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EXPERIENCE WITH MULTI-STEP TEST INPUTS FOR HELICOPTER
PARAMETER IDENTIFICATION

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EXPERIENCE WITH MULTI-STEP TEST INPUTS FOR HELICOPTER PARAMETER IDENTIFICATION

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ABSTRACT

This paper describes a test input design method which is intended to provide test signals having good properties for the purposes of helicopter parameter identification. For satisfactory identification test signals should be simple to apply, they should lead to long test records and thus to efficient estimation. They should also be robust in the sense that any minor variation from the ideal signal does not produce significant degradation of parameter estimates. Guidelines are presented concerning the features that should be present in the auto-spectrum of a desirable test signal. When account is taken of practical constraints on the form of input which can be applied, this design process leads to multi-step test signals. An optimal spectrum program can then be used for the synthesis of inputs of this particular form. Results of flight trials carried out at RAE (Bedford) are presented, and the effectiveness of this test input design approach is demonstrated. The paper includes a brief account of factors which arise in extending the work to involve optimal input design principles.

1.0 INTRODUCTION

The input applied to a helicopter for the purposes of system identification can have a substantial effect on the parameter estimates obtained. It is therefore important that a suitable input is chosen. Considerable effort has been directed towards input design for fixed-wing aircraft (e.g. [1],[2]), with good results, although relatively little work has been carried out to date on the design of rotorcraft inputs.

Before improved system identification inputs can be designed, it is essential to decide what makes one input better, or worse, than another. Inputs can then be compared to decide which is superior, and possible ways of designing inputs can be investigated.

In the current work, and in much of the work on input design reported in the past, the Cramer-Rao bound [3] has been used as a means for comparing inputs. The Cramer-Rao bound relates the variance of the parameter estimates to elements of the dispersion matrix, and for an efficient estimator states that,

$$\text{cov}(\theta) = D$$

where, θ is a vector containing the parameter estimates,
and D is the dispersion matrix.

Clearly, an input which gives a 'smaller' dispersion matrix, and thus parameter estimates with less variance from the true parameter values, is better than an input which gives a 'larger' dispersion matrix, and thus parameter estimates with greater variance. This has led to interest in developing algorithms that will design inputs which minimise some function of the dispersion matrix.

Care must be taken when using these algorithms since in many situations an efficient estimator does not exist, and the dispersion matrix then merely gives a lower bound on the covariance of the parameter estimates. In such cases, these input design algorithms can no longer be relied upon. However, inputs designed using the dispersion matrix are useful in situations where sufficiently long test records are available, since at least one class of estimators — maximum likelihood estimators — are asymptotically efficient.

Current rotorcraft identification work at the University of Glasgow is aimed principally at the validation of linearised flight mechanics models. Hence, any inputs used for identification must ensure that the modelling assumptions remain valid. The results of the identification can then be used to check that, given the modelling assumptions, the theoretical model is correct. The effect of violating the modelling assumptions can be studied separately, if required. Hence, any input design method needs to also take account of the various constraints on inputs for rotorcraft identification.

In section 2 of this paper experience with past rotorcraft identification inputs at the University of Glasgow is briefly described. A simple, but effective, input design method developed by the author is then given, together with a discussion of the results of flight trials of inputs designed using this method. These flight trials were performed using the Lynx helicopter at RAE (Bedford), with the inputs applied manually by the pilot via the normal controls. Section 3 contains a brief summary of more sophisticated design methods which are under development. Conclusions are then drawn in the final section.

2.0 SIMPLE MULTI-STEP INPUTS

2.1 Introduction

In past identification work at the University of Glasgow [4,5] the inputs used have consisted of doublets and 3211's (figure 1). The 3211 type of input in particular has also been used extensively elsewhere [2]. These have been used largely for historical reasons, and although useful results have been obtained, problems have been encountered in applying these inputs due to the difficulties inherent in the identification of rotorcraft. These include :

- a) The model involved is of high order, and the model states are tightly-coupled to each other. Hence, there are a large number of model parameters. For example, using an 8th order model, with 8 states and 4 control inputs, there are potentially 96 parameters to be identified.
- b) The model is obtained by linearising the system about an operating point. However, the system is poorly damped, and under some flight conditions can be unstable. Hence, typically only very short flight test records are obtained before the system departs too far from the operating point and the linear model becomes invalid.
- c) Finally, the signal-to-noise ratio of flight test data is poor due to the high vibration levels in rotorcraft.

With the Lynx helicopter at RAE (Bedford), the 3211 input has been found to give extremely short test records — typically only 3 seconds long — before the pilot is forced to recover control of the aircraft. Since the 3211 input itself is 7 seconds long, it is therefore not possible to fully apply the input in the available time.

The doublet input has been found to give significantly better results, producing test records of around 10 to 15 seconds duration before the response of the helicopter departs too far from the operating point of the linearised model, and so becomes non-linear. However, it has been found that this is still not long enough to allow the parameter estimates to converge.

