NUMERICAL PREDICTION OF THE AERODYNAMIC RESPONSE OF AEROFOILS SUBJECTED TO STEP INCREASES IN MACH NUMBER

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Abstract

This paper presents a preliminary study of the influence of step changes in Mach number on the aerodynamics of aerofoils. In order to obtain the unsteady response for the aerofoil, solutions of the Navier-Stokes equations were obtained using an implicit, high resolution finite volume approach based upon Osher's approximate Riemann solver. The instantaneous (step) change in Mach number was represented using a grid velocity term. Calculations were performed over a wide range of Mach numbers ($0.2 < M < 0.7$) for a NACA 0012 aerofoil at 2 degree's incidence with a Reynolds number based upon chord of $1x10^6$.

Preliminary analysis of the computed results indicates that the behaviour of the response function for step changes in Mach number closely resembles that for step changes in incidence. The initial response is dominated by an impulsive loading which decays rapidly. The initial peak loading is shown to be strongly Mach number dependent.

Nomenclature

c	chord length.
C_N	normal force coefficient.
C_m	pitching moment coefficient.
C_p	pressure coefficient.
E, F	numerical fluxes.
M	Mach number.
ΔM	step change in Mach number.
n	normal vector.
p	pressure.
Q	vector of conserved quantities.
Re_c	Reynolds number based upon chord.
S	surface area.
S	aerodynamic time, $S = \frac{tc}{U}$.
t	time.
U	freestream velocity.
u,v	Cartesian components of velocity.
V	volume.
x,y	Cartesian co-ordinate system.
α	angle of incidence.
ρ	density.

Subscripts

0	condition at $t < 0$
1	condition at $t=\infty$.
i	inviscid
L	left hand state
R	right hand state
v	viscous
∞	freestream condition.

Introduction

Despite considerable progress over many years the accurate prediction of the flowfield around a helicopter rotor in forward flight remains one of the most challenging problems in modern computational aerodynamics. Theoretical approaches must address the complex flow phenomena found around rotorcraft, these phenomena include transonic flow on the advancing blade tip, reversed flow over a significant proportion of the retreating blade, dynamic stall on the retreating blade tip and complex interactions of the rotor with the wake system of preceding blades.

In principal it is possible to solve fully the equations which govern such flows, but the large scale and closely coupled nature of the aerodynamics and structural dynamics makes such calculations impractical. In a more computationally tractable approach, see for example Sankar and Tung[1], the equations governing the aerodynamics and structural dynamics are solved separately within a loosely coupled iterative framework. Such methods show great promise, but the computational power required to compute the full unsteady three dimensional flowfield is considerable and consequently such methods

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remain too expensive for routine design calculations.

Instead an approach based upon blade element theory[2] has been developed in which the local static aerodynamic performance of aerofoil section is used to approximate the unsteady three dimensional flow around the helicopter rotor. Under normal flight conditions acceptable performance predictions can be obtained with blade element theory, but as the extremes of the flight envelope are approached unsteady effects become increasingly important and a quasi-steady model of the rotor aerodynamics can no longer be justified. Beddoes[3,4] and Leishman[5] have presented a more sophisticated treatment of this problem in which indicial functions are employed to represent the unsteady airloads generated by pitching oscillations. Beddoes[6,7] has extended the indicial formulation to include the important effects of flow separation and dynamic stall. The main advantage of such an approach is that for a given pitching motion the unsteady force and moment coefficients may be determined from steady performance data and a linear superposition of the results obtained for step changes in angle of attack.

The inability of current analysis codes to predict the unsteady behaviour of airloads on the advancing side of the rotor has been well documented[8]. Discrepancies between theory and measurement have been attributed in part to the quasi-steady treatment of the unsteady shock boundary layer interaction. In order to obtain an improved understanding of such interactions Shaw and Qin have used numerical solutions of the unsteady Navier-Stokes equations to investigate the aerodynamic behaviour of aerofoils which are subjected to inplane[9,10], pitching and combined inplane-pitching oscillations[11,12].

In Figures (1) and (2) calculated shock location and shock strength for an aerofoil performing an unsteady motion described by,

$$M = 0.5113(1 + 0.5263\sin(0.1976t))$$
$$\alpha = 0^\circ$$

are compared with results obtained using a quasi-steady assumption. It is clear from such comparisons that the widespread use of quasi-steady approximations to represent the unsteady effects of Mach number variations provides a poor representation of the underlying flow physics.

Van der Wall and Leischman[13,14] have investigated several theoretical approaches to the problem of computing force and moments in an incompressible oscillating freestream. They demonstrated that an indicial based approach (arbitrary motion theory) can match almost exactly results obtained using Isaac's exact theory when the angle of attack is constant.

