ON TIME-DOMAIN PARAMETER ESTIMATION TECHNIQUES APPLICABLE TO FIRST-PRINCIPLE ROTORCRAFT MODELS

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Abstract

In this paper, we consider the problem of parameter estimation from flight test data for rotorcraft vehicle models.

We describe two alternative parameter estimation classes of methods in the time domain, namely the recursive filtering and the batch optimization methods. Both classes of methods are formulated so as to be applicable to complex first-principle models of rotorcraft vehicles. In the recursive approach, we formulate a novel version of the Extended Kalman Filter which specifically accounts for the presence of unobservable states in the model. An important highlight of the proposed approach is that it does not require a reduced-order model of the system. In the case of the batch optimization methods, we present a formulation based on a new single-multiple shooting approach specifically designed for vehicle models with slow and fast solution components.

The paper is concluded by a preliminary assessment of the performance of the proposed procedures with the help of applications regarding manned rotorcraft vehicles.

1 INTRODUCTION

In this work, we consider the problem of parameter estimation from flight test data for rotorcraft vehicles. The objective of parameter estimation is to find values of the parameters in a given mathematical model such that the model-computed response best matches (in a statistical sense) the experimentally observed one.

In the literature, it is common practice to speak of system identification when in reality referring to the problem of parameter estimation. When performing system identification, one has the freedom to define both the structure of the model and its parameters, whereas when performing parameter estimation the model is prescribed and its parameters are the only free variables which can be used to minimize differences between the model-computed response and experimental data.

Much of the published works on rotorcraft system identification deals primarily with frequency domain approaches and linear models, an excellent review on the state-of-the-art being given in the recent Reference [1]. Typically, for each given problem, ad hoc linear models are postulated and expressed in terms of a limited number of states and unknown model parameters, which often represent stability and control derivatives synthetically accounting for global aerodynamic effects created by the various aerodynamic components of the vehicle (rotor(s), fuselage, lifting surfaces, etc.). Suitable methods are then formulated for extracting estimates of the unknown parameters from experimental observations.

On the other hand, multidisciplinary aeromechanical analysis tools are being developed at a fast pace from first principles. Modern analyses are based on comprehensive approaches, which aim at covering the widest possible range of vehicle geometries and configurations, analysis types (including performance, handling qualities, vibrations, loads, etc.) and flight conditions (maneuvers, trimmed flight, hover, taxing, etc.) [5, 23, 16, 2]. Hence, rotorcraft aeromechanical analyses are typically based on complex, highly non-linear, multi-field models. Reference [11] provides a review on the current aeromechanical modeling capabilities, and describes possible future needs and areas of required technological improvements.

From this discussion, it appears that there is a clear need to use modern parameter estimation techniques for supporting the current capabilities in the aeromechanical modeling of rotorcraft vehicles. In doing so, the estimation methods must be carefully chosen. In fact, models are now non-linear, formulated in the time domain and often implemented in highly complex simulation codes; this last aspect calls for a high level interaction with the estimation procedures in order to minimize code modifications. Furthermore, since in a comprehensive code most effects are modeled from first principles by the various coupled disciplinary sub-components, the to-be-estimated param-

eters must be chosen with attention. One possibility is to use unknown parameters to account for the defects of the analytical models, i.e. for the un-modeled or un-resolved physics. This way, the white box model implemented in a comprehensive analysis code is augmented with carefully chosen black box components which, if properly estimated, lead to an improved grey box model of the aircraft.

In the present work we focus on the time domain approach to parameter estimation and on non-linear vehicle models. Excellent reviews of time domain methodologies for parameter estimation are already available, most notably in Reference [15]. Reference [25] describes, among many different flight mechanics applications, the use of time domain parameter estimation for rotorcraft problems.

Here, we specifically consider methods which are applicable to unstable systems, since rotorcraft vehicles are typically unstable at least in certain flight conditions. Unstable vehicles must be operated in closed-loop, and this must be explicitly accounted for when formulating parameter estimation methods. Hence, after having more precisely defined the problem of parameter estimation for a generic comprehensive rotorcraft model in Section 2, we analyze the problem of parameter estimation for unstable vehicles in Section 3.

In Section 4 and 5 we describe the mathematical formulation of two alternative parameter estimation classes of methods in the time domain, namely the recursive filtering and the batch optimization methods. Both classes of methods are formulated so as to be applicable to complex first-principle models of rotorcraft vehicles.

In the recursive filtering case, we consider the Extended Kalman Filter (EKF) formulation. Here, the EKF algorithm is formulated in a novel way so as to account for the presence of unobservable states in the model. This way, the method can be applied to complex models which include both slow and fast scales, although measures are typically available only for the slow solution components. An important highlight of the proposed approach is that it does not require a reduced-order model of the system.

In the case of batch optimization methods, we consider the output error method and present a formulation based on a new single-multiple shooting approach specifically formulated for vehicle models with slow and fast solution components [8]. In this case, the basic idea is to use multiple shooting on the flight mechanics scales, and single shooting on the faster ones; this avoids the enforcement of the multiple shooting gluing constraints for the faster scales, which greatly enhances convergence and in turn reduces the computational cost.

The two classes of approaches described herein have characteristics which make them suitable candidates for the difficult problem of parameter estimation for complex first-principles models. On the other hand, the two methods have also important differences, so that one or the other might be favored depending on the application; a synergistic use of the two can also be foreseen.

In Section 6 we describe an application of the presented methods to rotorcraft vehicles, whereas conclusions and an outline of future activities are given in Section 7.

2 THE PROBLEM OF PARAMETER ESTIMATION

Given a system $\mathcal{S}$ (the plant) and a suitable model of it $\mathcal{M}(\boldsymbol{p})$, parameterized in terms of free quantities $\boldsymbol{p}$, the problem of parameter estimation is concerned with finding values of the parameters $\boldsymbol{p}$ such that the model outputs $\boldsymbol{y}$ best match in some given sense the measured quantities $\boldsymbol{z}$, when both plant and model are excited by the same inputs $\widetilde{\boldsymbol{\delta}} = \boldsymbol{\delta}$[1]. The problem is of a stochastic nature, since the plant is usually excited by a process noise $\widetilde{\boldsymbol{w}}$, while the observations are corrupted by a measurement noise $\widetilde{\boldsymbol{v}}$. This situation is illustrated in Fig. 1.
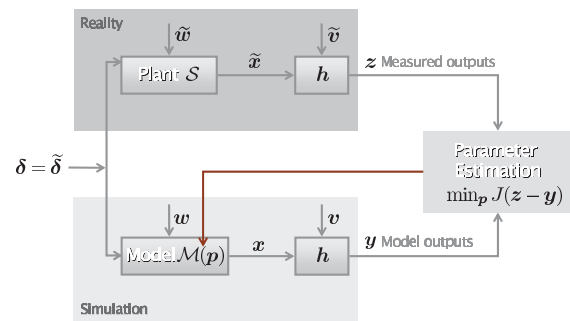

Figure 1: The problem of parameter estimation.

