TWENTY FIRST EUROPEAN ROTORCRAFT FORUM

Paper No IV.4

MATHEMATICAL MODEL OF HELICOPTER DYNAMICS IN EMERGENCY SITUATIONS

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August 30 - September 1, 1995

SAINT - PETERSBURG, RUSSIA

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Considered is mathematical modelling of a single rotor type helicopter emergency situations on the basis of nonlinear differential equations of spatial motion taking into account rotor and power plant dynamics. Aerodynamic forces and moments, acting on load bearing helicopter elements, are approximated by regression polynomials in a wide range of determining kinematic parameter variation.

Full mathematical model of helicopter motion in emergency situations is realized on a hard disk of a personal computer and is appropriate for utilization in operation conditions. Results of computations of balancing, typical transition regimes and maneuvers agree well with the corresponding flight tests data.

Introduction

At present helicopter flight dynamics mathematical modelling is a widespread method to solve a great many essential problems of project design, updating, and helicopter flight operation perfecting. However, mathematical modelling methods and tools are of particular importance for the analysis of the emergency situations, caused by failures, damages in combats, piloting mistakes, or by unfavourable atmosphere factors.

Mathematical modelling methods and tools allow having but limited information, given by emergency onboard recordes, to calculate essential non-registered parameters, to give a sound estimation of the emergency reasons and course, to approve some flight safety recommendation. Obviously, there is no possibility to use a linear system of disturbed motion equations when facing this type of problems, as well as to use assumptions of small and stationary change in the main kinematic parameters and helicopter dynamic characteristics. Thus, helicopter emergency simulation implies considerable complication and labouriousness of the calculating algorithm. Generally, a high-perfomance computer is necessary here.

On the other hand, mathematical modelling made near the place where a helicopter crashed would speed up the air accident investigation. In this case modelling program and results conform to the flight accident circumstances, an advanced version, technical conditions of the broken plants and units, and the analysis of the air accident data bank.

PC installed on-board of a flight laboratory, specially equipped for aircraft accident investigation in mobile conditions enable us to solve the problem. This is exatly the technique our Institute uses.

PC mathematical modelling of a spatial controllable motion in various emergency situations requires solution of a range of methodical problems, aimed at making a compromise between advisable simplicity and required accuracy of the modelling algorithm. The submitted paper dwells upon the above-mentioned items.

Mathematical model of helicopter flight dynamics

The canonical system of nonlinear differential equations, describing helicopter spatial motion is considered in the right-hand body axes of coordinates $OXYZ$ with the beginning in the mass center (figure 1) :

$$
\begin{aligned}
&m(\dot{V}_x - V_y\omega_z + V_z\omega_y) = T\sin\varepsilon - H\cos\varepsilon - X_P - G\sin\vartheta\cos\gamma,\\
&m(\dot{V}_y + V_x\omega_z - V_z\omega_x) = T\cos\varepsilon + H\sin\varepsilon + Y_P - G\cos\vartheta\cos\gamma,\\
&m(\dot{V}_z + V_y\omega_x - V_x\omega_y) = S - T_{TR} + Z_P + G\cos\vartheta\sin\gamma,\\
&I_x\dot{\omega}_x - I_{xy}(\dot{\omega}_y - \omega_x\omega_z) - (I_y - I_z)\omega_y\omega_z + K_R U_f l_f \omega\omega_z = M_{x_R} - T_{TR}y_{TR} + M_{x_P},\\
&I_y\dot{\omega}_y - I_{xy}(\dot{\omega}_x + \omega_y\omega_z) + (I_x - I_z)\omega_x\omega_z = M_e - T_{TR}(x_{TR} + x_g) + M_{y_P},\\
&I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y - I_{xy}({\omega_x}^2 - {\omega_y}^2) - K_R U_f l_f \omega\omega_x = M_{z_R} + M_{TR} + M_{z_P},\\
&I_\omega\dot{\omega} = M_e - M_t - K_{TR}M_{TR}
\end{aligned}
\qquad (1)
$$

where :

m, G, I_j – helicopter mass, the gravity force and moments of inertia accordingly;
I_ω, M_e – polar inertia moment of rotor and engines rotation moment;
U_f, l_f – static mass moment of rotor blades and the distance between the rotor rotation axis and the flapping hinge;
ω, K_R – the rotor rotation rate, and the number of the rotor blades;
$\omega_{x,y,z}$ – pitch, roll, and yaw rates;
ϑ, γ, ψ – pitch, roll, and yaw angles;
$V_{x,y,z}$ – translational flight speed projections on the body axes of coordinates;
T, H, S – thrust, longitudinal and lateral rotor forces;
M_{x_R}, M_{z_R}, M_t – the rotor lateral, longitudinal and torque air moments;
T_{TR}, M_{TR}, K_{TR} – the tail rotor thrust, torque air moment, and multiplication coefficient;
$X_P, Y_P, Z_P, M_{(x,y,z)_P}$ – the airframe aerodynamic forces and moments.

