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HELICOPTER ROTOR DYNAMICS AND AEROELASTICITY: SOME KEY IDEAS AND INSIGHTS

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Abstract

The purpose of this paper is to provide a detailed discussion of four important current topics in helicopter rotor dynamics and aeroelasticity. These topics are: (1) the role of geometric nonlinearities in rotary-wing aeroelasticity; (2) structural modeling, free vibration and aeroelastic analysis of composite rotor blades; (3) modeling of coupled rotor/fuselage aeromechanical problems and their active control; and (4) use of higher harmonic control (HHC) for vibration reduction in helicopter rotors in forward flight. Hopefully this discussion will provide an improved fundamental understanding of the current state of the art. Thus future research can be focused on problems which remain to be solved instead of producing marginal improvements on problems which are already understood.

Notation

a	= two dimensional lift curve slope
b	= semi-chord
A,B	= first order system and control matrices
C_{d0}	= profile drag coefficient
$\hat{e}_x, \hat{e}_y, \hat{e}_z$	= unit vector associated with the undeformed blade, Fig. 1.
$\hat{e}'_x, \hat{e}'_y, \hat{e}'_z$	= triad $\hat{e}_x, \hat{e}_y, \hat{e}_z$ after deformation, Fig. 1
e	= blade root offset
$\tilde{i}, \tilde{j}, \tilde{k}$	= unit vector in the direction x,y, and z respectively, Fig. 1
J	= cost functional
K_x, K_y, K_z	= flap, lag and torsion springs
l	= length of flexible portion of blade
M_F	= nondimensional fuselage mass
R	= blade radius
[S]	= transformation matrix between deformed and undeformed triads of unit vectors, Eq. (1)
S_{ij}	= elements of matrix [S]
t	= time
T	= n x m HHC transfer matrix
u,v,w	= components of the displacement of a point on the elastic axis of the blade, Fig. 1
W_z	= diagonal weighting matrix on vibrations
W_θ	= diagonal weighting matrix on control amplitudes

$W_{\Delta\theta}$	= diagonal weighting on control rate of change
x_A	= offset between blade aerodynamic center (A.C) and elastic axis (E,A)
x_I	= offset between blade center of mass (C.G) and elastic axis
x,y,z	= coordinates shown in Fig. 1
x,u	= system state and control vectors
Z	= n x 1 vector of vibration amplitudes
Z_0	= n x 1 vector of baseline vibrations
β_p	= precone angle
ε	= basis for order of magnitude, associated with typical elastic blade slope
θ_{HH}	= HHC control angle
$\Delta\theta(i)$	$= \theta(i) - \theta(i-1)$
θ	= total pitch angle
θ_0	= collective pitch
θ_{1s}, θ_{1c}	= cyclic pitch components
θ_{pk}	= pitch of the kth blade
$\theta_{0S}, \theta_{CS}, \theta_{SS}$	= amplitudes of HHC sine input in collective, longitudinal, and lateral control degrees of freedom
$\theta_{0C}, \theta_{CC}, \theta_{SC}$	= amplitudes of HHC cosine input in collective, longitudinal, and lateral control degrees of freedom
$\lambda, \lambda_{1C}, \lambda_{1S}$	= inflow ratio, and its cyclic components
μ	= advance ratio
ϕ	= rotation of cross section of blade around the elastic axis
ψ	= azimuth angle of blade, $\psi = \Omega t$
Ω	= speed of rotation
$\underline{\omega}_{HH}$	= HHC frequency
$\omega_{L1}, \omega_{T1}, \omega_{F1}$	= inplane, torsional and flapwise nondimensional fundamental frequency
$\bar{()}$	= nondimensionalized, quantity, frequencies by Ω, length by l
$\dot{()}$	$= \partial/\partial\psi$

1. Introduction

During the last two decades, since Loewy's [1] comprehensive review of rotary-wing dynamic and aeroelastic problems was published a vast body of published research, on these topics has appeared in the literature. This substantial body of research has also been discussed in a considerable number of survey papers which have emphasized various aspects of the rotary-wing aeroelastic stability and response problem together with the related vibration problems and their control by active and passive means. For convenience these survey papers [1-13] are cited in chronological order. Loewy's survey [1] was followed by a more restrictive survey by Dat [2] which discussed unsteady aerodynamic and vibration problems in forward flight. Hohenemser [3] discussed some aeromechanical stability problems as part of the broader field of flight mechanics. Friedmann [4,5] presented a detailed chronological discussion of rotary-wing aeroelasticity emphasizing the role of geometrical nonlinearities, due to moderate blade deflections, unsteady aerodynamics and forward flight. A similar discussion which was restricted to the case of hover and hingeless and bearingless rotors was also provided by Ormiston [6]. In addition to these papers which have emphasized primarily aeroelastic stability, two other surveys [7,8] have dealt exclusively with the vibration problem and its active and passive control in rotorcraft.

More recently Johnson [9,10] has published a comprehensive review paper which described both the aeroelastic stability and rotorcraft vibration problems in the context of dynamics of advanced rotor systems. Friedmann [11] has described the main developments between 1983-87 emphasizing new methods for formulating aeroelastic problems, treatment of the forward flight problem, coupled rotor/fuselage analyses, structural modeling and structural optimization, and the use of active controls for vibration reduction and stability augmentation. A more restrictive review emphasizing some practical design aspects capable of alleviating aeromechanical problems was presented by Miao [12]. Finally it should be noted that a very comprehensive research report [13] has been published recently which contains a detailed review of research carried out under Army/NASA sponsorship, between 1967 and 1987.

The purpose of this paper is not to present another review of the literature in this field. Instead the main objective here is to discuss four specific and important topics in rotor dynamics and aeroelasticity where the body of available research has reached a sufficient level of maturity for keys ideas to emerge. The emergence of these key concepts provides good insight on the course of research which needs to be conducted on these topics in the future.

The four topics which will be discussed in this paper are briefly described below:

1. *The role of geometric nonlinearities in rotary-wing aeroelasticity.* Methods for the effective formulation of the equations governing the rotary-wing aeroelastic problem are described and the importance of third and higher order geometrically nonlinear terms is discussed.
2. *Structural modeling, free vibration and aeroelastic analysis of composite rotor blades.* Available structural and structural dynamic models for composite rotor blades are discussed including geometrically nonlinear terms and general cross-sectional geometries.
3. *Modeling of coupled rotor/fuselage aeromechanical problems and their active control.* Various approaches for formulating such equations in forward flight are discussed. Various control approaches for stabilizing air resonance in hover and forward flight are considered. The active control of the flap-lag problem and coupled flap-lag-torsional problem are also briefly discussed.
4. *Use of higher harmonic control for vibration reduction in helicopter rotors in forward flight.* Available active control techniques for vibration reduction in forward flight are discussed. A new aeroelastic simulation capability for higher harmonic control is presented together with some important conclusions obtained from this research.

It is expected that the discussion of the four topics described above will provide an improved understanding of the state of the art. Thus future research can be focused on problems which remain to be solved.

2. The Role of Geometric Nonlinearities in Rotary-Wing Aeroelasticity

During the last twenty years it has been firmly established that geometrical nonlinearities, due to moderate blade deflections, play an important role in the aeroelastic stability analysis of hingeless and bearingless rotor blades [4-6,9-11,13]. The role of these terms for articulated blades is slightly less important but still significant [1,2,12-13].

As described in Refs. 5, 9-11, and 13 a number of beam theories accounting for moderate as well as large deflections of blades have been developed and are available. These theories can be divided into two basic categories: (a) those which are based on ordering schemes and are valid for moderate deflections, and (b) those which are valid for large deflections, and are not based on ordering schemes. The moderate deflection theories have been developed primarily between 1970-80 and the large deflection theories have been developed since 1980. It is therefore quite relevant to try and determine the significance of higher order geometrical nonlinearities, such as third order and higher order nonlinear terms, on the aeroelastic stability of hingeless and bearingless rotor blades in hover and forward flight.

