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REDUCED ORDER MODEL AND CLOSED LOOP VIBRATION CONTROL IN HELICOPTER FUSELAGES

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Abstract
This paper addresses the problem of vibration reduction in helicopter fuselages using the concept of Active Control of Structural Response (ACSR). Considering the large size of the coupled gearbox-flexible fuselage system dynamics, first a balanced realisation based order reduction is employed to reduce the size of the problem. Then using the reduced order model, a closed loop controller is designed to minimise the NB/rev vibratory levels in the fuselage with the constraint that the controller ensures stability of the original full order system. The controller design is based on the concept of disturbance rejection by internal model principle. Employing a four block representation of the problem and doubly coprime factorisation theory, a stable controller is designed for this multi-input-multi-output control problem. It is observed that this controller yields a closed loop transfer function which rejects the external disturbance not only at the desired frequency of NB/rev but also in its neighborhood. In addition, contrary to open loop control, the present technique of closed loop control reduces the vibratory levels both in the fuselage and gearbox.

Nomenclature

[A], [B],[C]	System matrix, control matrix and output matrix respectively
$[A_f]$,$[B_w]$,$[B_u]$	System matrix, external disturbance matrix and control matrix respectively of the full order model
$[A_1]$,$[B_{w1}]$, $[B_{ul}]$	System matrix, external disturbance matrix and control matrix respectively of controllable model
[A], $[B_1]$, $[B_2]$	System matrix, external disturbance matrix and control matrix respectively of the reduced model
C_i, C^i	Damping of the i-th gearbox mounting
$[C_y]$, $[C_z]$	Measurement and output matrices respectively
$[C_{y1}]$, $[C_{z1}]$	Measurement matrix and output matrix in transformed space respectively
$[C_1]$, $[C_2]$	Measurement matrix and output matrix in reduced space respectively
F_c , f	Control force
F_x, F_y, F_z	Vibratory forces at hub
G(s)	Transfer matrix in Laplace domain $[G(s)=C(sI-A)^{-1}B]$
$\{I_{xx}, I_{yy}, I_{zz}\}_{GB}$	Mass moment of inertia of gearbox
$\{I_{xx}, I_{yy}, I_{zz}\}_F$	Mass moment of inertia of fuselage
K	Controller transfer function or gain matrix
K_i, K^i	Stiffness of i-th gearbox mounting.
m_B	Rotor blade mass
m_F	Mass of fuselage
m_{GB}	Mass of gearbox
M_x, M_y, M_z	Vibratory moments at hub
NB	Number of blades in the rotor system
{p}	State vector
P	Controllability grammian
$\{q\}$	State vector in modal space
$\{\bar{q}\}$	State vector in transformed space
Q	Observability grammian
$\bar{T}$, T	Transformation matrices
T_{zw}	Closed loop transfer function relating output z and input disturbance w
{u}	Control input vector
{w}	External disturbance vector
$\{x_1\}$, $\{x_2\}$	Controllable (flexible) and uncontrollable (rigid-body) state vectors respectively
{y}	Measurement vector
{z}	Output vector
β_F	Structural damping coefficient
σ_i	Hankel singular values
Ω	Rotor angular velocity
ω_0	Frequency of external disturbance
(^)	Laplace transform

1. Introduction
The periodic variation of inertia and aerodynamic loads of main rotor system is the major source of vibration in helicopters and these loads increase with increase in forward speed. The vibratory rotor loads are transmitted to different parts of the fuselage through complicated load path and cause discomfort to pilot and crew, equipment deterioration, fatigue damage to the structure and

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increased maintenance cost; thereby restricting the operation and efficiency of the vehicle.

With increasing demand for high speed and high performance helicopters, along with improved system reliability and reduced maintenance costs, vibration reduction has become an important design criterion. The control schemes adopted so far to reduce vibration in helicopters can be broadly classified as either passive or active control technologies. Passive vibration control schemes include hub or blade mounted pendulum absorbers, anti-resonant vibration isolation devices like Dynamic Anti-resonant Vibration Isolation (DAVI), Anti-Resonant Isolation System (ARIS) and Liquid Inertia Vibration Eliminator (LIVE), structural modifications and structural optimisation. Since passive devices are turned to provide maximum vibration reduction at specific frequency, for any change in operating condition, their performance will degrade considerably. It is generally accepted that "jet-smooth ride" in helicopters would be possible in future only with the incorporation of active control schemes (Ref. 1). Active control methodologies include Higher Harmonic Control (HHC), Individual Blade Control (IBC), Active Flap Control (AFC) and Active Control of Structural Response (ACSR). While HHC, IBC and AFC control schemes are aimed at reducing the blade loads in rotating frame, ACSR is employed in the nonrotating frame to cancel the effect of vibratory hub loads on the fuselage. A comparison of active vibration control schemes is provided in Ref. 2.

The concept of ACSR scheme is based on the principle of superposition of two independent responses of a linear system, such that the total response is zero (Refs. 3-4). A schematic of a helicopter with ACSR is shown in Fig. 1. The rotor loads are transmitted to the fuselage through the gearbox support structure. The support structure is idealised as a spring, damper, and a control force generator. In passive scheme, the control force generator corresponds to a vibration absorber mass (as in ARIS), whereas in the case of ACSR, the control force generator can be an electro-hydraulic actuator or a small-piezo actuator (Ref. 1). Due to several advantages, incorporation of ACSR scheme in helicopters is being pursued vigorously by industries (Refs. 1, 3-5).

Some of the important aspects in practical implementation of active vibration control schemes are: (i) selection of sensor locations for vibration measurement; (ii) selection of actuator locations and (iii) formulation of closed-loop control scheme for vibration minimisation. In the case of ACSR scheme, the actuators are placed at the gearbox support structure, whereas the sensors have to be placed at optimal locations to maximise the effect of vibration control in fuselage. Recently, in Ref. 6, a systematic mathematical procedure, employing Fisher information matrix (Ref. 7) has been successfully applied to identify the optimal sensor locations for vibration reduction in helicopter fuselages. It was shown in Ref. 6 that irrespective of the excitation frequency, these optimally selected sensor locations experience high levels of vibration.

