

# SOME THOUGHTS ON DESIGN OPTIMIZATION OF TRANSPORT HELICOPTERS

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# ABSTRACT

Total operating cost per revenue seat-nautical mile is established as the object of design optimization, and various factors contributing to its minimization are examined. The role of cruise speed and total annual time available for revenue operations are reviewed first. Next, potential gains resulting from improved weight to equivalent drag ratios and compounding are investigated. The influence of aircraft purchase price and powerplant role in helicopter design optimization is studied, alongside the possibility of exchanging structural weight increases for lower cost per revenue seat-mile. Discussion of design decisions affecting total maintenance cost concludes this presentation.

# List of Symbols

| С                       | cost, \$  |                       | Subscripts                                       |
|-------------------------|---|-----------------------|--|
| ī,                      | avg. blade profile drag coefficient                   | 20                    | aircraft   |
| $\bar{c}_l$             | avg. rotor lift coefficient                           | cr                    | cruise   |
| D,                      | equivalent drag, lb.                                  | dpr                   | depreciation                                     |
| 'n.                     | coefficient, or factor                                | eng                   | engine   |
| L                       | distance between takeoff and<br>landing points, n.mi. | ex.eng<br>ex.fu<br>fu | except engines<br>except fuel<br>fuel            |
| n                       | integer   | ind                   | induced  |
| Т                       | time, hr  | ins                   | installed  |
| V                       | speed of flight kn                                    | insu                  | insurance  |
| ¥                       | speed of fingite, kit                                 | 0                     | baseline   |
| W                       | gross weight, lb                                      | rs                    | revenue seat                                     |
| WE                      | weight empty, lb                                      | rsm                   | revenue seat; n.mi                               |
| W                       | disc loading, psf                                     | tdm<br>toc            | total direct maintenance<br>total operating cost |
| w <sub>f</sub> ≡        | w/f; eq. flat plate area;                             | tot                   | total  |
|                         | (7) loading   | v                     | specu  |
| α                       | relative increment of helicopter<br>WE, minus engines | w                     | work   |
| <i>κ</i> <sub>w</sub> ≡ | $V_w/V_{cr}$ , ratio of work to cruise;               |                       | Superscripts                                     |
|                         | speeds  | hr                    | hourly   |
| μ                       | rotor advance ratio                                   | hr.rs                 | per hr & revenue seat                            |
| ρ                       | air density, slugs/cu.ft                              | Њ                     | pound  |
| ∑ ≡                     | $W/D_{\theta}$ , gross weight to eq. drag ratio       | rsm<br>WE             | revenue seat-mile<br>weight empty                |

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## 1. INTRODUCTION

# 1.1 General

Similar to other industrial products, optimization of rotary-wing aircraft is accomplished through continuous efforts of design organizations whose achievements are often inspired by new requirements and specifications developed either by procuring agencies or potential operators. In parallel with these practical, every-day activities, there has been a growing body of technical literature discussing various aspects of design and performance optimization of rotary-wing aircraft. As far as transport helicopters are concerned, as long as twenty-five years ago, Schnebly and Carlson<sup>1</sup> had already considered the problem of optimum design of helicopters of that type for maximum net profit. In more recent publications, a Soviet book by Tishchenko et al<sup>2</sup> should be mentioned as it is entirely devoted to a selection of transport helicopter and compound design parameters in order to optimize those machines with respect to either weight or economic criteria. This presentation is not a rigorous paper, but rather an informal talk, attempting to sketch in broad-brush strokes the possibilities and problems of transport helicopter design optimization. The main object of this latter is usually the best possible satisfaction of the actual or potential purchaser's requirements. Consequently often optimization criteria are explicitly specified by the customer. If not, the designer himself tries to establish them by asking hypothetical questions: what kind of helicopter with what performance, flying qualities, and economic characteristics should have the greatest appeal to the intended market.

Once these questions are answered, either a single criterion or a set of criteria can be formulated which, in turn, becomes the basis for developing the objective function or functional whose extremization becomes a goal of the formal optimization process.

It should be recalled at this point that transport helicopters are no different from other manmade objects, or man-controlled processes, in the respect that on the road towards optimal design various constraints will be encountered. Some of them may be physical (e.g. currently realizable strength of materials, airfoil maximum lift, or minimal drag coefficients), while others may reflect operational requirements (e.g. maximum disc loading), or requirements of a legislational nature (e.g. permissible noise level, or other environmental requirements).

## 1.2 Optimization Criterion

In a recent discussion of economic aspects of helicopters<sup>3</sup>, operators engaged in commercialscheduled, off-shore drilling, and executive transportation of people, unanimously indicated that low total operating cost per revenue seat-mile was the key to their business survival. That statement was accompanied by a strong plea to the manufacturers that everything possible be done to minimize that quantity. It appears hence that minimization of the overall cost per revenue seat-nautical-mile ( $C_{rsm}$ ) can be selected as an objective for design optimization of the people-transport helicopters.



Figure 1 Typical values of total operating cost per revenue seat-nautical mile

It is interesting to note that the present level of  $C_{rsm}$  (based on data from Ref. 3 plus two manufacturers' brochures, and expressed in 1978/79 dollars) is surprisingly uniform, being encompassed within approximate limits of 30 to 40 cents for a wide range of transport helicopters having from 6 to over 40 revenue seats. The only exception is the projected wide-body, HLH-based, helicopter capable of carrying over 200 passengers. In this case, a total operating cost per revenue seat-mile as low as 4 cents is visualized by Cameron<sup>3,4</sup>, although other experts expect that it may be somewhat higher but still quite low,  $C_{rsm} \approx 10 \text{ cts/rsm}$ .

In this talk, we want to sketch general directions in which design optimization should move in order to obtain a significant reduction of the  $C_{rsm}$ , as well as to identify at least some obstacles that may be encountered on that road. Furthermore, the sensitivity of  $C_{rsm}$  to the application of cost-saving policies to particular components forming the total operating cost per revenue seat-nautical-mile will be roughly indicated.