The source and structure of the geometrically nonlinear terms associated with *moderate blade deflections*, is conveniently illustrated by the transformation between the triad of unit vectors describing the deformed and undeformed state of a hingeless rotor blade, see Figs. 1a and 1b. Such a transformation, based on the assumption of small strains and finite rotations (slopes) has the following mathematical form [5]

$$\begin{Bmatrix} e'_x \\ e'_y \\ e'_z \end{Bmatrix} = [S] \begin{Bmatrix} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{Bmatrix} \tag{1}$$

where the elements of the transformation matrix [S] determine the accuracy or order of the theory. For example for a second order type theory S_{ij}'s are given by: (2)

$$S_{11} = 1; \quad S_{12} = v,_x; \quad S_{13} = w,_x$$

$$S_{21} = -(v,_x + \phi w,_x); \quad S_{22} = 1; \quad S_{23} = \phi$$

$$S_{31} = -(w,_{xx} - \phi v,_x); \quad S_{32} = -(\phi + v,_x w,_x); \quad S_{33} = 1$$

Transformations of this type combined with the Euler Bernoulli assumption have been used as the basis for moderate deflection beam theories which are suitable for the aeroelastic stability and response analysis of isotropic hingeless and bearingless rotor blades [5,11,13].

When such transformations are incorporated in the derivation of the inertia and aerodynamic operators associated with the rotary wing aeroelastic problem many, relatively small, nonlinear terms emerge. Such terms clearly represent considerable complication in the equations of motion from an algebraic point of view. The final equations of motion can be obtained using two different approaches [11].

One approach is based on generating the equations of motion in explicit form. The explicit approach is frequently combined with an ordering scheme [5] to "manage" the large number of terms associated with geometric nonlinearities. The purpose of the ordering scheme is to provide a rational basis for neglecting higher order terms in a consistent manner. A suitable *example* for an ordering scheme used in deriving the equations of motion for a hingeless rotor blade in forward flight is given below. Orders of magnitude are assigned to the various parameters of the problem in terms of elastic blade slopes which are assumed to be moderate, i.e., slopes are of order ε, with $0.10 \le \varepsilon \le 0.20$, and the assumption

$$0(1) + 0(\varepsilon^2) \cong 0(1) \tag{3}$$

is used when deriving the equations.

For the coupled flap-lag-torsional problem in forward flight the orders of magnitude for the relevant quantities are listed below:

$$w,_x = v,_x = \phi = 0(\varepsilon); \quad \lambda = \lambda_{1s} = \lambda_{1c} = \frac{w}{R} = \frac{v}{R} = 0(\varepsilon)$$

$$\frac{e}{R} = \frac{b}{R} = \beta_P = 0(\varepsilon); \quad \mu = 0(1); \quad \theta = \theta_{1s} = \theta_{1c} = 0(\varepsilon^{1/2}) \tag{4}$$

$$\frac{u}{R} = \bar{x}_I = \bar{x}_A = 0(\varepsilon^2); \quad c_{do/a} = 0(\varepsilon^{3/2}); \quad \frac{x}{l} = \frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial \psi} = 0(1)$$

It is important to note that ordering schemes are not unique, a somewhat different combination of parameters [14] can be also used as the basis of an ordering scheme for the same coupled flap-lag-torsional problem of a hingeless rotor blade in forward flight. Furthermore ordering schemes are based on common sense and experience with practical blade configuration thus their application requires both care and a certain degree of flexibility.

Ordering schemes are also convenient when using general purpose algebraic manipulators, such as MACSYMA [15] to derive equations of motion in explicit form. Since algebraic tasks are relegated to a computer, it is fairly easy to retain additional higher order terms. For example by changing the basic assumption in the ordering scheme to

$$0(1) + 0(\varepsilon^3) \cong 0(1) \tag{5}$$

one can include the next group of higher order terms. For this case the various elements of the transformation matrix S_{ij}, Eq (1), can be rewritten as [20]

$$S_{11} = 1 - \frac{1}{2}(v,_x^2 + w,_x^2); \quad S_{12} = v,_x; \quad S_{13} = w,_x$$

$$S_{21} = -(v,_x + \phi w,_x + \frac{1}{2} v,_x w,_x^2): \quad S_{22} = 1 - \frac{1}{2} v,_x^2 - \phi v,_x w,_{x;}$$

$$S_{23} = \phi - \frac{1}{2} w,_x^2 \phi; \quad S_{31} = -(w,_x - \phi v,_x - \frac{1}{2} v,_x^2 w,_x) \tag{6}$$

$$S_{32} = -(\phi + v,_x w,_x - \frac{1}{2} v,_x^2 \phi); \quad S_{33} = 1 - \frac{1}{2} w,_x^2$$

For this case explicit derivation of the equations of motion is feasible only when using symbolic algebraic manipulation on a computer.

Such explicit derivations of the equation of motion have a number of advantages. First it enables one to write out the equations of motion with considerable detail. Thus one can inspect the equations and identify the coupling terms introduced by retaining the geometrically nonlinear terms. Physical interpretation of such coupling terms facilitates the understanding of equations. Furthermore equations derived by various researchers can be compared with each other and differences between various formulations can be clarified. Another advantage is associated with the computational efficiency inherent in such formulations. Numerical implementation of blade stability and response calculations based on such equations are more efficient and computer storage requirements are reduced. This represents an advantage in structural optimization studies of rotor blades with aeroelastic constraints [16-18] which require repetitive evaluation of the objective functions and the constraints.

A second, and more recent, approach to generating rotary-wing equations of motion is based on the implicit approach [11]. In this approach the equations of motion are never explicitly written down and they are generated numerically by the computer during the solution process. This approach is particularly effective when combined with the finite element approach, for the spatial discretization of the equation. This approach allows the treatment of complicated configurations, provides considerable flexibility in the representation of the aerodynamic loads [19], and does not require the use of ordering schemes and the inherent approximations associated with such ordering schemes.

More recent studies use small strain and large rotation type of analyses [21-23] which utilize Rodrigues parameters to represent finite rotations. This approach when coupled with an implicit formulation completely eliminates the need for using an ordering scheme. Furthermore it also removes the need to use the torsional quasi-coordinate which was present in a previous formulation [24] and created complexity in the finite element models based on this formulation. Use of Euler angles combined with an implicit formulation is equally effective for representing the large displacement of both isotropic or composite beams [25]. These models are more consistent and mathematically more elegant than blade models based on ordering schemes. At the same time the incorporation of such models into a general analysis is more complicated. Thus to date the only aeroelastic analysis capability based on such general formulation is the GRASP program [26].

Based on the discussion presented above one can now assess the importance of third and higher order terms in rotordynamic models. For the case of aeroelastic stability of hingeless rotor blades in hover a recent study [27,28] has investigated the influence of third order nonlinear terms, similar to those in Eq. (6) when compared to second order nonlinear terms, similar to those in Eq. (2). It was concluded that for torsionally soft blades ($\omega_{T1} = 2.5$) and high collective pitch settings ($\theta_0 > 0.20$) the effect of these higher order terms on the stability boundaries was fairly small (less than 10%). Thus the effect of third order, and higher order nonlinear terms, on blade aeroelastic stability appears to be limited. The retention of higher order nonlinear terms can be useful in the study of mathematical properties of the transformations which relate the position vectors of the deformed and undeformed states of a blade undergoing nonlinear deformation [29].