The success of indicial theory in providing accurate representations of pitching oscillations and incompressible freestream velocity variations suggests that the study of the response of aerofoils subjected to step increases in Mach number may provide useful insight into the aerodynamics of aerofoils in a compressible oscillating freestream. The development of response functions for Mach number oscillations is hindered by the non-linear nature of the flowfield at high subsonic and moderate transonic Mach numbers. Further, the lack of a suitable experimental database prevents the synthesis of an empirical model.

A number of authors, for example Ballhaus and Goorjian[15] and Parameswaran and Baeder[16], have used solutions of the Navier-Stokes equations to determine indicial response functions for pitching aerofoils. In the current paper a numerical method, based upon the solution of the unsteady thin layer Navier-Stokes equations, has been employed to study the flow around a NACA 0012 aerofoil subjected to a step increase in Mach number. For the purposes of this study the boundary layer was assumed to be fully turbulent and the angle of incidence and Reynolds number based upon chord were fixed at $\alpha = 2^\circ$ and $Re_c = 1$ million respectively.

Numerical procedure

The Navier-Stokes equations express the principles of conservation of mass, momentum and energy for a fluid and can be written in integral form as,

$$\frac{\partial}{\partial t}\iiint_V Q\,dV + \iint_S F.\,n\,dS = 0 \qquad (1)$$

In order to represent the effects of rigid body motions (such as those which will be used to represent step increases in Mach number) Equation (1) is extended in the following manner. Consider the differential form of the one dimensional continuity equation, integrating for a control volume whose boundaries move over time we obtain,

$$\int_{x_1(t)}^{x_2(t)} \frac{\partial \rho}{\partial t} dx + \int_{x_1(t)}^{x_2(t)} \frac{\partial (\rho u)}{\partial x} dx = 0 \qquad (2)$$

Recognising that the derivative of $\int_{x_1(t)}^{x_2(t)} \rho dx$ can be rewritten in the following form,

$$\frac{d}{dt}\int_{x_1(t)}^{x_2(t)} \rho dx = \rho(x_2,t)\frac{dx_2}{dt} - \rho(x_1,t)\frac{dx_1}{dt} + \int_{x_1(t)}^{x_2(t)} \frac{\partial \rho}{\partial t} dx \qquad (3)$$

then, after some further manipulation, Equation (2) may be rewritten as follows,

$$\frac{d}{dt}\int_{x_1(t)}^{x_2(t)} \rho dx + \int_{x_1(t)}^{x_2(t)} \frac{\partial}{\partial x}\rho\left(u - \frac{dx}{dt}\right) dx = 0 \qquad (4)$$

in which $\frac{dx}{dt}$ is the velocity with which the control volume nodes move, referred to as the grid velocity. Similar results follow for the momentum and energy equations. The governing equations may therefore be rewritten for arbitrarily moving bodies by replacing the velocity in the convective flux terms of Equation (1) with the relative velocity of the fluid with respect to the moving grid.

A high resolution finite volume scheme based upon Osher's flux vector splitting[17] is employed for the spatial discretisation of the convective flux terms. In this procedure the numerical convective flux is evaluated using an approximate Riemann solver, which can be written as,

$$F_{i+\frac{1}{2},j} = \tfrac{1}{2}\left[F(Q_L) + F(Q_R)\right] - \tfrac{1}{2}\int_{q_L}^{q_R} \left|\frac{\partial F}{\partial Q}\right| dQ \qquad (5)$$

where the integration of the last term is carried out using a natural ordering of the sub-paths parallel to the eigenvectors of the flux Jacobian. Higher order spatial accuracy was obtained using MUSCL interpolation together with a flux limiter. Gauss' theorem is used to obtain the velocity and temperature gradients required for evaluation of the viscous flux terms. The Baldwin-Lomax algebraic turbulence model[18] was employed to provide a turbulent contribution to the viscosity.

After spatial discretisation the governing equations are reduced to a system of ordinary differential equations which are integrated in time using a first order Euler implicit scheme. One implicit step of the method can be written as,

$$\left[I + \frac{\partial E_i}{\partial Q} + \frac{\partial F_i}{\partial Q} + \frac{\partial F_v}{\partial Q}\right]\frac{\partial Q}{\partial t} = -\Delta t\left(\frac{\partial E_i}{\partial \xi} + \frac{\partial F_i}{\partial \eta} + \frac{\partial F_v}{\partial \eta}\right) \qquad (6)$$

Equation (6) represents a sparse, system of linear equations which is solved using Krylov subspace methods. In the current work a preconditioned variant of GMRES[19] was chosen.

Modelling step changes in Mach number

There are two main approaches which can be adopted in order to obtain the response of an aerofoil to a step change in Mach number. In the first approach a steady solution is first obtained at the initial Mach number. The aerofoil boundary conditions are then perturbed to produce the desired change in Mach number and the unsteady response is computed. There are a number of weaknesses inherent in such an approach. Of greatest concern is that the response which is obtained is not solely due to the step change in Mach number but includes the effects of an infinite acceleration (due to the discontinuous time derivative of Mach number). Additionally Parameswaran and Baeder[17] highlight a number of numerical problems associated with such an approach.