### 2. FIRST OVERVIEW OF THE PROBLEM

#### 2.1 Importance of Hourly Cost per Revenue-Seat and Work Speed

There is little doubt that the relationships linking the selection of basic design parameters with cost (taking into account various available options regarding layout details, component design, materials and manufacturing processes, etc.) are not always easily presentable in a simple analytical form. Nevertheless, they do exist and should be taken into consideration even if only the general trend between the influence of some design decisions on operating costs can be indicated.

For the moment, however, omitting all those details from our considerations, it may be stated that in the simplest form

$$C_{rsm} \approx C_{tac}^{hr} / V_w n_{rs} \tag{1}$$

$$C_{rsm} \approx (C_{toc}^{hr} / n_{rs}) / V_{w}$$
(12)

where  $C_{toc}^{hr}$  is the total operating cost per hour (say in S/hr),  $V_w$  is the work speed in knots that can be determined as a quotient of the doubled distance between the takeoff and landing points divided by the actual flying time plus ground time chargeable to commercial operations, and  $n_{rs}$  is the number of revenue seats. It should be noted that in all references to work speed and revenue utilization, we are considering a work cycle (e.g. a round trip, or two consecutive equal legs in a network) where pointto-point distance is L.



Figure 2. Character of the  $\kappa_w$  variation with flight distance and cruise speed plus sensitivity to ground time

From the design point of view, the cruise speed  $(V_{cr})$  and not the work speed is a significant aircraft characteristic. However,  $V_w$  and  $V_{cr}$  can be related as follows:

$$V_w = \kappa_w V_{cc} \qquad (2)$$

where  $\kappa_w$  is the work speed coefficient whose value for some assumed takeoff and landing procedures, ground time, and type of flight profile can be expressed as a function of the flight distance *L* and cruise speed  $\kappa_w = f(L, V_{cr})$ . A typical  $\kappa_w$  variation with both of these parameters is shown at the top of Fig. 2.

This relationship was developed under the following assumptions: Nominal allowances for the airborne portion of any leg were assumed at 25 seconds for takeoff, hover and acceleration to climb speed, one minute of climb (gaining one n.mi), one minute of maneuver, 30 seconds to descend and decelerate (gaining one n.mi), and 5 seconds to set down. All of this provides a convenient 3 minutes of "non-productive" flight time with two n.mi. gained. The ground time

used to develop the work rate coefficient,  $\kappa_w$ , was also nominal with a loading time of 5 minutes, unloading time of 2 minutes, and an increment of 3 minutes for fueling in each cycle. Thus the "base case" ground time becomes 17 minutes for the two-leg cycle.

The influence of ground-time variation on  $\kappa_w$  values for a very short (10 n.mi.) and longer (100 n.mi) point distances is shown in the lower part of Fig. 2. The implication of Fig. 2 from the design optimization viewpoint is quite clear. The  $\kappa_w$  coefficient can be recognized as some sort of operational "figure of merit", and thus should be maximized through design decisions leading to the reduction of ground time, which is the most significant parameter influencing the  $\kappa_w$  level, especially for very short haul operations.

This point is illustrated in Fig. 3. It is evident that a 3-minute saving in ground time per leg (6 min. per cycle from the baseline time of 17 minutes) is worth about 20 km of cruise speed for  $V_{cr} \approx 250$  km, and L = 100 n.mi, which drops to about 10 km for  $V_{cr} \approx 150$  km for the same distance. Very short haul (L = 10 n.mi)-achieving gains in  $V_w$  equivalent to those resulting from the 6-min. ground time reduction-would require an increase of cruise speed by over 50 km for  $V_{cr} \approx 200$  km and over 20 km for  $V_{cr} \approx 100$  km.

It can also be noted from Fig. 3 that for very short haul operations,  $V_w$  would vary little with cruise speed level. However, for L = 100 n.mi, and cruise speed  $100 < V_{cr} < 200$  kn,  $V_w$  appears as almost proportional to  $V_{cr}$  with  $\kappa_w \approx 0.75$ .

Taking into account Eq (2), Eq (1a) can be rewritten as follows:

$$C_{rsm} \approx (C_{toc}^{hr}/n_{rs})/\kappa_w V_{cr}$$
 (3)

A glance at the above relationship would lead one to a truism that in the fight for  $C_{rsm}$ minimization, the attack should go in two principal directions: (1) reduction of the hourly cost per revenue seat, and (2) increase of the cruise speed, along with the work speed coefficient; while remembering that  $C_{toc}^{hr}$  and  $V_{cr}$  are usually interrelated.

As far as current (and projected) levels of  $C_{toc}^{hr}/n_{rs}$  and corresponding cruise speeds are concerned, the lower part of Fig. 4 is presented as an illustration of those aspects, while in the upper portion of this figure, current prices per revenue seat are shown. There appears to be a trend toward a decrease of  $C_{toc}^{hr}/n_{rs}$  values with an increasing number of revenue seats, while the opposite is true as far as price per revenue seat is concerned.



Figure 3. Influence of ground time on the  $V_w$  vs.  $V_{cr}$  relationship



Figure 4. Typical hourly total operating cost (bottom) and price per revenue seat for presently operational and projected transport helicopters

### 2.2 Principal Components of Total Hourly Operating Cost



average annual utilization

#### 3. MAIN ELEMENTS OF DESIGN OPTIMIZATION

fuel and lubricants, and (5) crew. Typical percentile distribution of these cost items and their variation with average annual utilization of aircraft (based on manufacturers' data), as well as a trend of relative  $(C_{roc}^{hr}/n_{rs})$  value variation is shown in Fig. 5.

A glance at Fig. 5 would indicate two important facts: (1) except for crew costs, about 90% of the cost items making the  $(C_{toc}^{hr}/n_{rs})$  quantity are affected by design decisions, and (2) for a given aircraft, hourly cost per revenue seat with increasing annual utilization.

In order to gain a deeper insight into the role of design decisions regarding all of the above items, the following analysis will be made.