Obtaining sufficiently long test records is important, since the parameter estimates then have time to converge, resulting in improved estimates. Also, as mentioned above, long test records are required to give efficient estimation i.e. minimum variance estimation, which has the added advantage of allowing inputs to be designed using dispersion matrix criteria. Hence, as a first step it was decided to concentrate on trying to design inputs which would give longer test records while at the same time giving a reasonably 'small' dispersion matrix. The next step would then be to design inputs to strictly minimise some function of the dispersion matrix, while still giving sufficiently long test records. It is the results of the first step — inputs to give longer test records and 'small' dispersion matrix — that are covered in this paper.

Consideration of the robustness of the inputs was also included in the input designs. Robustness is an important, though often neglected, aspect of input design. Only an approximate model of the system involved is available beforehand, and so the inputs used must be insensitive to errors in the model. In addition, on the particular helicopter used in this work inputs are currently applied by the pilot via the normal controls, and so they must also be insensitive to being applied inaccurately. Since they are applied manually, they must also be kept relatively simple e.g. multi-steps. However, work is currently underway at RAE (Bedford) to develop an input device which will in the future allow inputs to be applied directly to the helicopter without pilot intervention [6].

2.2 Input Auto-Spectrum Design

The aim is to design an input which is robust, gives long linear test records, and which gives a reasonably 'small' dispersion matrix. Several general guidelines can be developed concerning the features that should be present in the auto-spectrum of such an inputs, as follows.

Typically, the transfer function between a given rotorcraft model state and a given control input contains large peaks. These peaks correspond to resonances in the system. If an input excites these resonances, then the response will be large, and will rapidly become non-linear, so leading to a short test record. Hence, by designing inputs which avoid exciting these resonances, longer test records can be obtained.

In addition, inputs should not contain a dc component. If a large dc component is present, then the aircraft response to the input will also contain a dc component. This is undesirable when using a model linearised about a particular operating point, since a dc component essentially means that the response is about a different operating point. If this operating point is significantly different from that used when linearising the model, then the model will be invalid.

Now, the next step is to consider how to obtain a reasonably 'small' dispersion matrix. Take the model,

$$\frac{dx(t)}{dt} = A x(t) + B u(t) \quad \text{where, } y(t) = \text{measured responses}$$

$$y(t) = C x(t) + \rho(t) \quad \begin{array}{l} \rho(t) = \text{Gaussian noise} \\ \text{with zero mean and} \\ \text{covariance, } R \end{array}$$

$$\begin{array}{l} u(t) = \text{control inputs} \\ x(t) = \text{model states} \\ A, B, C = \text{model matrices} \end{array}$$

Let θ be a vector containing the model parameters that are to be identified, and let the true values of these parameters be given by θ_0 .

Then, for sufficiently long test records, the time average dispersion matrix is given by [7],

$$D = M^{-1}$$

$$M = \int_0^{\infty} F^*(\omega) S_{VV}^{-1}(\omega) F(\omega) S_{uu}(\omega) d\omega$$

$$\begin{array}{ll} \text{where, } S_{uu}(\omega) &= \text{auto-spectrum of input } u(t) \\ S_{VV}(\omega) &= \text{auto-spectrum of noise } \rho(t) \\ F(\omega) &= dG(\omega)/d\theta|_{\theta=\theta_0} \\ G(\omega) &= C(j\omega - A)^{-1}B, \text{ transfer function matrix.} \end{array}$$

For white noise, $S_{VV}(\omega)$ is constant. In practical systems, $F(\omega)$ becomes negligible above some frequency, ω_c . Finally, for simplicity consider the scalar case of a single-input single-output system with only one parameter to be identified. This gives,

$$M = S_{VV}^{-1} \int_0^{\omega_c} |F(\omega)|^2 S_{uu}(\omega) d\omega$$

$$D = 1/M$$

Notice that the expression for M does not involve the phase of the input or the phase of $F(\omega)$. Hence, attention can be restricted to the magnitudes of the input and $F(\omega)$. It can then be seen that to obtain a small dispersion, D , a large M is needed i.e. the area under the curve $|F(\omega)|^2 S_{uu}(\omega)$ should be large.

Now, in order to obtain long test records, the input auto-spectrum, $S_{uu}(\omega)$ should avoid exciting the transfer function resonances. However, by exciting the frequencies away from the resonances, $|F(\omega)|^2 S_{uu}(\omega)$ can still be made to have a fairly large area.

Returning to the more general case of a vector model, exciting the frequencies away from the resonances will result in the elements of M being fairly large, and intuitively, this should lead to a 'small' dispersion matrix, D .

This argument concerning the dispersion matrix in particular, and the arguments concerning the other guidelines in general, are very crude. However, the system transfer functions are typically only known approximately, and $F(\omega)$ is usually even less well known. Hence, when designing inputs only the general characteristics of $F(\omega)$ can be relied upon to any extent, and therefore sophisticated algorithms using detailed knowledge of $F(\omega)$ are redundant. Of course, if $F(\omega)$ is better known, then more sophisticated algorithms become worthwhile.