Figure 1 cleares up the geometric parameters x_j, y_j, ε, and the positive directions of the forces and moments, acting on a helicopter.

Well-known in flight mechanics kinematic correlations of pitch, roll, and yaw angles and rates, as well as the equations of helicopter mass center trajectory in the normal earth coordinates are to be added to the equation system (1).

The main rotor moments differential equations, approximately taking into account its dynamic properties are the following:

$$\begin{aligned} \tau_x \dot{M}_{x_R} + M_{x_R} &= S y_g + M_{x_f} + \tfrac{1}{2} K_R U_f l_f \omega^2 (D_1 \eta + D_2 \chi), \\ \tau_z \dot{M}_{z_R} + M_{z_R} &= -T x_g + H y_g + M_{z_f} + \tfrac{1}{2} K_R U_f l_f \omega^2 (D_1 \chi - D_2 \eta) \end{aligned} \tag{2}$$

where :

M_{x_f}, M_{z_f} – the main rotor hub longitudinal and lateral moments, caused by the flapping hinges spacing;

η, χ – the swashplate deflection angles in the lateral and longitudinal planes;

D_1, D_2 – gearing ratio of the rotor and the swashplate resultant deflection angles;

τ_x, τ_z – dynamics time constants of the rotor total lateral and longitudinal moments.

In a general case of helicopter controlled slip flight aerodynamic forces in the rotor plane of rotation and the rotor hub moments are defined in the following way:

$$\begin{aligned} &H = H_0 \cos\beta_R + S_0 \sin\beta_R + T(D_1\chi - D_2\eta), \\ &S = S_0 \cos\beta_R - H_0 \sin\beta_R + T(D_1\eta + D_2\chi), \\ &M_{x_f} = M_{x_0} \cos\beta_R - M_{z_0} \sin\beta_R, \quad M_{z_f} = M_{z_0} \cos\beta_R + M_{x_0} \sin\beta_R \end{aligned} \tag{3}$$

The rotor angles of attack α_R and slip β_R accordingly:

$$\begin{aligned} &\alpha_R = -\arcsin \tfrac{V_{y_R}}{V_R} + (D_1\chi - D_2\eta)\cos\beta_R + (D_1\eta - D_2\chi)\sin\beta_R, \\ &\beta_R = \arctan(V_{z_a} + \omega_y x_g + \omega_x y_g)/(V_{x_a} - \omega_z y_g) \end{aligned} \tag{4}$$

The rotor flow about speed components in the body axes of coordinates $O_H X_H Y_H Z_H$ with the beginning in the hub center (figure 1) are :

$$\begin{aligned} V_{x_R} &= \sqrt{(V_{x_a}\cos\varepsilon - \omega_z y_g)^2 + (V_{z_a} + \omega_y x_g + \omega_x y_g)^2}, \\ V_{y_R} &= V_{y_a}\cos\varepsilon + V_{x_a}\sin\varepsilon - \omega_z x_g, \\ V_R &= \sqrt{{V_{x_R}}^2 + {V_{y_R}}^2} \end{aligned} \tag{5}$$

where $V_{x_a} = V_x - W_x$, $V_{y_a} = V_y - W_y$, $V_{z_a} = V_z - W_z$ are the helicopter airspeed components, W_j – wind speed components.

In a general case of a helicopter controlled flight with an operating autopilot the collective pitch of rotor φ_R, the pitch of the tail rotor φ_{TR}, the swashplate longitudinal deflection χ and the swashplate lateral deflection η are :

$$\begin{aligned} \varphi_R &= \varphi_{R_0} + \varphi_{R_1} + \varphi_{R_2}, \quad \varphi_{TR} = \varphi_{TR_0} + \varphi_{TR_1} + \varphi_{TR_2}, \\ \chi &= \chi_0 + \chi_1 + \chi_2, \quad \eta = \eta_0 + \eta_1 + \eta_2 \end{aligned} \tag{6}$$

The first items of system (6) are the deflections of a considered control body in helicopter steady trim condition; the second items are the pilot control actions; the trird ones are the work of the autopilot, included in the control system on the base of the differential scheme.

The following laws form the control signals in the autopilot channels :

$$\begin{aligned} \tau_\chi \dot{\chi}_2 + \chi_2 &= -i_\vartheta(\vartheta - \vartheta_i) - i_{\omega_z}\omega_z + i_V(V - V_0) + K_\chi \chi_1, \\ \tau_\eta \dot{\eta}_2 + \eta_2 &= -i_\gamma(\gamma - \gamma_0) - i_{\omega_x}\omega_x + K_\eta \eta_1, \\ \tau_{\varphi_R} \dot{\varphi}_{R_2} + \varphi_{R_2} &= -i_h(h - h_0) - i_{V_y}\dot{h}, \\ \tau_{\varphi_{TR}} \dot{\varphi}_{TR_2} + \varphi_{TR_2} &= i_\psi(\psi - \psi_0) + i_{\omega_y}\omega_y \end{aligned} \tag{7}$$

where: i_j, τ_j – the autopilot gearing ratios and the channels time constants;
$K_{\chi,\eta}$ – compensatory sensor coefficients;
h, V – helicopter hold height and hold speed.