#### 3.1 Basic Equation

Taking into consideration some calendar period long enough (say, one year) to encompass various anticipated (scheduled) and unanticipated (unscheduled) interruptions of the operation, the cost per revenue seat-mile  $(C_{rsm})$  can be expressed as follows:

$$C_{rsm} = \frac{(total yearly expenditure for helicopter operation)}{(annual work time) \times (work speed) \times (no. of revenue seats)}$$

Remembering Eq (2), the above expression can be expressed through the following formula:

$$C_{rsm} = \sum_{j=1}^{j=N} (C_1^{\gamma r} + C_2^{\gamma r} + ... + C_j^{\gamma r} \div ... + C_N^{\gamma r}) / T_w^{\gamma r} \kappa_w V_{cr} n_{rs}$$
(4)

where C's are various expenditures associated with rotorcraft operation during one year, and  $T_w^{yr}$  is the annual "work-time" (in hours). It is obvious that  $T_w^{\gamma r} \kappa_w V_{cr}$  represents the total distance flows by the aircraft during one year in revenue flights.

The aim of the optimization process is to make Eq (4) a minimum, not only through selection of basic design parameters (e.g. main rotor radius, number of blades, tip speed, and blade geometry including the airfoil shape); but also by such aspects as general layout, component design, type of material, manufacturing process, etc.

However, before starting the minimization process of Eq (4), the type of aircraft operation should be determined since that would influence the importance of the  $V_{er}$  parameter.

It also appears advisable to consider the number of revenue seats as invariable. Then, taking into account the five main cost items shown in Fig. 5 and remembering that both yearly expenditures for depreciation and insurance are proportional to the aircraft purchase  $price(C_{ac})$ , Eq (4) can be rewritten as follows:

$$C_{rsm} = \left\{ \left[ \left( k_{dpr}^{\gamma r} + k_{insu}^{\gamma r} \right) C_{ac} / n_{rs} \right] + \left( C_{tdm}^{\gamma r} / n_{rs} \right) + \left( C_{ci}^{\gamma r} / n_{rs} \right) + \left( C_{ci}^{\gamma r} / n_{rs} \right) \right\} / T_{w}^{\gamma r} \kappa_{w} V_{cr}$$
(5)

In the process of selecting an optimal aircraft for the assumed task, minimization of Eq (5) should be performed separately for each considered configuration (single-rotor, tandem, or side-by-side helicopters, compounds, etc.) and then the results compared.

However, in the present "broad-brush" approach, let us look at the most significant signposts pointing toward design optimization. This can be done by examining what can be done to maximize the denominator and to minimize the numerator of Eq (5). In addition, one should identify the inter-relationships existing between quantities appearing in the denominator and numerator of that equation.

# 3.2 Annual Work Time

It is obvious that  $\mathcal{T}''_w$  (in hours) can be assumed arbitrarily at some fixed number (say, 1500 hours). But since the influence of operating time on cost per hour is so important (Fig. 5), it is more interesting to find out what factors influence the maximum potential number of hours that an aircraft can be flown per year in revenue service.

Using an approach similar to that given in Ref. 2,  $\mathcal{T}_{w}^{\gamma r}$  can be expressed as follows:

$$T_{w}^{\prime r} = 365 \times 24 - (T_{M} + T_{NR} + T_{NRF})$$

where  $T_M$  = time aircraft is on the ground for maintenance,  $T_{NR}$  = ground time for reasons other than maintenance, and  $T_{NRF}$  = non-revenue flight time (training, aircraft positioning, etc.).

Taking a deeper look into the times appearing in Eq (6), it may be stated that  $T_M = T_{SM} + T_{US}$  = time for scheduled and unscheduled maintenance, respectively, where  $T_{SM}$  = scheduled maintenance time for removal and replacement of removable components plus time for work on non-removable portions of the aircraft, and time for pre- and postflight inspections, and all other routine maintenance.  $T_{US}$  = unscheduled maintenance time (similar to the scheduled items listed above, but in conjunction with problems due to unpredictable events). Next,  $T_{NR} = T_W + T_{DN} + T_H$ , where  $T_W$  = weather groundings,  $T_{DN}$  accounts for "dead-of-night"; for example, when passenger flights may not be feasible, and  $T_H$  accounts for reduced schedules due to holidays and weekends. Finally, the meaning of  $T_{NRF}$  is self-explanatory.

It is obvious that decisions not to operate the aircraft under marginal weather conditions or during night hours can be taken by the operator on the basis of specific considerations. But provision of the potential to perform revenue flights under those conditions rests with the designer, who should assure the necessary flying qualities, cockpit instrumentation, and navigational equipment. This in turn would increase the aircraft purchase price and thus influence not only the denominator, but also the numerator of Eq (5). It may also affect the maintenance cost, while its influence on the fuel cost through an increase in gross weight and parasite drag, would probably be negligible.

### 3.3 Influence of Cruise Speed and Fuel Cost on $C_{rsm}$

As was indicated before, the significance of cruise speed as far as the work speed value is concerned increases with the operational flight distance. However, it may be stated that in general, if cruise speed could be increased with little or no detrimental effect on the cost items, it would contribute to  $C_{rsm}$  reduction.



tio constraint is still with us (Fig. 6). As for the Mach part of that constraint, its upper limit would probably remain under the strong influence of noise abatement requirements. Although through swept blade tips with thin airfoils, the

It is obvious that, in reality, variation in  $V_{cr}$  may significantly affect the cost

items. But in addition to eco-

nomic aspects one should

realize that as far as increasing maximum flying speed

capabilities of pure helicopters

is concerned, the old advancing

tip Mach number-advance ra-

noise barrier has recently been

Figure 6. Advancing tip Mach number-advance ratio constraint

moved closer to M = 1.0, it will be (perhaps conservatively) assumed that the Mach limit is  $(M_t)_{90} \circ = 0.91$ . With respect to operationally acceptable advance ratio values, it appears that even for new rotor concepts with proper torsionally tuned blades, swept tips, etc.,  $\mu = 0.45$  can be assumed as a limit. For higher  $\mu$  values, compounding in thrust and probably, lift, would be required. Under these assumptions, for pure helicopters the speed limit for horizontal flight would be of the order of 190 knots.

