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STATISTICAL EVALUATION OF THE EMC SAFETY MARGIN AT SYSTEM LEVEL

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ABSTRACT

EMC equipment tests are well established and defined, while EMC system tests are still rather vague: in MIL-E-6051D dealing with EMC system testing only general guidelines are given with the definition of the safety margin. In a complex system such as an aircraft, the problem of the electromagnetic compatibility and safety margin have become more important especially with the introduction of electronic equipments into areas of the aircraft which directly relate to flight safety. Interference effects are not always repetitive and in many cases the malfunctions change randomly around an average level. Therefore a statistical evaluation of a safety margin was developed for the EMC investigation tests so that a number of parameters, from various avionic equipments could be monitored, when the aircraft onboard emissive equipments are activated. By use of this technique each equipment parameter may be monitored over a set period of time. Comparison can therefore be made, from this point of view, between each parameter with and without the emissive equipments activated in order to evaluate any drifting of any parameter due to EMI and obtain an EMC safety margin.

1. INTRODUCTION

In many works the need of a statistical model for EMC testing has been emphasized [1], [2] , [3] , because the interference effects are not always repetitive, but in many cases the malfucntions change randomly, around an average level. In a complex system as an aircraft, the problem of the definition of an EMC safety margin related to a statistical model is very important.

At the present EMC system tests are still rather vague and only general guidelines are given about the definition of a safety margin. The purpose of this work is to develope a mathematical model to perform an analysis of random interference effècts from the statistical point of view and to define a safety margin related to the characteristics of the equipment under test. Some considerations about the degree of uncertainty related to the safety margin evaluation are also given in order to reduce the error proability during the execution of the test at system level.

2. SAMPLE VALUE AND PARAMETER ESTIMATION

The two basic parameters of a random variable x which specify its central tendency and dispersion are the mean value and the variance μ_x and σ_x^2 :

$$\mu_x = E[x] = \int_{-\infty}^{+\infty} x\, p(x)\, dx \tag{1}$$

$$\sigma_x^2 = E\left[(x-\mu_x)^2\right] = \int_{-\infty}^{+\infty} (x-\mu_x)^2 p(x)\, dx \tag{2}$$

where p(x) is the probability density function of the variable x. An exact knowledge of the p(x) function will not generally be available. Hence one must be content with estimates of the mean value and variance based upon a finite number N of observed values :

$$\overline{x} = \hat{\mu}_x = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{3}$$

$$s^2 = \hat{\sigma}_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{x})^2 \tag{4}$$

The hats ($\wedge$) indicate that $\hat{\mu}_x$ and $\hat{\sigma}_x^2$ are used as estimators for the mean value and variance of the random variable x. Three principal factors can be used to determine the goodness of an estimator [4]:

a) estimator unbiased

$$E[\hat{\phi}] = \phi \tag{5}$$

where $\hat{\phi}$ is an estimator of ϕ.

b) estimator efficient

$$E\left[\left(\hat{\phi}_1-\phi\right)^2\right] \leqslant E\left[\left(\hat{\phi}_i-\phi\right)^2\right] \qquad (6)$$

where $\hat{\emptyset}_1$ is the estimator of interest and $\hat{\emptyset}_i$ is any other possible estimator of $\emptyset$.

c) estimator consistent

$$\lim_{N\to\infty} \mathrm{Prob}\left[\left(\hat{\phi}-\phi\right)\geqslant\varepsilon\right]=0,\forall\varepsilon>0 \qquad (7)$$

where N is the number of observed values and $\hat{\emptyset}$ is the estimator of $\emptyset$. It is desirable that the expected value of the estimator be equal to the parameter being established (estimator unbiased) and also that the mean square error of the estimator be smaller than for other possible estimator (estimator efficient). Moreover it is desirable that the estimator approach the parameter being estimated with a probability approaching unity as the sample size becomes large. The estimators $\hat{\mu}_x$ and $\hat{\sigma}_x^2$ are estimators unbiased, efficient and consistent for the mean value and variance of a random variable x [4].

3. SAMPLING DISTRIBUTIONS

Consider a random variable x with a probability distribution function p(x).

Let $x_1, x_2, \ldots\ldots, x_N$ be a sample of N observed values of x. Any quantity computed from these sample values will also be a random variable. For example, consider the mean value $\bar{x}$ and the variance S^2 of the sample. If a series of different samples of size N were selected from the same random variable x, the value of $\bar{x}$ and S^2 computed from each sample would generally be different. Hence $\bar{x}$ and S^2 are also the random variables with a probability distribution function $P(\bar{x})$ and $Q(S^2)$. These functions are called "Sampling distributions" of $\bar{x}$ and S^2.

If the variable x is normally distributed with a mean of μ_x and a variance of σ_x^2, the sampling distribution of the sample variance S^2 is given by [4]:

$$Q\left[\chi_n^2\right]=\left[2^{n/2}\Gamma\left(\frac{n}{2}\right)\right]^{-1}\left(\chi_n^2\right)^{\left[(n/2)-1\right]} exp\left(-\chi_n^2/2\right) \qquad (8)$$

where:

- Γ is the Gamma function
- $\chi_n^2 = nS^2/\sigma_x^2$

$Q[\chi_n^2]$ is the Chi-Square distribution function with n=N-1 degrees of freedom. Moreover the sampling distribution of the sample mean value $\bar{x}$ is given by |4|:

$$P[t_n] = \frac{\Gamma[(n+1)/2]}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)}\left[1+\frac{t_n^2}{n}\right]^{-(n+1)/2} \tag{9}$$

where:

- Γ is the Gamma function
- $t_n = \sqrt{N}\,(\bar{x}-\mu_x)/S$
- $n = N-1$

$P[t_n]$ is the Student distribution function with n=N-1 degrees of freedom.