In this paper use is made of the moving grid approach described above. In this approach the steady solution is again obtained at the initial flow conditions. The grid velocity is then set equal to the desired increase in Mach number and the unsteady response is computed. As the grid velocity is applied uniformly over the whole grid only the response to the step change in Mach number is computed. Strictly the application of a uniform grid velocity over the whole of the aerofoil chord implies that the response is for fore-aft motion rather than for a variation in freestream Mach number which should more properly be represented by a series of horizontally propagating gusts. However, theoretical results obtained by van der Wall and Leishman[14,15] suggest that the interpretation of the unsteady freestream as a fore-aft motion can be considered to be a good approximation over the frequency range considered to be important for rotorcraft applications.

Results

The method described in the previous sections has been used to study the response of a NACA 0012 aerofoil to step changes in Mach number. Solutions were computed for Mach numbers in the range $0.2 \leq M \leq 0.7$ at an angle of incidence of 2°. The flow was assumed to be fully turbulent with a Reynolds number based upon chord of $Re_c = 1$ million. Unless otherwise stated the calculations were performed with a time step size of 0.029089 on a grid containing 251 nodes in the flow direction (200 on the aerofoil surface) and 72 nodes in the aerofoil normal direction, a detail of this grid is shown in Figure (3).

Numerical tests

The sensitivity of the computed response function to the main numerical parameters (time step size, grid density and convergence criteria) was investigated to ensure that the governing partial differential equations are properly satisfied. For the purposes of these tests the aerodynamic response to a step increase in Mach number of 0.01 at M=0.5 was computed.

The influence of time step size, ΔS, upon the initial part of the calculated response functions for normal force and pitching moment coefficients is shown in Figures (4a) and 4(b) respectively. Calculations were performed for time step sizes of 0.058178, 0.029089 and 0.023271. The results indicate that for small time steps the solution can be considered independent of ΔS, while for larger values the unsteady behaviour of the flowfield is incorrectly modelled. In the remainder of this work ΔS has been chosen such that $\Delta S \leq 0.03$.

In Figure 4(b) the pitching moment response function contains small amplitude oscillations. These oscillations are thought to have arisen as a consequence of rounding errors in the calculation of pitching moment coefficient and as such are not physical. While such behaviour is undesirable for the purposes of the current discussion it is thought to be unimportant.

A grid dependency study was performed for computational meshes containing 153x48, 201x64 and 251x72 grid points, corresponding to 100, 150 and 200 nodes around the aerofoil surface respectively. Grid points were clustered towards the wall and wake cut line to provide adequate resolution of the shear layers in the boundary layer and wake, and close to the leading and trailing edges. A near uniform distribution was employed over the remainder of the chord. Details of the fine grid in the region of the aerofoil are shown in Figure (3).

Integrated force and moment coefficients from the initial steady state computations show considerable sensitivity to the grid density which is mirrored in the subsequent unsteady calculations, see for example Figures 5(a) and 5(b) which present a comparison of the force and moment response functions for the three grid levels. This sensitivity has been traced to the inability of the method to resolve the leading edge pressure peak adequately on the coarser grids, see Figure (6) in which the initial steady pressure distributions are compared. In view of these difficulties the fine grid was used throughout the remainder of this work.

In order to minimise the computational cost of the current method the system of linear equations which arises at each time step is not solved exactly. Instead an initial solution to the linear system is obtained using ADI factorisation, restarted GMRES is then applied in an iterative fashion to improve the solution. The linear system is considered to be solved when the ratio of the current solution error to the initial solution error has been reduced below some prescribed tolerance (typically four orders of magnitude). The tolerance imposed on the linear solver can therefore be considered as a measure of the factorisation error associated with the calculation and for this reason must be considered in conjunction with the time step size. It is expected that for a given 'overall' accuracy larger time steps will require that a much smaller tolerance be imposed. In the current work solver tolerances of 0.05, 0.005 and 0.0005 were considered in conjunction with a time step size of ΔS = 0.029089. The results presented in Figures 7(a) and 7(b) indicate that the solution is relatively insensitive to the value of this parameter. The value 0.0005 has been used throughout the remainder of this work.

<u>Response function calculations</u>

Calculated response functions for normal force and pitching moment coefficients are presented in Figures 8(a) and 8(b) respectively for step changes in Mach number. Here the normal force and pitching moment coefficients are normalised by their final steady values and the quantity S represents the non-dimensional aerodynamic time which can be interpreted as the number of chord lengths that the aerofoil has travelled.

