STOCHASTIC AEROELASTIC ANALYSIS OF eVTOL AIRCRAFT

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Abstract

An aeroelastic stability and response analysis of an eVTOL with distributed proprotors is performed with uncertainties in system parameters. First, an efficient aeroelastic model of eVTOL wing with distributed proprotors is developed. Stochastic aeroelastic response and stability analysis are then performed based on Monte Carlo Simulation method. Numerical simulations are performed for two cases: Isolated proprotor and wing with proprotors. The structural and aerodynamic parameters of wing, rotor and pylons are considered as random variables. Initial results of isolated proprotor stability analysis show that the randomness in structural and aerodynamic parameters has considerable impact on damping ratio. Next, aeroelastic model of wing with proprotor is developed. Dynamic stability of wing with single proprotor is performed with randomness in wing and proprotor structural parameters. Randomness in structural parameters have significant impact on the dynamic stability predictions. Aeroelastic model with proprotor and uncertainty formulation developed in this study can be extended to wing with multiple proprotors for further stochastic analysis of eVTOL aircraft.

1. NOTATION

R	Proprotor radius (m)
Θ	Proprotor pitch motion
Ψ	Proprotor yaw motion
Ω	Main rotor speed (rad/s)
C_Θ	Structural damping, rotor pitch motion (Nm-s/rad)
C_Ψ	Structural damping, rotor yaw motion (Nm-s/rad)
I_Θ	Inertia of rotor pitch motion (kg-m²)
I_Ψ	Inertia of rotor yaw motion (kg- m²)
I_x	Inertia coupling of rotor pitch-yaw motion
K_Θ	Stiffness of rotor pitch motion (Nm/rad)
K_Ψ	Stiffness of rotor yaw motion (Nm/rad)
R	Proprotor radius (m)
M_Θ	Aerodynamic moment, rotor pitch motion (Nm)
M_Ψ	Aerodynamic moment. rotor yaw motion (Nm)

2. INTRODUCTION

Electric Vertical Take-off and Landing (eVTOL) aircraft with distributed propellers show significant promise for the Urban Air Mobility Systems (UAMS). Recent advances in the battery technologies, electric motors, control systems and light weight composite structures has made the eVTOL physically realizable [1].

Most of the eVTOL aircraft configurations mainly falls under four types: eVTOL with seperate rotors for the vertical take off and propellers for forward flight as Lift+cruise configuration (L+C), tilting proprotors that can provide both lift and forward thrust as Vectored Thrust (VT), wingless multicopters (WM) and Rotorcraft [2]. However, the wings with distributed prop rotors configurations are most popular [2].

The eVTOLs with distribured proprotors, mostly the VT or L+C confiuration, has four to fourteen proprotors on the full span of main wing. The number and distribution of proprotors in the wing varies based on the flight mission requirements. Also, the rotors can be placed either on the leading edge or rear side of the wing, with the wing as unswept or swept, or with dihedral angle.

Currently, researchers and industries are focusing on identifying new phenomena, accurate modeling and analysis of these unconventional eVTOL configurations for safer and efficient design. Aeroelastic instabilities such as whirl-flutter are one of the major limiting factors of safety boundaries [3]. The crashes of two Lockheed L-188 C Electra II were due to the, then unknown phenomena, of Whirl Flutter (WF) instabilities. Even after the WF instability was known, still it lead to the crash of Beechcraft 1900C aircraft. Therefore, whirl flutter type instabilities have to be investigated for ongoing developments in unconventional eVTOL designs with distributed prop rotors.

Many studies have focused on developing models for WF stability analysis. These aeroelastic models vary from the prop-rotor with two Degree of Freedom (DoF) as pitch and yaw motions, to models with nine DoFs that include the individual blade flapping motions [4]. Similarly, the aerodynamic modeling varies from the simple momentum theory to advanced CFD based analysis [5].

A very few studies have focused on the distributed proprotors. Hoover and Shen [6] have studied the NASA XV-57 with 14 rotors on the wing, using multibody simulation approach. The parametric studies shown that the changes in tip proprotor properties can induce the instability in the system. However, in this study the wing is modelled with tip propellers while the inboard propeller doesn’t contribute to the stability of the system [6].