4. CONFIDENCE INTERVALS

The use of sample values as estimators for parameters of a random variable has been discussed previously . However those procedures result only in point estimates for a parameter of interest. A more meaningful procedure for estimating parameters of random variables involves the estimation of an interval, as opposed to a single point value, which will include the parameter being estimated with a known degree of uncertainty. Such an interval can be established if the sampling distribution of the estimator in question is known. For the case of the variance σ_x^2 based upon a sample variance S^2 computed from a sample of size N, a confidence interval can be established as follows:

$$\left[\frac{nS^2}{\chi^2_{n;\alpha/2}} \leq \sigma_x^2 < \frac{nS^2}{\chi^2_{n;1-\alpha/2}}\right] \tag{10}$$

where:

$$\int_{\chi^2_{n;\alpha/2}}^{\infty} Q[\chi_n^2]\,d\chi_n^2 = \mathrm{Prob}\left[\chi_n^2 > \chi^2_{n;\alpha/2}\right] = \frac{\alpha}{2}$$

The degree of trust associated with the confidence statement is $1-\alpha$ and it is called "confidence coefficient". Furthermore, if σ_x^2 is unknown, a confidence interval can still be established for the mean value μ_x based upon the sample values $\bar{x}$ and S as follows:

$$\left[x - \frac{s t_{n;\alpha/2}}{\sqrt{N}} \leq \mu_x < x + \frac{s t_{n;\alpha/2}}{\sqrt{N}} \right] \qquad (11)$$

where:

n=N-1

$$\int_{t_{n;\alpha/2}}^{\infty} P[t_n]\, dt_n = \mathrm{Prob}\left[t_n > t_{n;\alpha/2}\right] = \frac{\alpha}{2}$$

The degree of trust associated with the confidence statement is $1-\alpha$ and it is called "confidence coefficient".

5. TEST PROCEDURE AND SAFETY MARGIN EVALUATION

The procedure for estimating the interference margin related to a parameter of interest has been subdivided into three steps:

a) evaluation of sample mean value, sample variance and confidence intervals for the parameter of interest by means of the specification characteristics of the equipment under test

b) evaluation of sample mean value, sample variance and confidence intervals for the parameter of interest with minimum number of equipments switched on on the aircraft and interference generators switched off (minimum noise condition)

c) repetition of step b) with the same number of equipments switched on and interference generators switched on, one at a time on the aircraft.

The step a) allows to determine the confidence intervals |Lo, Uo| from the specification characteristics of the equipment under test. By means of the test procedure recorded in the step b) one may verify that the sample mean value and the variance, with minimum noise condition, falls into the confidence interval established in step a). Fig.1A,B shows a comparison between the confidence interval [Lo, Uo] and [Lo', Uo'] related to the specification limits of the equipment under test and the confidence interval [L1, U1] and [L1', U1'] obtained, for the same parameter, with minimum noise condition. An interference margin at system level may be defined as follows:

$$IM_{SYS} = 20 \log_{10}(L0/L1) \text{ , for } \bar{x}_N \leq \bar{x}_0 \qquad (12\text{ a})$$

$$IM_{SYS} = 20 \log_{10}(L0'/L1') \text{ , for } \sigma_N^2 \leq \sigma_0^2 \qquad (12\text{ b})$$

$$IM_{SYS} = 20 \log_{10}(U1/U0) \text{ , for } \bar{x}_N \geq \bar{x}_N \qquad (12\text{ c})$$

$$IM_{SYS} = 20 \log_{10}(U1'/U0') \text{ , for } \sigma_N^2 \geq \sigma_0^2 \qquad (12\text{ d})$$

An interference situation $(IM_{SYS} > 0)$ in the minimum noise condition is usually a clear symptom of an integration malfunction to be solved at system level. It is necessary to remove the cause of interference at system level before considering the effect of the interference generators (emissive equipment switched ON). When no interference occurs at system level ($IM_{SYS} \leq 0$) one may consider the effect of the emissive equipments (step c)). This measurement will still be referred to the confidence interval [Lo, Uo] and [Lo', Uo'] defined in step a) (see Fig. 1). Therefore an effective interference margin can be defined as follows:

$$IM_{EFF} = 20 \log_{10}(L0/L2) \text{ , for } \bar{x}_{N+I} \leq \bar{x}_0 \quad (13\text{ a})$$

$$IM_{EFF} = 20\log_{10}(L0'/L2') \text{ , for } \sigma^2_{N+I} \leq \sigma_0^2 \quad (13\text{ b})$$

$$IM_{EFF} = 20 \log_{10}(U2/U0) \text{ , for } \bar{x}_{N+I} \geq \bar{x}_0 \quad (13\text{ c})$$

$$IM_{EFF} = 20 \log_{10}(U2'/U0') \text{ , for } \sigma^2_{N+I} \geq \sigma_0^2 \quad (13\text{ d})$$

An interferent situation occurs when $IM_{EFF} > \emptyset$.