Conventionally the unsteady loading has been divided into contributions from non-circulatory (impulsive) and circulatory components. In the present calculations the initial loading is dominated by the non-circulatory component at low freestream Mach numbers. The influence of the impulsive contribution quickly diminishes with time as the pressure disturbances caused by the change in boundary conditions propagate away from the aerofoil. At the same time the loading due to the change in circulation caused by increased Mach number gradually approaches its new steady state value. The different time scales of the circulatory and non-circulatory responses leads to a distinctive second inflexion point in the loading. The behaviour of the computed normal force coefficient is broadly similar to that which has been calculated for step changes in incidence, see for example Parameswaran and Baeder[16].

The use of indicial techniques to calculate unsteady aerodynamic performance implicitly assumes that unsteady effects can be represented as a series of linear perturbations about a (non-linear) steady state solution. When determining the indicial response function numerically it is therefore important to choose a step size which is sufficiently small to ensure that the flow does not exhibit strong non-linearities. The influence of step size on the computed response function was investigated for the case $M=0.4+\Delta M$. The initial variation of normal force coefficient is shown in Figure (9) for step sizes of 0.05, 0.01 and 0.02. In Figure (9) the loading has been scaled by the step size. When this is done it is apparent that the initial loading is largely independent of ΔM, i.e. the initial loading is a linear function of the step change. The behaviour of the loading over longer time intervals is dominated by circulation effects and consequently the individual response curves have different asymptotic values to which they tend over longer periods of time.

As shown in Figures (8a) and (8b) the initial behaviour of the calculated response functions exhibits a strong dependence on freestream Mach number. For low freestream Mach numbers (for example 0.2) there is a large initial spike while in contrast at the higher freestream Mach numbers ($M=0.6$) there is no evidence of such behaviour. From acoustical considerations[20] the initial loading due to a step change in incidence or pitch rate is found to be inversely proportional to Mach number. This result follows from the fact that the local change in pressure coefficient due to an upwash, Δw, is given by,

$$\Delta C_p(x, S=0) = \frac{4}{M}\frac{\Delta w(x)}{V} \qquad (7)$$

It is not possible to cast the variation of pressure coefficient with step changes in Mach number in this form directly, however by analogy with Equation (7) the following functional form for the initial loading due to a step increase in Mach number is suggested,

$$\Delta C_P(x, S=0) = \frac{\Phi}{M_1}\frac{\Delta M(x)}{M_1} \qquad (8)$$

here Φ is an unknown function which is to be determined. In the current calculations the step change in Mach number is applied uniformly over the aerofoil chord and consequently the change in normal force coefficient can be obtained with relative ease from Equation (8). If the initial loading is assumed to be dominated by the non-circulatory component then Equation (8) suggests that the initial peak value of the response should behave in a linear manner with respect to the product of ΔM and $1/M^2$. The results presented in Figure (9) have already demonstrated that the computed normal force behaves linearly with ΔM over the short time period that the impulsive contribution dominates the overall load. In Figure (10) the peak value of the normal force coefficient divided by the step change in Mach number is plotted against the inverse of M^2. While the relationship between peak loading and $1/M^2$ does appear to be linear, the curves in Figure (10) show a strong dependence upon the direction (plus or minus) of the Mach number change. It is also observed that the calculated peak loading is not directly proportional $1/M^2$.

Conclusions

A numerical method for obtaining the indicial response functions of an aerofoil subjected to step changes in Mach number has been presented. The method is based upon the solution of the thin layer Reynolds averaged Navier-Stokes equations on moving grids. Computations of the normal force and pitching moment coefficient responses for a NACA 0012 aerofoil at 2° incidence have been presented for a wide range of subsonic Mach numbers. The calculated results indicate that the unsteady behaviour of the forces and moments closely resembles that for pitch rate and angle of attack variations. The initial behaviour of the response functions is dominated by an impulsive contribution which decays rapidly while more slowly varying circulatory effects dominate the later part of the time history.

Preliminary analysis of the results suggest that it may be possible to represent unsteady variations of Mach number (in the subsonic flow regime) using an indicial formulation which closely resembles that determined for angle of attack variations.

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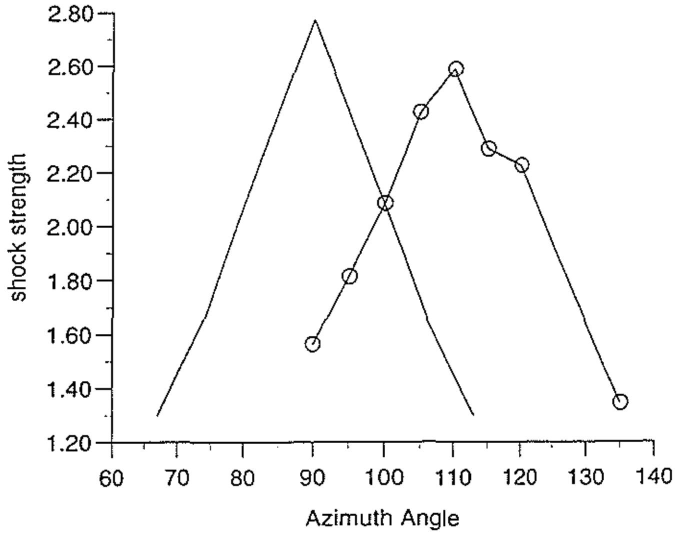