More precisely, consider a parametric flight mechanics vehicle model $\mathcal{M}(\boldsymbol{p})$, which includes structural and aerodynamic models of the vehicle components, using a multibody approach [5]. The dynamics of model $\mathcal{M}$ can in general be described in terms of a set of non-linear index 1-3 differential algebraic equations written as

$$\text{(1a)} \quad \boldsymbol{f}_{\text{sd}}(\dot{\boldsymbol{x}}_{\text{sd}}, \boldsymbol{x}_{\text{sd}}, \boldsymbol{\lambda}, \boldsymbol{x}_{\text{aero}}, \boldsymbol{\delta}, \boldsymbol{p}) = \boldsymbol{0},$$
$$\text{(1b)} \quad \boldsymbol{c}(\boldsymbol{x}_{\text{sd}}) = \boldsymbol{0},$$
$$\text{(1c)} \quad \boldsymbol{M}\dot{\boldsymbol{x}}_{\text{aero}} + \boldsymbol{L}\boldsymbol{x}_{\text{aero}} - \boldsymbol{\tau}(\boldsymbol{x}_{\text{sd}}, \boldsymbol{\delta}, \boldsymbol{p}) = \boldsymbol{0},$$

where $\boldsymbol{x}_{\text{sd}} \in \mathbb{R}^{n_{x_{\text{sd}}}}$ are the structural dynamics states (including states describing rigid and possibly flexible rotor(s), fuselage, engine, etc.), $\boldsymbol{\lambda} \in \mathbb{R}^{n_c}$ are

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[1]Here and in the following, quantities related to the plant (the true system, as opposed to a model of it) are indicated using the notation $\widetilde{(\cdot)}$.

constraint-enforcing Lagrange multipliers in a multibody vehicle model, $\boldsymbol{x}_{\mathrm{aero}} \in \mathbb{R}^{n_{x_{\mathrm{aero}}}}$ are aerodynamic states (e.g. dynamic inflow variables), $\boldsymbol{\delta} \in \mathbb{R}^{n_\delta}$ is the control input vector, and $\boldsymbol{p} \in \mathbb{R}^{n_{\mathrm{P}}}$ are the model parameters. Equations (1a) group together kinematic and dynamic equilibrium equations. Equations (1b) represent mechanical joint constraint equations in a multibody vehicle model, whereas Eqs. (1c) are the aerodynamic state equations (here written in a linear form resembling the one obtained using, for example, the classical Peters-He [22] dynamic inflow model, but which could have a more general non-linear form without affecting the subsequent discussion). Finally, the notation $\dot{(\cdot)} = \mathrm{d}(\cdot)/\mathrm{d}t$ indicates a derivative with respect to time t.

While the presence of differential algebraic governing equations does not pose any additional difficulty as far as the parameter estimation problem discussed here is concerned, for the sake of notational simplicity but with no loss of generality, in the following we will consider that Lagrange multipliers $\boldsymbol{\lambda}$ and redundant structural dynamics states can always be formally eliminated in favor of a minimal set of coordinates [13]. Therefore, the governing equations will be assumed to be of the ordinary differential type and will be simply expressed as

$$(2a) \qquad \boldsymbol{f}_{\mathrm{sd}}(\dot{\boldsymbol{x}}_{\mathrm{sd}}, \boldsymbol{x}_{\mathrm{sd}}, \boldsymbol{x}_{\mathrm{aero}}, \boldsymbol{\delta}, \boldsymbol{p}) = \boldsymbol{0},$$

$$(2b) \qquad \boldsymbol{M}\dot{\boldsymbol{x}}_{\mathrm{aero}} + \boldsymbol{L}\boldsymbol{x}_{\mathrm{aero}} - \boldsymbol{\tau}(\boldsymbol{x}_{\mathrm{sd}}, \boldsymbol{\delta}, \boldsymbol{p}) = \boldsymbol{0}.$$

It will be sometimes convenient to use a more synthetical form of the above equations, when we do not need to distinguish between structural dynamics and aerodynamic components; in those cases we will use the compact form

$$(3) \qquad \boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{\delta}, \boldsymbol{p}) = \boldsymbol{0},$$

where $\boldsymbol{x} = (\boldsymbol{x}_{\mathrm{sd}}^T, \boldsymbol{x}_{\mathrm{aero}}^T)^T$, $\boldsymbol{x} \in \mathbb{R}^{n_x}$, $n_x = n_{x_{\mathrm{sd}}} + n_{x_{\mathrm{aero}}}$, and $\boldsymbol{f}$ stacks together Eqs. (2a) and (2b).

For application to parameter estimation problems, we rewrite Eq. (3) as

$$(4) \qquad \boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{\delta}, \boldsymbol{p}) - \boldsymbol{F}\boldsymbol{w} = \boldsymbol{0},$$

where $\boldsymbol{w} \in \mathbb{R}^{n_x}$ is the process noise, a stochastic variable which represents the disturbances acting on the system (e.g., air turbulence) and any other modeling uncertainty. They are assumed to be zero-mean, Gaussian, white processes with power spectral density $\boldsymbol{Q}$. $\boldsymbol{F}$ denotes the (typically unknown) process noise distribution matrix, which is generally assumed to be an n_x–by–n_x diagonal matrix or a n_x–by–1 column vector.

The definition of output and measurement equations completes the formulation of the parameter estimation problem. In general, such equations take the following form:

$$(5a) \qquad \boldsymbol{y}(t_k) = \boldsymbol{h}\big(\boldsymbol{x}(t_k)\big),$$

$$(5b) \qquad \boldsymbol{z}(t_k) = \boldsymbol{y}(t_k) + \boldsymbol{v}(t_k),$$

with $k = 1, 2, \ldots, N$. Equation (5a) defines the model outputs $\boldsymbol{y} \in \mathbb{R}^{n_y}$ as a function of states, inputs, and parameters, whereas the measurements $\boldsymbol{z} \in \mathbb{R}^{n_y}$, affected by measurement noise $\boldsymbol{v} \in \mathbb{R}^{n_y}$ with covariance $\boldsymbol{R}_k = E[\boldsymbol{v}_k \boldsymbol{v}_k^T]$, $E[\cdot]$ being the expected value operator, are provided at N discrete sampling points t_k.

For the purpose of defining the model outputs, let us consider the following partitioning of the state vector $\boldsymbol{x}$:

$$(6) \qquad \boldsymbol{x} = \big(\boldsymbol{x}_{\mathrm{fm}}^T, \boldsymbol{x}_{\mathrm{oth}}^T\big)^T,$$

where $\boldsymbol{x}_{\mathrm{fm}}$ are the flight mechanics states, while $\boldsymbol{x}_{\mathrm{oth}}$ are all other remaining states in the model. The flight mechanics states are here defined as those describing the gross rigid body motion of the vehicle, i.e. they represent the position, orientation, linear and angular velocities of a body-attached (or floating, in the case of a flexible fuselage) reference frame.