In this case |L2, U2| and |L2', U2'| are the confidence intervals computed with emissive equipment switched on. Moreover L_i and U_i are the lower and upper limits of the confidence interval for the sample mean distribution function, while L'_i and U'_i are the lower and upper limits of the confidence interval for the sample variance.

6. GENERAL REMARKS

A. CONFIDENCE INTERVAL FROM EQUIPMENT SPECIFICATION

The evaluation of this confidence interval in easily performed for a fixed level signal, whose level and accuracy are shown. This signal may be represented as follows:

$$a(t) = K \pm \Delta K \tag{14}$$

where:

$K =$ nominal signal level
$\Delta K =$ maximum deviation

All the signal values are included in the range $[K- \Delta K; K+\Delta K]$. This range will be assumed as the confidence interval of the sample mean distribution, with a confidence coefficient $1-\alpha$ equal to unity and a sample mean value equal to K.

$$[Lo, Uo] \quad [K-\Delta K; K+\Delta K] \tag{15}$$

The maximum variance related to the signal of interest may be evaluated considering the following choice of signal samples:

$S_1 = K$
$S_2 = K+\Delta K$
$S_3 = K-\Delta K$

Therefore:

$$\sigma^2_{MAX} = \frac{1}{N-1}\left[\sum_{i=1}^{N} S_i^2 - N(\bar{S})^2\right] = \Delta K^2 \tag{16}$$

Where:

$N=3$

The minimum variance value is related to the case in which all the samples are equal to K. Therefore the minimum variance value will be equal to zero and the mean variance will be equal to $\Delta K^2/2$. Therefore the confidence interval of the sample variance related to the specification limits is as follows:

$$[Lo', Uo'] = [0; \Delta K^2] \tag{17}$$

and the confidence coefficient $1-\alpha$ is equal to unity.

B. SPECIFICATION DATA NOT AVAILABLE

When the equipment specification does not allow to obtain the mean value, variance and confidence interval performed in the previous paragraph, the measurement procedure can be carried out referring to the confidence interval calculated with minimum noise condition.

This is a worst case situation because the confidence intervals [L1, U1] and [L1' U1'] should be usually narrower ($IM_{SYS} < \emptyset$) than the confidence ontervals [Lo, Uo] and [Lo', Uo'] determined by means of the equipment specification. In this case one may define a statistical interference margin, as follows:

$$IM_{STAT.} = 20\log_{10} \frac{L1}{L2} \quad , \quad \bar{x}_{N+1} \leqslant \bar{x}_N \qquad (18a)$$

$$IM_{STAT.} = 20\log_{10} \frac{L1'}{L2'} \quad , \quad \sigma^2{}_{N+I} \leqslant \sigma_N{}^2 \qquad (18b)$$

$$IM_{STAT.} = 20\log_{10} \frac{U2}{U1} \quad , \quad \bar{x}_{N+I} \geqslant \bar{x}_N \qquad (18c)$$

$$IM_{STAT.} = 20\log_{10} \frac{U2'}{U1'} \quad , \quad \sigma^2_{N+I} \geqslant \sigma_N^2 \qquad (18d)$$

The statistical interference margin is related to the effective interference margin by means of the following relationship:

$$IM_{STAT.} = IM_{EFF} - IM_{SYS} \qquad (19)$$

High probability of an interference exists when $IM_{STAT.} > \emptyset$.

C. TEST EXECUTION TIME

The test procedure summarized in the previous pages requires the execution of a large number of measurements. In case it is necessary to reduce the measurement number, only one mean value (or variance) measurement can be made during the test. The effective interference margin can be defined as follows:

$$IM_{EFF} = 20\log_{10} \frac{Lo}{x_{N+I}} \quad , \quad \bar{x}_{N+I} \leqslant \bar{x}_o \qquad (20a)$$

$$IM_{EFF} = 20\log_{10} \frac{\bar{x}_{N+I}}{Uo} \quad , \quad \bar{x}_{N+1} \geqslant \bar{x}_o \qquad (20b)$$

$$IM_{EFF} = 20\log_{10} \frac{Lo'}{\sigma^2{}_{N+I}} \quad , \quad \sigma^2_{N+I} \leqslant \sigma^2_o \qquad (20c)$$

$$IM_{EFF} = 20\log_{10} \frac{\sigma^2{}_{N+I}}{Uo'} \quad , \quad \sigma^2{}_{N+I} \geqslant \sigma^2_o \qquad (20d)$$

In this case the statistical distribution of the values $\bar{x}_{N+I}$ and $\sigma^2{}_{N+I}$ will be neglected (see Fig. 2).

D. INTERFERENCE MARGIN PROBABILITY

The following error can occur when the statistical evaluation of the interference margin is performed. The mean value μ_x of a sample of N independent observations of a random variable is assumed as an estimator of the true mean value μ_o. Now the sample value μ_x will probably not come out exactly equal to μ_o because of the sampling variability associated with μ_x. In order to establish the probability of this error, it is necessary to specify some deviation of the true parameter μ_o from the assumed parameter μ_x. If the true mean value were in fact

$$\mu_o = \mu_x \pm d \qquad (21)$$

an error would occur with probability β, if the sample value μ_x falls below the upper limit or above the lower limit of the confidence interval (see Fig. 3). By means of analytical analysis |4|, the probability β is related to the samples number N, as follows:

$$N = \left[\frac{S\,(t_{n;\alpha/2} + t_{n;\beta})}{d} \right]^2 \qquad (22)$$

where:

S = standard deviation
n = N-1
$1-\alpha$ = confidence coefficient
d = maximum deviation

The previous considerations are also valid when the sample variance σ_x^2 is assumed as an estimator of the true variance: $\sigma_o^2 = \sigma_x^2 + d_S^2$.
It follows that:

$$N = 1 + \frac{d_S^2}{\sigma_x^2} \left[\frac{\chi^2_{n;\beta}\ \chi^2_{n;\alpha/2}}{\chi^2_{n;\alpha/2} - \chi^2_{n;\beta}} \right] \qquad (23)$$

The error probability β is still related to the samples number N. Another error can occur because the area related to the confidence interval is not equal to unity. The parameter under test can assume values out-of-confidence interval with probability α. Therefore α is the probability of this second type of error.
The interference margin is computed with a uncertainty related to an error probability equal to:

$$\text{Prob. [ERROR]} = \text{Prob. [(TYPE 1 ERROR) U (TYPE 2 ERROR)]} =$$
$$= \alpha + \beta - \alpha\beta \qquad (24)$$

7. STATISTICAL ANALYSIS APPLICATION

Some experimental results shall be described in this section to emphasize the advantages of the statistical analysis technique summarized in the previous pages.
In this case in the aircraft there is the advantage of using the on board main computer which for a selected number of parameters has the possibility of measuring and also displaying the following data:

- mean value
- standard deviation

The test integration program (TIP) was run on main computer and each parameter was sampled with and without emissive equipment activated and with the following input data:

- sample number: 500
- confidence coefficient: 95%
- time repetition of sampling: 30 sec.
- data requested for each sampling: $\bar{x}$ and S.

A performance safety margin typical of the parameters under test was calculated according to the expressions (20).
Some examples of the test results are reported in Fig. 4.5.6. The system under test is in this case the air data computer (ADC) and the emissive equipment is the UHF transmitter. The ADC parameters controlled by test integration program (TIP) are:

True air speed (TAS)
Calibrated air speed (CALAS)

One may be noted that an interferent situation occurs (P.S.M. $>$ 0) when the UHF transmitter is activated. The test has been performed as described in section 6-C in order to reduce the test execution time.
In absence of specification data the P.S.M. has been computed referring to the confidence interval evaluated with minimum noise condition ($\bar{x}_N$).

8. CONCLUSION

The test procedure described in the previous sections may be performed monitoring some parameters of interest one at a time. In many cases a parameter θ of interest is related to the other "sub parameters" $\gamma_1, \gamma_2, \ldots, \gamma_N$ by means of a functional relationship. For example:

$$\theta = g\,(\gamma_1, \gamma_2, \ldots, \gamma_N) \qquad (25)$$

when the parameter θ is interferred ($IM_{EFF} > \emptyset$), the following situations are possible:

- the interference is due to the transfer function $g(\gamma_1, \gamma_2, \ldots, \gamma_N)$ of the equipment.
- the interference is due to one or plus "subparameters" $\gamma_1, \gamma_2, \ldots, \gamma_N$. In this case the equipment not produces further interference effects.
- the interference is due to the "subparameters" and to the equipment transfer function at the same time.

Therefore in order to detect the cause of an interferent situation it is necessary to show the transfer function of the equipment under test and to test, with the emissive equipment activated, some or all the "subparameters" γ_i. In many cases these are not always accessibles for the test or the transfer function $g(\gamma_i)$ is unknown. Therefore it is necessary in this case to determine an analysis technique which allow to discover the actual interference cause. This problem has not been solved at the present and the purpose of this section is to emphasize the need to go back to the effective cause of a malfucntion, so that this will be avoided easily.

REFERENCES

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SAFETY MARGIN DEFINITION AND EVALUATION