Autopilot signals maximum control bodies deflections amount up to 20% of the control bodies design range.

The twin rotor powerplant moment of rotation, determining the rotor and tail rotor rotation frequency with coefficient of power utilization ζ_N is :

$$M_e = \frac{2N_e\zeta_N}{\omega} \tag{8}$$

In order to calculate the required power N_e we use the mathematical model of a powerplant with the following main parameters :

$$\omega = \omega_0 + \Delta\omega, \quad n = n_0 + \Delta n, \quad Q = Q_0 + \Delta Q \tag{9}$$

The first items of system (9) are the nominal rotor rate ω; nominal turbocompressor rotor frequency of rotation n; and the nominal fuel consumption of the turboshaft engine Q. The quantities are defined by setting the free turbine governor at a steady flight regime of a helicopter. These static engine characteristics depend mainly on the position of the collective pitch control lever. The following polynomials give a simple and accurate description of them :

$$\begin{aligned} \lambda &= \lambda_0 + \lambda_1\varphi_R + \lambda_2\varphi_R^2, \\ n_0 &= m_0 + m_1\lambda, \\ Q_0 &= q_0 + q_1 n_0 + q_2 {n_0}^2 + q_3 {n_0}^3 + q_4 {n_0}^4, \\ N_{e_0} &= e_0 + e_1 n_0 + e_2 {n_0}^2 + e_3 {n_0}^3 \end{aligned} \tag{10}$$

where λ is a position of the engine throttle lever.

To avoid temperature and air pressure influence upon engine parameters n_0, Q_0, N_{e_0}, parameters $\bar{n}_0, \bar{Q}_0, \bar{N}_{e_0}$ are used relative to the conditions of standard atmosphere.

The amounts of the approximation polynomials coefficients (10) are calculated by means of the standard procedures using known operational characteristics of specific engines and the correction grip position.

The second items of system (9) show the change in the considered parameters at transient to a new steady powerplant operational mode. These engine dynamic characteristics are determined by the rotor governor, and described by the following system of two differential equations :

$$\begin{aligned} \tau_Q\dot{\bar{Q}} + \bar{Q} &= \bar{Q}_0 - K_c\Delta\omega, \\ \tau_n\dot{\bar{n}} + \bar{n} &= \bar{n}_0 + K_\varphi\Delta\bar{Q} \end{aligned} \tag{11}$$

The governor static coefficient K_c can be assumed as a constant for a considered engine. K_φ is a coefficient, defining the governor re-setting at a change of the rotor collective pitch, it is the turbocompressor rotor rotation speed polynomial function :

$$K_\varphi = a_0 + a_1\bar{n} + a_2\bar{n}^2 + a_3\bar{n}^3 \tag{12}$$

Time constants τ_Q, τ_n are to be chosen considering the best approximation of the modelled transient process to the experimental data.

Engine responsiveness and throttling are described by the conditions :

$$\dot{Q}_{min} \leq \dot{Q} \leq \dot{Q}_{max}; \qquad \dot{n}_{min} \leq \dot{n} \leq \dot{n}_{max} \tag{13}$$

Besides, the work of the engine ultimate envelope protection device is described by the inequalities :

$$n \leq n_{max}; \qquad Q_{min} \leq Q \leq Q_{max} \tag{14}$$

Thus, the considered powerplant mathematical model is nonlinear and covers all modes. This is principally when emergency situations are being investigated, caused by powerplant failures, or rotor intensive overspeeding and overload take place.

Rotor and tail rotor aerodynamic characteristics

Aerodynamic forces and moments for a helicopter of a definite type are calculated using the results obtained in the Mil Design Bureau. On the base of nonlinear experimental blade profile polars, i.e. accounting for the flow stall and compressibility, a file of total rotor aerodynamic characteristics for a helicopter of Mi-8 type is obtained in a wide range of kinematic parameters $V = 0...250km/h$, $\varphi_R = 1...14°$, $\alpha_R = -30... + 30°$. The file is obtained by means of numerical integration of blades flap motion equations, calculation of forces and moments, acting on every blade, and composition of the forces and moments, acting on the rotor.