With reference to Eq. (5a), we define the output vector as

$$(7) \qquad \boldsymbol{y} = \big(\boldsymbol{x}_{\mathrm{fm}}^T, \boldsymbol{a}_{\mathrm{fm}}^T\big)^T,$$

i.e. we take as model outputs the flight mechanics states $\boldsymbol{x}_{\mathrm{fm}}$ and the flight mechanics accelerations $\boldsymbol{a}_{\mathrm{fm}}$, where

$$(8) \qquad \boldsymbol{a}_{\mathrm{fm}} = \dot{\boldsymbol{x}}_{\mathrm{fm}} = \boldsymbol{g}_{\mathrm{fm}}(\boldsymbol{x}_{\mathrm{fm}}, \boldsymbol{\delta}, \boldsymbol{p}).$$

With this choice, function $\boldsymbol{h}$ in Eq. (5a) is defined as

$$(9) \qquad \boldsymbol{h} = \big((\boldsymbol{H}_{\mathrm{fm}}\boldsymbol{x})^T, \boldsymbol{g}_{\mathrm{fm}}^T\big)^T.$$

3 PARAMETER ESTIMATION TECHNIQUES FOR UNSTABLE SYSTEMS

Any technique developed for parameter estimation of rotorcraft models from experimental data must account for the fact that these vehicles are typically unstable, at least in certain flight conditions. For this reason, rotorcraft vehicles usually operate in closed-loop, under the action of an output feedback mechanism implemented through a flight control system (FCS). Hence, in the case of rotorcraft vehicles, experimental data used for parameter estimation is gathered in closed-loop. This fact has important consequences on the process of parameter estimation.

This situation is illustrated in Fig. 2. Plant $\mathcal{S}$ is driven by an input $\widetilde{\boldsymbol{\delta}}$, output by the vehicle FCS, and by a stochastic disturbance $\widetilde{\boldsymbol{w}}$. In turn, the output $\widetilde{\boldsymbol{\delta}}$ of the FCS is driven by two inputs: the pilot input $\boldsymbol{\delta}_p$

and the system outputs $\widetilde{y}_{\text{FCS}}$, which are functions of the plant states $\widetilde{x}$, i.e. $\widetilde{y}_{\text{FCS}} = h_{\text{FCS}}(\widetilde{x})$. During the experiment, some measured outputs z are gathered, where $z = h(\widetilde{x})$; the measurements are affected by measurement noise $\widetilde{v}$.
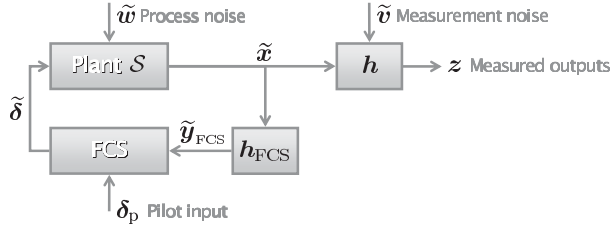

Figure 2: Gathering of flight test data in closed-loop.

Figure 3: Parameter estimation using the direct approach.

Parameter estimation performed from data collected in closed-loop is termed in the literature *closed-loop* estimation. Closed-loop parameter estimation is the only method that can be used in most practical cases when dealing with unstable aircrafts for obvious safety reasons. Furthermore, it should be noted that open-loop flight testing also suffers from its own severe drawbacks, most notably from the fact that data can only be gathered for short time spans, before the system excessively drift from the starting trim point. Hence, in the following we will only consider closed-loop estimation.

Reference [12] reviews several strategies for closed-loop parameter estimation, illustrating the theoretical conditions under which consistent estimates are possible. In summary, there are three possible approaches to unstable system identification:

- *Indirect approach.* Parameter estimation is performed by using a closed-loop model based on the explicit knowledge of the feedback controller. Unfortunately, a detailed model of the FCS might not always be available, for example because covered by proprietary rights; furthermore, modeling approximations in the FCS will inevitably affect the estimation results. These are two of the most important drawbacks of this approach.

- *Direct approach.* In this case, parameter estimation is performed by ignoring the feedback, and using the sole measurements of plant inputs and outputs. This situation is illustrated in Fig. 3.

  Observing the figure, we notice that in this case one feeds the plant model $\mathcal{M}(p)$ directly with measured inputs δ. Hence, at the price of gathering the control inputs down-stream the FCS (as opposed to the direct case, when one needs only to measure the up-stream values δ_p), one has the advantage that no knowledge of the FCS is required. Therefore, there is no possible effect of
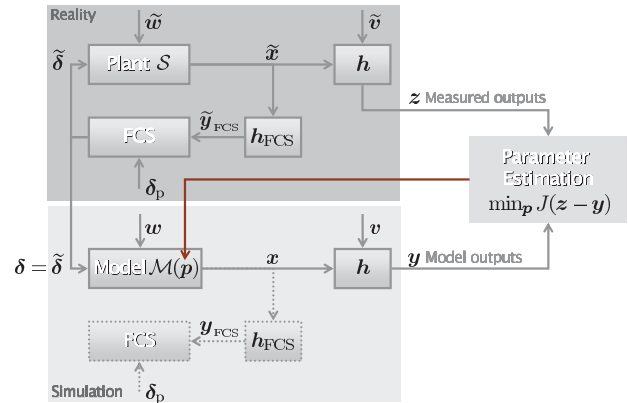
regulator modeling approximations on the quality of the results.

However, even this approach has its own drawbacks. In fact, it can be shown that it is necessary to have a sufficient signal to noise ratio $\delta_p / \widetilde{w}$ [15]. Since usually δ_p can not be too large to avoid non-linear effects, this implies that $\widetilde{w}$ has to be small, so that flight tests have to be conducted in calm, low turbulence air.

Another potential difficulty, which to the best of our knowledge has not been previously noticed in the literature, is due to the fact that the input δ fed into the model was computed by the plant FCS. This signal was computed during the flight test so as to stabilize the plant; however, in general there is no guarantee that the same signal will also stabilize the model, especially when the parameter estimates are still far from convergence. This might in principle lead to instabilities for those parameter estimation methods which require the integration of the model equations, for example through shooting. Therefore, it might be convenient to use stabilized time marching, for example through filtering or multiple shooting [15].

- *Joint input-output approach.* Using this method, one regards the system inputs and outputs as the combined outputs of an augmented system, driven by some extra inputs and noise. It can be shown [12] that joint input-output methods can be regarded as the combined direct identification of both the open-loop system and the regulator.

Given the necessity of perfect knowledge of the FCS for the indirect approach, in the remainder of this work we adopt the direct one as the method of choice for the present research effort.

In the next sections, we formulate two alternative time domain parameter estimation methods, applicable to non-linear models of rotorcraft vehicles. Both

are based on the direct approach discussed above, and hence can deal with unstable vehicles and with experimental data gathered in closed-loop. Furthermore, both methods are based on a stochastic approach and hence can deal with the presence of process and measurement noise.