L_N = CONFIDENCE INTERVAL LOWER LIMIT

U_N = CONFIDENCE INTERVAL UPPER LIMIT

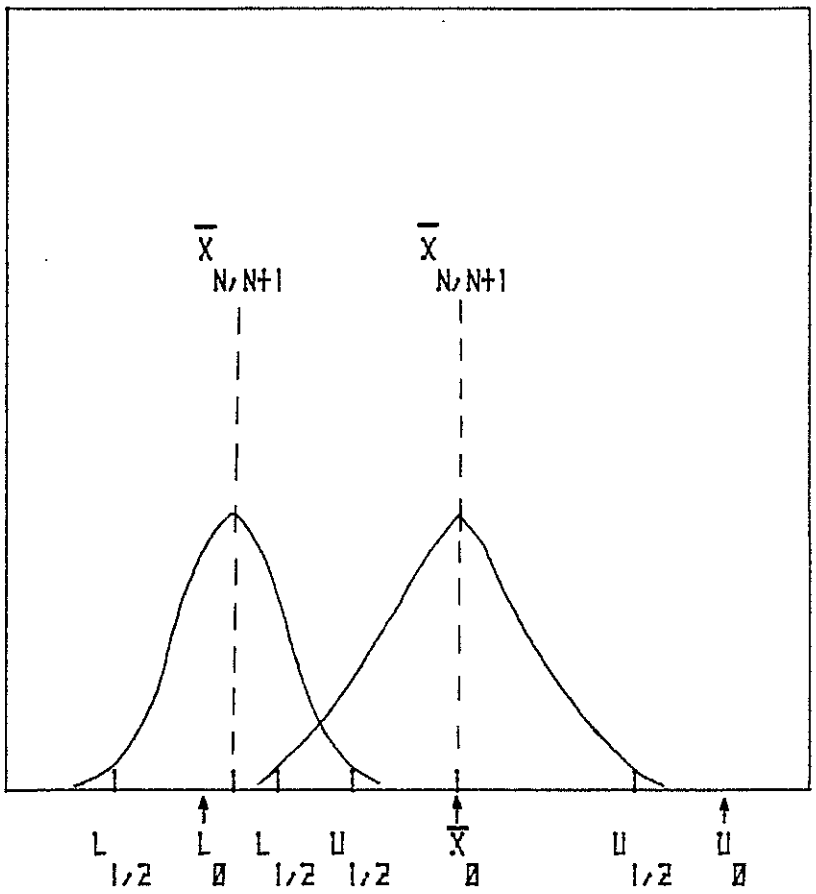


FIG. 1 A

SAFETY MARGIN DEFINITION AND EVALUATION