Another interesting question which can be asked is whether dynamic models of rotor blades which are based on nonlinear beam kinematics with finite large rotations are superior to previous moderate rotations theories, based on ordering schemes? There is no unique answer to this intriguing question. When one uses criteria based on mathematical elegance, consistency and accuracy large deflection theories have a slight advantage because the structural and inertia operators obtained when using these theories are slightly more accurate. However, it is essential to note that in order to complete the formulation of aeroelastic problem the unsteady aerodynamic loads have to be combined with the inertia and the structural parts. Previous reviews of the unsteady aerodynamics used in aeroelastic problems [2,5,9-11,13] have clearly indicated that the aerodynamic theories used for this purpose are *linear* theories, except when one attempts to model dynamic stall, or transonic effects. Therefore the combination of kinematical model which contains third order nonlinear terms, with a linear incompressible aerodynamic theory such as Greenberg's theory [27,28] does not produce necessarily a consistent aeroelastic model. This point is well illustrated by a recent study by Hodges, Kwon and Shankar [30] where it was shown that by combining three dimensional tip loss and unsteady inflow effects with a conventional moderate deflection theory remarkable agreement between theoretical and experimental results were obtained. In Fig. 2, taken from Ref. 30, the lead-lag damping of a stiff-in-plane hingeless rotor blade, with configuration parameters chosen from the experimental model rotor with soft pitch flexure, zero precone and droop, is shown for various collective pitch angles.

Since both theoretical predictions have the same structural model, the differences are due to aerodynamics. Clearly the results with the three dimensional unsteady aerodynamics, denoted as panel method in Fig. 2, give much better agreement with the experimental data than those based on two dimensional quasi-steady aerodynamics, denoted 2-D theory. This example indicates that the key to substantial improvements in the aeroelastic modeling capability of rotor blades is linked to improved unsteady aerodynamic models, and not to the retention of third and higher order geometrically nonlinear terms.

Another basic inconsistency in the formulation of beam models for moderate and large deflections is associated with the fact that frequently excessive preoccupation with geometrically nonlinear terms leads to the neglect of simple structural effects such as those associated with Timoshenko beams, namely shear and rotary inertia.

Using a finite element model capable of capturing Timoshenko beam effects in an accurate manner [31] and calculating the influence of Timoshenko beam effects on the first three flapwise and first three in-plane of rotation frequencies of graphite-epoxy beam having the cross-sectional dimensions shown in Fig. 3, with a length of 15 feet and rotating at $\Omega = 400$ RPM, yields the results presented in Table 1. It is evident that for this particular case the influence of shear and rotary inertia on the lead-lag or in-plane of rotation modes, and in particular the frequency of the second and third mode can be significant. Thus incorporation of such effects could be of equal importance to third order geometrical nonlinearities. Obviously the incorporation of true anisotropy, present in composite rotor blades is a much more important topic. This topic is the subject matter of the next section.

3. Structural Modeling, Free Vibration and Aeroelastic Analysis of Composite Rotor Blades

Most of the structural models developed to date have been restricted to isotropic material properties. On the other hand modern helicopter blades are frequently built of composites. To remedy this situation a substantial share of the recent studies in this field has been aimed at the development of models which are suitable for the structural and aeroelastic analysis of composite rotor blades. The important attributes of such a structural model, require the capability to represent transverse shear deformation, cross-sectional warping and elastic coupling, in addition to an adequate representation of geometric nonlinearities. Rotor blades are typically modeled as a beam. In a beam theory, the deformations of the cross-section, both in and out of the plane, are assumed to be either small or neglected. Therefore, an approach commonly used in the available structural models for composite rotor blade analysis is to determine the cross section warping functions, shear center location and cross sectional properties based on a linear theory. The linear, two-dimensional analysis for the cross-section is decoupled from the nonlinear, one-dimensional global analysis for the beam and can be done once for each cross section of a non-uniform beam. This decoupling is usually assumed in the literature without rigorous proof. The discussion of composite rotor blade structural modeling can, therefore, be divided into two categories: (1) Modeling approaches which lead to the determination of the stiffness properties of arbitrary blade cross sections. Anisotropic materials and the composite nature of the blade are taken into account in this category. (2) Structural models which use an one-dimensional beam kinematics suitable for composite rotor blade analysis. A typical structural model in this category should include geometric nonlinearities, pretwist, transverse shear deformation and cross section warping. Many of the existing composite rotor blade models in category (1) were reviewed in detail in a recent review paper by Hodges [32]. Our objective here is not to duplicate Hodges' review, but to consider the subject from a slightly different perspective.

Mansfield and Sobey [33] initiated the first pioneering study of this difficult subject. They developed the stiffness properties of a fiber composite tube subjected to coupled bending, torsion, and extension. Transverse shear and warping of the cross section was not included in the model. This model is too primitive to be suitable for composite rotor blade aeroelastic analysis. However, the authors made some attempts to explore the potential of this model for the innovative idea of aeroelastic tailoring.

Rehfield [34] used a similar approach but included out-of-plane warping and transverse shear deformation. This was strictly a static theory for a single cell, thin walled, closed cross section composite, with arbitrary layup, undergoing small displacements. This model was suitable for preliminary design, or frequency tailoring studies, as well as linear free vibration analysis of nonrotating composite beams. Pretwist, dynamic effects and geometric nonlinearities which are known to play a major role in helicopter rotor dynamics are not accounted for. This relatively simple theory was correlated by Nixon [35] with experimental data and by Hodges, Nixon and Rehfield [36] with a NASTRAN finite element analysis of a beam model with a single closed cell. Wörndle [37] developed a linear, two-dimensional finite element model to calculate the cross section warping functions of a composite beam under transverse and torsional shear. With these warping functions, the shear center locations and the stiffness properties of the cross section could be calculated. The cross section can have arbitrary shape but the material properties were restricted to monoclinic.

A more general model for calculating the shear center and the stiffness properties of an arbitrarily shaped composite cross section was developed by Kosmatka [38]. He used a two-dimensional isoparametric eight node quadrilateral finite element model to obtain the St. Venant solution of the cross-section warping functions of a tip loaded composite cantilever beam with an arbitrary cross section. The beam was assumed to be prismatic (axially uniform), nonhomogeneous, and anisotropic. The blade material was generally orthotropic, i.e., orthotropic material whose material principal axes are oriented arbitrarily. Therefore, the beam behaves in an anisotropic manner. Subsequently this model was combined with a companion moderate deflection beam theory suitable for the structural dynamic analysis of advanced prop-fan blades and helicopter rotors, which will be discussed later in this section.

Giavotto, et al. [39] also formulated a two-dimensional finite element model for determining cross section warping functions, shear center location and stiffness properties. A special feature of this formulation is that the resulting equations have both extremity solutions and central solutions. The central solutions correspond to the warping displacements due to applied loads without considering end effects, while the extremity solutions correspond to the warping displacements due to end effects. Subsequently this work was extended by Borri and Merlini [40] to include the so-called geometric section stiffness associated with large displacement formulations. Bauchau [41] developed a beam theory for anisotropic materials based on the assumption that the cross section of the beam does not deform in its own plane. The out-of-plane cross section warping was expressed in terms of the so-called eigenwarpings. This theory is valid for thin-walled, closed, multi-celled beams with transversely isotropic material properties. Subsequently it was extended by Bauchau, Coffenberry and Rehfield [42] to allow for general orthotropic material properties.

All the studies described in this section up to this point employed a separate two-dimensional analysis to determine the cross sectional warping functions and stiffness properties. For non-uniform beams, such a two-dimensional analysis is carried out once for each cross section. A new approach developed by Lee and Kim [43] and Stemple and Lee [44] uses a finite element formulation which can represent thin-walled beams with arbitrary cross sections, general spanwise taper and planform distributions and allows arbitrary cross section warping. This was accomplished by distributing warping nodes over the cross section situated at the node of regular beam type finite element. Thus the treatment of the cross section warping is coupled with the treatment of the beam bending, torsion and extension. This formulation considered only the out-of-plane warping and linear problems, or small deflection problems.