All the above studies focusing on aeroelastic response and stability of the eVTOL are performed in the deterministic framework. However, the parametric values can vary significantly during the operating conditions due to the changes in material properties, loosening of mechanical joints, and aerodynamic conditions, etc. Beran et.al [7] have discussed the effects of uncertainty in aeroelastic modeling and simulations. In general, uncertainties are typically classified as the Epistemic Uncertainty (EU) and Aleatory Uncertainties (AU). Epistemic uncertainty arises due to the lack of accurate mathematical modeling of physical phenomena or uncaptured phenomenon. Aleatory uncertainty arises due to the randomness associated with the parametric values used in the numeric simulations. These aleatory uncertainties arise due to manufacturing tolerances, degradation during its life time and unexpected operating environments. These uncertainties has to be accounted in the early stages of design and analysis for safer and efficient performance.

Aeroelastic systems are highly sensitive to randomness in system parameter values due to the nonlinearities, and fluid structure interactions, and interacting coupled subsystem components [7]. Murugan, et.al. [8,9,10] have studied the effect of material and aerodynamic parameters on the helicopter aeroelastic response, stability and flight performance. Numerical results have shown that the uncertainties or randomness in material properties have significant impact on the aeroelastic response and stability. As discussed above, all the eVTOL design and analysis studies are performed in a deterministic framework. However, the effect of uncertainty in system parameters can have significant impact on the eVTOL designs and therefore has to be investigated.

In this study, a computationally efficient aeroelastic model of eVTOL wing with distributed proprotors is developed initally. Lagrangian formulation and modal methods are used to develop the aeroelastic equation. Aerodynamic loads are derived based on the strip theory. Stochastic aeroelastic response and stability analysis are then performed based on Monte Carlo Simulation method [8]. Numerical simulations are performed for two cases: Isolated proprotor, and elastic wing with single proprotor. The structural and aerodynamic parameters of wing, rotor and pylons are considered as random variables. Aeroelastic model and stochastic numerical results are discussed in the following sections.

3. eVTOL AEROELASTIC MODEL

In this section, the aeroelastic models of the eVTOL with distributed proprotors are developed. The models are developed in three stages and uncertainty analysis is performed in each stage for better understanding of the uncertainty effects on each subsystem performance and its propagation to the global final wing with two proprotors. Therefore, the three levels of aeroelastic models are: a) Isolated proprotor, b) Elastic wing with single proprotor, and c) Elastic wing with two distributed proprotors.

The structural and aerodynamic parameter values associated with the eVTOL aeroelastic models are considered as random variables. That is, these parameters are considered to vary from their nominal values for a chosen standard deviation and probalistic distribution function. Stochastic aeroelastic stability and aeroelastic response analysis is then performed with Monte Carlo simulation method. The two levels of aeroelastic model of eVTOL is described below.

1.1. Isolated proprotor

A simple model of isolated proprotor with two degree of freedom is described in Ref. [11, 12]. In this model, the rotor is considered to have rigid blades spinning at speed (Ω), and subjected to an axial flow condition with velocity (V) as shown in Fig. 1.

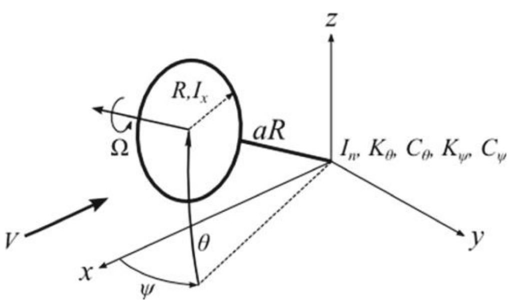

Fig. 1 Isolated proprotor model

The rotor is allowed to oscillate in pitch and yaw, about an effective pivot point with moment of Inertia I_Θ, and I_Ψ, respectively. The rotor is attached to the wing via springs with stiffness K_Θ and K_Ψ. Aerodynamic loads are derived based on the strip theory. The equations of motion are given in Eq. (1).

$$\begin{bmatrix} I_n & 0 \\ 0 & I_n \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} C_\theta & -I_x\Omega \\ I_x\Omega & C_\psi \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} K_\theta & 0 \\ 0 & K_\psi \end{bmatrix}\begin{bmatrix} \theta \\ \psi \end{bmatrix} = \begin{bmatrix} M_\theta \\ M_\psi \end{bmatrix}$$