L'_N = CONFIDENCE INTERVAL LOWER LIMIT

U'_N = CONFIDENCE INTERVAL UPPER LIMIT

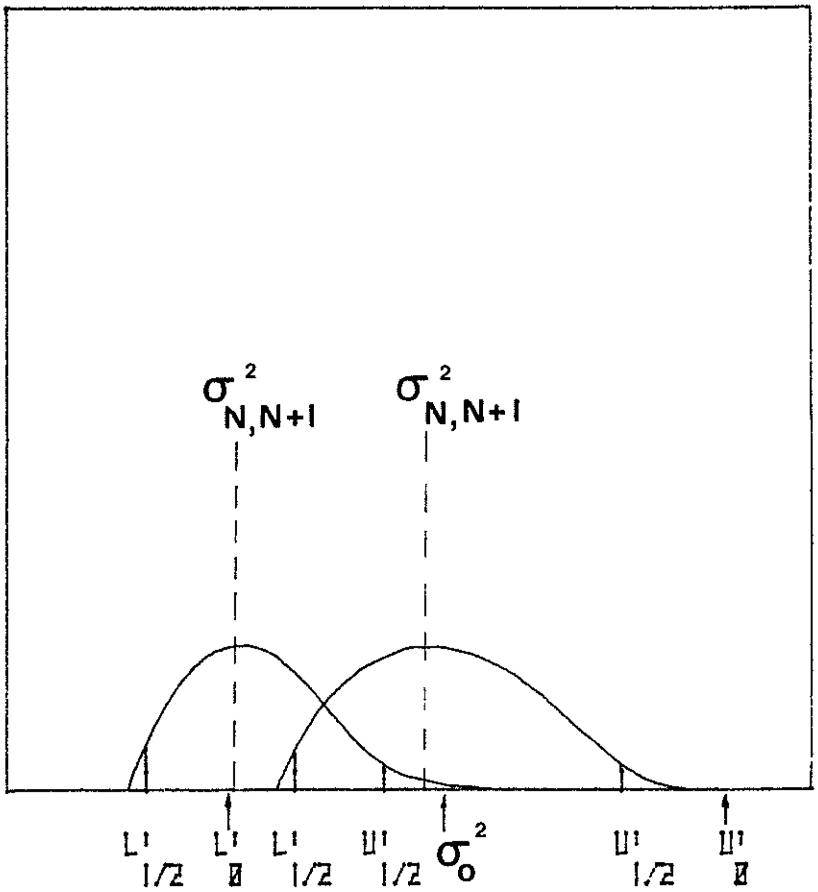


FIG. 1 B

SAFETY MARGIN DEFINITION AND EVALUATION

EXECUTION TIME REDUCTION