The main difference between the two methods is the way gathered data points are processed. In fact, the first approach is of a *recursive* nature and transforms the unknown parameters into dynamic variables, which are integrated to steady state through a relaxation process by treating one data point at a time. The second one, on the other hand, is a *batch* method, which processes all data points simultaneously in order to yield an estimate of the unknown model parameters.

4   A   RECURSIVE   APPROACH: ADAPTIVE KALMAN FILTERING

The basic idea behind parameter estimation using recursive filtering is to promote the unknown model parameters p to the role of states. This leads to a new augmented state space model, and transforms the parameter estimation problem into a state estimation one. The augmented system can be written as

(10a) $\qquad \boldsymbol{f}_{\mathrm{a}}(\dot{\boldsymbol{x}}_{\mathrm{a}}, \boldsymbol{x}_{\mathrm{a}}, \boldsymbol{\delta}) - \boldsymbol{F}_{\mathrm{a}} \boldsymbol{w}_{\mathrm{a}} = 0,$

(10b) $\qquad\qquad \boldsymbol{y}(t_k) = \boldsymbol{h}_{\mathrm{a}}\big(\boldsymbol{x}_{\mathrm{a}}(t_k)\big),$

(10c) $\qquad\qquad \boldsymbol{z}(t_k) = \boldsymbol{y}(t_k) + \boldsymbol{v}(t_k),$

with $k = 1, 2, \ldots, N$.

Equation (10a) represents the augmented model dynamics and, more precisely, it is composed of two coupled differential equations:

(11a) $\qquad \boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{\delta}, \boldsymbol{p}) - \boldsymbol{F}\boldsymbol{w} = \boldsymbol{0},$

(11b) $\qquad\qquad \dot{\boldsymbol{p}} = \boldsymbol{0}.$

The first, Eq. (11a), describes the coupled structural dynamics and fluid dynamics components of the vehicle model, both being affected by a process noise w through the noise distribution matrix $\boldsymbol{F}$. The second, Eq. (11b), is the parameter dynamics evolution equation. For problems with time varying parameters, one may add a process noise term to the right hand side which is responsible for exciting the temporal variations of the parameter vector. Given these dynamic equations, the augmented state vector is defined as $\boldsymbol{x}_{\mathrm{a}} = (\boldsymbol{x}^T, \boldsymbol{p}^T)^T$ and stacks together the vehicle state vector together with the model parameters. Finally, Eqs. (10b) and (10c) define model outputs y and measurements z, respectively (see Section 2).

The state estimation problem (10) can be solved with a number of different filtering techniques. The problem is of a non-linear nature, since model parameters often appear non-linearly in first-principle vehicle models. Among the different possible choices

of non-linear filtering techniques, in this work, we use the extended Kalman filter (EKF), which amounts to an approximate generalization of the Kalman filter to non-linear systems obtained by linearizing the dynamics at each time step. This filter has found wide applicability to several state estimation problems for its simplicity and demonstrated effectiveness in many practical cases. We refer to Reference [14] for the formulation of the EKF. The implementation of this method developed in the present work performs the required linearizations by using perturbations with centered differences.

Given the recursive nature of the approach, the implementation of the EKF to problem (10) leads to a time stepping procedure which processes one measurement sample at a time and leads to a time sequence of model parameter estimates. In fact, given control inputs $\boldsymbol{\delta}$, obtained for example by smooth interpolation of the recorded values $\boldsymbol{\delta}(t_k)$, the augmented equations of motion are integrated on each sampling interval $[t_{k-1}, t_k]$ to yield an augmented state prediction $\hat{\boldsymbol{x}}_{\mathrm{a},k}^-$ and the corresponding outputs $\hat{\boldsymbol{y}}_k^-$. Notice that the integration time step used for marching the model equations forward in time on each sampling interval may be smaller than the sampling step $t_k - t_{k-1}$, because of accuracy or stability requirements during the numerical integration of the model equations. For example, this might be crucial in the case of a vehicle model $\mathcal{M}$ with fast dynamic components which need small time steps to be resolved, as in the case of vehicle models with flap blade or gimbal rotor states. Next, at each sampling instant the augmented state predictions are updated based on the innovations $\big(\boldsymbol{z}_k - \hat{\boldsymbol{y}}_k^-\big)$ as

(12) $\qquad \hat{\boldsymbol{x}}_{\mathrm{a},k}^+ = \hat{\boldsymbol{x}}_{\mathrm{a},k}^- + \boldsymbol{K}_{\mathrm{a},k}\big(\boldsymbol{z}_k - \hat{\boldsymbol{y}}_k^-\big),$

where $\boldsymbol{K}_{\mathrm{a},k}$ is a time-varying gain matrix, which is propagated forward in time together with state estimates based on the covariances of the estimation error, and of the process and measurement noise.

4.1   Filter Design

The filter design developed for this work is based on multi-time scale arguments. The basic observation is that complex rotorcraft vehicle models include both slow flight mechanics scales and faster aero-elastic ones, although measures are typically available only for the slow solution components describing the gross vehicle motion. As a result, all state variables whose dynamics are characterized by frequencies above the flight mechanics ones cannot be reconstructed from the available data. Hence, these states, being unobservable, are of no interest for the problem at hand.

We make use of this fact and reconstruct the sole states associated with the slow scales, for which an

accurate estimation is possible. This is conceptually equivalent to reducing the system to its reachable and observable parts, although a key feature of the proposed approach is that no reduced-order model of the system is required [24]. We call the resulting estimation algorithm *selective* Kalman filter.

Let us consider the following partitioning of the augmented state vector:

$$(13) \qquad x_{\mathrm{a}} = \left(x_{\mathrm{S}}^T, x_{\mathrm{F}}^T \right)^T,$$

where x_{S} and x_{F} are the states associated with slow and fast scales, respectively. With this partitioning of the state vector, the linearized output equations write

$$(14) \qquad \delta y = H_{\mathrm{a}} \delta x_{\mathrm{a}},$$

where

$$(15) \qquad H_{\mathrm{a}} = [H_{\mathrm{S}} \quad H_{\mathrm{F}}].$$

As previously noticed, in the present application the outputs y are related to the slow scale solution components (cfr. Eq. (7)). The proposed approach is based on neglecting the effects of the fast scales on the slow outputs, which, with the current notation, implies setting

$$(16) \qquad H_{\mathrm{F}} \equiv 0.$$

This is reasonable in most flight mechanics applications of interest here, where the fast scales are related to rotor degrees of freedom, including rigid and flexible states, and aerodynamic states, which can be considered as decoupled from the slow gross rigid body motion of the vehicle. A possible exception to this situation is represented by the regressive lag mode (frequency $1 - \omega_L/\Omega$) of a hingeless rotor/proprotor, which for certain rotor speeds might reach frequencies well below 1 Hz. If the mode is well damped, it does not however pose any difficulty to the application of the proposed procedure.