The relative cost share of fuel and lubricants (in this text, simply called "fuel cost") can, of course, change drastically with the increasing price of petroleum products (in Fig. 5, a low, at this writing, S0.60/gal was assumed for fuel); nevertheless, the designer's decisions regarding overall rotorcraft aerodynamic effectiveness as measured by the gross weight to the equivalent weight ratio  $(W/D_e) = f(V)$ , engine selection with due consideration of cost, sfc, and  $W/n_{rs}$  ratio still remains of prime importance<sup>5</sup>.

As far as operational cruise speed is concerned, it is of course appealing from the fuel economy point of view to have it close to that corresponding to  $\Sigma_{max} \equiv (W/D_e)_{max}$  (which will be called  $V_{\Sigma max}$ ), since fuel consumed per revenue seat-mile can be expressed as follows:

$$W^{rsm}_{f_{ij}} = (W/n_{rs}) sfc/325 \Sigma_{y} \kappa_{w}$$
<sup>(7)</sup>

where  $\Sigma_y \equiv W V/325 SHP$  (see Ch III, Ref. 7).

Variation of  $\Sigma_{\nu}$  vs speed at SL, std for presently operating transport helicopters as well as those representing the 1980 technology level is shown in Fig. 7. Here, an auxiliary grid of constant (W/SHP) lines is marked as well as the constraint to the pure helicopter flight speed. For comparison, the  $(W/D_{e}) = f(V)$  for a compound transport helicopter of the tail-ring type (courtesy of D.N. Meyers of Piasecki Aircraft Co.) is also shown.

Eq (7) indicates that if there were no other fuel expenditures (i.e. for ground run, hover, climb, etc.) and if the variation in sfc could be neglected, then flying the helicopter at  $V_{\Sigma max}$  would be synonymous with the minimization of the fuel contributing to the total  $C_{rsm}$  value. However, there are other components of the hourly costs which are, or may be, assumed in the first approximation to be independent of the cruise speed.



Figure 7 Gross weight to the equivalent drag ratio for transport helicopters and a compound, shown vs flightspeed at S/L, std.

Thus the question may be asked whether it would be beneficial to increase the cruise speed beyond  $V_{\Sigma max}$ , and at the expense of higher fuel consumption (i.e. its cost as well), to operate at a more favorable speed from the overall cost-per-revenue seat-mile viewpoint.

To answer this question, Eq (5) is first rewritten as follows:

$$C_{rsm} = \frac{(C_{ex,fu}^{hr}/n_{rs})}{\kappa_w V_{cr}} + \frac{C_{fu}^{lb}(W/n_{rs})sfc}{325\kappa_w \Sigma_y}$$
(8)

where  $V_{cr}$  is in knots, and the new symbols are  $C_{ex,fu}^{hr}$  = hourly total operating cost of all items except fuel and lubricants, and  $C_{fu}^{lb}$  = cost of fuel and lubricants per pound. All the sfc values are assumed to incorporate hourly consumption of lubricants per hp.

It is readily seen that the first term of Eq (8) decreases with increasing  $V_{cr}$ , while the second one increases for  $V_{cr} > V_{\Sigma max}$ . It may be expected hence that at some  $V_{cr} > V_{\Sigma max}$ ,  $C_{rsm}$  would *i* reach its minimal value.

To get some idea about the influence of the fuel cost on the  $V_{cr}$  values minimizing  $C_{rsm}$ , let us investigate the following relationship:

$$C_{rsm}/(C_{rsm})_{\Sigma max} = f(V_{cr}) \tag{9}$$

where  $(C_{rsm})_{\Sigma max}$  is the total cost per revenue seat-nautical mile at  $V_{cr} = V_{\Sigma max}$ .

To simplify the matter it will be assumed that the magnitude of the installed power is governed by other requirements such as hovering and/or service ceiling with one engine inoperative, and thus flying at speeds  $V_{\Sigma max} < V_{cr} < V_{max}$  would not demand any increase in the installed power. Remembering also that  $(C_{rsm})_{\Sigma max} = (C_{toc})_{\Sigma max}/n_{rs} \kappa_w V_{\Sigma max}$ , Eq (9) can be expressed as

$$\frac{C_{rsm}}{(C_{rsm})_{\Sigma max}} = \frac{C_{ex,fu}^{hr}}{(C_{toc}^{hr})_{\Sigma max}} \frac{V_{\Sigma max}}{V_{cr}} + \frac{C_{fu}^{lb} sfc V_{\Sigma max}}{325 \Sigma_{v} (C_{toc}^{hr}/W)_{\Sigma max}}$$
(92)

The character of the variation with cruise speed of the ratio expressed by Eq (9a) as well as the first and second term of that equation is shown in Fig. 8; assuming  $\Sigma_{\nu}$  = f(V) as the upper boundary of the early 1970 helicopters, and  $C'_{fu}^{b} =$ 0,2 \$/lb. For a lower fuel cost (0.1 \$/lb), the  $(C_{rsm})/(C_{rsm})_{\Sigma max}$  ratio is also shown. It can be seen from this figure how the cost of fuel influences the optimal cruise speed. At low fuel costs  $V_{opt}$  tends to shift from  $V_{\Sigma max}$  toward higher speeds, while with the increasing cost of the combustible (and lubricating) mater-



Figure 8. Example of cruise speed influence on relative cost per revenue seat-mile for two levels of fuel cost

ial, it comes closer to  $V_{\Sigma max}$ .

At this point one may say that the selection of cruise speed represents an operator's decision, while the price of petroleum products also lies completely outside of the designer's sphere of authority. Hence where are the inputs that can be made through design decisions during the design optimization? There are many of them. Even the practicality of selecting high cruise speeds by the operator for economic reasons depends on the ability of the helicopter to fly at speeds close to the powerlimited  $V_{max}$  without encountering excessive cabin vibration and high periodic structural loads.

However, the most important aspects of design optimization with respect to the fuel-cost share in  $C_{rsm}$  is the assurance of high  $(W/D_e)$  values at the highest possible flying speeds and when doing so, to adversely affect as little as possible such other components of the total hourly operational cost as those reflecting on the aircraft purchase and maintenance costs.