For proper use of the mentioned file as well as for taking into account the outside air real density in comparison with the calculated one and considering the rotor specific modes, the aerodynamic forces and moments, acting on the rotor should be presented in the following way :

$$\begin{aligned}
&T = c\bar{T}_V\bar{T}_g(T' + T^{\omega_x}\omega_{x_R} + T^{\omega_z}\omega_{z_R}), \qquad H_0 = c(H' + H^{\omega_z}\omega_{z_R} + H^{\omega_x}\omega_{x_R}),\\
&S_0 = c(S' + S^{\omega_x}\omega_{x_R} + S^{\omega_z}\omega_{z_R}), \qquad M_{x_0} = c(M_x{'} - \mu\omega\omega_{x_R}),\\
&M_{z_0} = c(M_z' - \mu\omega\omega_{z_R}),\\
&M_t = c(M_t' + \triangle M_t)
\end{aligned} \tag{15}$$

where : $c = ((\omega - \omega_y)/\omega_0)\bar{\rho}$, $\qquad \mu = (K_R U_f l_f(\bar{k}\gamma_R + 8))/2\gamma_R(1 + \bar{k}^2)$,
$\bar{\rho} = \rho_h/\rho_0$ – relative air density,
γ_R – blade mass characteristic,
$\bar{k}$ – blade flapping compensator coefficient.

The quantities marked $'$ in (15) are obtained by the above-mentioned method for a steady straight-and-level flight of Mi-8 helicopter without slip in the standard boundless atmosphere at a height of $h_0 = 500m$. The direct rotary derivatives $H^{\omega_z}, S^{\omega_x}$ of rotor damping were obtained in the analogous way whereas cross rotation derivatives $H^{\omega_x}, S^{\omega_z}$ are rather small and taken constant without any considerable mistake.

The rotary derivative T^{ω_z} describes unsteady dynamic stall delay from the blades at a helicopter pitching-up at a considerable angular rate. The rotary derivative T^{ω_x} is rather small and accounts for the angle of attack change in the blades profile at helicopter rotation about the longitudinal axis.

At the same time all rotor rotary derivatives account for the angular rate influence upon the aerodynamic characteristics :

$$\begin{aligned} \omega_{x_R} &= \omega_x \cos\beta_R + \omega_z \sin\beta_R, \\ \omega_{z_R} &= \omega_z \cos\beta_R - \omega_x \sin\beta_R \end{aligned} \tag{16}$$

$\bar{T}_V$ describes abnormal change in the rotor thrust at vortex mode, and in the case of constant required power, can be approximated on the base of the calculated and experimental data in the following way :

$$\begin{aligned} \bar{T}_V &\approx 1 + 0.16 \sin 1.4w \le 1, \\ w &= 0.3(\dot{h} + \varphi_R + 4) \le 0 \end{aligned} \tag{17}$$

$\dot{h}$ is given in m/s, whereas φ_R – in degrees.

Following the nature of the vortex ring, the $\bar{T}_V$ function domain is defined at $1 < \varphi_R < 10°$, $-4 < \dot{h} < -20m/c$.

The following formula (considering both calculated and experimental data) approximates the half-empiric function of the shield influence upon the rotor thrust $\bar{T}_g$

$$\bar{T}_g \approx \left\{1 + 0.2e^{-0.18[h+10(1-\bar{\rho}^2)]+7V_R}\right\}\sqrt{\cos\vartheta\cos\gamma} \tag{18}$$

where $\bar{V}_R = V_R/\omega R$ is the relative speed of the rotor flow about, R is the radius of the rotor.

Following the nature of the shield influence, the $\bar{T}_g \ge 1$ function domain is defined at $h < R$ and $V_R < 70km/h$. The geometric height of helicopter flight h is measured from the landing gear wheel to the underlying surface.

$\triangle M_t$ implies the difference between the torque air moment values, obtained in the calculations, mentioned above, at a constant circle Mach number 0.65 and its real value M_0. At the same time $\triangle M_t$ depends on three determining parameters $\bar{V}_R$, T, and $\triangle M_0 = M_0 - 0.65$ in a rather complicated manner.

In analogous way the tail rotor aerodynamic characteristics T_{TR}, M_{TR} are defined in the following way :

$$T_{TR} = (T'_{TR} + \triangle T_{TR})\Big(\omega/\omega_0\Big) \tag{19}$$

where : $\triangle T_{TR}(\varphi_{TR}, \alpha_{TR}, V_{TR}) \le 0$ is a half-empiric function of the vortex ring mode influence upon the tail rotor thrust.

Usually the rotor and tail rotor aerodynamic characteristics T', T^{ω_z}, H', H^{ω_z}, S', S^{ω_x}, M'_x, M'_z, M'_t, $\triangle M_t$, T_{TR}, $\triangle T_{TR}$, M_{TR} are presented as tables with a given step of the determining parameters. At the numerical integration of helicopter motion the equations interpolation polynomials are used to address the tables. However, but stationary large-powered computers are able to satisfy the memory and fast-response requirements this kind of calculations features.