Given the partitioning of the augmented state vector, the covariance matrix of the estimation uncertainty, which is computed as

$$(17) \qquad P_{\mathrm{a}} = E[\hat{e}_{\mathrm{a}} \hat{e}_{\mathrm{a}}^T],$$

$\hat{e}_{\mathrm{a}} = \hat{x}_{\mathrm{a}} - x_{\mathrm{a}}$ being the reconstruction error, can be partitioned as

$$(18) \qquad P_{\mathrm{a}} = \begin{bmatrix} P_{\mathrm{SS}} & P_{\mathrm{SF}} \\ P_{\mathrm{FS}} & P_{\mathrm{FF}} \end{bmatrix}.$$

We further assume that the error on fast scales is uncorrelated to the one on slow scales, i.e.

$$(19) \qquad P_{\mathrm{SF}} = P_{\mathrm{FS}} \equiv 0,$$

which is coherent with hypothesis (16).

Under these assumptions, the Kalman gain matrix,

$$(20) \qquad K_{\mathrm{a}} = P_{\mathrm{a}} H_{\mathrm{a}}^T \left(H_{\mathrm{a}} P_{\mathrm{a}} H_{\mathrm{a}}^T + R \right)^{-1},$$

has the following form:

$$(21) \qquad K_{\mathrm{a}} = \begin{bmatrix} K_{\mathrm{S}} \\ 0 \end{bmatrix},$$

where

$$(22) \qquad K_{\mathrm{S}} = P_{\mathrm{SS}} H_{\mathrm{S}}^T \left(H_{\mathrm{S}} P_{\mathrm{SS}} H_{\mathrm{S}}^T + R \right)^{-1}.$$

This way, the filter effectively ignores the fast scale variables. In fact, only slow scale states (e.g. x_{fm}, p) are updated by slow scale innovations, while the fast scale states remain unaffected. This behavior is due to the two hypotheses expressed above by Eqs. (16,19), without which fast scale innovations would affect slow state estimates. It was verified experimentally that, in the present application, this has the effect of corrupting the estimates to the point of leading to the divergence of the filter, since such fast scale innovations do not carry enough information content on the slow scale solution components.

4.2 Implementation Issues

4.2.1 Adaptive Filtering

The implementation of the Kalman filter requires knowledge of the process and measurement noise covariances. Proper choices of these matrices is crucial for good filter performance. In fact, the use of poor statistics can lead to large estimation errors or even to the divergence of the estimates.

To alleviate the need for careful tuning of these parameters, which is sometimes problematic, one can use an adaptive filtering method [18]. The approach used in this work is based on Reference [19]. The basic idea is to estimate the unknown time-varying noise statistics simultaneously with the system state using a buffer of past data.

4.2.2 Enforcement of Constraints

The enforcement of constraints on the estimated augmented states during filtering was recently described in Reference [10] and references therein. This may be important to ensure that the estimated values of the parameters remain within physically meaningful limits.

In the case of simple bounds on the parameters, a simpler although heuristic approach is to use clipping: if the current update of a parameter exceeds the allowed bounds, it is reset to the bound value, otherwise the update is accepted. This however may slow down convergence. The present implementation uses clipping to approximately incorporate the effects of bounds on the model parameters.

5 BATCH APPROACHES: THE OUTPUT ERROR AND FILTER ERROR METHODS

In batch methods, all data points are used simultaneously (as opposed to the recursive approach described above) so as to estimate the model parameters. The formulation of such methods therefore leads to an optimization problem defined over the whole duration of the data gathering experiment. There are two main methods in such category: the output error method (OEM) and the filter error method (FEM). In the following, we describe the OEM, which is slightly simpler but also typically more robust, while we refer to [15] for the FEM for reasons of space limitations. Since the two methods share between themselves most features, the description of the OEM gives the opportunity to describe some novel features of the implementation that we have developed in this research to increase the robustness and efficiency of the algorithms, specifically regarding a novel hybrid shooting designed for dealing effectively with rotorcraft vehicle models with fast solution components.

5.1 The Output Error Method

In the output error method (OEM), all measures are used simultaneously (as opposed to the recursive approach described above) so as to estimate the model parameters. The formulation of such method therefore leads to an optimization problem defined over the whole duration of the data gathering experiment.

Under the assumption that the system is corrupted by measurement noise only, the resulting optimization problem writes

$$(23a) \qquad \min_{\boldsymbol{p}} \quad J^{\mathrm{id}},$$

$$(23b) \qquad \text{s.t.:}\ \boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{\delta}, \boldsymbol{p}) = 0, \quad \forall t \in [t_{k-1}, t_k],$$

$$(23c) \qquad \boldsymbol{y}(t_k) = \boldsymbol{h}(\boldsymbol{x}(t_k)),$$

$$(23d) \qquad \boldsymbol{g}(\boldsymbol{p}) \leq \boldsymbol{0},$$

where J^{id} is a cost function which measures in a statistical sense the match between model outputs $\boldsymbol{y}$ and measured quantities $\boldsymbol{z}$. Depending on the definition of J^{id}, we have the following methods:

- *Maximum Likelihood*:

  $$(24) \qquad\qquad J^{\mathrm{id}} = \det(\boldsymbol{R}),$$

  where $\boldsymbol{R}$ is the maximum likelihood estimate of the measurement noise covariance matrix,

  $$(25)\ \ \boldsymbol{R} = \frac{1}{N} \sum_{k=1}^{N} \big(\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k)\big)\big(\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k)\big)^T.$$

  The method seeks to maximize the probability density of observed variables given the model parameters. This can be readily transformed into the equivalent but simpler to handle problem of minimizing the logarithm of the density function [15] and leads to the definition of the optimization cost function in Eq. (24).

- *Weighted Least Squares*:
  (26)

  $$J^{\mathrm{id}} = \frac{1}{2} \sum_{k=1}^{N} \big(\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k)\big)\boldsymbol{W}\big(\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k)\big)^T,$$

  where $\boldsymbol{W}$ is a weight matrix. This method can be seen as a particular case of the Maximum Likelihood method when the measurement noise covariance matrix $\boldsymbol{R}$ is known [15]. In this case, $\boldsymbol{W} = \boldsymbol{R}^{-1}$.

In practice, the method can be used as follows. At first, one guesses a value of the weights; once a solution has been obtained based on this choice of $\boldsymbol{W}$, matrix $\boldsymbol{R}$ is computed using Eq. (25). Next, the weighting matrix is initialized to the inverse of $\boldsymbol{R}$, and another solution is computed. The iterations are continued until convergence. This way one recovers the maximum likelihood solution by solving a succession of weighted least squares problems.