Since the frequency of dominant component of fuselage vibration in helicopters is always NB/rev (where NB is the number of blades in the main rotor system), all vibration control schemes aim to minimise the NB/rev component of fuselage vibration. While applying ACSR schemes of vibration reduction, the control forces were evaluated by minimising a cost function in Refs. 3-6. In Ref. 8, the control forces were evaluated by equating the total steady state force across the servo actuators to zero. It is noted in Refs. 6 and 8 that on reducing fuselage vibration, ACSR scheme of vibration minimisation increases the gearbox vibratory levels (Ref. 6) and hub loads (Ref. 8). In Ref. 9, while describing methods of active control of vibration in helicopters, the authors have highlighted the applicability of internal model principle of disturbance rejection for the design of closed loop controllers for vibration minimisation in helicopter fuselages. The idea of disturbance rejection of fixed frequency (NB/rev) signal is based on the internal model principle (Ref. 10) wherein a suitable reduplicated model simulating the dynamic characteristics of the disturbance signal is incorporated in feedback path. The purpose of internal model is to provide closed loop transmission zeros which cancel the poles of the disturbance signal.

Dynamic analysis of complex structures, in general, involves several stages of model order reduction, namely, (i) the distributed parameter system with infinite degrees of freedom is reduced to a manageable finite element model with a few thousand degrees of freedom; and (ii) in the next stage using undamped free vibration modes obtained from an eigen analysis of the finite element model a further reduction in the model order is achieved by modal transformation with truncated number of modes (considering only the first few modes of interest). This reduced order model is then used for response and stability analysis. In the case of helicopters, due to high modal density of the fuselage structural modes, even in the modal space a large number of modes have to be considered particularly for the vibration analysis. It is known that a dynamic model with large number of degrees of freedom will lead to numerical difficulties and high computational costs. In addition for closed loop control the design of controller for a high order system will be difficult and the controller will have reduced efficiency. It is always preferable to have a low order system for the design of compensator; with the constraint that the controller designed for the small order system will ensure stability when incorporated in the full order system. (Note: In this paper full order system implies the system in modal space).

The technique known as model order reduction via balanced state space representation has been developed by Moore (Ref. 11). This technique is highly suitable for large systems incorporating multivariable control. In analysing model reduction for flexible space structure, it was shown in Ref. 12 that balanced realisation based order reduction is superior to modal truncation based order reduction particularly when the natural frequencies are closely spaced. A brief description of this technique is provided here for convenience (Refs. 13 and 14). A proper way to reduce the order of a dynamic system for control purposes is to delete those states which are least controllable and observable. For a systematic approach to delete the least observable and controllable states, one requires a measure of controllability and observability. It is known that the singular values of controllability and observability grammians define a measure of controllability and observability in certain directions of the state space. Since the grammians are variant under a coordinate transformation, it is shown that there exists a coordinate system in which the grammians are equal and diagonal. The corresponding space is denoted as the balanced space. A reduced order model of the system can be obtained by deleting the least controllable and observable states in the balanced space. Then for the reduced model, a suitable controller can be designed. Of course, the efficiency of the compensator has to be established by analysing the full order system with the controller designed for low order system.

The aim of this present study is to address the problem of vibration minimization using ACSR scheme in helicopter fuselages, by integrating several independent concepts in a novel manner. The main objectives of this study are:

- Formulate a reduced order model for the coupled gearbox/fuselage helicopter model using balanced realization;
- Design a closed loop controller for vibration minimization using ACSR scheme, by disturbance rejection approach. For this multi-input-multi-output control problem, the controller is obtained by employing four block representation and doubly coprime factorisation theory (Ref. 15);
- Evaluate the efficiency of the controller for the full order model.

2. Mathematical Formulation

The mathematical formulation consists of: a) equations of motion of the coupled gearbox/fuselage dynamical system, b) balanced realisation based order reduction and c) design of closed loop controller for disturbance rejection. A brief description of these items is presented in the following. The details of the formulation can be found in Refs. 16-17.

2.1 Equations of motion

A simplified dynamic model of a coupled rotor-gearbox-fuselage system is shown in Fig. 1. The gearbox is supported on the top of the fuselage at four nodes. Rotor blade dynamics are not included in the model; however, the vibratory rotor loads are assumed to be acting at the top of the gearbox. The gearbox support is represented as a combination of linear spring, viscous damper and an active force generator for vibration minimization. Several simplifying assumptions have been made while formulating the equations of motion.

- The gearbox is assumed to be rigid and has only vertical translation, pitch and roll degrees of freedom.
- The fuselage is assumed to be undergoing rigid-body vertical translation, pitch and roll motions as well flexible deformation due to elastic modes.
- Rigid body motions of fuselage and gearbox are assumed to be small.
- Products of inertia of the gearbox and fuselage are assumed to be zero.
- Gearbox supports are assumed to be uniaxial members, providing forces only in vertical (z) direction.

The equations of motion of coupled gearbox-fuselage system can be grouped into three sets. Set I corresponds to the rigid-body equations of the gearbox; Set II presents the rigid-body equations of the fuselage; and Set III represents the equations of motion of the elastic modes of the fuselage. The elastic modes have been obtained from an eigen analysis of a 3-dimensional finite element model of the fuselage. The details of the formulation of the equations are given in Ref. 16.

The equations of motion of the coupled gearbox-fuselage system can be written in state space form as

$$
\begin{aligned}
\{\dot{q}\} &= A_f\{q\} + B_w\{w\} + B_u\{u\} \\
\{z\} &= C_z\{q\} \qquad (1) \\
\{y\} &= C_y\{q\}
\end{aligned}
$$

Where, {q} denotes the state vector of modal degrees of freedom; {w} is the vibratory hub loads (or external disturbance); {u} represents actuator forces (or control forces) acting at four locations at the top of the fuselage; {z} is the output vector to be minimised; and {y} represents the response at preselected sensor locations which are used for feedback to obtain the control forces. The size of the state vector is 52X1, corresponding to three rigid-body modes of the gearbox + three rigid-body modes of the fuselage + 20 flexible modes of the fuselage. Figure 2 shows a block diagram of the system represented by Eq. 1 and Fig. 3 represents the closed loop system.