(1)

The aerodynamic forces on the right hand side are given as

$$M_\theta = \frac{N_B}{2}K_aR\left[-(A_3+a^2A_1)\frac{\dot{\theta}}{\Omega} - A_2'\psi + aA_1'\theta\right]$$

$$M_\psi = \frac{N_B}{2}K_aR\left[-(A_3+a^2A_1)\frac{\dot{\psi}}{\Omega} + A_2'\theta + aA_1'\psi\right]$$

$$K_a = \frac{1}{2}\rho c_{l,\alpha}R^4\Omega^2$$

(2)

The constants in above aerodynamic loads are given as

$$A_1 = \frac{c}{R}\int_0^1 \frac{\mu^2}{\sqrt{\mu^2+\eta^2}}d\eta$$

$$A_1' = \mu A_1$$

$$A_2' = \frac{c}{R}\int_0^1 \frac{\mu^2\eta^2}{\sqrt{\mu^2+\eta^2}}d\eta$$

$$A_3 = \frac{c}{R}\int_0^1 \frac{\eta^4}{\sqrt{\mu^2+\eta^2}}d\eta$$

$$\mu = \frac{V}{\Omega R}$$

(3)

Initially, this simple aeroelastic model is used to understand the effect of uncertainties in pylon stiffnesses, inertia, rotor speed, rotor inertia and aerodynamic values on the whirl flutter stability of single proprotor.

1.2. Elastic wing with single proprotor

In this section, an aeroelastic model of the wing coupled with single proprotor is discussed [3]. The aeroelastic model is shown in Fig. 2 and 3. The flexibility of the elastic wing is considered into the equation of motions. The bending and torsional motion of wing is modeled with spring stiffness. The proprotor is considered to have yaw and pitch motions, as the previous case. Lagrangian formulation is used to derive the equation of motion. The final equation with four degrees of freedom is given below in Eq. (4).

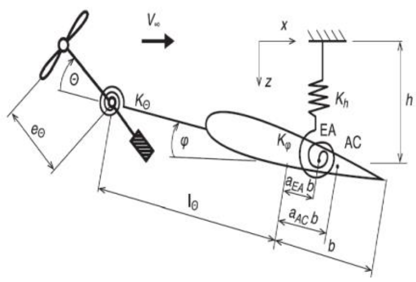

Fig. 2 Wing and proprotor model, side view

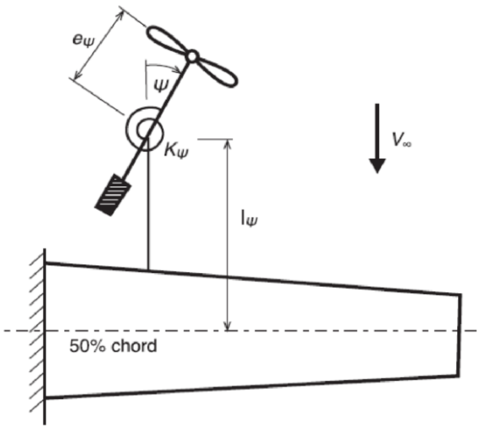

Fig. 3 Wing and proprotor model, top view

$$\begin{aligned}
&J_{\Theta P}\left(\ddot{q}_\Theta+\gamma_\Theta\frac{\omega_\Theta^2}{\omega}\,\dot{q}_\Theta+\omega_\Theta^2 q_\Theta\right)+J_{\Theta\Psi}\Omega\,\dot{q}_\Psi+S_{\Theta P}\,\ddot{q}_b+J_{\Theta\varphi P}\,\ddot{q}_\varphi=Q_\Theta\\
&-J_{\Theta\Psi}\Omega\,\dot{q}_\Theta+J_{\Psi P}\left(\ddot{q}_\Psi+\gamma_\Psi\frac{\omega_\Psi^2}{\omega}\,\dot{q}_\Theta+\omega_\Psi^2 q_\Psi\right)-J_{\Psi\varphi}\Omega\,\dot{q}_\varphi=Q_\Psi\\
&S_{\Theta P}\,\ddot{q}_\Theta+M_{WP}\left(\ddot{q}_b+\gamma_b\frac{\omega_b^2}{\omega}\,\dot{q}_b+\omega_b^2\,q_b\right)+S_{\varphi WP}\,\ddot{q}_\varphi=Q_b\\
&J_{\Theta\varphi P}\,\ddot{q}_\Theta+J_{\Psi\varphi}\Omega\,\dot{q}_\Psi+S_{\varphi WP}\,\ddot{q}_b+J_{\varphi WP}\left(\ddot{q}_\varphi+\gamma_\varphi\frac{\omega_\varphi^2}{\omega}\dot{q}_\varphi+\omega_\varphi^2 q_\varphi\right)=Q_\varphi
\end{aligned} \tag{4}$$