Finally, the robustness of the inputs has to be considered. Since the model is not known exactly, the frequencies of the resonances are not known exactly. To allow for errors in these, inputs should avoid exciting a range of frequencies around the predicted position of each resonance. This will also make the inputs less sensitive to errors introduced during the application of the inputs by the pilot, since errors in the input's auto-spectrum can then be tolerated to a greater extent.

So, by intuitive arguments, the features that should be present in the auto-spectrum of a reasonable input have been determined. These are :

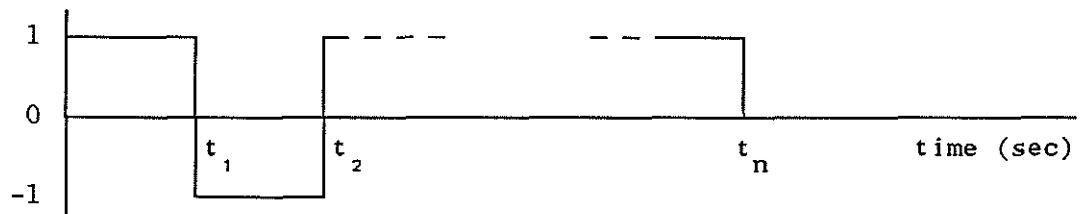
- a) Auto-spectrum should avoid exciting transfer-function resonances, to give longer test records.
- b) Auto-spectrum should also avoid exciting frequencies around the resonances, to give robustness.
- c) Auto-spectrum should excite the remaining frequencies, however, to give a reasonably 'small' dispersion matrix.

It is of interest to note that if there is no constraint on the magnitude of the system's response to an input, then features (a) and (b) can be discarded. The resulting input is then the standard wide-bandwidth type, such as an impulse or a pseudo-random binary input, which is in widespread use for identification.

2.3 Optimal Spectrum Program

Now that the criteria for designing the auto-spectrum of an reasonable input have been obtained, the next stage is to produce an input with the desired auto-spectrum characteristics. Since inputs must be applied by the pilot, they must be kept relatively simple e.g. a sequence of steps. A program has therefore been written that will generate the binary multi-step input whose auto-spectrum best meets a given specification.

Consider the general aperiodic binary multi-step input,



The Fourier Transform of this input is,

$$F(\omega) = \frac{1}{j\omega} \left[1 + 2 \sum_{i=1}^{n-1} (-1)^i \exp(-j\omega t_i) + (-1)^n \exp(-j\omega t_n) \right]$$

where,

$F(\omega)$ is the Fourier Transform

ω is the frequency in radians/second.

$j = \sqrt{-1}$

$n+1$ is the number of steps in the input.

t_i is the time in seconds of the i^{th} step in the input; $t_0 = 0$ sec.

Now define the cost function,

$$I = \sum_{k=1}^m a_k |F(\omega_k)|^2$$

where,

a_k are constants, $k=1, 2, \dots, m$

ω_k are frequencies in rads/sec, $k=1, 2, \dots, m$

The optimal spectrum program takes the following information as input :

- 1) The number of steps, n in the input.
- 2) The number, m of constants in the cost function, I
- 3) The values of the weightings, a_k and the frequencies, ω_k in the cost function, I .

The program will then calculate the times, t_i of the steps in the input that will result in the cost function, I being maximised. Specifying a large, positive a_k results in an input with a large auto-spectrum component at frequency ω_k . Conversely, specifying a large negative a_k results in an input with a small auto-spectrum component at the corresponding frequency, ω_k .

This permits the straightforward synthesis of inputs with auto-spectra meeting the guidelines developed in § 2.2. The optimal spectrum program, in conjunction with the auto-spectra guidelines, therefore forms a simple input design approach.

2.4 Flight Trials

In order to assess the input design method presented above, inputs were designed for the Lynx helicopter at RAE (Bedford), and subsequently underwent flight trials.

Inputs were designed for application to the longitudinal cyclic control of the Lynx at 80 knots level flight, and attention was restricted to the parameters of the Pitching Moment Equation of the model [8]. This equation contains 7 parameters, as follows,

$$\frac{dq(t)}{dt} = M_u u(t) + M_w w(t) + M_q q(t) + M_v v(t) + M_p p(t) + M_{\eta_{1s}} \eta_{1s} + M_{\eta_{1c}} \eta_{1c}$$

where,

$u(t)$ = longitudinal velocity $w(t)$ = vertical velocity

$q(t)$ = pitch rate $v(t)$ = lateral velocity

$p(t)$ = roll rate

$\eta_{1s}(t)$ = longitudinal cyclic control input

$\eta_{1c}(t)$ = lateral cyclic control input

$M_u, M_w, M_q, M_v, M_p, M_{\eta_{1s}}, M_{\eta_{1c}}$ are the parameters to be estimated in the system identification.

A frequency-domain equation-error program developed by Black [9] was used to perform the identification.