Recently, Stemple and Lee [45] have extended this formulation to allow for large deflection static and free vibration analysis of rotating composite beams. The disadvantage of this approach is that the analysis is more expensive than those whose cross section analysis is decoupled from the nonlinear beam analysis. Furthermore the numerical results obtained from this theory were not compared to other available theories, thus the validity of the theory still remains to be determined.

The structural theories discussed so far emphasize the modeling approach associated with category (1), where the emphasis is on determining the shear center, warping and cross-sectional properties of the composite cross section. For category (2) structural modeling, where the emphasis is one-dimensional beam kinematics suitable for the analysis of composite rotor blades, two types of theories are available depending on the level of geometric nonlinearity being retained in the one-dimensional beam kinematics. The first type is based on a moderate deflection type of theory while the second type is capable of modeling large deflections. Moderate deflection theories usually rely on an ordering scheme to limit the magnitude of blade displacements and rotations. While large deflection theories do not utilize an ordering scheme to limit the magnitude of blade displacements and rotations. For such theories the only assumption used to neglect higher order terms is the assumption that the strains are small.

In rotary wing aeroelasticity moderate deflection theories are usually adequate provided that a consistent ordering scheme is used. The first aeroelastic model for a composite rotor blade in hover was presented in a comprehensive study by Hong and Chopra [46]. In this specialized model, the blade was treated as a single-cell, laminated box beam composed of an arbitrary lay-up of composite plies. The strain-displacement relations for moderate deflections were taken from Hodges and Dowell [24], which does not include the effect of transverse shear deformations. Each lamina of the laminate was assumed to have orthotropic material properties. The equations of motion were obtained using Hamilton's principle. A finite element model was used to discretize the equations of motion. Numerical results for the coupled flap-lag-torsional behavior of hingeless rotor blades clearly illustrated the strong coupling effects introduced by the composite nature of the blade. These coupling terms which depend on fiber orientation have a strong influence on blade stability boundaries in hover. Subsequently this analysis was extended to the modeling of composite bearingless rotor blades in hover [47] and a systematic study was carried out to identify the importance of the stiffness coupling terms on blade stability with fiber orientation and for different configurations. In this model the composite flexbeam of the bearingless rotor blade was represented by an I-section consisting of three laminates. Each laminate is composed of an arbitrary lay-up of composite plies. The outboard main blade and the torque tube were assumed to be made of isotropic materials. Thus this model represents a somewhat idealized model for a composite bearingless rotor blade. In addition to aeroelastic stability studies of composite rotor blades in hover, Panda and Chopra [48] also studied the aeroelastic stability and response of hingeless composite rotor blades in forward flight using the structural model presented in Ref. 46. It was found that ply orientation is effective in reducing both blade response and hub shears.

A more comprehensive analysis for the structural dynamic modeling of composite advanced prop-fan blades, which with some modifications, is also suitable for the general modeling of curved, pretwisted composite rotor blades was developed by Kosmatka [38]. The cross section geometry of the blade is arbitrary, and the associated cross section stiffness properties and shear center location can be obtained from the accompanying linear two-dimensional finite element model which has been discussed earlier in this section. In the one-dimensional, nonlinear analysis, the curved pretwisted blade was modeled by a series of straight beam elements which are aligned with the curved line of shear centers of the blade. Each beam finite element was derived using Hamilton's principle and the following basic assumptions: the beam has an arbitrary amount of pretwist, undergoes moderate deflections, is composed of generally orthotropic materials, has an arbitrary cross sectional shape, and rotates about a vector in space. Numerical results for frequencies and mode shapes obtained from this structural dynamic model were in good agreement with modal tests on conventional and advanced propellers [49,50]. Bauchau and Hong [51-53] have developed a series of large deflection composite beam models which are intended for rotor blade structural dynamic and aeroelastic analysis. The first of these models [51] used a finite element approach combined with a general global coordinate system. While the model was general it also proved itself to be computationally inefficient. A second version of this model [52] used a curvilinear coordinate system and the resulting finite element model was found to be computationally more efficient than the first one. However some deficiencies associated with the derivation of the strain displacement relation were noted by Hong in his dissertation [54]. After additional improvements in the model a final version of this theory which is capable of modeling naturally curved and twisted beams undergoing large displacements and rotations and small strains was developed. The kinematics of this theory is an extension of the common approach, using the definition of Green strains, to include effects such as small initial curvature, transverse shear deformations and out-of-plane warpings. The fundamental assumptions in the kinematics are the indeformability of the cross-section in its own plane and a revised small strain assumption. In this revised small strain assumption, both axial and shearing strains are still neglected compared to unity, however, no assumption is made about the relative magnitude between the axial and shearing strains. Therefore, the second order shear strain coupling terms in the axial strain expression are retained under this revised small strain assumption. The commonly used small strain assumption, which include an additional assumption that the axial and shearing strains are of the same order of magnitude, was often successfully used in beam models with isotropic or slightly anisotropic materials. However, Bauchau and Hong [53] showed that it might not be adequate for beams with highly anisotropic material by comparing the analytical and experimental results of a thin-walled kevlar beam. The strain-displacement relations were derived using an implicit formulation with seven unknown functions which depend on the space coordinate, namely three displacement components, three rotation parameters (Euler angles were used), and the amplitude of the torsional warping. The three rotation parameters, which are used to describe the large rotations from the undeformed to the deformed triad of unit vectors, are defined implicitly in the rotation matrix instead of appearing explicitly in the strain-displacement relations. An extension of this model for free vibration analysis can be found in Hong's dissertation [54]. Using this model a number of cases testing the static large deflection capability and the free vibration capability of the model were computed. However an aeroelastic analysis of a rotor blade, based on this model, is not available to date.

Recently Minguet and Dugundji [55,56] have developed a large deflection composite blade model for static [55] and free vibration [56] analysis. Large deflections are accounted for by using the Euler angles to describe the transformation between a global and local coordinate system after deformation. However, transverse shear deformation, and cross section warping were not incorporated in this model. Thus the model is more suitable for the study of flat composite strips than actual rotor blades.

Atilgan and Hodges [57] have recently presented a theory for nonhomogeneous, anisotropic beams undergoing large global rotation, small local rotation and small strain. They used a perturbation procedure to obtain a linear two-dimensional cross section analysis which is decoupled from the nonlinear one-dimensional global analysis. The nonlinear beam kinematics was based on a study by Danielson and Hodges [58]. The nonlinear beam kinematics, which defines a conjugate stress measure, describes a constitutive relationship and develops the momentum balance conditions, was based on Atluri [59]. The cross section warping, both near the ends (boundary layer solutions) and away from the ends (St. Venant solutions) was obtained in terms of force and moment strains from a linear two-dimensional equation. The three force strains include axial and both transverse shear strains at the reference line; the three moment strains are torsion and curvature quantities [58]. Since the strain components from the nonlinear beam kinematics are expressed in terms of force and moment strains, and cross section warpings, it is possible to obtain a one-dimensional system of equations in terms of force and moment strains only by substituting the solutions for the cross sectional warpings into the strain expressions. However, the authors abandoned this procedure as being too tedious and unnecessary. Instead, they chose to follow the same approach as Giavotto, et al. [39], which solves the cross section warping functions in terms of force and moment stress resultants. A geometric stiffness matrix was also formulated in this model. This matrix and the material stiffness matrix were intended to be used in an incremental updated Lagrangian formulation for large deflections. A general mixed variational principle was used in the formulation of the one-dimensional, geometrically nonlinear global analysis. No numerical results were presented in this paper. This study draws rather heavily on the material presented in Refs. 39 and 59 and its principal contribution seems to be the derivation of a geometric stiffness matrix.