As to the value of the flying speed corresponding to  $V_{\Sigma max}$ , it can be obtained from such classical expressions as those in Ch III, Ref. 7, and can be approximately expressed in knots as

$$V_{\Sigma max} \approx 1.64 \sqrt[4]{k_{ind}k_{\gamma}^{2} w w_{f}/\rho^{2}}$$
(10)

where  $k_{ind}$  = the induced power coefficient (it may be assumed as  $k_{ind} \approx 1.1$  for single rotors and  $k_{ind} \approx 1.8$  for tandems);  $k_y$  = the download factor in forward flight, w is the disc loading,  $w_f \equiv W/f$ is the equivalent flat plate area (f) loading, and  $\rho$  is the air density.

The maximum value of the gross weight to the equivalent drag ratio becomes

$$\Sigma_{max} = 1/k_{tot} \left[ k_v \sqrt{k_{ind} w/w_f} + (3/4)\mu(1 + 4.7\mu^2)(\bar{c}_d/\bar{c}_l) \right]$$
(11)

where the new symbols are  $\mu \equiv 1.69 V_{\Sigma max}/V_t$ ,  $V_t$  being rotor tip speed in fps;  $\overline{c}_d/\overline{c_l}$  is the ratio of the average profile drag coefficient to the average lift coefficient (defined as  $\overline{c}_l = 6w/\sigma \rho V_r^2$ ); and  $k_{tot} \equiv SHP/RHP.$ 

It can be seen from Eq (10) and Fig. 9 that as far as design aspects are concerned, an increase in both disc and equivalent flat plate area loading pushes  $V_{\Sigma max}$  higher. From a configurational viewpoint, tandems for the same values of the  $ww_f$  product would exhibit higher  $V_{\Sigma max}$  levels because of the higher  $k_{ind}$  values (but  $\Sigma_{max}$  will obviously be lower). It appears that compounding would tend to move  $V_{\Sigma max}$  toward higher cruise speed values (see Fig. 7).

With respect to the  $\Sigma_{max}$  values, Eq (11) indicates that a fractional reduction of the disc loading contributes as much to the improvement of  $\Sigma_{max}$  as an equal increase in  $w_f$ , representing aerodynamic cleanness of the airframe (including hub).

However, because of detrimental effects of too-low disc loadings (large radius of the lifting rotor) the main drive in design optimization would probably go towards increasing  $w_f$ . Furthermore,



Figure 9 Flying speeds for  $\Sigma_{max}$ , and  $\Sigma_{max}$  values vs.  $w_f$  at selected w and  $\overline{c}_{d}/\overline{c}_{l}$  levels

it can be seen from Fig. 9 (top) that within practical limits of  $8 \le w \le 12 psf$ , the variation of the disc loading would not significantly affect the  $\Sigma_{max}$  values. It can also be seen that the airfoil characteristics reflected in the  $\overline{c}_d/\overline{c}_l$  ratios are quite important, and reduction in that ratio should considerably improve the  $\Sigma_{max}$  values.

# 3.4 Influence of $\Sigma_{max}$ Values, and $\Sigma = f(V)$ Character on $C_{rsm}$

In order to get an idea regarding the influence of the  $\Sigma_{max}$  values and character of the  $\Sigma = f(V)$  variation on  $C_{rsm}$ , the ratio  $C_{rsm}/(C_{rsm})_0$  will be examined; where  $C_{rsm}$  refers to a new, and  $(C_{rsm})_0$  refers to the previously studied baseline aircraft (Fig. 8). This ratio will be examined for helicopters representing advanced technology (upper boundary of  $\Sigma = f(V)$  for 1980 technology) and for the compound whose  $\Sigma = f(V)$  is shown in Fig. 7.

Using Eq (8) and remembering that the cost per revenue seat-mile of the baseline aircraft can be expressed as

$$(C_{rsm})_{o} = (C^{hr}_{toc})_{o}/n_{rso} \times_{wo} V_{opto},$$

the desired ratio becomes

$$\frac{C_{rsm}}{C_{rsm_o}} = \frac{(C_{ox,fu}^{hr}/n_{rs})\kappa_{w_o}V_{opt_o}}{\kappa_{w}V_{cr}(C_{toc}^{hr}/n_{rs})_o} + \frac{[C_{fu}^{b}/(C_{toc}^{hr}/n_{rs})_o](W/n_{rs})sfc\kappa_{w_o}V_{opt_o}}{325\kappa_{w}\Sigma_{v}}$$
(12)

where  $V_{opto}$  is the flying speed minimizing  $C_{rsm}$  for the baseline aircraft (e.g.  $V_{opt} = 140 \text{ kn}$  for the higher fuel cost of  $C_{fu}^{lb} = \$0.2/lb$  (Fig. 10)).

5-10

It can be seen from Fig. 10 that if the improvements in the  $\Sigma = f(V)$  postulated for the 1980 technology helicopters can be realized without increasing the values of  $(C_{ex,fu}^{hr}/n_{rs})$  and  $(W/n_{rs})$ , then reductions of up to 25% over the current Crsm levels can be expected. Even assuming a 5% increase in the hourly non-fuel cost per revenue sear, and an equal increase in the gross-weight to the number of revenue seats ratio, the basic picture remains the same. One should note, however, that in order to realize these improvements, the helicopter should be capable of being operationally flow (low cabin vibration level and periodic structural loads) at cruise speeds close to 180 kn. It should also be mentioned that in plotting Fig. 10, it was



Figure 10. Illustration of the influence of higher  $\Sigma$  values for advanced helicopters and compounding on relative cost per revenue seat-mile

assumed that  $\kappa_w = \kappa_{wo}$  in spite of an increase in  $V_{cr}$ . This assumption obviously becomes more reliable as the operation flight distances increase (Fig. 2).

As far as compounds are concerned, Fig. 10 also indicates that if by some chance, both  $(C_{ex.fu}^{hr}/n_{rs})$  and  $(W/n_{rs})$  could be kept at the same values as for helicopters of a similar technology level, then gains in  $C_{rsm}$  of up to 25% could be realized through compounding. However, taking a more realistic viewpoint, the compounding would bring some penalties in both  $(C_{ex.fu}^{hr}/n_{rs})$  and  $(W/n_{rs})$ , one can see that with a 20% increase in those quantities, compounds may still show a small operating cost advantage over pure helicopters. But should the penalties go still higher, the advantages may evaporate.

