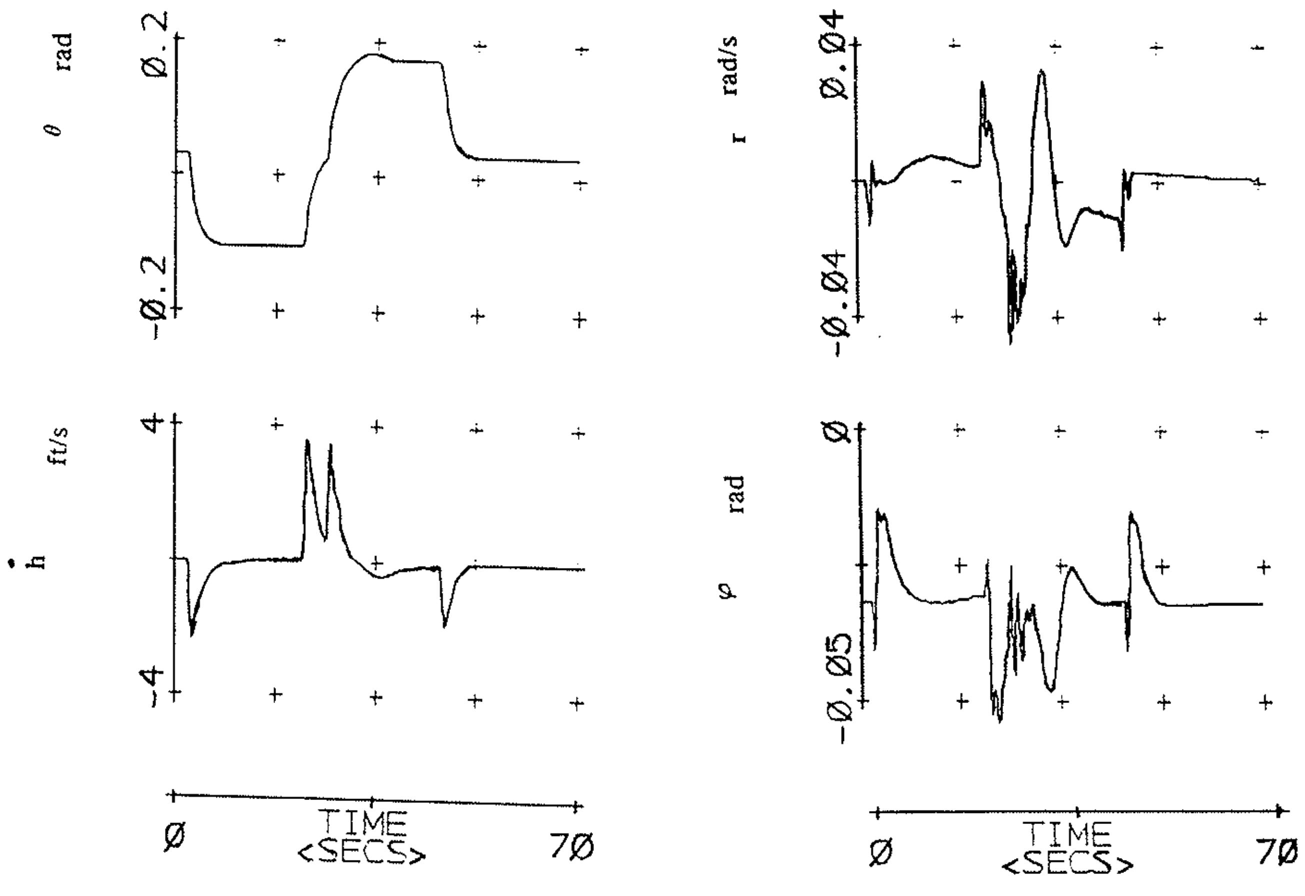

Figure 13: Response of system with scheduled controller. Non-linear simulation involving sequence of positive and negative steps for pitch angle.