In order to enable personal computers to calculate helicopter dynamics problems in emergency situations the author of the presented paper approximates the rotor and tail rotor aerodynamic characteristics with the regression polynomials. A regression experiment was carried out to develop the desired regression model. The experiment was based on purposeful variation and manipulation of the main kinematic patameters φ_R,α_R, V_R (φ_{TR},α_{TR}, V_{TR} accordingly), and showed the nature of their influence upon the approximated aerodynamic characteristics. As a result there was found an optimum structure and rational utilization of the regression model for φ_j, $A_j = V_j \cos\alpha_j$, $B_j = V_j \sin\alpha_j$ $(j = R, TR)$ composition arguments of the main aerodynamic characteristics T_j, H, S, M_j and for φ_j, α_j, V_j initial arguments of the auxiliary characteristics $\triangle M_t$ and $\triangle T_{TR}$.

The reliability of the regression model was evaluated by means of the Fisher criterion with calculation of the residual dispersion. In the case both the regression model and the phase space region are selected properly, i.e. a real combination of possible emergency values of the determining parameters φ_j, α_j, V_j is found, the approximation accuracy (figure 2) is quite satisfactory, taking into account the aim of the presented research. At the same time the structure of the regression polynomials (3-d order maximum) is convenient for PC modelling.

Airframe aerodynamic characteristics

There is an agreed-up way to present aerodynamic forces and moments, acting on a helicopter airframe :

$$\begin{array}{ll} X_P = \frac{1}{2}\rho C_{x_P}\pi R^2 V_a^2, & Y_P = \frac{1}{2}\rho C_{y_p}\pi R^2 V_a^2 - \triangle Y_P, \\ Z_P = \frac{1}{2}\rho C_{z_P}\pi R^2 V_a^2, & M_{(x,y,z)_P} = \frac{1}{2}\rho m_{(x,y,z)_P}\pi R^3 V_a^2 \end{array} \tag{20}$$

The quantity $\triangle Y_p \approx 0.018T$ approximately provides for the tail rotor loss of thrust by vertical air blowing at helicopter hover mode.

The coefficients of aerodynamic forces C_{jp} and moments m_{jp} are usually obtained in the wind tunnel tests as results of the airframe model air blowing in the operating range of angles of attack α_p and slip angles β_p, which is $\pm 20°$ as a rule. However, in emergency situations caused, for example, by the directional control system failure, or by a considerable loss of the tail rotor aerodynamic efficiency, a helicopter may well turn into the air flow at a random angle. There are emergency situations when the airframe angle of attack significantly exceeds the limiting experimental value.

In order to determine the airframe aerodynamic characteristics in the $\pm\frac{\pi}{4}$ angle of attack range and in the full circle range of the slip angles we have developed a half-empiric method, based on experimental air blowings of elementary bodies as well as on the circulation and flow separation theory. As a result, we have the following expressions of the aerodynamic forces and moments :

$$\begin{aligned} C_{x_p} &= C_{x_0}\cos^2(\alpha_p + \alpha_0)\,|\cos\beta_p|\cos\beta_p, \\ C_{y_p} &= C_{r_y}[C_{y_p}^{\alpha} + (1 - C_{y_p}^{\alpha})\,|\sin\alpha_p|]\sin\alpha_p\cos^2\beta_p, \\ C_{z_p} &= -C_{r_z}[C_{z_p}^{\beta} + (1 - C_{z_p}^{\beta})\,|\sin\beta_p|]\sin\beta_p\cos^2\alpha_p, \\ m_{x_p} &= -\bar{l}_1 C_{z_p}, \\ m_{y_p} &= (\bar{K}_z - \bar{K}_x)\bar{W}\cos^2\alpha_p\sin 2\beta_p + \bar{l}_2 C_{z_p}, \\ m_{z_p} &= [m_{z_f} + \triangle m_{z_p} + \tfrac{F_s}{\pi R^3}(x_g + x_s)K_s C_{y_s}]\cos\beta_p \end{aligned} \tag{21}$$

The known (for a considered helicopter) dependences of the fuselage longitudinal moment coefficient m_{z_f} and the tailplane lift C_{y_s} (considering the tailplane braking coefficient K_s) on the corresponding angles of attack is approximated by the polynomials of 3-4 order with extrapolation in the domain of $|\alpha_j| \approx 40°$ analogously with the experimental data for similar aerodynamic configurations.

Besides, here :

C_{x0}, C_{rj} are the minimum drag coefficient at the longitudinal flow about at the angle of attack α_0 and the airframe drag coefficient at the lateral flow about along O_y and O_z axes;

$C^{\alpha}_{y_p}, C^{\beta}_{z_p}$ are the partial derivatives at zero angle of attack and slip angle accordingly;

$\bar{K}_j, \bar{W}$ are air virtual mass coefficients along the OX and OZ axes, and relative volume of the air, displaced by the airframe;

$\bar{l}_j$ is relative aerodynamic forces arms referenced to the rotor radius;

F_s, X_s are the tailplane area and the distance between its center of pressure and the helicopter center of mass.