The optimization problem (23) is subjected to a number of constraints. The first set of constraints is represented by the model equations (23b). Given the control inputs $\boldsymbol{\delta}$, obtained by smooth interpolation (for example using cubic splines) of the values $\boldsymbol{\delta}(t_k)$ recorded during the flight test, and the current estimates of the parameters $\boldsymbol{p}$, these equations can be integrated on each sampling interval $[t_{k-1}, t_k]$ to yield the state predictions $\boldsymbol{x}(t_k)$; Eqs. (23c) define the corresponding outputs. Finally, inequality (23d) enforces possible constraints on the model parameters. Such constraints ensure that the estimated parameters lie within acceptable bounds and do not take at convergence values which are non-physical.

Problem (23) is a constrained non-linear optimization problem, whose unknowns are the free parameters $\boldsymbol{p}$, together with the states $\boldsymbol{x}$, and the outputs $\boldsymbol{y}$. One simple way to solve this problem is to eliminate $\boldsymbol{x}$ (and in turn $\boldsymbol{y}$) by shooting. In other words, we use the fact that, given a set of initial conditions, it is possible to integrate Eqs. (23b) on each sampling step, which in turn enables one to compute the corresponding outputs. Since there are no terminal or internal constraints on these quantities (as opposed to, for example, optimal control problems), the integration of these equations has the effect of completely eliminating them from the problem, which therefore becomes a standard non-linear programming problem (NLP) in the sole discrete unknown parameters $\boldsymbol{p}$. The resulting NLP problem can be efficiently solved using

the sequential quadratic programming (SQP) method, with Jacobians computed through centered finite differencing by perturbation of the unknowns [4].

Such methods are designed to deal effectively with equality and inequality constraints, so that there is no difficulty in handling parameter constraints (Eq. (23d)). We remark that the ability to include such constraints in a straightforward manner and at no additional complexity is an important highlight of the present approach, since this helps to guarantee that the solution for the unknown parameters stays within admissible limits.

We further remark the fact that such an optimization based approach to parameter estimation is formally extremely similar to the problem of trajectory optimization, an optimal control approach to the solution of maneuvering problems which finds applicability in rotorcraft flight mechanics, see Reference [9]. In fact, in a parameter estimation problem the inputs are known, while parameters should be computed so as to best match given measures; on the other hand, in trajectory optimization problems the model parameters are assumed to be known, while the control inputs should be computed so as to minimize or maximize an index of performance. In both cases, one is lead to the solution of a constrained optimization problem.

We have exploited this fact for developing a general purpose software program named STOP (System Identification and Trajectory Optimization Program). The code is capable of solving both classes of problems using a suite of numerical methods and algorithms that cover a broad range of vehicle models of varying complexity; the code has an internal vehicle model and is also coupled with several external rotorcraft simulators, including the commercial code FLIGHTLAB [2].

In STOP, the optimization problem (parameter estimation or trajectory optimization) is transformed into a NLP problem by discretization in the temporal domain; this can be done either by transcription or by shooting [7, 6]. In this work we use a multiple shooting approach, which is briefly described next.

5.2 The Multiple Shooting Method

Multiple shooting is often advocated as a better, although somewhat empirical, solution than single shooting, especially when dealing with unstable systems as in the present case [15]. In fact, in optimal control problems, multiple shooting is often the only way to avoid solution blow up caused by the dramatic amplification of small perturbations [6].

We consider a partition of the time domain $I = [t_0, t_N]$ given by $t_0 = \tau_0 < \tau_1 < \ldots < \tau_M = t_N$ with $I^m = [\tau_m, \tau_{m+1}]$, $m = (0, M-1)$, where each I^m is a shooting segment (see Fig. 4). The resulting NLP
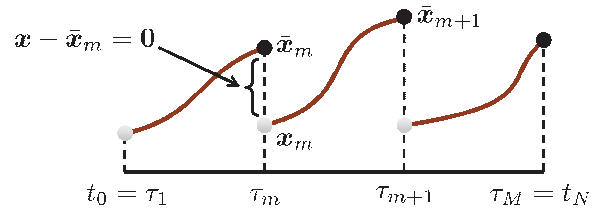

Figure 4: Basic principle of the multiple shooting method.

problem is defined as follows. First, the set of NLP variables are chosen as:

$$(27) \qquad \bar{p} = (x^T_{m=(0,M)}, p^T)^T,$$

where $x^T_{m=(0,M)}$ are the discrete values of the states at the interfaces between shooting segments. Next, the governing equations (23b) are marched in time within each shooting segment I^m, starting from the initial conditions provided by the values of the states x_m at the left boundary of the segment. The effect of the forward integration is to generate a discrete time history of states and corresponding outputs within I^m, which we label, respectively, x_i^m and y_i^m, $i = (1, N^m)$, where N^m is the number of sampling time instants in that segment. Here again, the integration time step used for marching the model equations forward in time on each sampling interval $[t_{k-1}, t_k]$ may be smaller than the sampling step $t_k - t_{k-1}$, to account for fast dynamic components in the model. The last value of the states sequence is named $\bar{x}_{m+1} = x_{N^m}^m$, and represents the prediction of the state variables at the right boundary of the shooting segment. Segments are then glued together by imposing the following equality constraints

$$(28) \qquad x_m - \bar{x}_m = 0, \qquad m = (2, M).$$

The cost J^{id} of Eq. (23a) is evaluated using the segment time histories y_i^m. Next, the gluing conditions (28) are used to express the set of equality constraints generated by the discretization of the equations of motion (23b).

5.3 The Single-Multiple Shooting Approach

In this section we describe some novel features of the implementation of the OEM that we have developed in this research to increase the robustness and efficiency of the algorithms, specifically regarding a novel hybrid shooting designed for dealing effectively with rotorcraft vehicle models with fast solution components. In fact, such models are seldom used in the solution of optimization problems because it is often hard to provide the required accuracy within a

reasonable computation time, while avoiding numerical instabilities due to the complex nonlinear rotor model [17].

The reason for this is twofold: on one hand, one needs to use a small integration time step length to correctly resolve the high frequency components of the solution within a given accuracy. For rotorcraft models of the form (3), this implies a computational cost associated with the time-marching of the vehicle equations of motion (which represents the main contribution to the total cost of one iteration of the solution process), since, at every time step, a Newton-like method should be used to correctly evaluate the mutually influencing dynamics of the model. To obtain the total cost of one evaluation of the gluing constraints (28), this time must be multiplied by the number of perturbations of the unknown states needed for the evaluation of the Jacobian matrix of the constraints. Clearly, as the number of model states increases, the computational cost grows accordingly.

On the other hand, one has to guarantee the continuity of the rotor states by imposing the proper gluing constraints. We have observed that the satisfaction of such constraints can be particularly difficult and usually ends up dominating the problem. This is not surprising, since the rotor generates most of the aerodynamic forces acting on the vehicle and even small variations in its states may imply large variations in the resulting forces, which hinders the satisfaction of the gluing constraints.

We have found that these problems can be alleviated by using multi-time scale arguments [8]. In fact, the rotor states (both structural and aerodynamic) are significantly faster than the flight mechanics ones. Thus, since the multiple shooting treatment of these fast states is the main cause of the two aforementioned issues, i.e. raise in computational cost and difficulty in satisfying gluing constraints, one can think of treating slow and fast scales using different methods.