Figure 2 shows pseudo theoretical transfer functions of the helicopter. At the flight condition used, the Lynx helicopter is unstable. Hence, the impulse response of the helicopter diverges to infinity as time goes to infinity. The transfer functions are therefore given by the Laplace transform of the impulse response rather than the Fourier transform, since the Fourier transform cannot be used with a divergent function (see e.g. [10]). However, the results shown in figure 2 were obtained by taking the Fourier transform of the first 60 seconds of the theoretical impulse response of the Lynx. Hence, they are not the true transfer functions. Nevertheless, they reflect the characteristics of the system, and are sufficient for the present purposes. The eight model states shown in figure 2 are,

- | | |
|-------------------------------|--------------------------|
| 1. longitudinal velocity, u | 2. lateral velocity, v |
| 3. vertical velocity, w | 4. pitch angle, θ |
| 5. roll angle, ϕ | 6. pitch rate, q |
| 7. roll rate, p | 8. yaw rate, r |

It can be seen that there is a large peak in the pseudo magnitude transfer functions around 0.3 rads/sec, and that there is also a rapid change in phase. This corresponds to the unstable phugoid-type mode of the Lynx, which has a theoretical natural frequency of 0.36 rads/sec.

The optimal spectrum program was therefore used with weightings chosen to ensure the following :

- a) The input auto-spectrum did not contain a dc component.
- b) The input auto-spectrum avoided exciting the resonance at around 0.3 rads/sec.
- c) The input auto-spectrum excited the frequencies between 2 and 3 rads/sec.

It was decided that the input auto-spectrum should not excite frequencies above about 3 rads/sec since previous experience with the Puma at RAE (Bedford) suggested that the theoretical model was only valid up to around 3 rads/sec. Above this frequency, rotor dynamics appear to dominate the response, and these are not included in the simplified model that is being used.

Using these weightings, an input consisting of 5 steps was generated by the optimal spectrum program (see figure 3). It can be seen that the auto-spectrum of this input has little power below 1 rad/sec, and that most of the power is concentrated between 1 and 3 rads/sec, as desired.

However, the timings of the steps in this input are complex, and it is therefore unrealistic to expect a pilot to apply such an input. Hence, this input was modified to give two versions with much simpler timings. This should be straightforward, since the original input was designed to be robust, and in particular to withstand being applied inaccurately by the pilot. Hence, moderate changes in the timings of the input should have only a small effect.

The modified inputs (figure 4) were called the *double-doublet* and the *1221*. Care was taken to ensure that these inputs contained no dc component. It can be seen that the auto-spectra of these inputs are very similar to that of the original input. However, certain differences are apparent.

In the case of the double-doublet, it can be seen that the input contains little power below 2 rad/sec. Hence, it can accommodate a larger error in the predicted frequency of the unstable resonance of the Lynx than the original input. However, the double-doublet contains slightly more power below 1 rad/sec than the original input, and so may excite the unstable mode more and so give shorter test records.

In contrast, the 1221 has less power below 0.75 rad/sec than the original input, and so should give longer test records. However, since the region around the unstable resonance where the input power is low is smaller than in the original input, the 1221 is more sensitive to errors in the frequency of the resonance. Hence, the double-doublet and the 1221 complement each other, and the original input is a compromise between the two.

From the flight trials, it was found that the 1221 input was not sufficiently robust, and the large peak in its auto-spectrum at around 1 rads/sec produced too large an excitation, typically giving less than 20 seconds of test data before the helicopter response became non-linear. In contrast, the double-doublet gave relatively long test records - typically more than 30 seconds of data were obtained before the response became non-linear. This input was also found to give consistent and reproducible results in several test runs, indicating its robustness.

Figure 5 shows the variation of the parameter estimates with the length of the test record used for one of the double-doublet test runs. It can be seen that the estimates of M_q , M_p , $M_{\eta_{1s}}$, and $M_{\eta_{1c}}$ appear to have converged after about 28 seconds. The estimates of M_u , M_v , and M_w also appear to reach a plateau after 28 seconds. However, they then start to vary rapidly again after about 32 seconds, corresponding to non-linearities becoming significant.

Notice that 28 seconds is the length of approximately two cycles of the slowest mode of the Lynx. Intuitively, it is perhaps reasonable to expect that a few cycles of this mode will be needed before its parameters can be properly identified.

It can be seen that, as mentioned before, unless an input provides fairly long test records the parameter estimates will not converge, and poor estimates will result. Recall that long records are also necessary to give efficient estimation i.e. with the minimum parameter variance (given by the dispersion matrix). Figure 6 shows the variation of the standard-deviations of the parameter estimates with the length of test record. It can be seen that these appear to have converged for M_q , M_p , $M_{\eta_{1s}}$, and $M_{\eta_{1c}}$, but not for M_u , M_v , and M_w . Hence, still longer records would perhaps be desirable.

2.5 Conclusions

In conclusion, these flight trials have demonstrated the effectiveness of the input design method presented, despite its simplicity. The importance of obtaining sufficiently long test records has been shown, both in terms of the convergence of the parameter estimates and in terms of convergence of the parameter variances to give efficient estimation.

It is interesting to take note of the results concerning the 1221 and double-doublet inputs. These inputs complement each other, as discussed above — the 1221 theoretically produces less excitation of the unstable resonance, while the double-doublet is more robust to errors in the theoretical model, and to pilot errors in applying the input. Since the double-doublet has been found to give the best results, it appears that robustness has been the deciding factor.