It should be emphasized at this point that the above-discussed potential gains in  $C_{rsm}$  resulting from either improved aerodynamic effectiveness of pure helicopters or through compounding are realizable only for longer flight distances, where cruise is the predominant regime of flight. For very short-haul operations, those improvements would obviously be of little value.

#### 3.5 Aircraft Purchase Price

As indicated in Fig. 5, the depreciation (amortization) and insurance costs represent approximately 35 to 55% of the hourly operating cost, depending on annual utilization. Both of these items are proportional to the aircraft purchase price ( $C_{ac}$ ), which on a yearly basis (see Eq. 5) are

$$C_{dpr}^{\gamma r} = k_{dpr}^{\gamma r} C_{ac}$$
 and  $C_{insu}^{\gamma r} = k_{insy}^{\gamma r} C_{ac}$ 

where  $k_{dpr}^{\gamma r}$  is usually taken as equal to 1/10 or 1/12. With respect to  $k_{lnsu}^{\gamma r}$ , it should be strongly emphasized that it depends on the safety of operation. This means that such aspects as structural integrity and flying qualities definitely represent cost items. Furthermore, as clearly visible from U.S. experience, the premiums paid for hull and casualty damages offer the operator little protection from the possible disastrous loss of public confidence in aircraft that have been exposed to an unfortunate series of accidents.

A glance at Fig. 4 (top) indicates that the present average  $(C_{ac}/n_{rs})$  values obtained from data in Ref. 5 (given in 1978/79 dollars) are included for medium and very large size helicopters within limits of \$100,000 <  $C_{ac}/n_{rs} < $200,000$ , while one small machine shows  $C_{ac}/n_{rs} \approx $57,000$ .



Figure 11. Installed power and gross weight per revenue seat, and installed power loading for existing and projected transport helicopters



Figure 12. Trends in specific weights and specific 1979 prices of modern turboshaft engines

One should remember that outside the designer-controlled inputs there are many economic manufacturing and marketing factors which could influence the  $(C_{ac}/n_{rs})$  level; for instance, position of the currently offered aircraft along the learning curve, sharing of development costs with the military or other government agencies, technical and organizational level of production facilities, labor cost. etc. Nevertheless, it is the designer who through his inputs and decisions creates the technical basis for the aircraft purchase cost. In that domain, there are so many aspects of the designers' contributions to design optimization (configuration, layout, simplicity of design, matching the design to the manufacturing process, etc.) that discussion of only the most important ones would require a volume similar in scope to Ref. 2

The designer most directly exercises his influence on shaping the concept and cost of such major assemblies as airframe and dynamic system. However, the powerplant, navigational equipment, and instrumentation represents a somewhat different case. For instance, in the powerplant case, the designer contributes to the aircraft optimization by deciding the number of engines and then selecting the most suitable engine from those actually being marketed or anticipated in the near future. These decisions affect a relatively small fraction of the aircraft purchase cost.

As can be seen from Fig. 11 (bottom), on the average, the installed horsepower per revenue seat amounts to  $(SHP_{ins}/n_{rs}) \approx 150 \ hp/rs^5$ . Fig. 12 (based on

data provided by R. Semple of Boeing Vertol) indicates that in the 1500-4000 hp rated power range, the engine cost per horsepower is  $C_{eng}/hp \approx \$90/hp$ . This means that the powerplant cost per revenue seat is approximately  $C_{eng}/n_{rs} \approx \$13,500/rs$ , which amounts to about 10% to 12% of the medium transport helicopter price.

However, one should realize that the importance of engine selection in helicopter design optimization goes well beyond the above-indicated percentage since, through overhaul and replacement costs it affects other important items of yearly expenditures which are reflected in the total direct maintenance cost. For this reason, special attention will be paid to the powerplant aspects.

#### 3.6 Powerplant Role in Helicopter Optimization

With respect to powerplants, the designer is usually faced with the following: (1) establishment of the total level of installed power, and (2) distribution of that power according to the number of engines (usually two or three).

In principle, the total power installed can be dictated by one of the following requirements which, in the design optimization process, become constraints: (1) maximum flying speed, (2) service ceiling with one engine inoperative, and (3) hovering OGE, or prescribed rate of vertical climb under given conditions of pressure altitude and temperature.

For helicopters with  $\Sigma = f(V)$  characteristics anticipated for the 1980s (Fig. 7), the installed power level would probably be determined by either condition (2) or (3). For single-rotor helicopters the  $SHP_{ins}$  value is usually dictated by the hovering requirements, while for tandems, it is dictated by the service ceiling requirements. The current levels of installed power loading shown in Fig. 11<sup>5</sup> (center) indicate a wide scatter in that value with  $4 < W/SHP_{ins} < 7 lb/hp$ . Fig. 13 was prepared to show the single-rotor helicopter  $W/SHP_{ins}$  values resulting from the hovering requirement at 4000 ft. std., and 4000 ft, 95°F (plotted vs disc loading on the left side); and those on the right as dictated by the service ceiling (R/C = 180 fpm) of 12,000 ft, std. The influence of the  $\overline{c}_d/\overline{c}_l$ ratio is also indicated by assuming it equal to 1/40 and 1/60.

A comparison of  $W/SHP_{ins}$  values from Figs. 11 and 13 with those marked on Fig. 7 indicates that for pure helicopters, once the power installed loading resulting from the OGE hovering conditions at 4000 feet are satisfied, there should be no problem of achieving flight speeds up to about 190 knots (the advancing blade blade tip Mach number – advance ratio constraint). Also it appears that even for the two-engine configurations, the one-engine-out condition does not dictate the  $(W/SHP_{ins})$  levels. Consequently, in striving for power loading, maximization efforts should center around hovering and/or vertical climb aspects. Here disc loading (Fig 13, left-side)



Figure 13. Examples of installed power loading for single-rotor helicopters, resulting from hovering and service ceiling with the one-engine-out requirement

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appears to be the strongest design parameter, while the influence of the  $c_d/c_l$  ratio is somewhat smaller. It should be remembered, however, that although a reduction of w leads to higher  $(W/SHP_{ins})$  values it may cause structural weight increases and thus, higher  $(W/n_{rs})$  values. This in turn could lead to higher power installed per revenue seat since  $(SHP_{ins}/n_{rs}) = (W/n_{rs})/(W/SHP_{ins})$ . These consequences of disc loading reduction should be weighed in the optimization process.