The aerodynamic forces and moments are derived based on strip theory. The stochastic aeroelastic response and stability analysis, with the uncertainty in structural and aerodynamic parameters, are performed with the above model.

1.3. Wing with multiple distributed proprotor

A typical eVTOL based on vectored thrust or Lift – cruise configurations will have wing with four or more distributed proprotors on the span of the wing. For example, the NASA Maxwell X-57 configuration is shown in Fig. 4 [2,13]. The above formulation for aeroelastic model can be extended to the multiple rotors and tip rotors. However, in this study the uncertainty analysis is performed with the elastic wing with single prop rotor.

Fig. 4 eVTOL wing with distributed rotors [Ref. 13]

2. NUMERICAL RESULTS

In this section, the numerical results for the isolated proprotor and elastic wing coupled with the proprotor is, discussed. The baseline values, of isolated prop rotor a are given in Ref [12]. However, the numerical parameters for coupled wing with proprotors are assumed to be similar to that of NASA Maxwell XV15.

2.1. Uncertain isolated-proprotor stability

Stochastic aeroelastic simulations are performed with the equation given in Eq. (1). The aeroelastic equation in second order form is converted into the first order state space form as given below in Eq. (5).

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X}=\mathbf{0}$$
$$\mathbf{X}=\begin{bmatrix}\theta\\ \psi\end{bmatrix} \tag{5}$$

$$\mathbf{J}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}$$

Eigenvalue analysis is then performed to find the eigenvalues of the state space matrix, J. The frequency and the damping ratio of the isolated rotor is then given as

$$\omega=\sqrt{\mathrm{Re}\,(\lambda)^2+\mathrm{Im}\,(\lambda)^2}$$
$$\zeta=\frac{-\mathrm{Re}\,(\lambda)}{\omega} \tag{6}$$