In Eq. 1, rigid-body modes and flexible modes are highly coupled. Since the control forces are internal forces to the total system, from a control theoretic point of view, rigid body modes are weakly controllable by actuator control inputs. Due to this reason, there are difficulties in deriving a suitable control law. Practical difficulties arise due to ill-conditioning while computing grammians and they lead to very high values of gain in the resultant control law (Ref. 18). A practical way to avoid this difficulty is to separate the rigid-body modes from the other flexible modes before proceeding towards designing control law. The decoupling of modes into uncontrollable rigid-body modes and controllable flexible modes is carried out by the following step-by-step procedure.

1. Find a transformation $\overline{T}$ of the state space as $\{q\} = \overline{T}\{\overline{q}\}$ to convert Eq. 1 to the form

$$\{\dot{\overline{q}}\} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \{\overline{q}\} + \begin{bmatrix} B_{w1} \\ B_{w2} \end{bmatrix} \{f\} + \begin{bmatrix} B_{u1} \\ B_{u2} \end{bmatrix} \{u\}$$
$$\{z\} = C_z \overline{T}\{\overline{q}\}$$
$$\{y\} = C_y \overline{T}\{\overline{q}\}$$

Where A_2 is a 6X6 matrix whose eigenvalues (=zero) correspond to the rigid body modes.

2. Partition $\{\overline{q}\}$ into controllable state vector $\{x_1\}$ and uncontrollable vector $\{x_2\}$ and also partition all the corresponding matrices.

3. Expressing the model pertaining to the controllable (flexible) part of the dynamics as

$$\begin{aligned} \{\dot{x}_1\} &= A_1\{x_1\} + B_{w1}\{f\} + B_{u1}\{u\} \\ \{z\} &= C_{z1}\{x_1\} \\ \{y\} &= C_{y1}\{x_1\} \end{aligned} \qquad (2)$$

It may be noted that the output vector {y} and measurement vector {z} in Eq. 2 contain the effects due to only the flexible (or controllable) part of the dynamics of the system. The size of the model given in Eq. 2 is 46-th order which is still very high from the point of view of control design problems. In addition all the states of this model will have different levels of controllability and observability. Hence it is desirable to reduce the order of this model by eliminating weakly controllable and observable states (note that here weak or strong is only relative) in comparison to other states.

2.2 Balanced realisation based order reduction

A balanced realization based order reduction is applied to Eq. 2 to reduce the size of the problem particularly for the design of closed loop control law. A brief description of the concept of balanced realization is provide for the sake of convenience. The details of the procedure can be found in Refs. 11, 13 and 14.

Let a stable linear system with transfer matrix $G(s)$ be realized in state space form as

$$\begin{aligned} \{\dot{p}\} &= A\{p\} + B\{u\} \\ \{y\} &= C\{p\} \end{aligned} \qquad (3)$$

The transfer matrix $G(s)$ in Laplace domain between input $\{u\}$ and output $\{y\}$ of the system given by Eq. 3 is denoted as $G(s) = (A, B, C)$ and this notation implies that $G(s) = C(sI - A)^{-1}B$. The Controllability Grammian for the state equation (Eq. 3) is defined as the matrix

$$P = \int_0^\infty e^{At} BB^T e^{A^T t}\, dt$$

and can be shown to be the positive semi-definite solution of the Liapunov equation

$$AP + PA^T + BB^T = 0$$

Similarly, the Observability Grammian of Eq. 3 is defined as the matrix

$$Q = \int_0^\infty e^{A^T t} C^T C e^{At}\, dt$$

and is the positive semi-definite solution of the Liapunov equation

$$QA + A^T Q + C^T C = 0$$

The eigenvalues of P and Q are always non-negative. (Since P and Q are square symmetric matrices, their eigenvalues are identical to their singular values. It may be noted that singular values of any matrix provide a measure of its closeness to being singular or rank deficiency.) If P and Q have zero eigenvalues, then it indicates that there is lower order realization of the transfer matrix $G(s)$. On the other hand, Controllability and Observability Grammian matrices P and Q are positive definite iff (A,B,C) is minimal realization (Note: Minimal realization implies that the ranks of the observability and controllability matrices are equal to the order of the system). The magnitudes of the eigenvalues of P indicate the relative dominance of corresponding states being influenced by the input (in other words the input-to-state influence); similarly the magnitude of the eigenvalues of Q indicate the dominance of the corresponding states in influencing the output (i.e., state-to-output influence). For a general realization given in Eq. 3, the eigenvalues of Controllability and Observability Grammians will not be same. Therefore, the states which are influential in input-to-state relation (i.e., controllability aspect) may not be influential in state-to-output relation (i.e., observability aspect) and vice-versa. From this viewpoint, the eigenvalues of the Grammians P and Q of an arbitrary realization do not give a correct picture of how much dominant a particular state is, in the input-output response relation. Such a picture is correctly provided by the balanced realization in which both P and Q are equal and diagonal. Therefore if a reduced order system $G_1(s) = (A_1, B_1, C_1)$ is obtained by removing from Eq. 3 (general realization) those state variables corresponding to very small eigenvalues of either P or Q, the two frequency responses $G(s)$ and $G_1(s)$ do not necessarily have close gains and phases. On the other hand, if a reduced order system is obtained from a balanced realization, then the frequency responses of the original system and the reduced system will have close gains and phases. The reason being that in balanced realization case, the reduced order system is obtained by deleting those states which are equally weak in both controllability and observability aspects. Since Controllability and Observability Grammians vary under a coordinate transformation, the balanced realization is obtained by performing a coordinate transformation. The details of the procedure can be found in Refs. 17 and 19.