From this review of the literature on available structural models capable of representing composite rotor blades it is evident that this has been an active area of research during the last five years. Yet it is remarkable to note that despite the availability of such models the only published body of research which actually contains aeroelastic stability and response type of results is that published by Chopra and his associates [46-48]. This is somewhat disappointing since the potential for aeroelastic tailoring for composite rotor blades has been demonstrated [46]. A typical result, Fig. 4, taken from Ref. 48, shows that for a four bladed hingeless soft-in-plane composite rotor, where the composite single cell box has symmetric laminates, the ply orientation angle, Λ_v^s, for the vertical walls of the box has a very significant effect on the peak-to-peak value of nondimensional vertical hub shears. This quantity is indicative of the vibration levels experienced at the hub. It is interesting to note that both positive and negative ply angles reduce the oscillatory hub shear, however positive angles of 30° are more effective in reducing vibration levels. In view of the positive nature of coupling effects present in composite blades one would expect to see more studies of this type which take advantage of the vibratory load reduction potential present in composite blades.

Instead of the excessive preoccupation with arbitrarily large deflection theories for composite beams it appears to be more sensible to derive realistic composite blade models, based on moderate deflection theories, and use them to explore the potential for designing composite rotor blades which have low vibration levels and good aeroelastic stability margins. Based on the present state-of-the-art it appears that a moderate deflection type of beam theory combined with a cross-sectional analysis of the composite blade presented in Refs. 38 or 39 would be both suitable and adequate for this purpose.

4. Modeling of Coupled Rotor/Fuselage Aeromechanical Problems and their Active Control

The aeromechanical instability of a helicopter, on the ground or in flight, is caused by coupling between the rotor and body degrees of freedom. this instability is commonly denoted air resonance when the helicopter is in flight and ground resonance when the helicopter is on the ground. The physical phenomenon associated with this instability is quite complex. The rotor lead-lag regressing mode usually couples with the body pitch or roll to cause an instability. The nature of the coupling which is both aerodynamic and inertial is introduced in the rotor by body or support motion. The importance of developing a mathematically consistent model capable of representing the coupled rotor/fuselage dynamic system has already been discussed in previous reviews [5,6,9-11,13]. With the use of algebraic symbolic manipulative programs or implicit formulations [26,60] sophisticated mathematical models for coupled rotor/fuselage aeromechanical problems can be formulated and solved.

The primary purpose of this section is to discuss a number of modeling aspects for this class of problems which have been shown to be important by recent research. Subsequently research on the use of active controls to stabilize this class of aeromechanical problems is also described.

Many previous air resonance studies [6,9-11,13] were limited by simplifying assumptions such as hover, blades which were assumed to be torsionally rigid or aerodynamic loads based on quasisteady assumptions. Recent studies [61,62] have clarified the role of unsteady aerodynamics, forward flight and torsional flexibility on air resonance. Furthermore the mathematical model derived for this coupled rotor fuselage system had also a provision for including an active controller capable of suppressing air-resonance.

The mathematical model of the rotor/fuselage system of Ref. 61 and 62, and its salient features are described next. The fuselage is represented as a rigid body with five degrees of freedom, where three of these are linear translations and two are angular positions of pitch and roll (Fig. 5). Yaw is ignored since its effect in the air resonance problem is known to be small. A simple offset hinged spring restrained rigid blade model is used to represent a hingeless rotor blade (Fig. 6). This assumption simplifies the equations of motion, while retaining the essential features of the air resonance problem. In this model, the blade elasticity is concentrated at a single point called the hinge offset point, and torsional springs are used to represent this flexibility. The dynamic behavior of the rotor blade is represented by three degrees of freedom for each blade, which are flap, lag, and torsion motions. The aerodynamic loads of the rotor blades are based on quasi-steady Greenberg's theory, which is a two dimensional potential flow strip theory. Compressibility and dynamic stall effects are neglected, though they could be important at high advance ratios. Unsteady aerodynamic effects, which are created by the time dependent wake shed by the airfoil as it undergoes arbitrary time dependent motion, are accounted for by using a dynamic inflow model. This simple model uses a third order set of linear differential equations driven by perturbations in the aerodynamic thrust, roll moment, and pitch moment at the rotor hub. The three states of these equations describe the behavior of perturbations in the induced inflow through the rotor plane. The dynamic inflow model coefficients used are those of Ref. 63.

The equations of motion of the coupled rotor/fuselage system are very large and contain geometrically nonlinear terms due to moderate blade deflections in the aerodynamic, inertial, and structural forces. Furthermore, the coupled rotor/fuselage equations have additional complexity due to the presence of the fuselage degrees of freedom. To reduce the equations to a manageable size, an ordering scheme is used in the derivation of the equations of motion to systematically remove the higher order nonlinear terms. The ordering scheme is based on Eq. (3). For this class of problems ε represents the slopes of the deflections of the blades, which usually are of an order of magnitude which is less than .15. The blade degrees of freedom are assigned an order of ε, while the fuselage degrees of freedom are of order $\varepsilon^{3/2}$, the various other parameters have the order of magnitude given in Eqs. (4). A symbolic manipulation program is then used to generate the nonlinear set of equations of the rotor/fuselage system using the ordering scheme. Five fuselage equations result of which three enforce the fuselage translational equilibrium and two enforce the roll and pitch equilibrium. The three resulting rotor blade equations are associated with flap, lag, and torsional motions of each blade. Also, the aerodynamic thrust and roll moments at the hub center are determined for the perturbation aerodynamics in the dynamic inflow equation. All of these equations can be found in detail in Ref. 62.

An active control device to suppress the air resonance instability through a conventional swashplate is also incorporated in this mathematical model. The pitch of the k-th rotor blade is given by the expression

$$\theta_{pk} = (\theta_0 + \Delta\theta_0) + (\theta_{1c} + \Delta\theta_{1c})\cos(\psi_k) + (\theta_{1s} + \Delta\theta_{1s})\sin(\psi_k) \tag{7}$$

The various pitch terms with the symbol Δ are small and these represent the active control inputs, while those without Δ are the inputs necessary to trim the vehicle.

The stability of the system is determined through the linearization of the equations of motion about a blade equilibrium solution and the helicopter trim solution. The helicopter trim and equilibrium solution are extracted simultaneously using harmonic balance for a straight and level flight condition [61]. After linearization, a multi-blade coordinate transformation is applied, which transforms the set of rotating blade degrees of freedom to a set of hub fixed non-rotating coordinates. This transformation is introduced to take advantage of the favorable properties of the non-rotating coordinate representation. The original representation has periodic coefficients with a fundamental frequency of unity, however, the transformed system has coefficients with a higher fundamental frequency. These higher frequency periodic terms have a reduced influence on the behavior of the system and can be ignored in some analyses at low advance ratios [5]. In hover, the original system has period coefficients with a frequency of unity, but the transformed system has constant coefficients.

Once the transformation is carried out, the system is rewritten in first order form.

$$\dot{x} = A(\psi)x + B(\psi)u \tag{8}$$

The fundamental frequency of the coefficient matrices depends on the number of rotor blades. For an odd bladed system the fundamental frequency is N_b per revolution, while for an even bladed system the fundamental frequency is $N_b/2$ per revolution. Stability can now be determined using either an eigenvalue analysis or Floquet theory for the periodic problem in forward flight. An approximate stability analysis in forward flight is also possible by performing an eigen-analysis on the constant coefficient portion of the system matrices in Eq. (8).

The mathematical model was carefully tested by comparing results to other investigator's analytical and experimental results. The correlation with these results was good and verified that the effects of torsion, unsteady aerodynamics, and forward flight were accurately represented in the model [61,63].

This mathematical model was used to analyze the behavior of a four bladed hingeless helicopter somewhat similar to the MBB 105 helicopter. The nominal configuration differs from the MBB 105 in that it has an *unstable air resonance mode.* The system has 37 states. The five body degrees of freedom and the twelve rotor degrees of freedom (three degrees of freedom for each blade) produce 34 position and rate states. The dynamic inflow model augments the system with three more states giving a total system order of 37. Thus even this relatively simple model requires a considerable number of states (or degrees of freedom).