Figure (2) Comparison of computed unsteady and quasi-steady shock strength

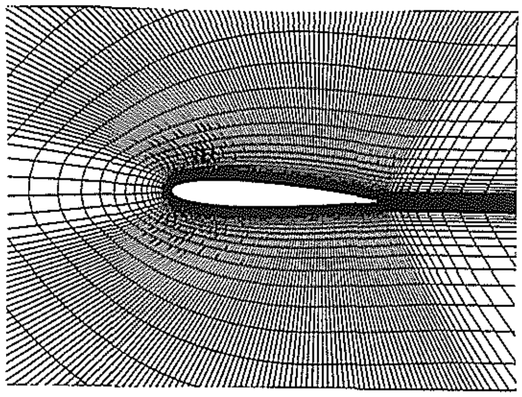


Figure (3) Computational grid

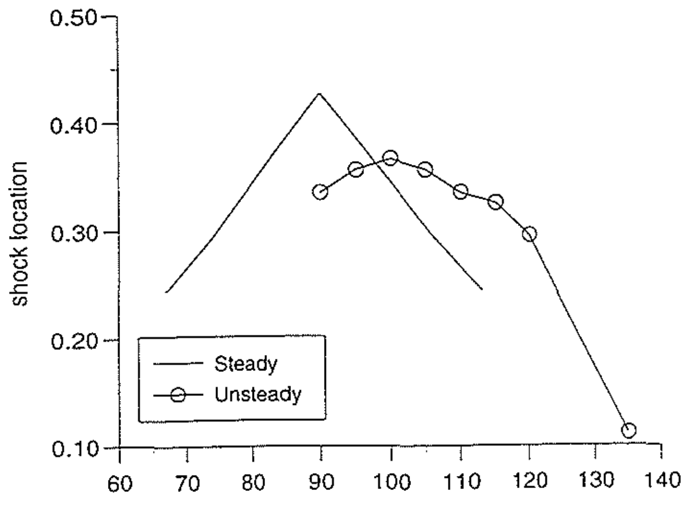


Figure (1) Comparison of computed unsteady and quasi-steady shock location

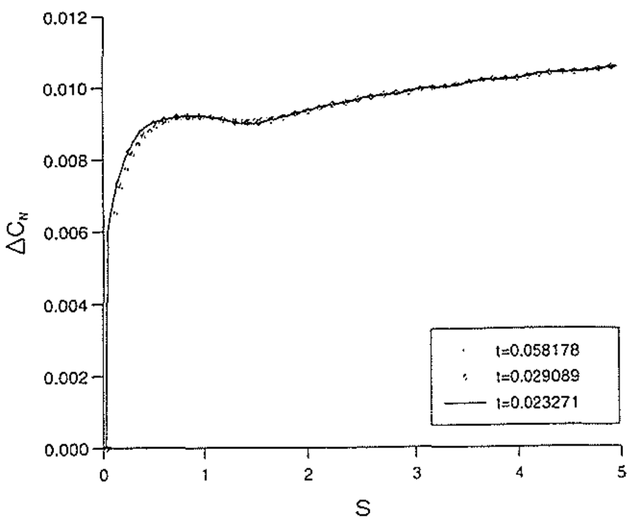


Figure 4(a) Sensitivity of normal force response to time step size (M=0.50+0.01)

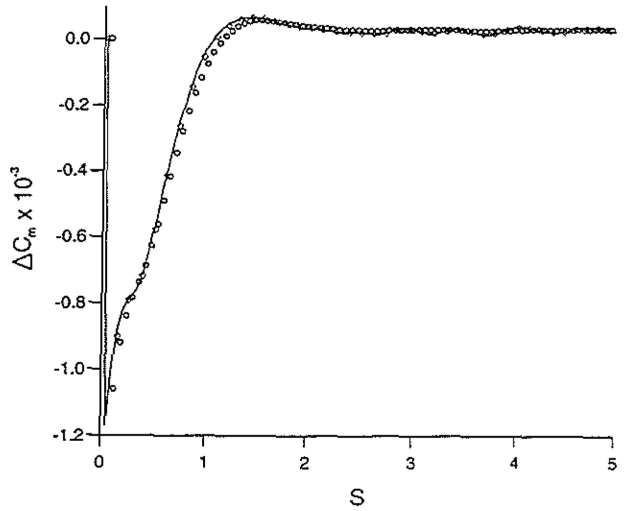


Figure 4(b) Sensitivity of pitching moment response to time step size (M=0.50+0.01)