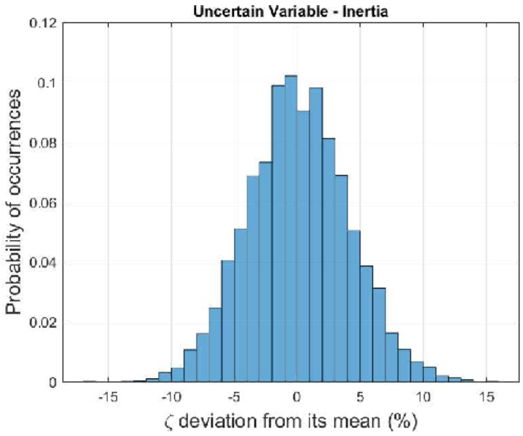

Fig5. Effect of uncertain inertia on isolated rotor stability

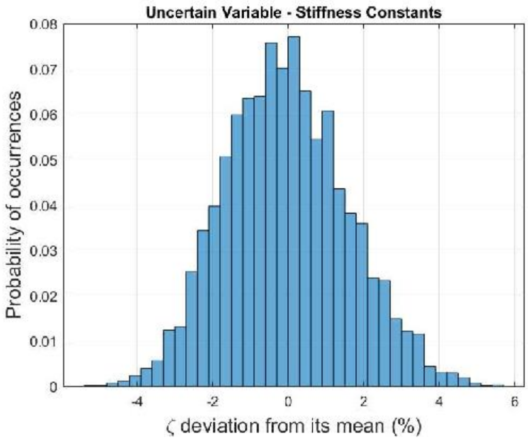

Fig 6. Effect of uncertain stiffness on isolated rotor stability

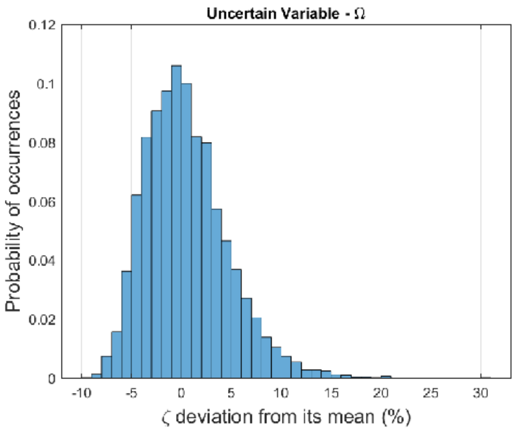

Fig 7. Effect of uncertain rotor speed on isolated rotor stability

Stochastic aeroelastic simulations are performed with the equation given in Eq. (1). The aeroelastic equation in second order form is converted into the first order state space form as given in Eq. (5). Eigenvalue analysis is then performed to find the eigenvalues of the state space matrix, J. The frequency and the damping ratio of the isolated rotor is then given as in Eq. (6).

The dynamic aeroelastic stability, ie, whirl flutter stability analysis is now performed with the aerodynamic loads. The aerodynamic parameters such as the lift slope ($C_{l,\alpha}$) and air density (ρ) along with the inertia parameters of rotor (Ix, In), stiffness parameters of the pylon and rotor speed (Omega) are taken as the uncertain variables. A coefficient of variation of 5% and a Gaussian distribution is considered for each variable. The uncertainty analysis is performed with the bench top rotor values given in Ref. [12]. The results are shown in Figures 5 to 9.

Uncertainty analysis is performed with Monte Carlo simulations with 5000 samples. The effect of uncertainty due to lift curve slope on the damping ratio is shown in Fig 8. The uncertainty due to the air density on the damping ratio is given in Fig 9. After the addition of aerodynamic parameters, effect of uncertainty in rotor and pylon inertia, on the damping ratio is shown in Fig 5. Similarly, the effect of uncertainty in rotor and pylon stiffness of yaw and pitch motion, on the damping ratio is shown in Fig 6. Finally, the uncertainty in rotor angular speed on the damping ratio is shown in Fig. 7. The numerical results clearly show the whirl flutter stability is highly susceptible to uncertainties. Specifically, the uncertainty in lift curve slope of rotor airfoils, density of the flow around rotor and rotor speed has significant impact on the whirl flutter stability.

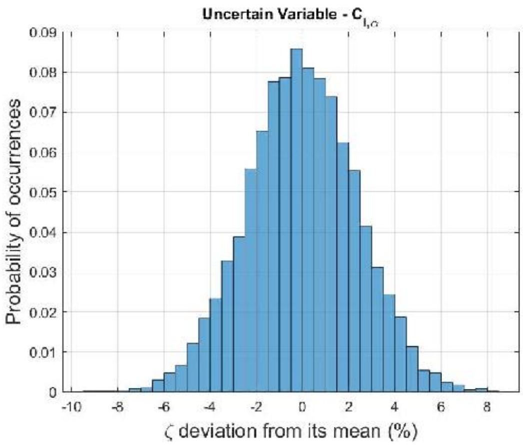

Fig 8. Effect of uncertain lift curve slope on whirl flutter stability

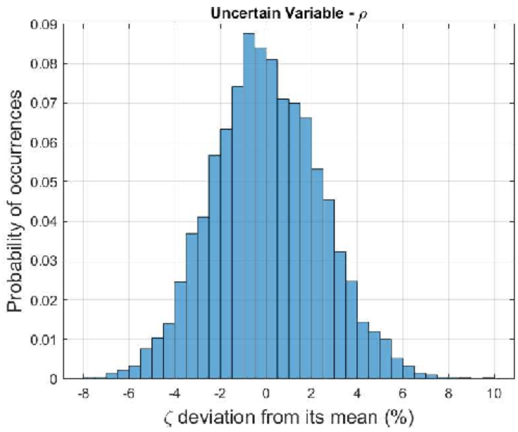

Fig 9. Effect of uncertain air density whirl flutter stability

2.2. Uncertain Wing-Proprotor stability

Now, the effect of uncertainties on the dynamic stability of the coupled wing prop rotor system is performed. The aeroelastic equation of motion of wing rotor coupled system, given in Eq (4), is used for uncertainty analysis similar to the isolated rotor case.