More specifically, STOP uses a multiple shooting approach for the slow states. This is crucial, since with single shooting small changes early in the trajectory can produce dramatic effects at the end of it [3]; clearly, the problem is exacerbated when analyzing unstable systems, which is often the case when considering rotorcraft vehicles. Hence, the multiple shooting treatment of slow scales avoids the blow up of the solution.

On the contrary, the code treats the fast scales using a single shooting approach, as depicted in Fig. 5. This does not compromise the robustness of the procedure, since fast scales will not diverge if slow ones do not; hence, the stabilizing effect produced by the multiple shooting treatment of slow scales is felt also at the level of the fast ones.

With such a hybrid single-multiple shooting approach, the size of the resulting NLP problem is sub-
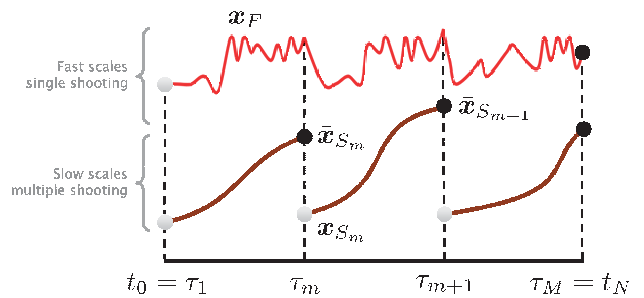

Figure 5: Hybrid single-multiple shooting approach.

stantially reduced and so is the total computational cost. Furthermore, there are no gluing constraints to be enforced for the fast rotor states, since only the slow states need to be glued together at the shooting interfaces. This has the effect of greatly increasing the robustness of the procedure, and the convergence speed.

The detailed mathematical formulation of the single-multiple shooting method is given in Reference [8].

6 NUMERICAL APPLICATIONS

We consider a Level 2 fidelity model [21] of a medium size helicopter implemented in the general purpose rotorcraft flight simulator FLIGHTLAB [2].

In modern comprehensive models, much of the physics is based on first-principle modeling (e.g. geometrically exact non-linear beam models, dynamic wake models, etc.) and/or experimental data (e.g. lifting lines using experimental airfoil data). This raises the issue of the choice of the to-be-estimated model parameters. There is evidence in the literature that the correct prediction of the aerodynamics associated with real flow features, such as interference effects, is an important contributing factor to the overall fidelity of rotorcraft flight dynamic simulations (see, e.g., Reference [20]).

In this work, model parameters are considered to be the coefficients of a rotor-fuselage, rotor-aerodynamic surface empirical interference model. Such empirical corrective models represent whatever aerodynamic loads are acting on the vehicle which are not fully captured or adequately resolved by the implemented analytical models.

6.1 An empirical Rotor-Fuselage, Rotor-Aerodynamic Surface Interference Model

Airloads produced by aerodynamic surfaces are computed using the lifting line theory with experimental airfoil data tables. Lift, drag, and pitch moment coefficients are obtained using a 2-D linear interpolation

as a function of angle of attack, α, and control surface deflection δ. Similarly, look-up tables are used to compute the aerodynamic forces and moment coefficients of the fuselage as a function of angle of attack, α, and angle of sideslip, β.

Consider a generic aerodynamic coefficient C whose values are given in tabular form as a function of two variables, i.e. $C = C_{\text{table}}(x_1, x_2)$, where $x_1 = \alpha$, $x_2 = \delta, \beta$. The rotor interference is modeled using corrective factors K to modify the table entries as:

$$(29) \quad C_{\text{table}} = C_{\text{table}} + K_{x_1} C_{\text{table}}(x_1, 0) + K_{x_2} C_{\text{table}}(0, x_2) + K_0.$$

Force and moment coefficients are then computed by interpolation of the modified data.

6.2 Results
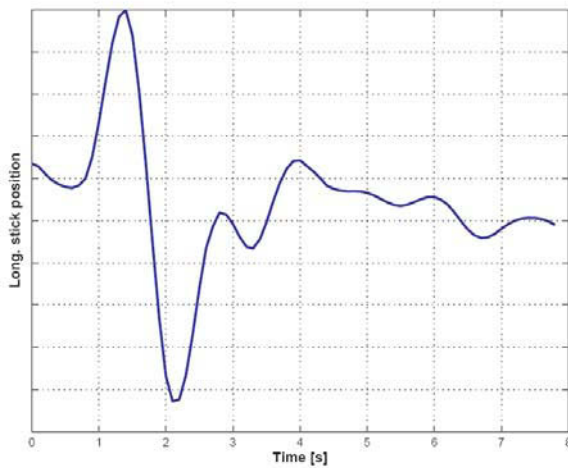
Figure 6: Longitudinal stick command input

We present results for a longitudinal stick doublet maneuver in forward flight (see Fig. 6). With reference to the interference model of Eq. (29), estimation is performed for the corrective factors K_α of the horizontal stabilizer lift coefficient and K_0 for the fuselage lift coefficient, i.e.

$$(30) \quad \boldsymbol{p} = (K_{\alpha \, \text{H-Stab}}, K_{0 \, \text{Fus}})^T.$$

We apply both the EKF and the OEM method to the solution of the estimation problem. For the design of the EKF, we consider only the states pertaining to the longitudinal motion of the vehicle, i.e.

$$(31) \quad \boldsymbol{x}_S = (\theta, u, w, q)^T,$$

where θ is the pitch attitude, u, v are the linear velocity components along the x- and z-axis in a body-attached frame, and q is the pitch rate. The OEM is run using the merit function J^{id} of Eq. (26) with a diagonal weight matrix $\boldsymbol{W}$, whose entries are chosen so

that during the optimization process non-dimensional variables of order $\mathcal{O}(1)$ are used, i.e.

$$(32) \quad \boldsymbol{W} = \text{diag}(W_i), \quad i = 1, \dots, n_y,$$

where $W_i^{-1} = \max_k \{|z_i(t_k)|\}$, and $|\cdot|$ is the absolute value operator. For both methods, we define the output vector $\boldsymbol{y}$ as:

$$(33) \quad \boldsymbol{y} = (\theta, u, w, q, a_x, a_z, \dot{q})^T,$$

where a_x, a_z are the linear accelerations along the x- and z-axis in a body-attached frame, and $\dot{q}$ is the pitch acceleration.

Figure 7 and 8 compare, respectively, the plant and the model-predicted pitch rate and acceleration time histories using the interference model with the estimated parameters (solid line) and prior to estimation (dash-dotted line). Both methods yield parameter estimates which improve the matching between system and model-predicted responses, although there is much room for improvement.

7 CONCLUSIONS AND FUTURE WORK

In this work we have formulated two alternative classes of methods for the time domain parameter estimation of first-principle rotorcraft models from flight test data, namely the batch optimization and adaptive recursive filtering methods. The batch OEM and FEM algorithms have been implemented in the general purpose software program STOP, which is a unified platform for optimization problems in rotorcraft flight mechanics capable of also supporting trajectory optimization problems.