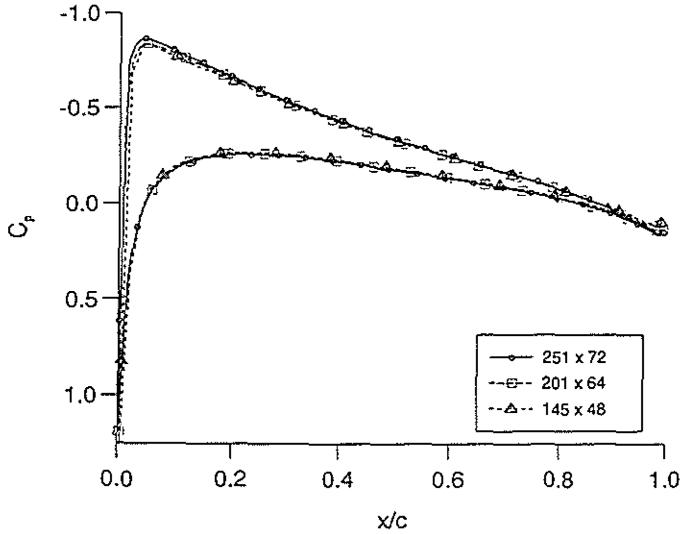


Figure (6) Sensitivity of computed pressure distribution to grid density (M=0.50)

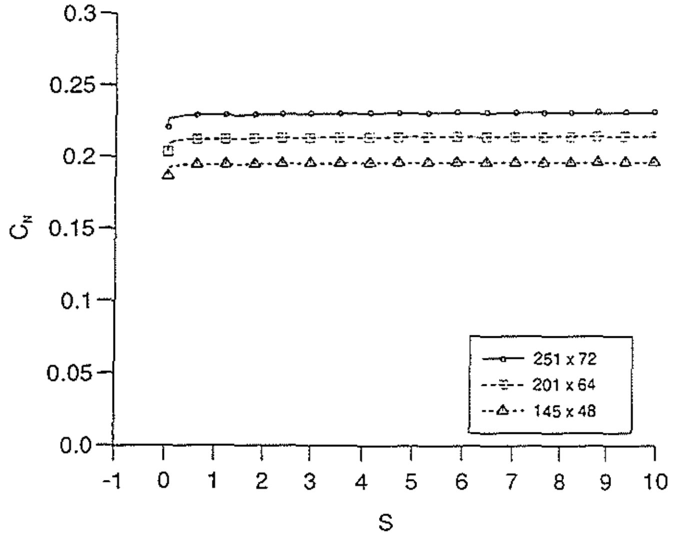


Figure 5(a) Sensitivity of normal force response to grid density (M=0.50+0.01)

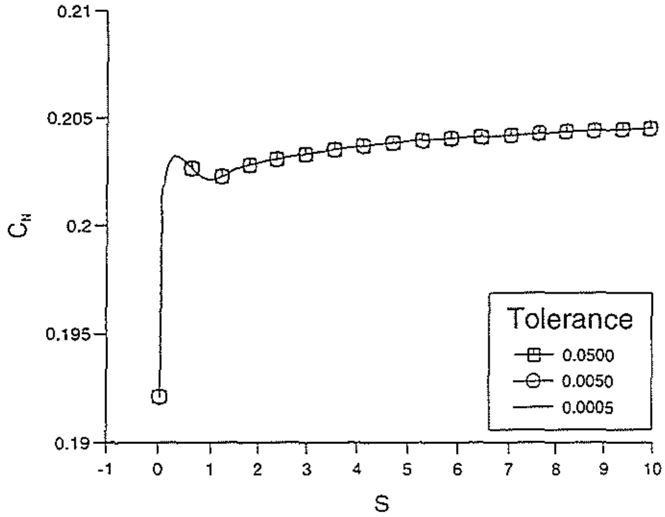


Figure 7(a) Sensitivity of normal force response to solver tolerance (M=0.50+0.01)

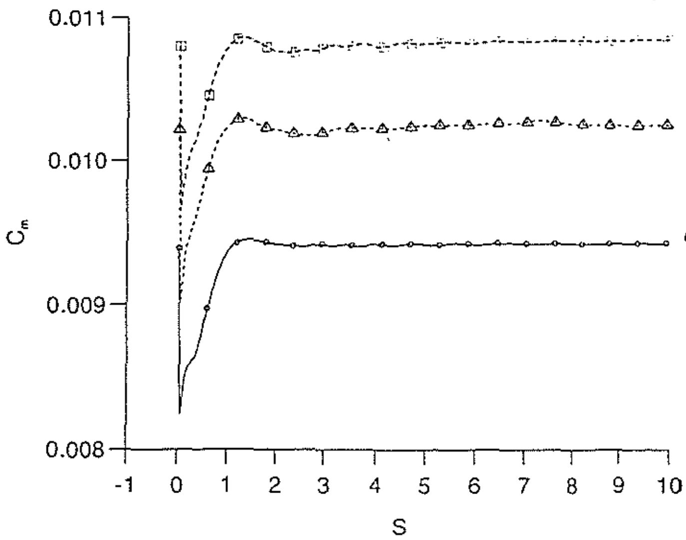


Figure 5(b) Sensitivity of pitching moment response to grid density (M=0.50+0.01)