In the isolated rotor case, the numerical values of bench top prototype are used. In this section, the numerical values, representative of a NASA Maxwell X-57 is used. Also, the six parameters are considered as random variables. Initially, the stiffness of wing and propellers, mass of wing and propeller, elastic axis of wing, moment of inertia of propeller, and the speed of the propellers are considered as random variables. The MCS are performed for individual and combined variables cases. The results are shown in Figures 10 to 12. The numerical results show that stability varies from the -100 to 400 % deviation from the deterministic value. And the distribution also shows a non-gaussian type distribution. However, this variation is due to the less value of damping in the system. Further the aerodynamic loads are not included in the uncertainty analysis. Uncertainty analysis of the wing aerodynamic loads of wing and propellers are considered as the work in progress. The uncertainty formulation developed in this work can be extended to eVTOL wing with multiple proprotors.

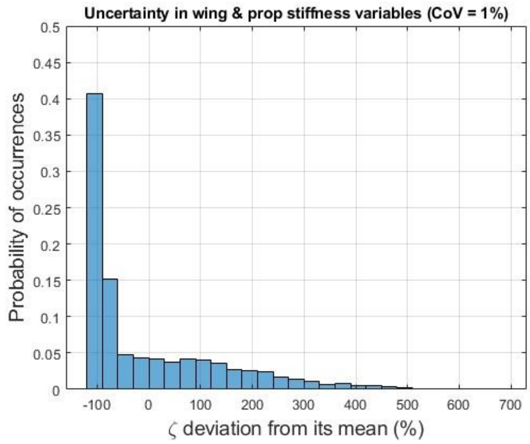

Fig10. Effect of uncertain stiffness on wing propeller dynamic stability

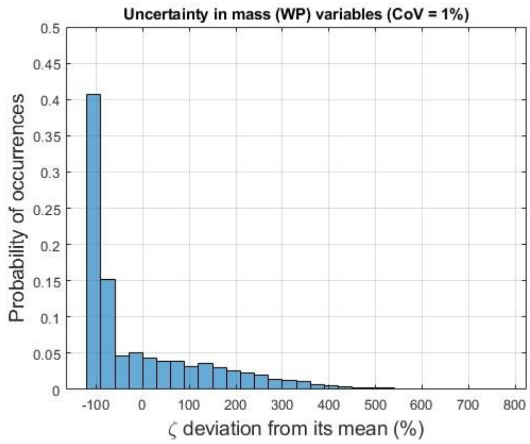

Fig11. Effect of uncertain mass on wing propeller dynamic stability

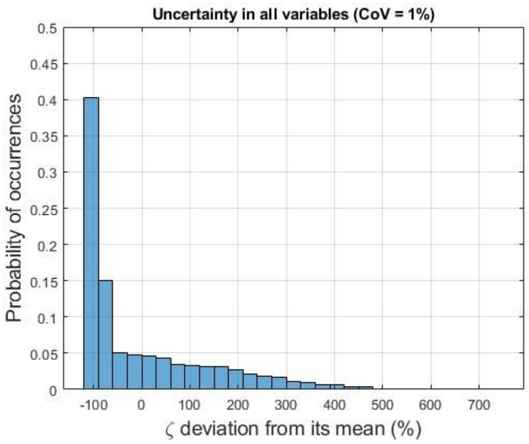

Fig12. Effect of uncertain variables (six) on wing dynamic propeller stability

3. CONCLUSIONS

A stochastic aeroelastic stability and response analysis of an eVTOL with isolated and distributed proprotors is performed with uncertainties in system parameters. First, an efficient aeroelastic model of isolated proprotor and wing with distributed proprotors are developed. Stochastic aeroelastic response and stability analysis are then performed based on Monte Carlo Simulation method. Effect of uncertainties in the structural and aerodynamic parameters of wing, rotor and pylons on the stability and wing response are quantified.