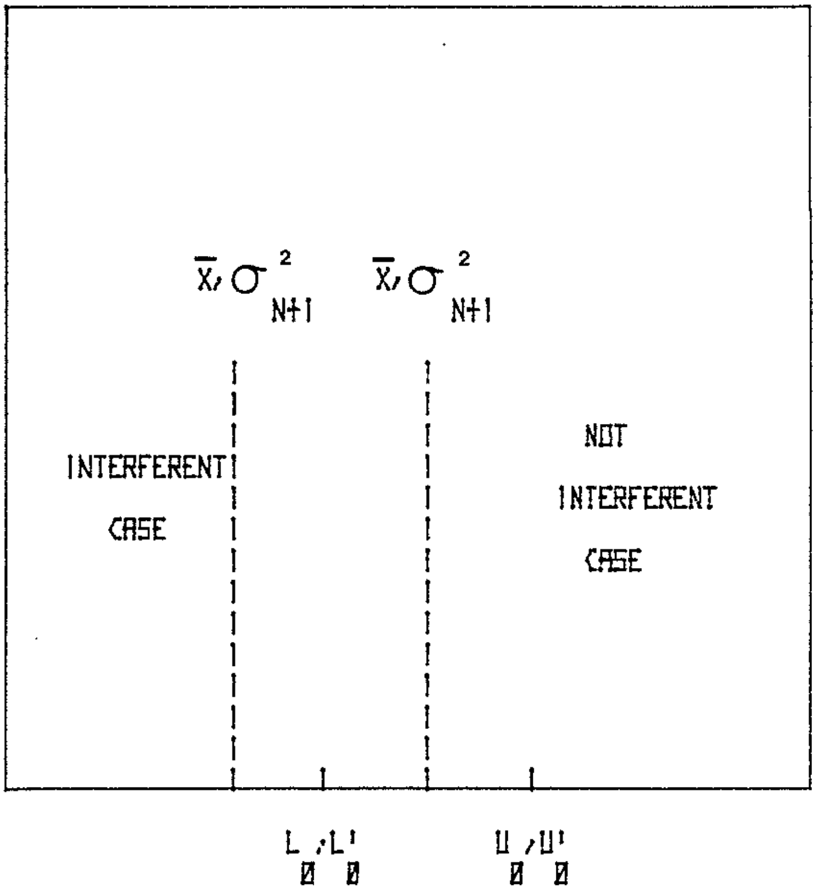


FIG. 2

SAFETY MARGIN ERROR PROBABILITY

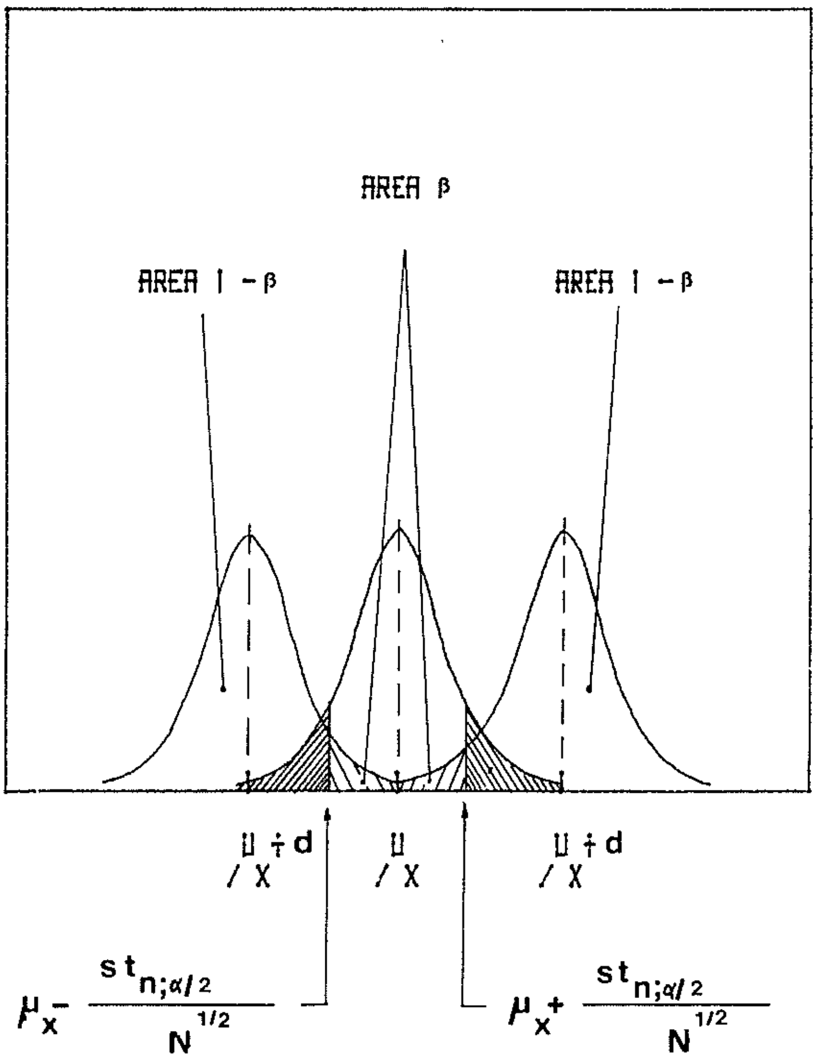
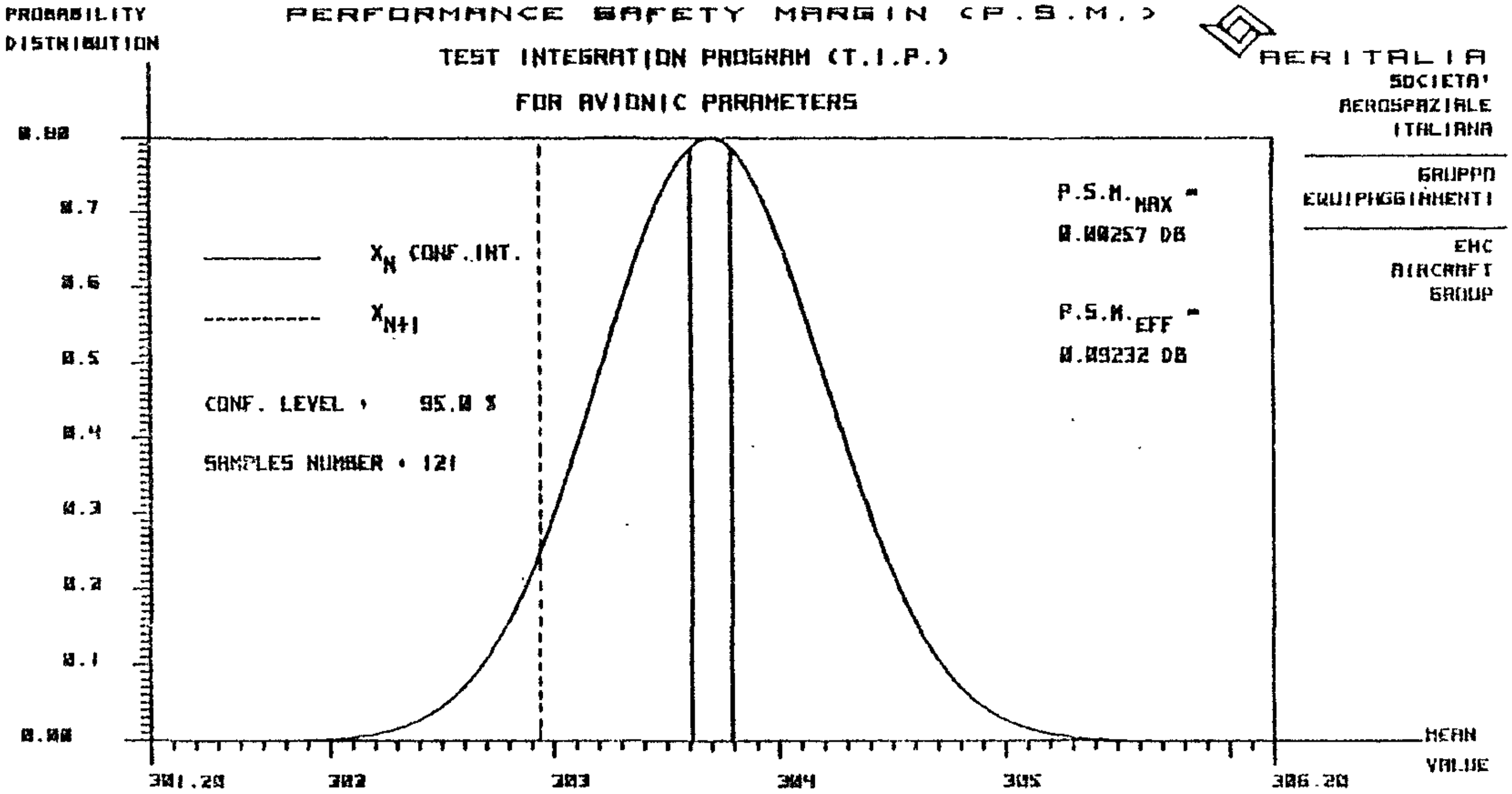