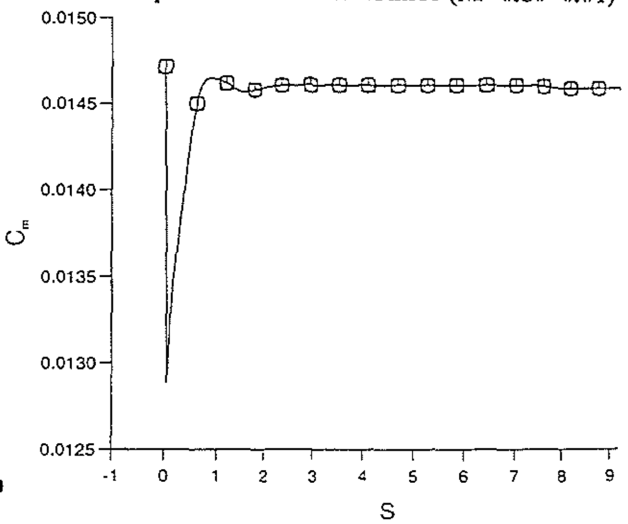
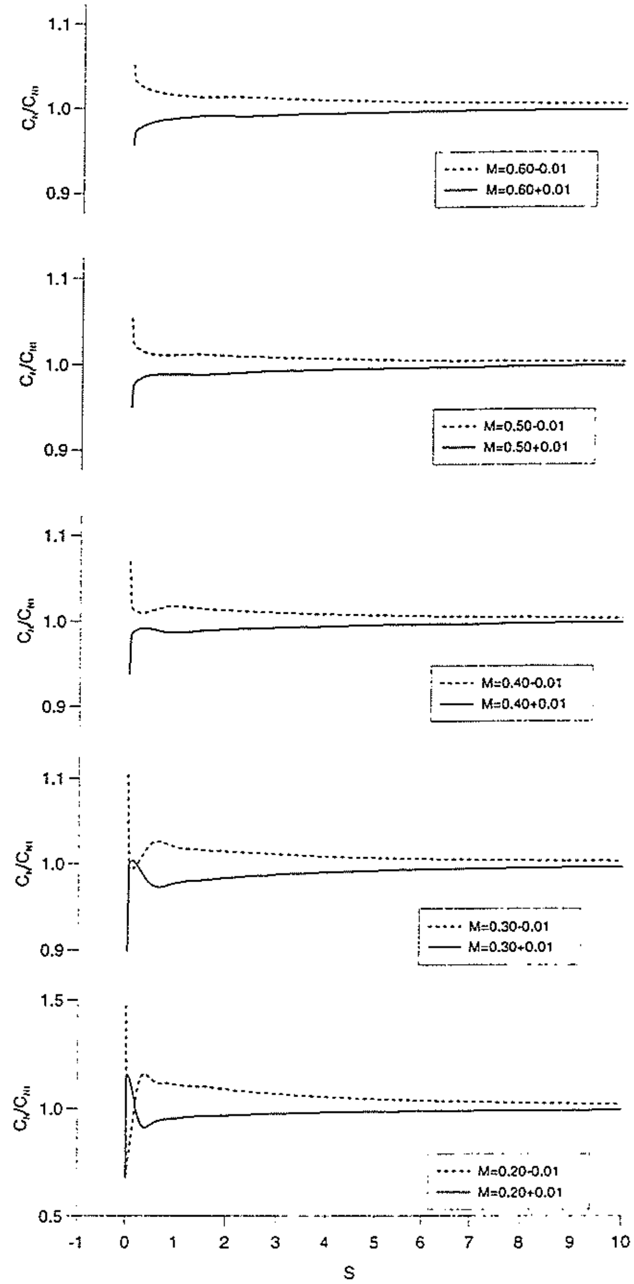


Figure 7(b) Sensitivity of pitching moment response to solver tolerance (M=0.50+0.01)