As mentioned before the lead lag regressing mode is the critical degree of freedom associated with air resonance therefore the essential features of this instability are described by damping plots for this particular degree of freedom.

Figure 7 illustrates the influence of unsteady aerodynamics as well as the effect of periodic coefficients (or forward flight) on the lead-lag regressing mode damping of the open loop configuration. The two sets of curves represent air resonance damping of the configuration with quasi-steady aerodynamics and with dynamic inflow at various advance ratios. Dynamic inflow captures primarily the low frequency unsteady aerodynamic effect which is important for rotor/fuselage aeromechanical problems such as air resonance. The stabilizing effect in forward flight, which is evident in the figure, is consistent with behavior observed in previous studies [5,11]. For hover, the system has constant coefficients and thus the constant coefficient approximation and the periodic system produce the same results, as is clearly evident in the figure. It is also shown in the figure that the effect of periodic coefficients is relatively minor. The quasisteady aerodynamic model produces a more stable system than the model which includes the unsteady aerodynamic effects as represented by the dynamic inflow model. It is also worthwhile mentioning that considerable differences between the two models exist particularly in low advance ratios.

Figure 8 shows that neglecting the torsional degree of freedom on the nominal configuration increases the instability of the lead-lag regressing mode. The trend of the two curves also tends to diverge at high advance ratios. The addition of torsion also tends to amplify the effect of the periodic terms. At high values of advance ratio, the flap-lag-torsion model shows a much greater difference between the constant and periodic stability analysis than does the flap-lag analysis.

Coupled rotor fuselage analyses based on an implicit formulation developed by Done et al. [60] have modeled the effect of swashplate flexibility and control system stiffness on this class of problems. The limited number of numerical results presented [60] do not convey a clear picture of the influence of these additional parameters on the air resonance problem.

Another recent study by Loewy and Zotto [64] studied the effect of rotor shaft flexibility and associated rotor control coupling on the ground/air resonance of helicopters. This is of particular interest for certain types of advanced helicopters which have a relatively flexible shaft. The analysis was formulated using a Lagrangian approach and the equations were derived using symbolic manipulation based on the MACSYMA package. The equations are based initially on large Euler angles however subsequently the equations were simplified using an ordering scheme. The blades were offset hinged spring restrained with flap and lag degrees of freedom for each blade, and a four bladed rotor was considered. The model included a total of 12 degrees of freedom. Two cyclic flap modes, two cyclic lag modes, fuselage pitch and roll, fuselage center of mass translation in two directions, hub translation due to shaft flexibility in two directions, and shaft bending slope in two directions. The blades were assumed to be torsionally stiff. Only hover was considered, the aerodynamic loads were based on simple strip theory, and the periodic coefficients weres eliminated using multiblade coordinates. Numerical results were obtained for a configuration similar to the OH-58D helicopter with a four bladed articulated rotor. The effect of shaft flexibility and associated coupling terms, together with the influence of a mass simulating a mast mounted sight on air and ground resonance stability boundaries was studied with considerable detail. It was found that flexibility/control coupling adds new modes of instability for ground resonance, however this was found to be a weak instability which was eliminated by small amounts of structural damping. Air resonance type of instabilities are more susceptible to shaft flexibility/control coupling effects and small amount of structural damping fails to stabilize the coupled rotor/fuselage system. Therefore for certain combinations of parameters shaft/blade pitch coupling effects have to be designed carefully to insure the stability of the air resonance mode.

The air resonance stability of hingeless rotors in forward flight was also studied in Ref. 76. The body of research available on coupled rotor/fuselage aeromechanical problems has reached a remarkable level of maturity in a fairly short period of time. It is also evident that reliable models for this class of problem should contain coupled flap-lag-torsional blades models combined with a fuselage which has pitch and roll as well as two translational degrees of freedom. For certain configurations coupling effects introduced by swashplate flexibility, shaft bending and pitch link flexibility should be also incorporated in the model so as to obtain reliable stability boundaries. The aerodynamic representation which is most suitable for this class of problem is the dynamic inflow model [63]. Up to advance ratios of $\mu = 0.40$ the role of periodic coefficients is fairly small, and usually the most unstable cases for air resonance occur for the case of hover. For soft-in-plane hingeless helicopter configurations and articulated blade configurations forward flight usually introduces a stabilizing effect.

The derivation of coupled rotor/fuselage analyses or models has become fairly routine when using computer algebra or implicit formulations. Therefore studies based on very simple models which contain fewer degrees of freedom than those retained in Refs. 61-62 and 64 will rarely serve a useful purpose, since the validity of such models is questionable.

Improved understanding of aeromechanical phenomena such as air and ground resonance has also raised the possibility of eliminating or suppressing such instabilities using active controls. One possible means of stabilizing or augmenting stability of air resonance is through an active controller operating with a conventional swashplate. This approach is feasible from a practical point of view only if it is simple to implement since it must compete against the straightforward mechanical solution to this problem based on lag dampers. Such an active controller would need sensing and actuating devices leading to an expensive system. However, with the inevitable introduction of other active control devices such as higher harmonic control (HHC) for vibration suppression [65-67] this argument is considerable weakened. Vibration control requires sensors and actuators with bandwidths well above the 1/rev frequency. Since the air resonance instability results in an unstable lead-lag regressing mode (i.e., the mode associated with the $[1 - \omega_{L1}]$ frequency) these devices would also be sufficient for air resonance control. Thus, sensing and actuator hardware, which may be already available, could be used for additional purposes below the frequency range intended for the available vibration control objective.

Research in the active control of air and ground resonance has been limited to a few studies [68-70], where various theoretical active control studies were presented. The helicopter models used in these studies were quite limited since important effects such as torsional flexibility of the rotor blades, forward flight, and unsteady aerodynamics were all neglected. Furthermore, the studies dealing with the active control of air resonance did not adequately demonstrate the ability of the control schemes to operate through the wide range of operating conditions which can normally be encountered.

A comprehensive study which demonstrated the feasibility of designing a simple active controller capable of suppressing air resonance throughout the flight envelope representative of a wide range of operating conditions which may be encountered by a helicopter, with hingeless blades, has been completed recently [61,62,71]. The coupled rotor/fuselage model representing this system was briefly described at the beginning of this section. The equations of motion describing this system are represented by Eq. (8), and the pitch of the k-th blade, determined by the controller is expressed by Eq. (7).

The control studies undertaken consisted of two stages. In the first stage [61] the coupled rotor/fuselage system was combined with linear quadratic optimal control theory to design *full state feedback* controllers. These controllers were then used to evaluate the importance of various modeling effects on the closed loop damping of the unstable air resonance mode. It was found that periodic terms in the model seem to play only a small role at advance ratios less than, $\mu = 0.40$. With this in mind and considering the cost of extracting periodic optimal gains, it seems reasonable to neglect the periodic terms in the initial stage of controller development. Knowing that the constant model is a reasonable approximation also allows the use of many other control design techniques. The results also indicate that unsteady aerodynamics and blade torsional flexibility seem to be important modeling effects that should be included in a controller design model. Significant errors of between 25 and 50 percent in closed loop lead-lag damping could result in these effects are not included. The collective control input seems to have little influence in controlling air resonance at high advance ratios, so it was felt to be unnecessary for the complete control task. Finally, partial state feedback of the body states does not seem to be a reasonable approach to controlling air resonance. Poor lead-lag damping results and lead-lag progressive mode excitation is a possible consequence. This particular conclusion contradicts the behavior observed for articulated rotors in Ref. 69.

Since full state feedback is not practical and the first stage of the research showed that partial state feedback is not reliable, the second stage of this research [62,71] was based on more advanced control system synthesis techniques to design a suitable controller.