When designing inputs, the degree of robustness required should therefore be taken carefully into account.

3.0 OPTIMAL INPUTS

The input design method described above is relatively crude, but with the introduction in the near future of a control input device [6] it should become profitable to use more sophisticated input design methods.

In the literature, various algorithms have been studied which will design inputs to minimise the trace or determinant of the dispersion matrix. These algorithms can be roughly divided into those which work in the time-domain, and those which work in the frequency-domain, each of which has particular advantages. Time-domain algorithms are more complex, but they allow any length of test record to be catered for. While frequency-domain algorithms are simpler, but assume that test records are long compared to the time-constants of the system being dealt with. For the current application it is felt that the frequency-domain algorithms are more suitable, for the following reasons :

- a) Unless relatively long test records are available, an efficient estimator may not exist, and hence designing inputs using the dispersion matrix is unreliable, as mentioned above. There is therefore no advantage in using the time-domain approach.
- b) The frequency-domain methods are simpler.
- c) Past identification work at the University of Glasgow has concentrated on using frequency-domain methods of parameter estimation, and frequency-domain input design therefore complements this.

However, designing rotorcraft identification inputs involves several complications that have not been present in the applications reported to date of frequency-domain input design. In particular, the rotorcraft plant is unstable, and so the Fourier Transform cannot be used directly. Rotorcraft inputs are also subject to major constraints e.g. they must produce a response which does not depart too far from the operating point of the linearised model being identified, or the response will become non-linear. These constraints must therefore be included in any input optimisation algorithms. Work is therefore required in order to develop suitable algorithms.

4.0 CONCLUSIONS

It has been found that even a simple input design method has produced worthwhile results, and the double-doublet input has proved to be significantly better than previous identification inputs. Work is therefore now underway to investigate more sophisticated input design methods which optimise the dispersion matrix.

This input design work is being carried out at the University of Glasgow in collaboration with RAE (Bedford) as part of a continuing drive for the development of robust parameter identification techniques which can be used for the validation of flight mechanics models.

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Figure 1

Time histories and auto-spectra of doublet and 3211 inputs.