C_{rj} coefficients are determined by the method of compartments with interpolation of the fuselage aerodynamic characteristics from the corresponding ones of elementary configuration bodies, using the data obtained in wind tunnel air blowings of these bodies.

The airframe centers of pressure (sailing centers) are placed, as a rule, below and back of the helicopter center of mass, which determines signs in the equations of moments.

Δm_{z_p} coefficient includes so-called "spoon" in the characteristic of the longitudinal trim at a low flight speed. The spoon is attributed to the change of the vortex sheet influence upon the blades flapping and the tailplane flow about. $\Delta m_{z_p}(V_a)$ function is determined from the trim curves of the helicopter of a considered type, and is approximated with a polynomial of 4-5 order at a limited range of air speed $0 < V_a < 100 km/h$.

The calculation results given by the advanced technique agree well with the corresponding experimental data of the airframe model circle air blowing (figure 3). They faithfully show the laws of airframe aerodynamic characteristics at supercritical angles of slip and attack, which are usually beyond consideration of helicopter flight dynamics.

Evaluation of the mathematical model

The most evident and efficient way to set up correspondence between a mathematical model and a real subject is comparison of the main motion parameters obtained in modelling with the ones obtained in a real flight.

Figure 4 shows the calculated trim characteristics (dash lines) of Mi-8 helicopter for the flight speed compared with flight tests data, given in experimental values spread. One can easily see that covergence of the compared parameters is quite satisfactory.

Figure 5 gives two characteristic examples of transient processes — time variations in the main motion parameters of Mi-8 helicopter, written during flight and represented at simulation with maximum possible accuracy of controls deflection a pilot is able to provide. Here the covergence of the corresponding calculated (dash lines) and experimental (solid lines) data comes up to the existing in flight dynamics notions.

After repeated analogous comparison one can draw a conclusion that the developed mathematical model stands the test for both statics and dynamics.

The mathematical model has been being updated for more than ten years while the theory of helicopter flight was being developed, and new experimental data were being obtained. The mathematical model has been efficiently utilized in both civil and military aviation of the former Soviet Union and several foreign airline companies.

As the matter of fact, the optical-mechanical emergency onboard recorder, built into the majority of the helicopters of this type, keep only six analog parameters of flight, and more than that, the recording is not always reliable. Obviously, this is not sufficient for ascertaining the reason and the course of the accident.

Mathematical modelling of the helicopter terminal flight portion, having objective information of the onboard recorder and the air accident conditions, i.e. in essence, the reverse problem of flight dynamics with deficient information, is in itself, very complicated scientific problem which is beyond the presented paper consideration.

Conclusions

1. The developed mathematical model of single rotor helicopter spatial controlled flight includes :
   - interinfluence of all degrees of freedom;
   - nonlinearity of the aerodynamic forces and moments, acting on a helicopter in a wide range of the determining kinematic parameters, including supercritical stall modes;
   - aerodynamic effects caused by shield and vortex ring;
   - the main rotor and servomotor dynamic properties;
   - transient processes of a change in operational mode and turboshaft engine control.
2. Helicopter static and dynamic characteristics obtained in calculations by means of the developed mathematical model agree well with the corresponding experimental characteristics.
3. Due to effective approximation of the aerodynamic characteristics with regression polynomials, the mathematical model of helicopter flight dynamics in emergency situations is realized on PC hard disks, that allows its prompt utilization in the case of an air accident investigation.

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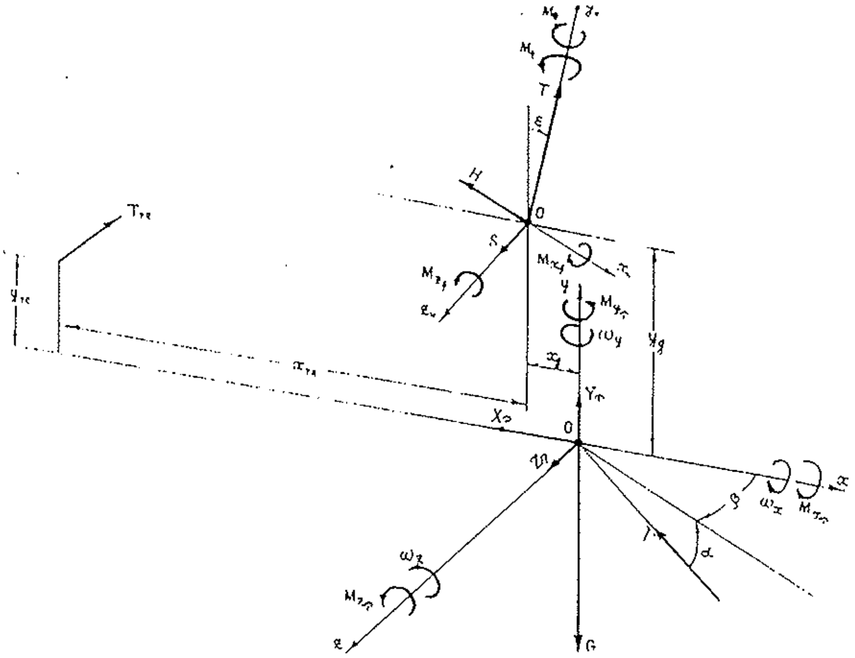