It is to be noted that during coordinate transformation the eigenvalues of grammians P and Q change; whereas the eigenvalues of the product PQ are invariant. The positive roots $\{\sigma_1, \sigma_2, ...\sigma_n\}$ of the eigenvalues of PQ are called as Hankel singular values of (A,B,C). If $\sigma_i > \sigma_k$ then it can be said that the state variable x_i is more influential than x_k in the input-output response relation. Therefore the state variables of the balanced realization play an important role in understanding the relative significance of all the states of the state space model. A reduced order model of the original system (A,B,C) (having G(s) as transfer function) can thus be obtained by deleting the equations corresponding to the dynamics of those state variables which are of weak significance. Let the state variables are ordered in such a way that $\sigma_1 \geq \sigma_2 \geq .... \geq \sigma_n$. Let ($A_1$, B_1, C_1) be the reduced model obtained by removing those state variables corresponding to $\sigma_{r+1} .....\sigma_n$. Then the error in the frequency response between the original transfer function G(s) and the reduced system transfer function G_1(s) is given by

$$\| G(s) - G_1(s) \|_{H_\infty} \leq 2(\sigma_{r+1} + ... + \sigma_n)$$

In practice, the parameter r is chosen in such a way as to make the right hand side of the above inequality as small as possible.

2.3 Control law for disturbance rejection

The reduced order model can be symbolically written as

$$\begin{aligned} \{\dot{x}\} &= A\{x\} + B_1\{w\} + B_2\{u\} \\ \{z\} &= C_1\{x\} \\ \{y\} &= C_2\{x\} \end{aligned} \tag{7}$$

Where $\{x\},\{w\},\{u\},\{z\}$ and $\{y\}$ denote respectively the reduced order state vector, disturbance input, control input, measured output and measurements for feedback control. A block diagram, as shown in Fig. 3 can represent the above set of equations. For the present problem on vibration minimisation in helicopter fuselage, $\{z\}$ represents the vibratory levels at pre-selected optimal sensor locations to be minimised. In general, the measurement quantities $\{y\}$ used for feedback can be different from the quantities to be minimised. However in the present study, the measurement quantities are assumed to be same as $\{z\}$. Hence in Eq. (7), one has $C_1 = C_2$. Taking Laplace transform of Eq. (7) and assuming zero initial conditions, the standard four block representation can be written as

$$\begin{bmatrix} \hat{z} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} \hat{P}_1 & \hat{P}_2 \\ \hat{P}_3 & \hat{P}_4 \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{u} \end{bmatrix} \tag{8}$$

Where the variables with hat symbol are the Laplace transforms of the corresponding time domain quantities. For the present case, one has

$$\begin{aligned} \hat{P}_1 &= C_1(sI - A)^{-1}B_1 \\ \hat{P}_2 &= C_1(sI - A)^{-1}B_2 \\ \hat{P}_3 &= C_2(sI - A)^{-1}B_1 \\ \hat{P}_4 &= C_2(sI - A)^{-1}B_2 \end{aligned} \tag{9}$$

For the four block representation of the reduced order system, the vibration minimisation problem is cast as follows: Design a closed loop controller with transfer function K such that the effect of the external disturbance $\{w\}$ is asymptotically reduced to zero in the output {z} while providing closed loop stability. (i.e., Determine a controller $\hat{u} = \hat{K}\hat{y}$ such that the closed loop system with the four block plant is stable). The controller is obtained as a solution of the following problem.

2.3.1 Disturbance rejection problem

The closed loop transfer function between input {w} and output {z} can be expressed as

$$\hat{z} = \left[\hat{P}_1 + \hat{P}_2\hat{K}(I - \hat{P}_4\hat{K})^{-1}\hat{P}_3\right]\hat{w} \qquad (10)$$
$$\hat{z} = T_{zw}\hat{w}$$

The idea of disturbance rejection requires to satisfy the condition that the transfer function T_{zw} has a zero at the frequency of the external disturbance {w}, which is $\omega_0 = NB/rev$, i.e., $T_{zw}(j\omega_0) = 0$. Since T_{zw} is a nonlinear function of the closed loop gain $\hat{K}$, solving the above equation for $\hat{K}$ is difficult.

A solution to the above problem can be found by using the factorisation theory of feedback system synthesis (Ref. 15). In this approach the set of all controllers which provide closed loop stability are given by the following equivalent formulae

$$\hat{K} = (Y - M\overline{Q})(X - N\overline{Q})^{-1}$$
or (11)
$$\hat{K} = (\tilde{X} - \overline{Q}\tilde{N})^{-1}(\tilde{Y} - \overline{Q}\tilde{M})$$

Where, $N, M, \tilde{N}, \tilde{M}$ are stable transfer matrices and are obtained from doubly coprime factorisation of $\hat{P}_4$; $X, Y, \tilde{X}, \tilde{Y}$ satisfy Bezout identity; and $\overline{Q}$ is an arbitrary stable proper transfer matrix of conformable size. The advantage of the above formula is that on substitution in Eq. (10), the transfer matrix of the stable closed loop system can be expressed as

$$T_{zw} = T_1 - T_2\overline{Q}T_3 \qquad (12)$$

The details of the formulation and the expressions of T_1, T_2 and T_3 (which are known quantities) can be found in Refs. 15 and 17. In the modified form given by Eq. (12), the transfer matrix T_{zw} is linearly related to the matrix $\overline{Q}$. Therefore, the requirement on asymptotic disturbance rejection on vibration minimisation is satisfied by finding a stable transfer function $\overline{Q}$ such that

$$T_1(\pm j\omega_0) - T_2(\pm j\omega_0)\overline{Q}(\pm j\omega_0)T_3(\pm j\omega_0) = 0 \qquad (13)$$

Any transfer matrix $\overline{Q}(s)$ which is stable, proper and satisfying the above equation, thus provides a closed loop controller $\hat{K}$ which meets the requirement of disturbance rejection at the specified frequency ω_0. In the present study, the matrix $\overline{Q}$ is evaluated by representing each element (say ij-th element) by a second order stable transfer function of the form

$$\overline{Q}_{ij} = \frac{s^2 + as + b}{(s+1)^2} \qquad (14)$$

The two unknown quantities a and b are solved by first substituting $s = \pm j\omega_0$ in $\overline{Q}$ and formulating two sets of algebraic equations by equating each element of the matrix Eq. 13 to zero, separately for $+j\omega_0$ and $-j\omega_0$.