The parameter estimation methods considered in this research effort have notable differences but many common features. Batch methods are one-shot approaches that process all available data simultaneously to arrive at an estimate of the parameters. They are typically associated with a higher computational cost and are very strongly non-linear problems which may experience difficult convergence; however when they converge they typically provide rather reliable estimates. Recursive methods, on the other hand, process one sample data point at a time, and hence sweep rather swiftly through the data sets, to the point of often being applicable to real-time estimation problems for systems with time-varying parameters. The unknown parameters are however transformed into dynamic variables, and the relaxation towards steady state values is not always easy to achieve.

In the formulation of the two approaches, we have paid particular attention at ensuring common crucial features to both, which in fact:

- Use data gathered in closed-loop, which is crucial for the analysis of unstable vehicles as helicopters and tilt-rotors, at least in certain flight conditions.
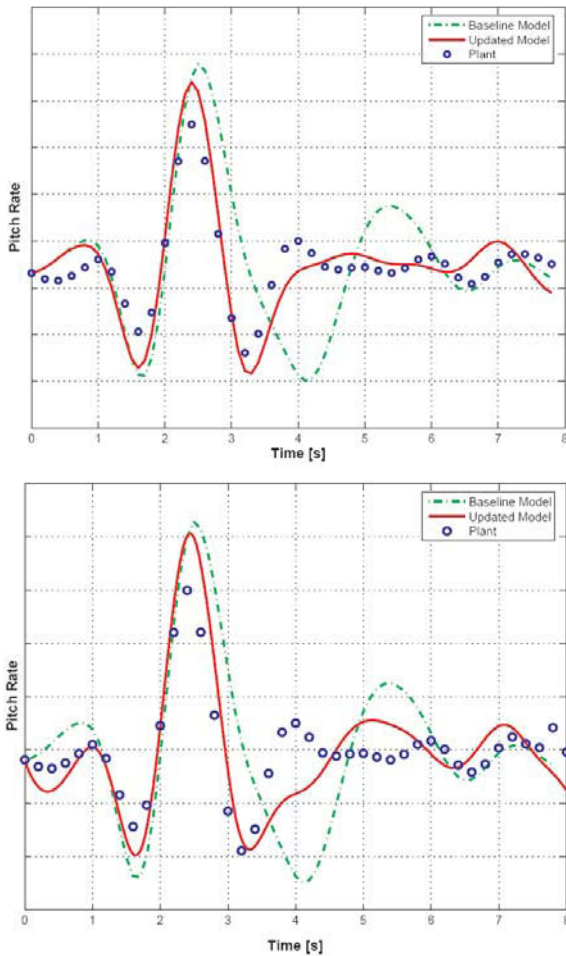
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Figure 7: System and model predicted pitch rate time history for a longitudinal stick doublet maneuver for a medium size helicopter in forward flight: with interference model (solid line), without interference model (dash-dotted line). Up: EKF; bottom: OEM.
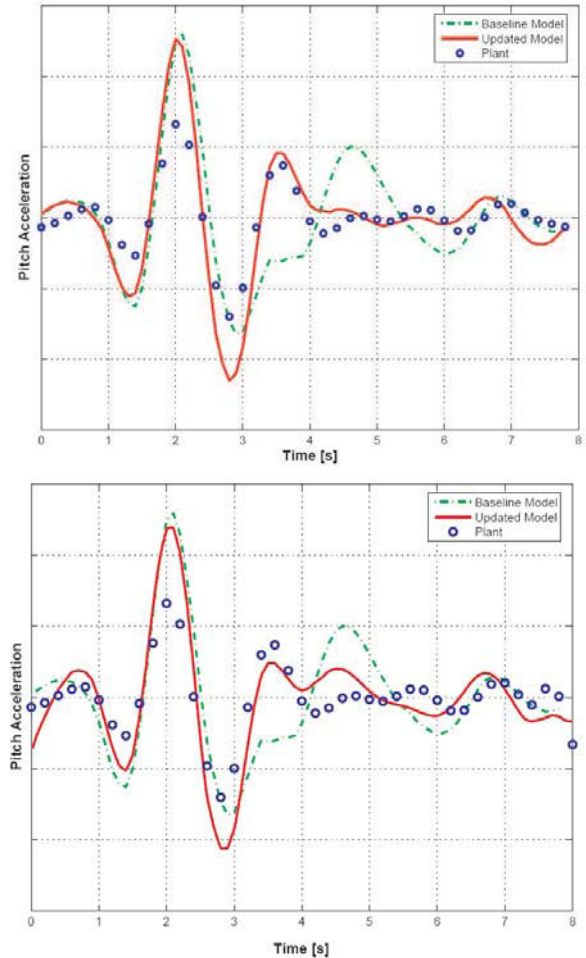

Figure 8: System and model predicted pitch acceleration time history for a longitudinal stick doublet maneuver for a medium size helicopter in forward flight: with interference model (solid line), without interference model (dash-dotted line). Up: EKF; bottom: OEM.

- Require no knowledge of the FCS, which might not be available altogether or which might affect the quality of the results when only partial knowledge is available or when modeling approximations are made (for example, neglecting saturation, free-play or other sources of non-linearities in the regulator).

- Are statistically based, and can deal with both process (e.g. in the case of flight testing in turbulent air) and unavoidable measurement noise.

- Can deal with unmeasurable model states, which is for example the case whenever the model includes aerodynamic states.

- Can be used in conjunction with first-principle flight mechanics models of the vehicle, including solution components characterized by fast time scales.

- Minimize potential problems due to fact that the FCS command was generated in flight to stabilize the plant, and hence might not be a stabilizing command for the model, using filtering to keep the model integration in close proximity of the datum.

- Provide estimates of all necessary statistics as part of the solution, without relying on prior knowledge of the noise, since this is typically unavailable or difficult to obtain and may require extensive manual tuning.

Given their common features and differences, we speculate that the two classes of methods can be profitably used in a synergistic way. Since the recursive approach is fast, it can be used for creating at low computational cost reasonable values of the model parameters. Next, these values can be used as ini-

tial guesses for the more computationally demanding batch approach.

The work program for our future activities in parameter estimation of rotorcraft vehicles calls for:

- Further validation and extensive testing of the procedures with the help of representative rotorcraft parameter estimation problems. With regard to this aspect, we will begin shortly a parameter estimation campaign using small hobby helicopters, following the guidelines provided by the feasibility study described in Reference [26].

- Leveraging on the fact that STOP code implements both trajectory optimization and parameter estimation solution procedures, we intend to work on the definition of optimal maneuvers for parameter estimation. The idea would be in this case to formulate trajectory optimization problems which maximize the identifiability of a given set of parameters, while operating within the flight envelope constraints of the vehicle. This might help in the definition of advanced flight testing procedures which go beyond the classical sequences of doublets and 3-2-1-1 (or 1-1-2-3) sequences.

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