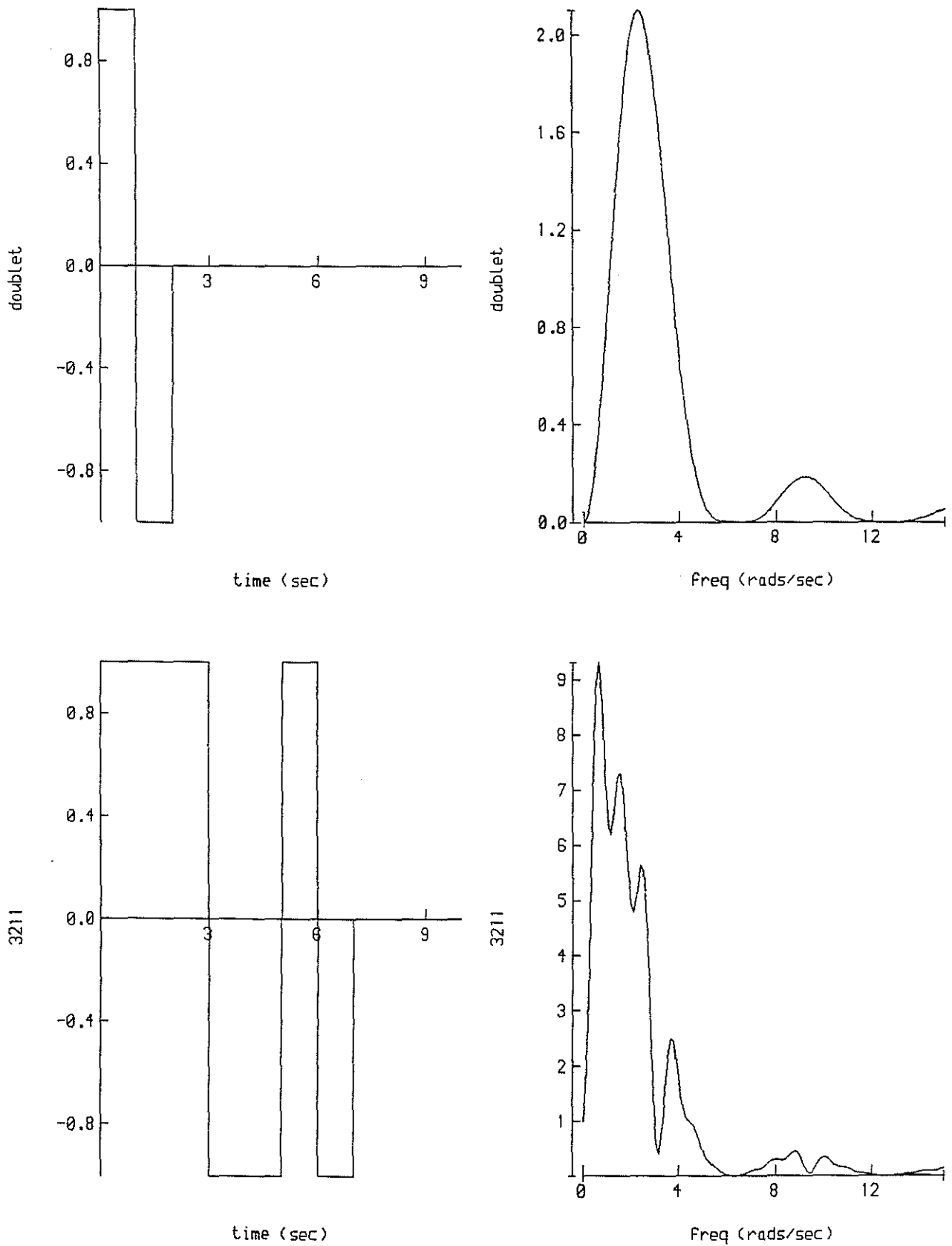


Figure 2
Transfer functions between the longitudinal cyclic input and the 8 model states of the theoretical model. 8th order model used for Lynx at 80 knots level flight.

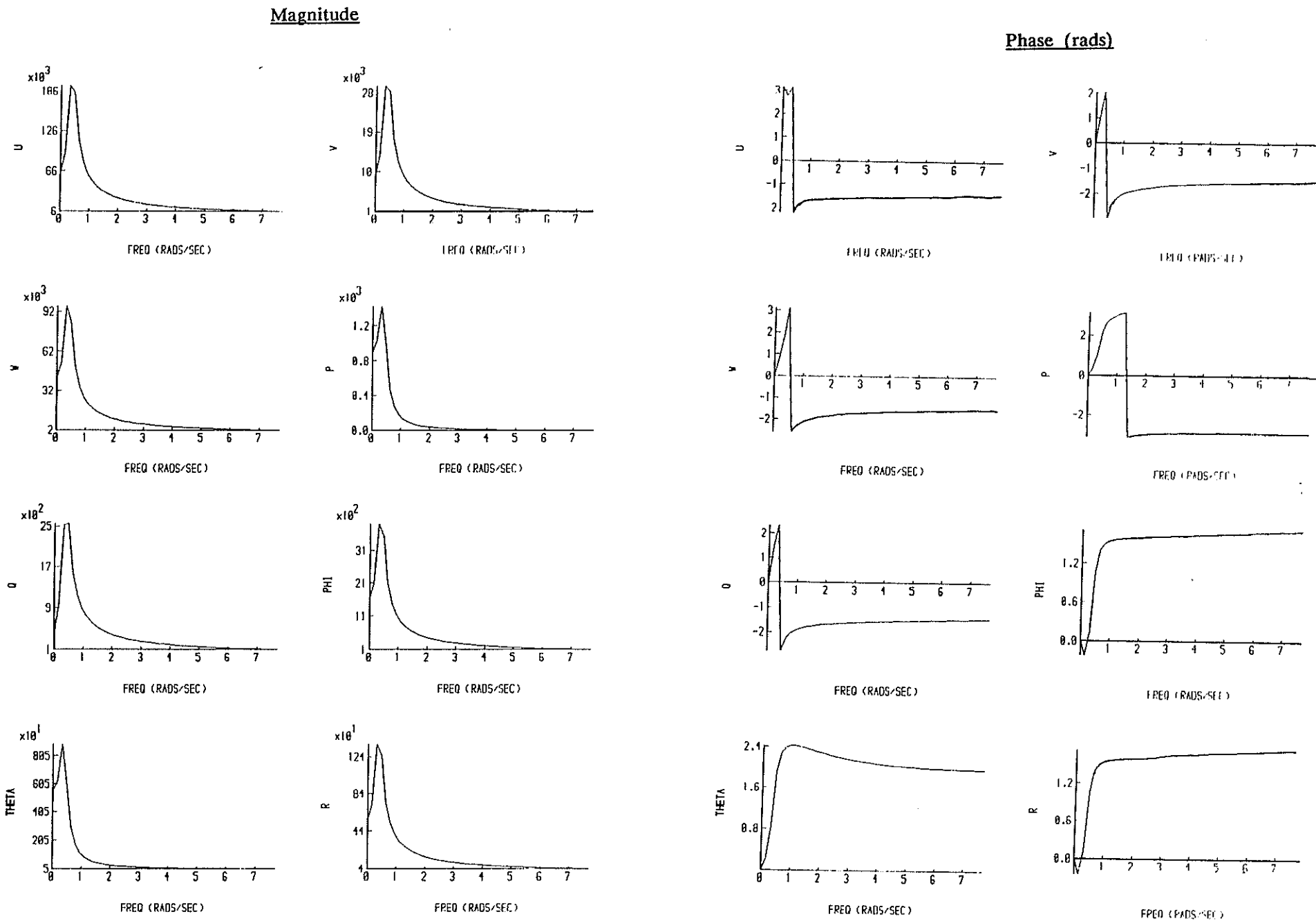


Figure 3

Time history and auto-spectrum of the input generated by the optimal spectrum program in accordance with the weightings given in Table 1.