3. Results and Discussion

Using the dynamic model of the coupled gearbox-flexible fuselage system shown in Fig. 2, an order reduction based on balanced realisation approach is performed. Then a closed loop controller is designed using the reduced order model. The controller design is based on disturbance rejection scheme, using doubly coprime factoraisation theory. The output measurements used for the controller design are the vibratory loads at pre-selected sensor locations. The effectiveness of the controller is evaluated by performing the vibratory response of the full order system incorporating the closed loop controller designed for the reduced order model.

Figure 4 shows a finite element model of the helicopter fuselage. The length, height and width of the model are respectively 8.25m, 2m and 3m. the fuselage is 4m long, having a width of 2.5m and a height of 1.5m. The tail boom length is 4.25m and the span of the horizontal stabilizer is 3m. Lumped masses representing two engines, tail gearbox, and two end plates are also attached to the structure at appropriate nodes. The total number of nodes and the degrees of freedom are 64 and 356 respectively. The details of the structural properties and other data are given in Ref. 20. It is shown in Ref. 20 that the undamped natural frequencies and mode shapes of this model are similar to those of a realistic helicopter.

Assuming that the main rotor system consists of four blades, the vibratory hub loads will have a nondimensional excitation frequency of

4/rev. For the fuselage model, the nondimensional frequency of the 20-th flexible mode is 6.41 (Ref. 20), which is 50% more than the excitation frequency (4/rev) of the hub loads. The coupled gearbox-fuselage model shown in Fig. 2, has the gearbox mounted on the roof of the fuselage at four nodes (39, 48, 46 and 37). The vibration analysis is performed by applying vibratory loads at the top of the gearbox. Since the vibratory load in the vertical direction is more predominant, without loss of generality, it is assumed that the sensors measure only the vertical (z) component of fuselage vibration. The total number of degrees of freedom is 26; these include three rigid body modes (pitch, roll and heave) each for the gearbox and fuselage, and 20 flexible modes of the fuselage. In state space form the order of the system is 52 and this model is treated as the full order system. The data used for the analysis are given in Table 1.

3.1 Sensor locations

A key aspect in vibration control is the choice of sensor locations for measurement of vibratory levels in the fuselage and for feedback in the closed loop control scheme. In Ref. 6, following the mathematical procedure involving Fisher Information matrix and Effective Independence Distribution Vector (EIDV), 23 optimal sensor locations were identified. In was also shown in Ref. 6 that irrespective of the excitation frequency, these optimally selected sensor locations measure high levels of vibration. Assuming a 4/rev vibratory force in the z-direction, baseline vibratory levels in the fuselage at all nodes are shown in Fig. 5. Node number 0 refers to the gearbox c.g. location. The arrows in the figure indicate the locations of the 23 optimal sensors. The peak vibratory response occurs at node 33. Even though, optimal selection procedure does not identify node 33, there are two sensors at nodes 32 and 34 measuring the second highest level of response.

In the present study, several sets of 5 sensor locations are considered for order reduction and closed loop vibration control. The reason for choosing 5 sensors is to provide redundancy for the closed loop problem in determining the control forces for the four control actuators. However for conciseness, results corresponding to only one set 5 of sensors are presented here. The sensor locations are 8, 17, 23, 34 and 39. These locations are selected from the optimal set of 23 sensor locations providing high levels of baseline vibratory response, as shown in Fig. 5.

3.2 Reduced order model

The 52-nd order full system is first decomposed into controllable (46-th order) and uncontrollable (6-th order) subsystems, by performing a transformation of states to obtain block diagonal form of system matrix, as described in Sec. 2.1. The uncontrollable subsystem corresponds to rigid body modes having zero eigenvalues.

It may be noted that the reduced order model depends on the measurement matrix C_y, which is related to sensor locations. The Hankel singular values of the grammians in balanced space of the controllable 46-th order system are observed to vary in the range 0.5 to 0.002, indicating the effectiveness of the states in the input-output relation. Initially, (by trial) the system was truncated to a 10-th order model. This 10-th order model provided a good approximation of the frequency response of the original system. But the controller designed for the 10-th order reduced model could not effectively stabilise the full order system. The reason could be attributed to spill over instability. Then the size of the reduced order model is increased one at a time till closed loop stability of the full order system is ensured. An 18-th order model is found to provide both a good approximation to the frequency response and a stable controller for the full order system. A comparison of the frequency response (both gain and phase) of the 18-th order reduced model (+6 uncontrollable rigid body modes) with that of the full order system (46 controllable +6 uncontrollable states) at the five nodes (8,17, 23, 34 and 39) is shown in Fig. 6. At these five sensor locations, the frequency response of the reduced order model matches very well with that of the full order system up to a frequency 5/rev. Considering that the controller will be designed to reduce the fuselage vibration at the excitation frequency of 4/rev, it can be stated that in the frequency range of interest the 18-th order reduced model (+6 rigid body uncontrollable modes) is an excellent approximation to the 52-nd order full system.

3.3 Closed loop vibration control

Using the reduced order model and following the disturbance rejection approach based on factorisation theory, a controller is designed to minimise the vibration in the fuselage. This controller is incorporated in the feedback loop of the full order system and the vibratory level of the fuselage is calculated at all nodes. The results are presented in the following.