Figure 1. Calculation scheme

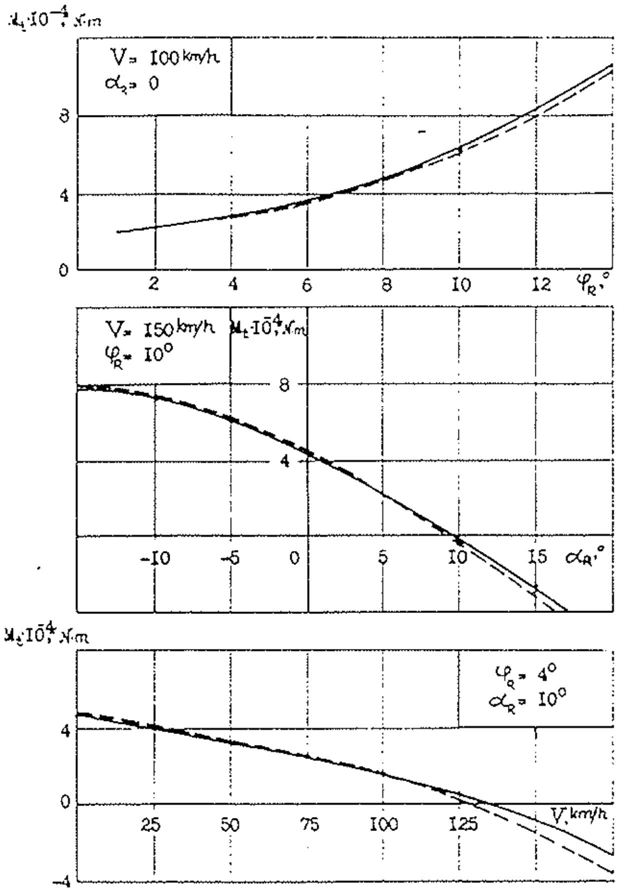

Figure 2. Polynomial approximation (dash lines) of the rotor torque air moment dependency on the kinematic parameters

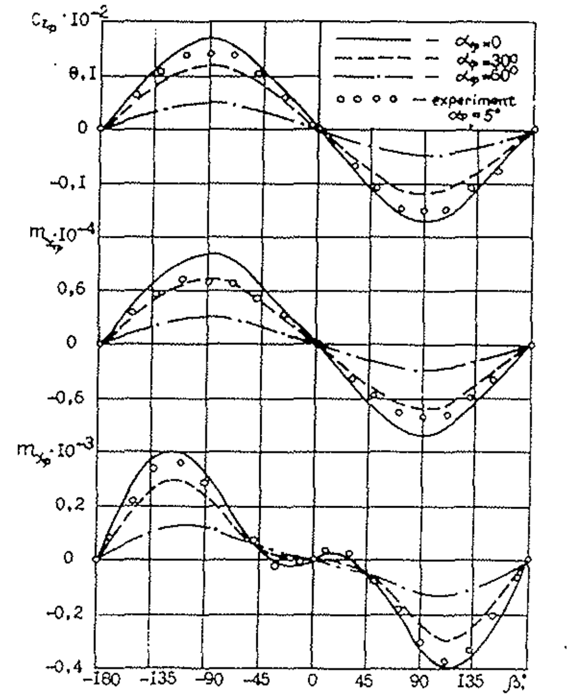

Figure 3. Dependency of Mi-24 helicopter model airframe aerodynamic characteristics on the angles of slip and attack