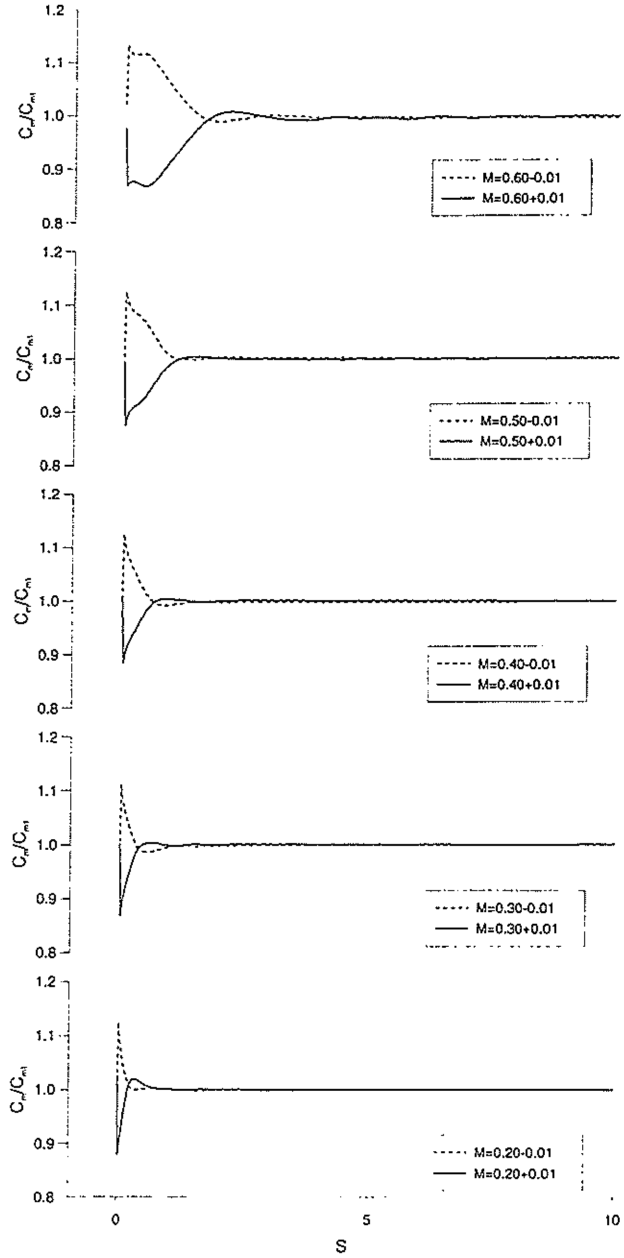
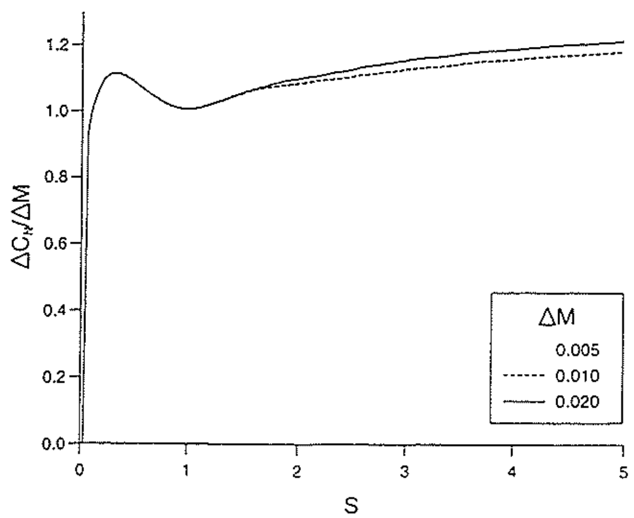


Figure 8(a) Calculated normal force coefficient response ($\Delta M=\pm0.01$)

Figure (8b) Calculated pitching moment coefficient response ($\Delta M=\pm0.01$)



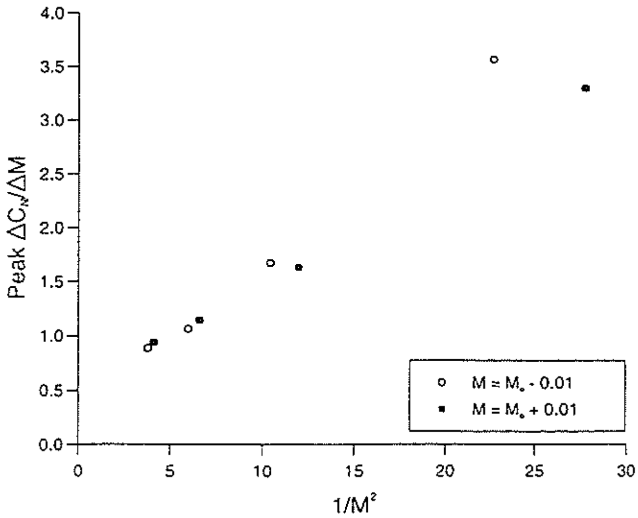


Figure (9) Sensitivity of response to step change in Mach number ($M=0.4+\Delta M$)

Figure (10) Correlation of peak normal force with Mach number ($\Delta M=0.01$)