Comparison of baseline and controlled vibratory levels at all the 64 nodes is shown in Fig. 7. The arrows in the figure indicate the locations of the five sensor locations. Node number 0 indicates the c.g. of the gearbox. For the baseline configuration, the peak vibratory level is 0.27g at node location 33 and the lowest level is at node 57 with a value of 0.00017g. With closed loop control, the peak vibratory level is reduced to 0.036g at node 33. Tough closed loop control reduces the vibratory levels in the fuselage substantially, there is an increase in vibratory response at the tail portion. For example, at node 64, the vibratory level is increased from a baseline value of 0.0032g to 0.0145g. The reason for this increase can be attributed to not having a sensor in the tail portion. It is interesting to note that with closed loop control, vibratory level at the gearbox c.g (node 0) is reduced from a baseline value of 0.06g to 0.019g. This observation seems to be contrary to the vibration reduction schemes using open loop control (Refs. 6 and 8), where a reduction in fuselage vibration increases the vibratory level in the gearbox (or hub). In the present case of closed loop control scheme, the vibratory levels at both fuselage and gearbox are reduced simultaneously.

The frequency response of the gain for the uncontrolled and the controlled full order system at the five sensor locations are shown in Fig. 8. It can be seen that the controller is effective in reducing the vibration not only at the desired frequency of 4/rev but also in the neighbourhood of the desired frequency, indicating robustness of the control scheme.

4. Concluding remarks

Application of balanced realisation based order reduction has been carried out to obtain a reduced order model for a coupled gearbox-flexible fuselage dynamical system. Using the reduced order model, a closed loop controller is developed using disturbance rejection approach based on internal model principle and stable coprime factorisation theory. Most important conclusions of this study are summarised below.

- A 46^{th} order controllable subsystem of the coupled gearbox-flexible fuselage model is reduced to an 18^{th} order model. The frequency response of the reduced model closely matches the full order system in the frequency range of interest.
- The controller designed for the reduced order model provided a substantial reduction in fuselage vibratory levels. The controller is found to provide vibration reduction not only at the desired frequency but also in its neighbourhood.
- Contrary to open loop control, it is observed that closed loop control reduces not only fuselage vibration but also the vibratory level in the gearbox.

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Table 1: Data used for analysis

Reference quantities for nondimensionalisation
m_b=65kg, R=6m, Ω =32 rad/sec
Nondimensional quantities
K_i =60.01, C_i =0.033, m_F =33.846, m_{GB} =4.615
I_{xxF} =0.6838, I_{yyF} =2.735, I_{xxGB} = I_{yyGB} =0.0171
$F_z / (m_b \Omega^2 R) = 0.0001$
Fuselage c.m from nose of fuselage (origin):
x =0.5632, y =0, z = 0.0833
Gearbox c.m from nose of fuselage (origin):
x=0.5632, y = 0, z = 0.3333
Structural damping of fuselage elastic modes
β_F =0.005

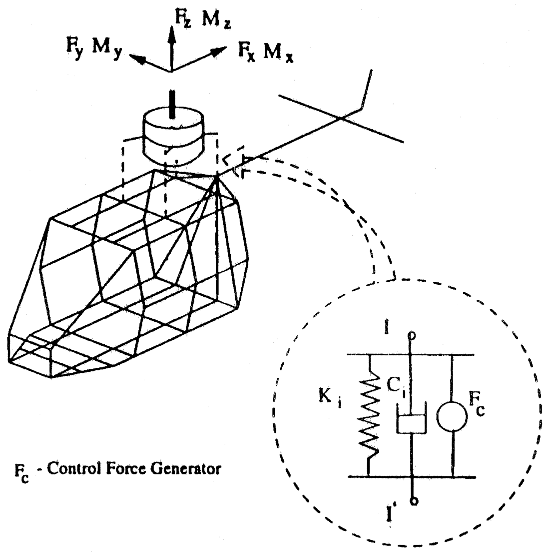


Figure 1

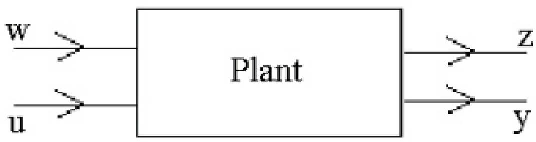


Figure 2

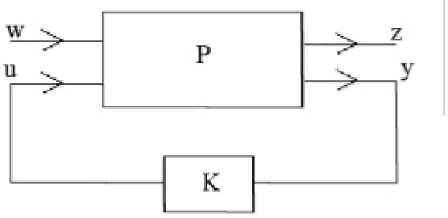
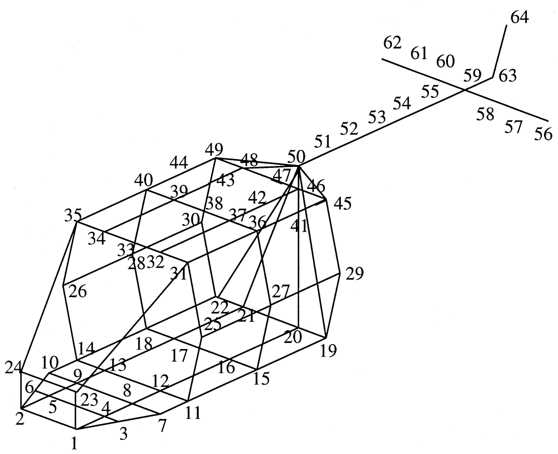