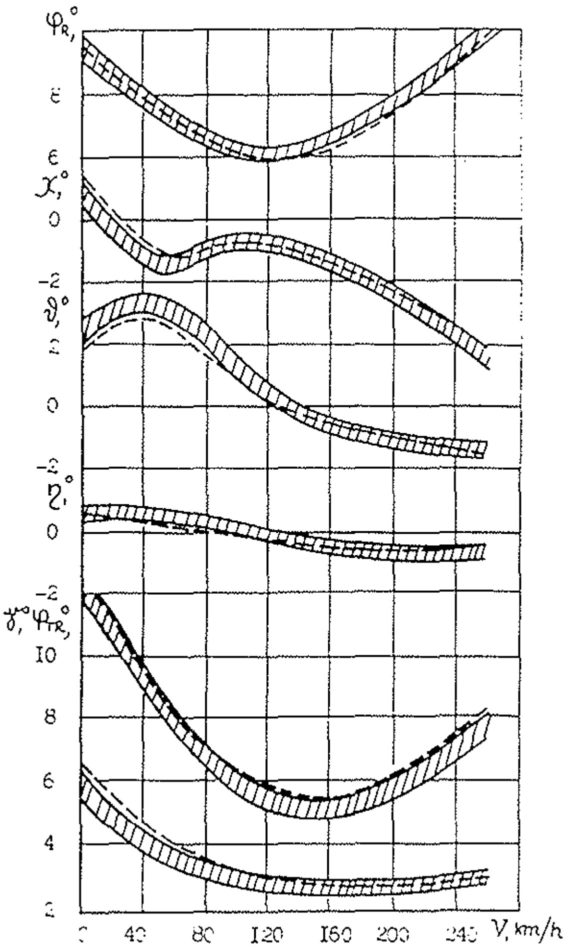

Figure 4. Flight speed trim characteristics of Mi-8 helicopter

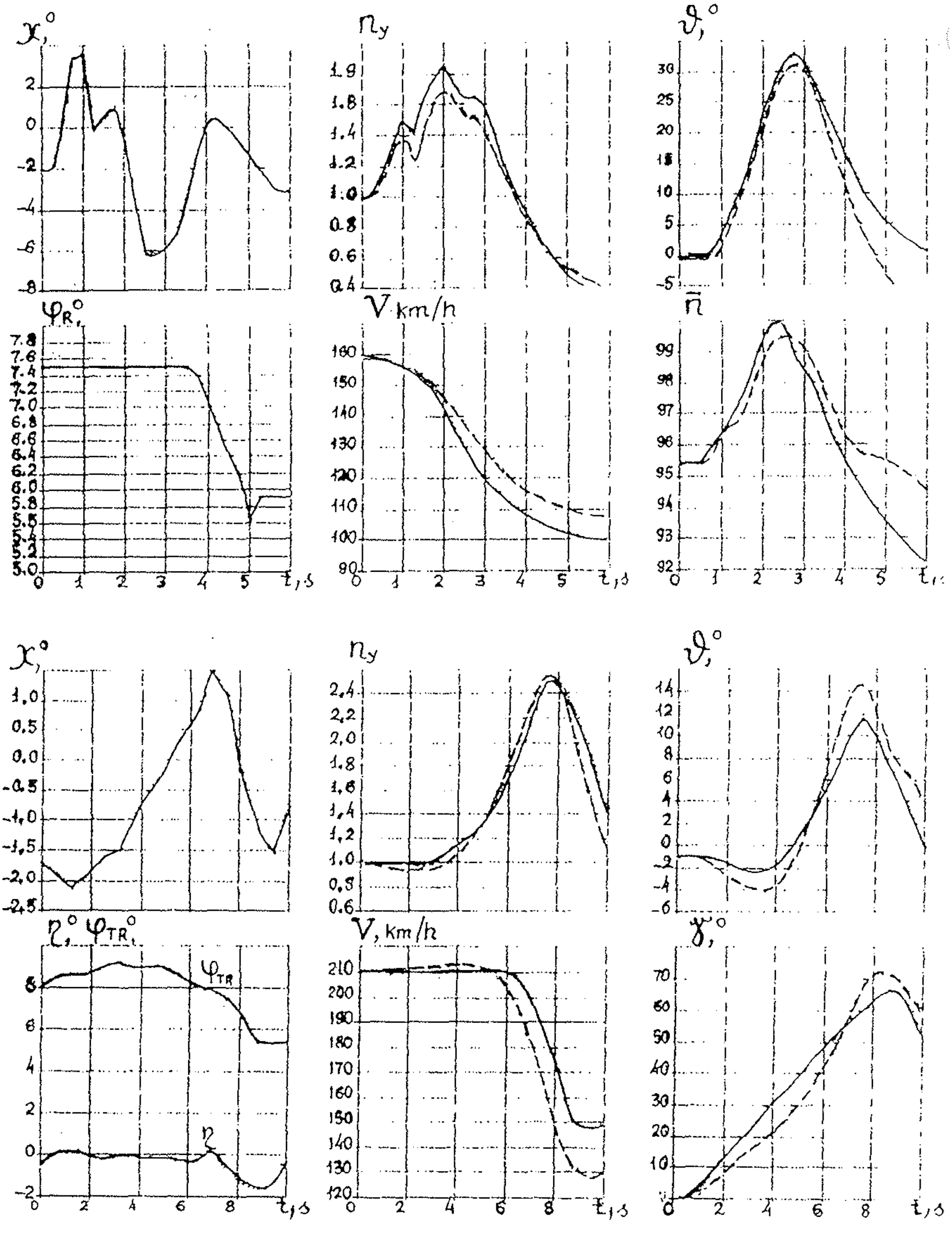

Figure 5. Dynamic response of Mi-8 helicopter to the control bodies deflection :
—— - flight tests; – – – – modelling