The controller aimed at suppressing air resonance in the flight envelope of the helicopter is based on an optimal state estimator in conjunction with optimal feedback gains [72]. A constant coefficient model is assumed since the results obtained in the first stage indicated that a periodic model was unnecessary. The objective in this portion of the study was to design a controller at an operating condition and require it to function adequately at off design conditions, corresponding to the entire flight envelop of the helicopter. Furthermore in all applications, the design model and the actual plant to be controlled will have unavoidable differences due to the limitations associated with formulating models of physical systems. To overcome these difficulties, the multivariable frequency domain design methods of Refs. 73,74 and 75 were used. This allows interpretation of the design process using frequency domain concepts and accounts for the possibility of high frequency modeling error. All this can be accomplished while retaining the structure of the state space approach. The technique, which is based on transfer function singular values, proved to be particularly effective in resolving problems that would not be obvious if only the covariance and weight matrices were used in the design process. To select the design loop shapes, Loop Transfer Recovery was used, which can be interpreted as an optimization balancing system performance requirements and the requirement of stability in presence of modeling errors.

The controller used a single roll rate measurement and both the sine and cosine swashplate inputs. This configuration is particularly simple since the measurement is taken from a non-rotating frame of (the fuselage) reference avoiding the need to send signals across the rotor head. Using sine and cosine inputs is also simple and can be accomplished through a conventional swashplate mechanism. A constant four mode design model consisting of the body roll and pitch modes and the lead-lag regressing and progressing modes was found to be quite practical for control design. The controller was shown to stabilize the system throughout a wide range of loading conditions and forward flight speeds and it required small inputs of the order of three degrees or less.

Some typical results obtained in this study, for the same four bladed hingeless rotor configuration for which results were presented in Figs. 7 and 8, are presented next. The open loop lead-lag regressing mode damping of the helicopter configuration throughout its flight regime is presented in Fig. 9. The horizontal axis is the advance ratio, while the vertical axis is the fuselage mass M_F nondimensionalized by the blade mass of 52 kg. A nondimensional fuselage mass of 32 plus four blades corresponds to the nominal total mass of 1872 kg. The figure indicates that the system experiences an air resonance instability throughout most of the flight regime. Marginal stability exists at an advance ratio greater than $\mu = 0.35$ and the point of deepest instability is at $M_F = 30$ and in the vicinity of hover. Figure 10 shows the same system after the controller designed according to the methodology [71] discussed above, has been applied on the helicopter. From the figure it is clear that the lead lag regressing mode is stable throughout the whole flight regime, and its stability is lowest in the neighborhood of $M_F = 23$ and $\mu = 0.11$.

The body of research described above indicates that during the last five years considerable progress has been made in understanding the role of active controls as a potential means for stabilizing air and ground resonance. It should be noted that air resonance is a relatively mild instability, while ground resonance is substantially stronger thus the former is easier to control. It is interesting to note that partial state feedback of the body states does not seem to be a reasonable approach to controlling air resonance in four bladed hingeless rotors [61]. This is probably due to the fact that for hingeless rotors the coupling between the blade and fuselage degrees of freedom is much stronger. A relatively simple controller using only a single body roll rate measurement and two swashplate control inputs (sine and cosine) was shown to stabilize the air resonance *throughout the whole flight envelope* when using modern state of the art control system design techniques [62,71]. It is precisely this robustness of the modern control system synthesis techniques employed which make them attractive when compared to more classical control techniques employed by Ham and his associates in their research on the use of individual-blade-control for stability augmentation [77].

Finally it should be noted that while the air resonance instability proved itself to be fairly easy to control, the active control of more powerful instabilities such as the fuselage induced flap-lag instability, and the coupled flap-lag-torsional instability of stiff-in-plane hingeless rotor blades in forward flight, which was also studied in Ref. 62, proved itself to be much more difficult to control.

5. The Use of Higher Harmonic Controls for Vibration Reduction of Helicopter Rotors in Forward Flight

The use of high frequency blade pitch inputs, referred to as higher harmonic control (HHC), to reduce helicopter vibrations has been investigated in a number of studies. Aircraft flight tests [67,78,79], wind tunnel tests [80-83], and analytical simulations [84-91] have shown that HHC is capable of substantial reduction in helicopter vibration levels encountered in forward flight.

Furthermore the literature in this field has been surveyed in a number of review articles [92,11]. Particularly noteworthy is Johnson's review article [92], which described the state of the art up to 1982 and a recent paper by Shaw et al. [82] which provides an excellent perspective on this important topic. Currently it is well understood that HHC produces vibration reduction by modifying the unsteady aerodynamic loads on the rotor blades.

It is quite remarkable that a significant portion of the research in this area involved wind tunnel testing [80-83] and flight testing [67,78,79]. While a number of analytical studies were carried out [84-91] they were less comprehensive than the tests. Previous analytical studies have generally relied on simple analog [87] or frequency domain [87] models of the helicopter response. Other studies [84-86,90,91] were based on using a fairly old aeroelastic response code, the G400 [93,94] for simulation purposes. The model used in these studies did not have a consistent representation of the geometrical nonlinearities due to moderate deflections. Other shortcomings of this simulation capacity were the lack of time domain aerodynamics needed for capturing high frequency unsteady aerodynamic effects as well as a step by step time integration method which precluded the calculation of direct stability information in forward flight, such as provided by Floquet theory.

Recently a comprehensive aeroelastic simulating capability has been developed [95-97] and used to study a number of fundamental issues in higher harmonic control. The principal topics studied using this new aeroelastic simulation capability are listed below:

1. Comparison of the effectiveness of deterministic and cautious controllers based on local and global HHC models in reducing vibratory hub shears.
2. Comparison of the response of HHC of roughly equivalent articulated and hingeless rotors.
3. Evaluation of changes in hub moments when hub shears are minimized using HHC. Similar calculations are done when HHC is used to try to reduce both hub moments and hub shears simultaneously. These studies are done for both articulated and hingeless blades.
4. Determination of the influence of HHC on the aeroelastic stability margins of the blade in forward flight.
5. Comparison of the relative additional power requirements experienced when HHC is applied to two similar rotor configurations, one with articulated and one with hingeless blades.

A brief description of the aeroelastic simulation capability, the implementation of HHC and few important conclusions obtained in the course of these studies [95-97] are presented next.

The coupled flap-lag-torsional equations of motion which serve as the basis of this analysis are similar to those derived in Ref. 98. They contain geometrically nonlinear terms due to moderate blade deflections as illustrated in Fig. 1. These equations form the basis of an implicit flap-lag-torsional undergoing small strains and moderate deflections. Thus the equations contain geometrically nonlinear terms in the structural, inertia, and aerodynamic operators associated with this aeroelastic problem. The partial dependence in the equations is eliminated by using a Galerkin type finite element method [19]. A modal coordinate transformation, using six rotating coupled modes, is performed to reduce the number of degrees of freedom. These modes are calculated at a fixed value of collective pitch which depends only on advance ratio. For the configurations considered, the six lowest modes are usually the first three flap, first two lead-lag, and the fundamental torsional modes. The ordinary differential equations are solved using quasilinearization in an iterative manner to obtain the periodic equilibrium position in forward flight, for a propulsive trim type flight condition.

The inertia loads are determined by using D'Alembert's principle. An implicit formulation for the aerodynamic loads is used. At each iteration an approximation to the blade response is produced. This response is then used to generate numerical values of the modeling quantities needed in expressions for the aerodynamic loads to be used in the next iteration. The equations are linearized by writing perturbation equations about the nonlinear equilibrium position. Stability is determined by using Floquet theory.