Figure 3



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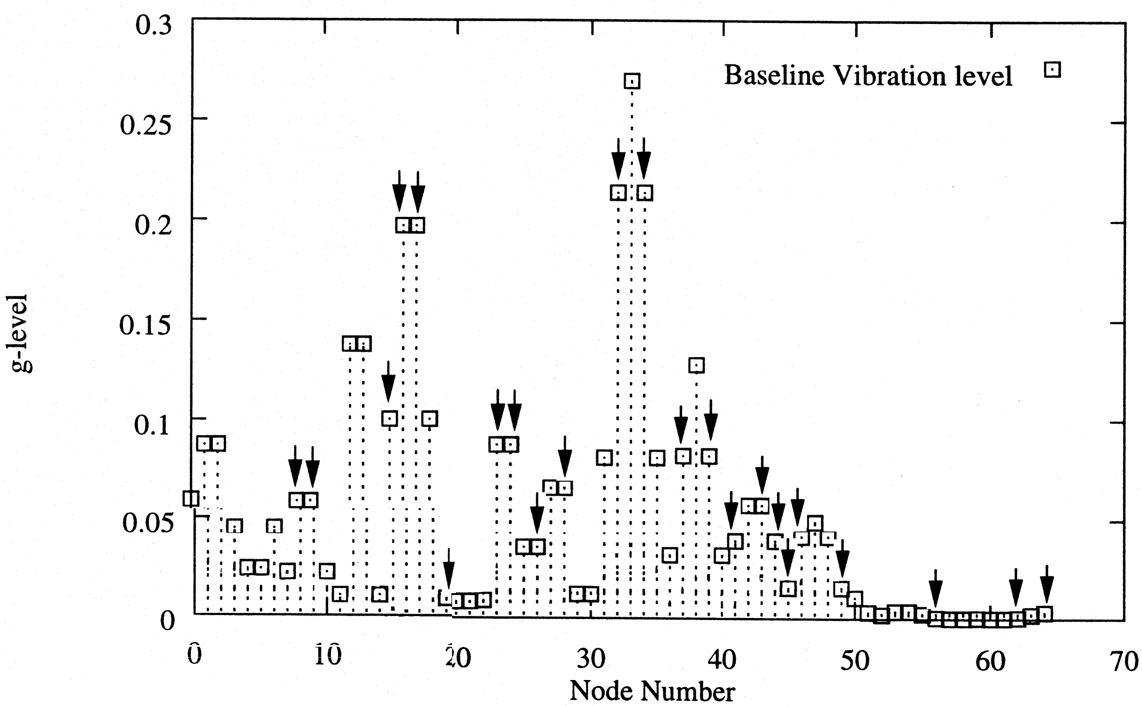

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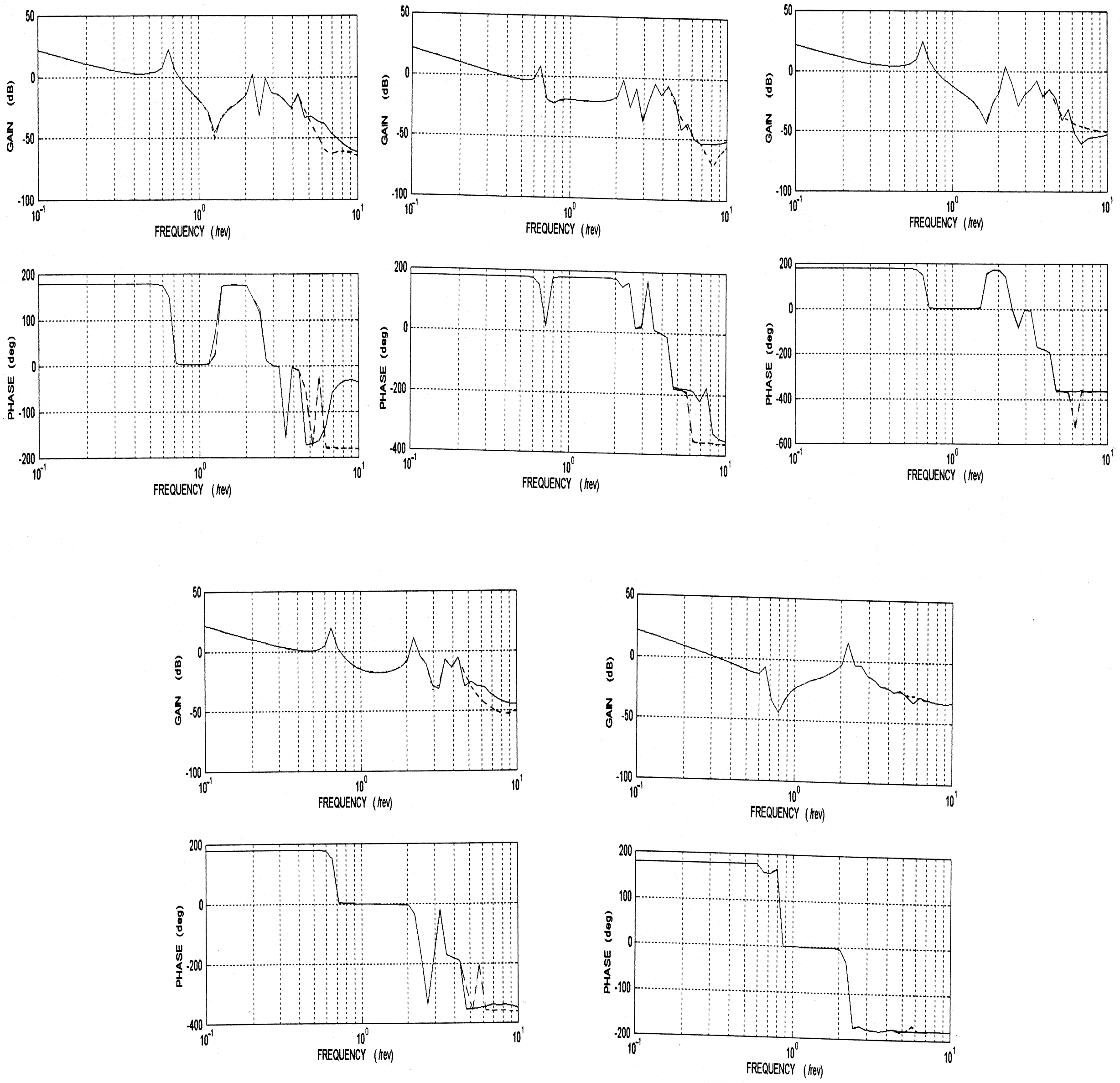