FIG. 3



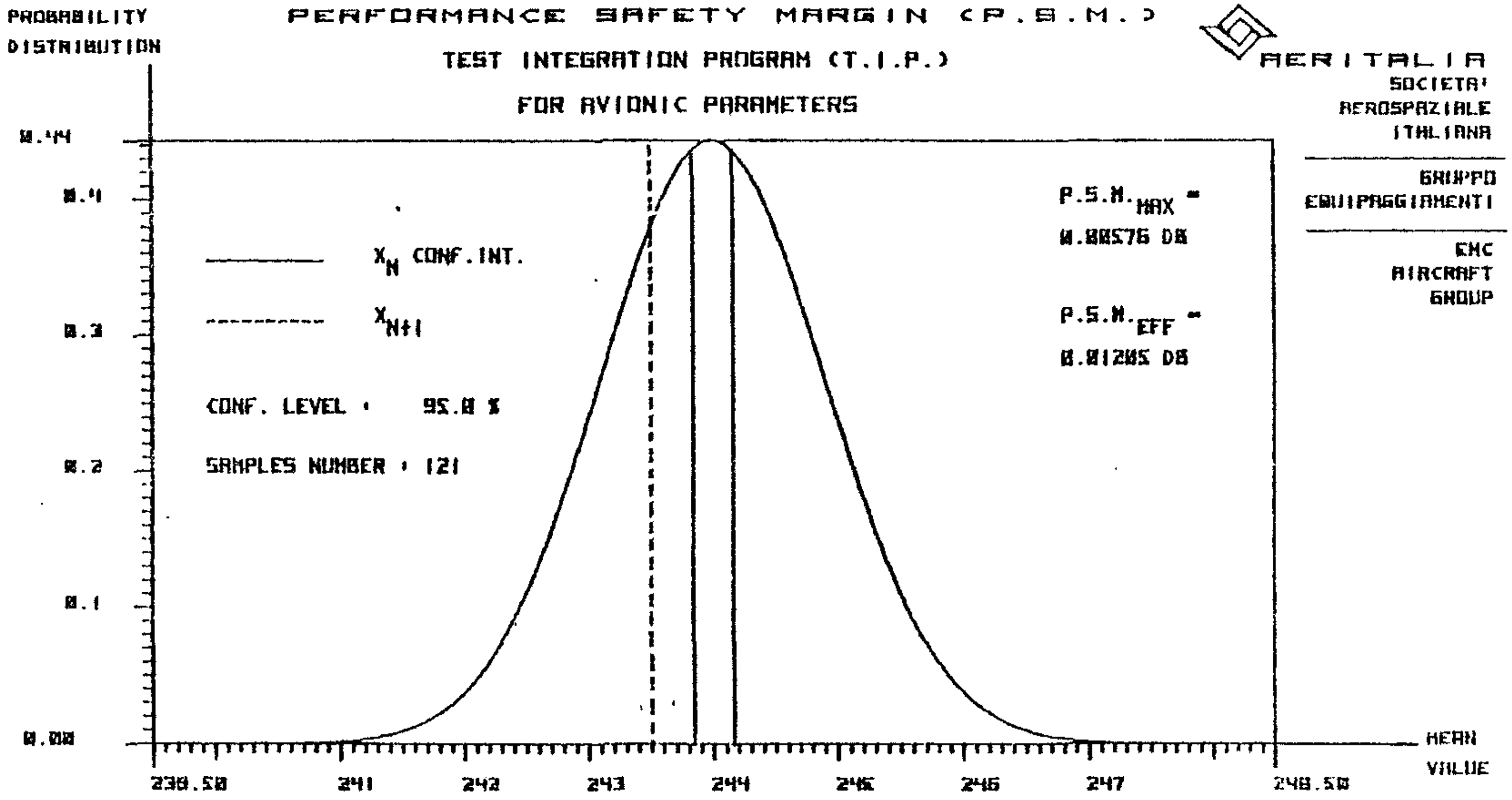
SYSTEM UNDER TEST : AIR DATA

PARAMETER UNDER TEST : CALAS

AIRCRAFT TEST COND. : A/C SETTING AS RUN A

EMISSIVE EQUIPMENT : UHF LOW. 225 MHZ

FIG. 4



SYSTEM UNDER TEST : AIR DATA

PARAMETER UNDER TEST : CALAS

AIRCRAFT TEST COND. : A/C SETTING AS RUN A

EMISSIVE EQUIPMENT : UHF LOW. 385 MHZ

FIG. 5

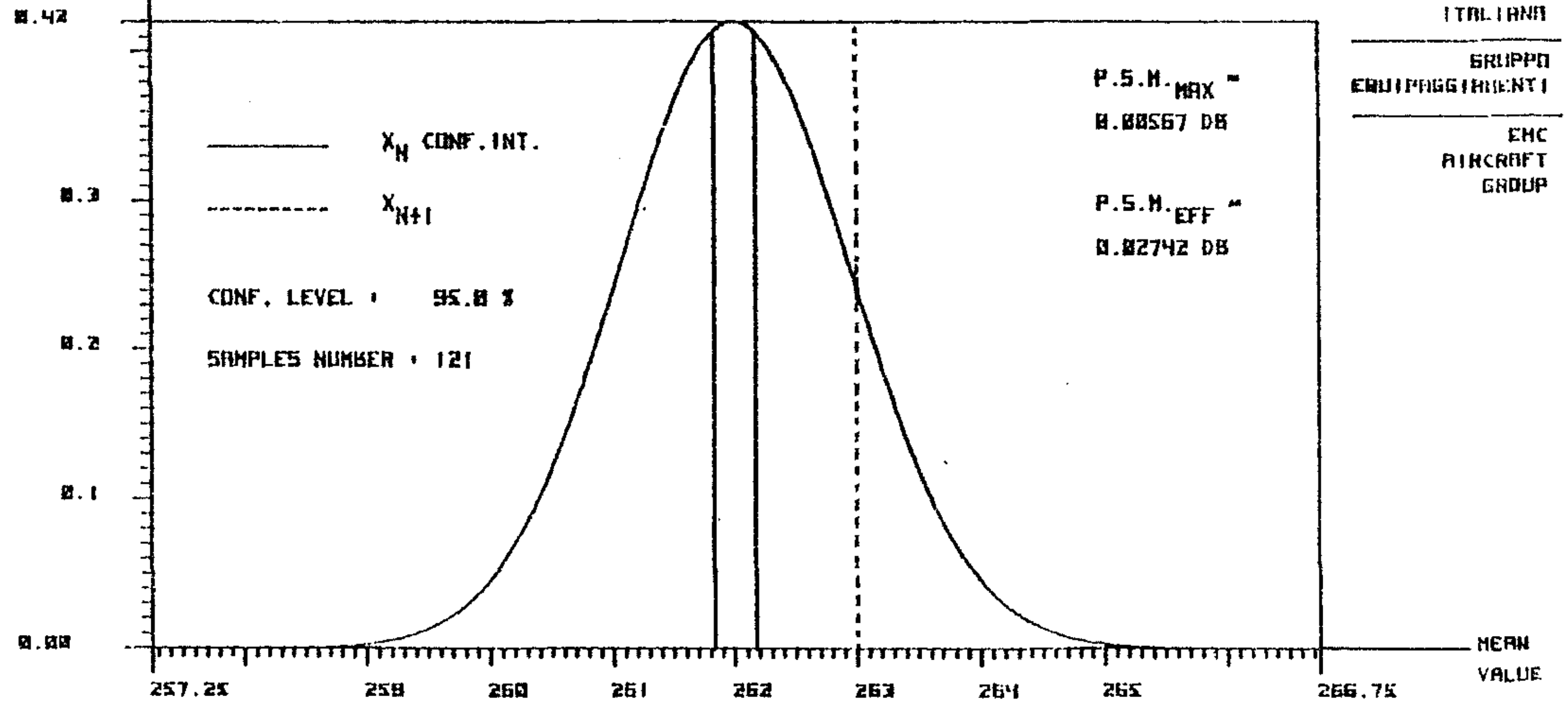
PROBABILITY DISTRIBUTION

PERFORMANCE SAFETY MARGIN (P.S.M.)

TEST INTEGRATION PROGRAM (T.I.P.)

FOR AVIONIC PARAMETERS

AERITALIA
SOCIETA' AEROSPAZIALE ITALIANA
GRUPPO EQUIPAGGIAMENTI
EMC AIRCRAFT GROUP



SYSTEM UNDER TEST : AIR DATA

PARAMETER UNDER TEST : TAS

AIRCRAFT TEST COND. : A/C SETTING AS RUN A

EMISSIVE EQUIPMENT : UHF LOW. 390 MHZ

FIG. 6