1. Initially, an isolated proprotor stability and whirl-flutter analysis is performed. The stability results show that the uncertainty in structural parameters has considerable impact on damping ratio of proprotor. The uncertainties in lift curve slope, flow density, and rotor speed have significant impact on the whirl flutter stability. Also, the Gaussian distribution of uncertain input variables result in non-Gaussian type distributions of the damping ratio which needs further investigation.

2. An aeroelastic model of flexible wing coupled with single is developed. Dynamic stability of this wing coupled with proprotor is performed with randomness in system parameters. Numerical results show that the uncertainty in stiffness, mass and rotor speed parameters have significant impact on the dynamic stability of the coupled wing prop rotor system. A randomness of CoV of 1 % in the structural variables results in -100 to 400% change in the system damping. This can be due to the low damping available in the system, without any aerodynamic loads or structural damping. Aeroelastic stability analysis of coupled wing and prop rotor with uncertainties in both the structural and aerodynamic parameters are considered as the future work.

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-Nil-

5. REFERENCES

1. Brelje, Benjamin J., and Joaquim RRA Martins. "Electric, Hybrid, And Turboelectric Fixed-Wing Aircraft: A Review Of Concepts, Models, And Design Approaches." *Progress in Aerospace Sciences,* Vol. 104, 2019, pp. 1-19.

2. Garrow, Laurie A., Brian J. German, and Caroline E. Leonard. "Urban Air Mobility: A Comprehensive Review And Comparative Analysis With Autonomous And Electric Ground Transportation For Informing Future Research." *Transportation Research Part C: Emerging Technologies,* Vol. 132, 2021, pp. 1-31.

3. Cecrdle, J, *Whirl Flutter of Turboprop Aircraft Structures*, Elsevier, Cambridge-UK, 2015.

4. Yeo, Hyeonsoo, and Andrew R. Kreshock., "Whirl Flutter Investigation of Hingeless Proprotors," *Journal of Aircraft*, Vol. 57, (4) 2020, pp. 586-596.

5. Cecrdle, J., "Analysis of Twin Turboprop Aircraft Whirl-Flutter Stability Boundaries," *Journal of Aircraft,* Vol. 49, (6), 2012, pp. 1718-1725.

6. Hoover, Christian B., and Jinwei Shen., "Parametric Study of Propeller Whirl Flutter Stability with Full-Span Model of X-57 Maxwell Aircraft," *Journal of Aircraft* 55, (6), 2018), pp. 2530-2537.

7. P. Beran, B. Stanford, C. Schrock, "Uncertainty Quantification In Aeroelasticity," *Annual. Review of Fluid Mech*, Vol. 49, (1), 2017, pp. 361-386.

8. Murugan, Senthil, Ranjan Ganguli, and Dinesh Kumar Harursampath, “Stochastic Aeroelastic Analysis Of Composite Helicopter Rotor,“ *Journal of The American Helicopter Society,* Vol. 56, (1), 2011, pp. 12001-120013.

9. Murugan, Senthil, Rajib Chowdhury, S. Adhikari, and M. I. Friswell, “Helicopter Aeroelastic Analysis With Spatially Uncertain Rotor Blade Properties," *Aerospace Science and Technology,* Vol. 16, (1), 2012, pp. 29-39.

10. Siva, C., M. Senthil Murugan, and Ranjan Ganguli, “Uncertainty Quantification In Helicopter Performance Using Monte Carlo Simulations”, *Journal of Aircraft*, Vol. 48, (5) 2011, pp. 1503-1511.

11. Bielawa, Richard L. *Rotary Wing Structural Dynamics and Aeroelasticity*. American Institute of Aeronautics and Astronautics, Washington, 2006.

12. Mair, Christopher, Djamel Rezgui, and Branislav Titurus. "Nonlinear Stability Analysis Of Whirl Flutter In A Rotor-Nacelle System." *Nonlinear dynamics,* Vol. 94, 2018, pp. 2013-2032.

13. Massey, Steven J., Carol D. Wieseman, Bret Stanford, and Jennifer Heeg. "Aeroelastic analysis of a distributed electric propulsion wing." In 58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, p. 0413. 2017.

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