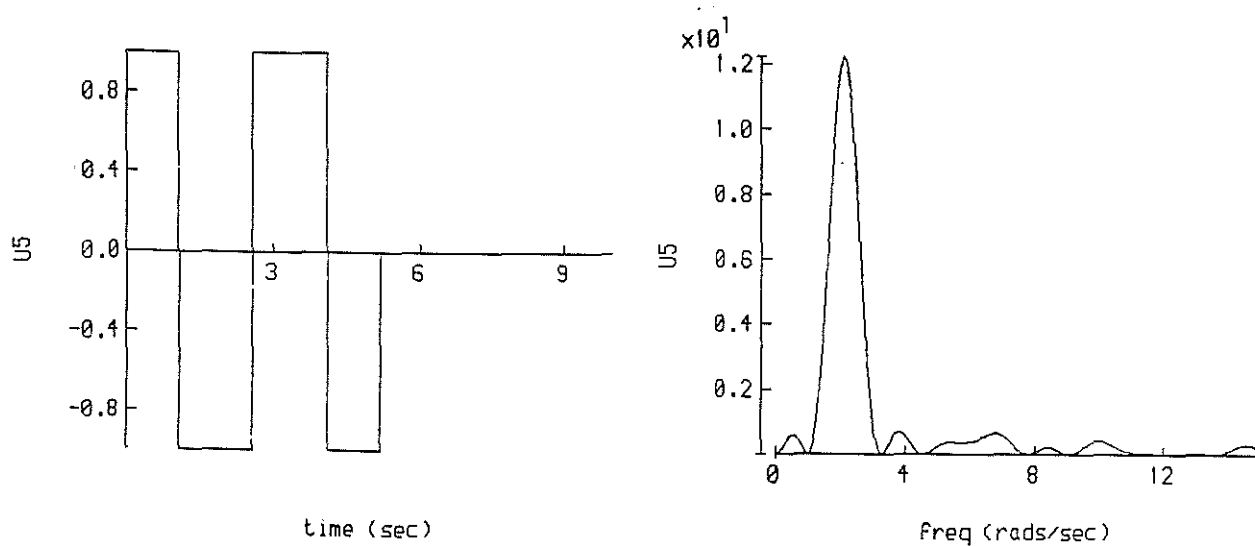


Figure 4

Time histories and auto-spectra of double-doublet and 1221 inputs.