- - - - - Reduced Order System ——— Full Order System

Figure 6 Comparison of Frequency Response of Full and Reduced Order Models

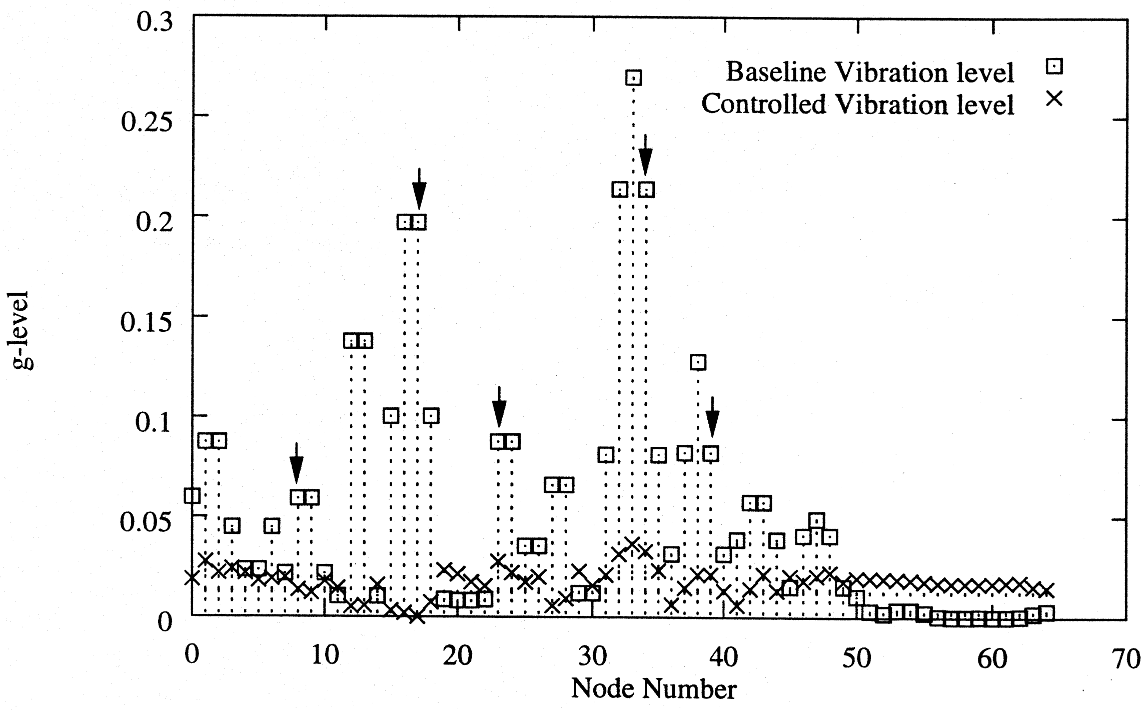
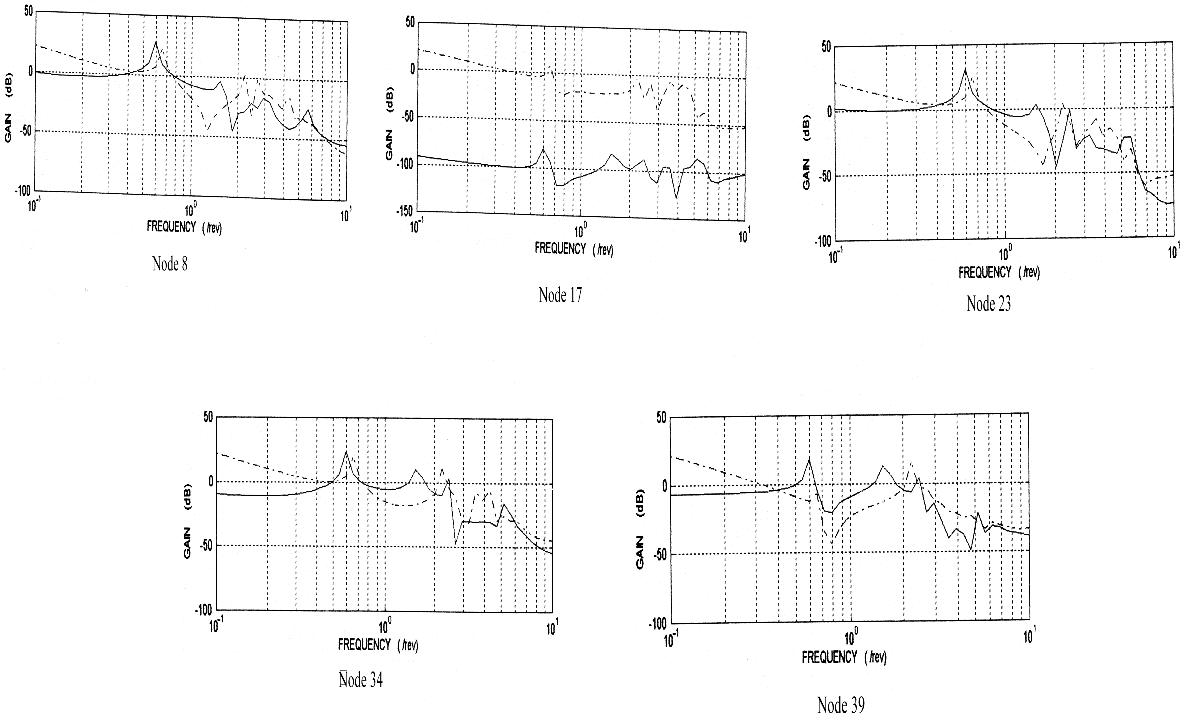

Baseline Vibration level
Controlled Vibration level
g-level
Node Number
0.3
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
60
70

Figure 7



___ _ ___ _ Uncontrolled System ___________ Controlled System

Figure 8 Frequency Response of Uncontrolled and Closed Loop Controlled Full Order System