Fig. 13 seems to imply that for single-rotor helicopters, and typical hovering and service ceiling with one-engine-out requirements, a decision regarding two vs. three engines will be influenced by factors other than loss of one engine. A glance at Fig. 12 will indicate that both the specific cost  $(C_{eng}/SHP)$  and specific weight  $(W_{eng}/SHP)$  of the engine increase with decreasing rated power. Hence, it appears that from the above points of view, splitting the installed power required into two rather than three units appears more advantageous. It is obvious that in very large helicopters the availability of high-powered engines and limits to power that may be transferred through a single mesh of the transmission system may force the designer to select more than two engines.

There are obviously other aspects both positive (e.g. possibility of hovering IGE with one engine out) and negative (e.g. maintenance) of the more than two-engine configuration. Assuming that the appropriate engine-out performance may be realized with two engines, it would seem with regard to maintenance that the logic to limit the installation to two powerplants is compelling. Admittedly there is greater ease in removing a smaller engine, but the complexities of additional engines in terms of duplicated systems (accessories, functional subsystems, and controls) can only increase direct maintenance costs. The chief remaining advantage with regard to maintainability of additional powerplants is portability of unserviceable engines, providing that the aircraft can be ferried safely with one engine out. In total, however, maintenance considerations favor minimizing the number of powerplants.

### 3.7 Exchange of Structural Weight Increase for Lower Crsm

A question that often comes to one's mind during design optimization is whether it would be possible to reduce the cost of various helicopter components by increasing their weight, and what influence the so-obtained cost saving would have on the  $C_{rsm}$ . When trying this approach, one should be aware of the danger of runaway "weight inflation" which may occur in the psychological climate of even slight relaxation of the strict low-structural-weight standards. Many examples can be quoted when such relaxation of weight consciousness later resulted in costly, and not always successful, crash weight-reduction programs. Assuming, however, that through very strict controls by the designer, helicopter weight empty, except engines, can be increased to and not beyond the intended limit of  $WE_{ex.eng} = (1 + \alpha)WE_{ex.eng_0}$ , than, at least on paper, an exercise can be performed examining how the associated reduction of the weight-empty cost to  $C_{ex.eng}^{WE} = (1 - \beta)C_{ex.eng_0}^{WE}$ will influence the  $C_{rsm}$  decrease.

An increase of the helicopter weight empty, except engines, by a fraction  $\alpha$  would lead to a new  $(W/n_{cc})$  value, which can be approximately expressed as

$$\frac{W}{n_{rs}} = (1+\alpha) \frac{WE_{ex.eng}}{n_{rs}} + \frac{W_{eng}}{n_{rs}} \div \frac{W_{fu}}{n_{rs}} + \frac{W_{crew}}{n_{rs}} + W_{pass}$$
(13)

where the new symbol  $W_{pass}$  = average passenger weight, assumed as 180 lb. In Eq (13), the engine weight contribution can be written as  $(W_{eng}/n_{rs}) = [(W/n_{rs})/(W/SHP_{ins})] (W_{eng}/SHP_{ins})$  and fuel weight per  $n_{rs}$  can be computed by assuming it is equal to a quantity required; say, for 2.5 hours of cruise flight:  $(W_{fu}/n_{rs}) = 2.5(W/n_{rs})V_{cr} sfc/325\Sigma_v$ . Substituting these quantities into Eq (13) and solving for  $(W/n_{rs})$  the following is obtained:

$$\frac{W}{n_{rs}} = \frac{(1+\alpha)(WE_{ex.eng}/n_{rs}) + (W_{crew}/n_{rs}) + 180}{1 - 2.5(sfc V_{cr}/325\Sigma_{v}) - [(W_{eng}/SHP_{ins})/(W/SHP_{ins})]}$$
(14)

Dividing both sides of Eq (14) by  $(W/n_{rs})_{a=0}$ , the relative increase of  $(W/n_{rs}) = f(\alpha)$  can be obtained. An example of this ratio is shown in Fig. 14 where it was calculated under the following assumptions:  $(WE_{ex.eng}/n_{rs})_{a=0} = 480 \ lb/rs$ ;  $(W/SHP_{ins}) = 6.0 \ lb/hp$ ;  $(W_{eng}/SHP_{ins}) = 0.18 \ lb/hp$ ;  $V_{cr} = 140 \ kn$ ;  $\Sigma_{v} = 4.5$ ;  $sfc = 0.5 \ lb/hp.hr$ ; and  $(W_{crew}/n_{rs}) = 20 \ lb/rs$ . Under these assumptions  $(W/n_{rs})_{a=0} = 800 \ lb/rs$ , which is

typical for contemporary transports (Fig. 11, top).

Next, the relative variation of cost per revenue seat-mile with respect to  $(C_{rsm})_{a=0}$  was computed, postulating various levels of decrease ( $\beta = -\alpha$ ,  $\beta = -2\alpha$ , and  $\beta =$  $-3\alpha$ ) in the purchase cost of the helicopter without engines. The results shown in Fig. 14 were computed under the following assumptions: initial cost of depreciation and insurance per flight hour and revenue seat of helicopter without engines = \$13.50/hr.rs, which decreases with  $\alpha$  by a factor  $(1 - \alpha)$ ,  $(1 - 2\alpha)$ , or  $(1 - 3\alpha)$ . Depreciation and insurance cost due to engines =





\$1.85/hr.rs, which increases with  $\alpha$  at the rate of the  $(W/n_{rs})$  increase. Fuel (and lubricant) cost = \$10.10/hr.rs, which was calculated for \$0.2/lb (solid lines in Fig. 14), and \$0.1/lb (broken lines). It increases with  $\alpha$  proportionally to the  $(W/n_{rs})$  changes. The total direct maintenance cost was assumed independent of  $\alpha$ ; and amounts to \$10.40/hr.rs = const. The crew cost was also assumed invariant and is equal to \$4.00/hr.rs. For  $V_w = \kappa_w V_{cr} = 0.75 \times 140 = 105$  kn and the higher fuel cost,  $(C_{rsm})_{\alpha=0} = $0.38/rsm$ , which is typical (Fig. 1).