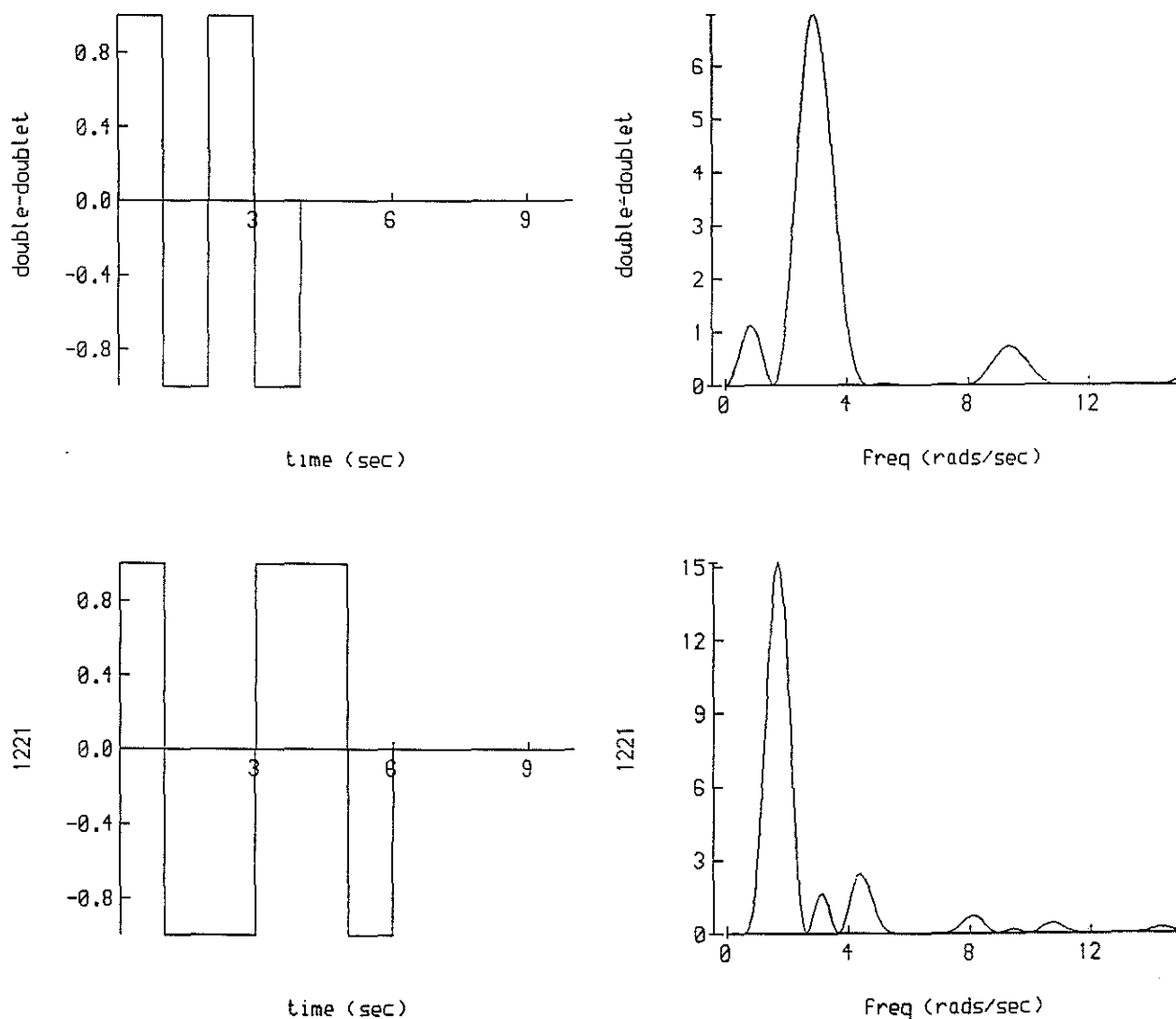


Figure 5

Variation of the estimates of the parameters of the Pitching Moment Equation with the length of test record used (Lynx helicopter: flight 190/12).

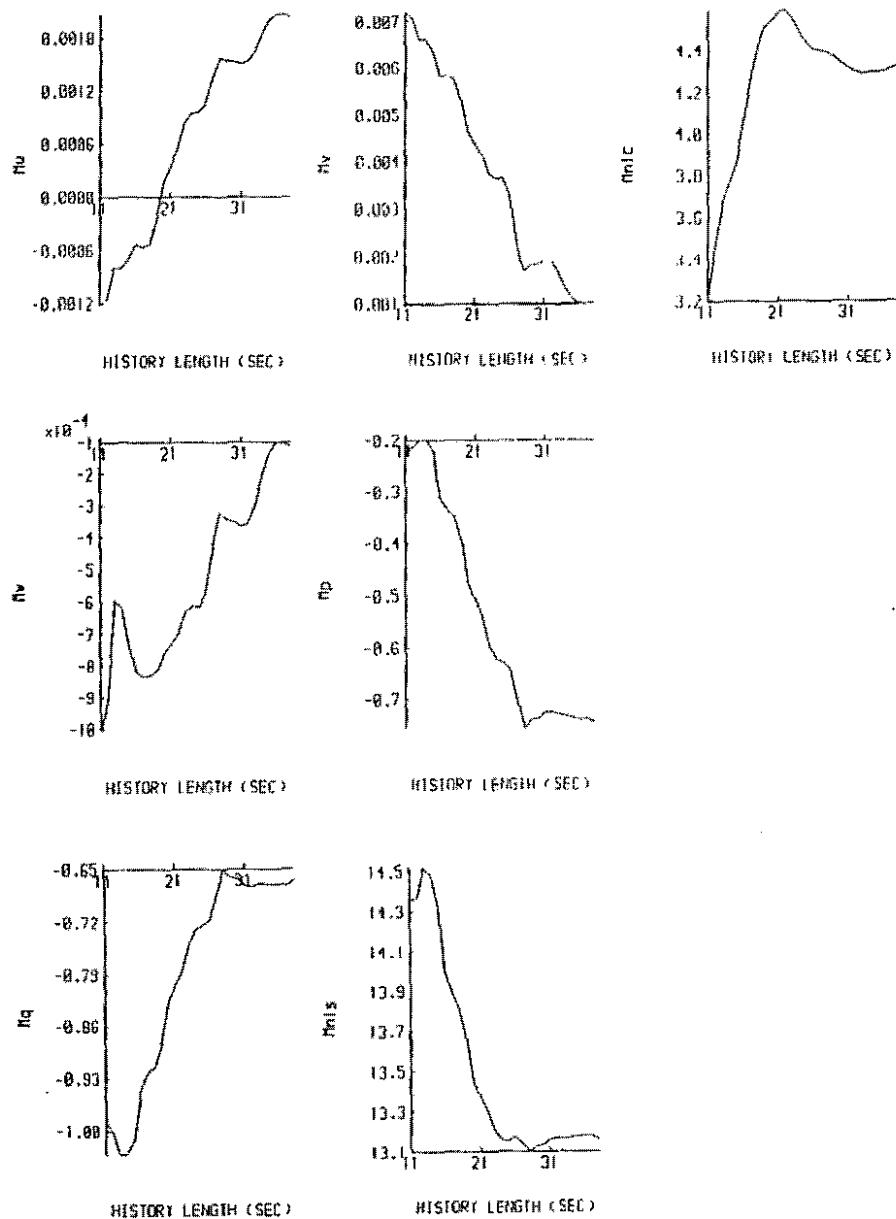


Figure 6

Variation of the Pitching Moment Equation parameter standard deviations with the length of test record used (Lynx helicopter: flight 190/12)