It can also be seen from Fig. 14 that some decrease in the total operating cost per revenue seat-mile can be achieved if an increase of the weight empty of a helicopter minus engines can be exchanged for a cost reduction of the affected components. However, those gains remain modest, unless really spectacular price reductions can be linked to structural weight increases. It should also be noted that overall  $C_{rsm}$  reductions are obtained at the expense of a higher fuel consumption (for this reason, gains are higher for the lower fuel cost), which may not be a popular idea in an energy-conscious environment<sup>6</sup>.

### 3.8 Design Decisions Affecting Total Hourly Maintenance Cost

As in the case of the helicopter purchase price, total maintenance expenditures represent a considerable share of the hourly operating cost (Fig. 5). Consequently it should be of interest to review the role of at least some design decisions on the process of  $C_{rsm}$  minimization. The total direct maintenance cost is strongly affected by such economic factors beyond the designers control as labor, pay-scale level, etc., but as in the case of the purchase price the designers through their decisions regarding such aspects as accessibility, ease of removal of components and their reliability plus maintainability, etc., exert a decisive influence on the total direct maintenance cost level.

It will be recalled that the standard for comparing aircraft maintenance cost is the ATA formula based on regression of historic data showing the relationship between labor, weight, materials, and price of the aircraft. While this method has been useful over the years for in-service aircraft of the same technical generation, it offers no solution to the questions that the designer must consider,

particularly if he is willing to depart from the beaten path of design for maintenance. For example, he might ask, "If I made this aircraft completely accessible so as to permit 'instant' inspection, would the labor saving offset the weight penalty?" The ATA formula provides no answer or even a hint as to the trades, since it simply says "if it weighs more, it will require more labor." And similarly, with regard to materials cost, if the designer specified the most expensive parts of the utmost reliability, the formula would indicate unacceptably high materials cost. Having no formula for innovative design, the designer must resort to gathering data on each definable element of the maintenance system, determining its maintainability characteristics: size, weight, reliability (cyclic and/or hourly) price, complexity, exposure, etc., and then aggregate the resulting hourly and/or cyclic cost by subsystem for the aircraft configuration under consideration. Each candidate design is then tested against a base (preferably on an existing operational type) where component and total maintenance characteristics and costs are known. In this largely judgmental process, with all the tedious analysis of components, care must be taken to detect interactions between subsystems that can modify the projected maintenance characteristics, and thereby impact the maintenance cost. Throughout the design for maintenance procedure, the design team must be noting the consequences of those decisions on the ultimate suitability of the aircraft in its operational role. Accessibility not only influences weight, but can also affect drag and thereby performance (payload, range, and speed) and with the additional considerations of manufacturing complexity the accessibility decision, as only one small example, also impacts price and consequentially depreciation and insurance.

A deeper insight into the determination of maintenance cost can be found in Ref. 2. Our recommendation for this complex situation is a multi-routine computer program which tests the designs for economic merit in performing its expected task. This, however, must be the subject of another paper.

### 4. CONCLUDING REMARKS

Design optimization of transport helicopters is a game played with several control variables in the presence of overriding constraints and requirements (Table I). The goal of that game is the development of the most marketable product for potential customers. The lowest possible total operating cost per revenue-seat and nautical mile appears as the strongest single optimization criterion for transport rotary-wing aircraft. However, there is no single favored path leading directly to that optimization goal. Consequently, the attack should be carried out simultaneously on several cost-influencing targets. An example of gains in total operating cost resulting from a multi-prong attack on the technology front is shown in Fig. 15, reproduced from a recent presentation by Ellis and Walls<sup>8</sup>.

It should be noted however that in a complete optimization cycle technology advances are combined with selection of optimal design parameters. In this respect, the designer's intuition and experience can be helped through development of functional relationships linking control variables with various  $C_{rsm}$  components, and then finding optimum values of those variables through such techniques as multivariable search<sup>9</sup>; and others, so well summarized in Ref. 10.

In order to better prepare future designers for their professional life, design optimization reflecting real-world requirements should be more strongly emphasized than at present in the academic engineering curricula. Finally, perhaps, it would be advisable to devote one whole session of the next European Forum to design optimization problems.

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| BASIC ASSUMPTIONS<br>AND REQUIREMENTS | CONTROL VARIABLES                     | CONSTRAINTS   |
|---------------------------------------|---------------------------------------|---|
| • CONFIGURATION                       | ROTOR RADIUS     (DISC LOADING)       | MAX. DISC LOADING<br>(DOWNWASH VELOCITY)                      |
| • NUMBER OF REVENUE                   | *                                     |   |
| SEATS                                 | NUMBER OF BLADES                      | AVAILABLE ENGINES   |
| • CREW                                | COMPLETE BLADE<br>GEOMETRY (AIRFOIL.  | <ul> <li>LIMITS FOR ENGINE RPM</li> <li>VARIATIONS</li> </ul> |
| HOVERING OR                           | PLANFORM, TWIST)                      |   |
| VERTICAL CLIMB                        |                                       | PERMISSIBLE BLADE   |
|                                       | TIP SPEED                             | DROOP   |
| • ONE-ENGINE-OUT                      |                                       |   |
| SERVICE CEILING                       | EQUIV. FLAT PLATE                     | NOISE IN HOVER AND  |
|                                       | AREA LOADING                          | FORWARD FLIGHT  |
| MAXIMUM SPEED OF                      |                                       |   |
| FLIGHT                                | • NUMBER OF ENGINES                   | • VIBRATION   |
| • RANGE                               | TYPE OF CONSTRUCTION<br>AND MATERIALS | STRENGTH OF MATERIALS   |
| BAD WEATHER AND                       |                                       | CERTIFICATION REGULA-   |
| NIGHT OPERATION                       | DESIGN DETAILS                        | TIONS   |

Table I Principal Elements of a Design Optimization Circle

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Figure 15. Possible gains in total operating cost through advanced technology

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