The unsteady aerodynamics are finite-state time-domain aerodynamics presented in Ref. 99 which are an improved version of the theory developed by Dinyavari and Friedmann [100]. Stall and compressibility effects are neglected. The aeroelastic model is combined with a new trim procedure [95,96] which provides fully coupled simultaneous solution to both the aeroelastic response and the trim problem.

The rotor dynamic model has provisions for inclusion of HHC pitch changes in the structural, inertial, and aerodynamic portions of the model to allow HHC to be input to the model as 4/rev. sine and cosine components in each of the collective, longitudinal, and lateral control degrees of freedom. The general HHC input may be expressed as:

$$\begin{aligned}\theta_{HH} &= [\theta_{0S} \sin \bar{\omega}_{HH}\psi + \theta_{0C} \cos\bar{\omega}_{HH}\psi] \\ &+ [\theta_{CS} \sin \bar{\omega}_{HH}\psi + \theta_{CC} \cos \bar{\omega}_{HH}\psi]\cos\psi \\ &+ [\theta_{SS} \sin \bar{\omega}_{HH}\psi + \theta_{SC} \cos \bar{\omega}_{HH}\psi]\sin\psi \end{aligned} \tag{9}$$

where θ_{0C}, θ_{0S}, θ_{CS}, θ_{CC}, θ_{SS}, and θ_{SC} are independent of ψ. The hub shears to be minimized are calculated by integrating the inertia and aerodynamic loads due to a given blade response over the blade. The loads due to the four blades are then combined and the total loads are transformed to the non-rotating coordinate system. Fourier series representations of these loads are then found and the 4/rev. components extracted. Alternately to calculate blade root loads the Fourier analysis is done in the rotating system at the blade root.

Operating the electro-hydraulic actuators needed to implement HHC will require power from the helicopter powerplant. In addition, the helicopter rotor may require more or less power than at the baseline condition because of the additional aerodynamic loads which are imposed on it by the HHC inputs. The total power used to operate the rotor consists of the power needed to drive the rotor and the power needed to input control pitch at the blade root.

The various contributions to the power required to actuate the blade were calculated with a level of care and detail [96,97] which was not available in previous studies.

The vast majority of all HHC investigations to date have used linear optimal control solutions based on a quadratic cost functional. Therefore this approach was also used in Refs. 95-97. Minimum variance control is based on the minimization of a cost functional which is the expected value of a weighted sum of the mean squares of the control and vibration variables.

Minimum variance controllers are obtained by minimization of the cost functional:

$$J = E\left\{Z^T(i)W_Z Z(i) + \theta^T(i)W_\theta\theta(i) + \Delta\theta^T(i)W_{\Delta\theta}\Delta\theta(i)\right\} \tag{10}$$

Typically Z, θ, and $\Delta\theta$ consist of the sine and cosine components of the N/rev. vibrations and HHC inputs. The weightings of each of these parameters may be changed to make it more or less important than the other components.

The minimum variance controllers are obtained by taking the partial derivative of J with respect to $\theta(i)$ and setting this equal to zero:

$$\frac{\partial J}{\partial\theta(i)} = 0 \tag{11}$$

The resulting set of equations may be solved for the optimal HHC input $^*\theta(i)$.

The form of the resulting algorithm will depend on whether the global or local system model is used and on whether a deterministic or cautious controller is desired.

The global model of the helicopter response to HHC assumes linearity over the entire range of control application:

$$Z(i+1) = Z_0 + T\theta(i) \tag{12}$$

The vibration vector Z at step $i+1$ is equal to the baseline vibration Z_0 plus the product of the transfer matrix T and the control vector θ at step i. This implies that T, the transfer matrix relating HHC inputs to vibration outputs, is independent of $\theta(i)$.

The local model of the helicopter response to HHC is a linearization of the response about the response to the current value of the controller vector:

$$Z(i+1) = Z(i) + T[\theta(i+1) - \theta(i)] \tag{13}$$

or:

$$\Delta Z(i+1) = T\Delta\theta(i+1) \tag{14}$$

The vibration vector Z at step $i+1$ is equal to the vibration vector at step i plus the product of the transfer matrix and the difference in the control vector from step i to step $i+1$. This allows for variation of the transfer matrix T with input θ.

A deterministic and cautious minimum variance controller can be programmed into two algorithms, one for the local and one for the global HHC model [90,92].

A few selected results together with a summary of the most important conclusions obtained in the course of this study [95,97] are presented below.

An interesting test of the ability of the controllers to adapt to changing flight conditions was performed by introducing a step change in advance ratio from $\mu = 0.30$ to $\mu = 0.35$. This was done by starting with a converged optimal solution and its response at $\mu = 0.30$, changing the propulsive trim values to those for $\mu = 0.35$, and proceeding with iterative control calculations and quasilinearization solution. Results for a four-bladed soft-in-plane hingeless rotor are shown in Figs. 11 and 12.

The iteration history of 4/rev. hub shears for the deterministic and cautious local control are shown in Fig. 11 and the iteration history for the global control are shown in Fig. 12. Application of HHC eliminated essentially all the 4/rev. hub shears within five iterations. With the local controller, the vertical shears rise slowly after the second iteration indicating that the transfer matrix identification has not been ideal. For lack of space the control inputs are not shown here. However examination of these inputs [95] reveals that when the step change in advance ratio was applied to the local controller there were large oscillations in the calculated control inputs and the resultant hub shears. On the other hand when a step change in μ was applied to the global controller, the oscillatory behavior in the required control input was reduced substantially and the controller moved fairly smoothly from an initial large increase in shears toward a minimum. A comparison of the three shear components and their baseline values for the local and global controllers is given in Fig. 13. It is evident that the global controller has been more successful in reducing shears.

Another interesting result is associated with the instantaneous control power as a function of azimuth which was evaluated for both the hingeless rotor and a comparable articulated rotor, at an advance ratio of $\mu = 0.30$. Figures 14 and 15 show the variation in instantaneous trim power, HHC power, and total control power respectively for the articulated and hingeless blades.

For the hingeless blade the HHC power contributes relatively much more to the total control power as can be observed from the nature of these curves which tend to follow one another closely in Fig. 15. As is evident from comparing the vertical scales, the maximum power excursions are on the order of five times larger for the hingeless blade than for the articulated blade and in addition are much sharper. The peaks are biased toward positive power values so that the total control power over one revolution is much higher for the hingeless than the articulated blade.

The most important conclusions obtained from these studies were:

1. Overall blade response was compared for a baseline no HHC condition and the optimal reduced vibration condition. Principal differences were in the torsional and flap response. Overall response magnitudes changed little but large 4/rev. components were introduced to modify the airloads and cancel out vibrations.
2. A global controller was used in comparison of the effects of HHC on roughly equivalent articulated and hingeless rotors. Shears were successfully reduced for both rotors by a HHC algorithm which minimized just hub shears. As shears were reduced there were large increases in hub moments for the hingeles rotor but only moderate increases for the articulated rotor. Much larger HHC angles were required to reduce shears for the hingeless rotor.
3. Attempts to reduce both hub shears and moments were not very successful for either blade. With the articulated blade moments were already low and were only decreased slightly while shears increased slightly. For the hingeless blade the large moments decreased greatly but only at the expense of poor shear reduction. These results indicate that vibration reductions with HHC may be more difficult in hingeless rotor configurations than for articulated configurations.
4. Application of HHC to the hingeless rotor lead to an increase in required power of 1.44% while for the articulated rotor this increase was only 0.18%. The required power increase for the hingeless rotor was somewhat mitigated by a 0.2% increase in rotor thrust.
5. Overall blade aeroelastic stability margins were not significantly degraded by application of HHC for either the articulated or hingeless blade.

It is interesting to note that two, somewhat similar studies, on the application of higher harmonic control to hingeless rotor systems were also recently completed by Nguyen and Chopra [101,102]. These studies were based on an advanced HHC aeroelastic simulation capability. The analysis utilities finite element in both space and time. A nonlinear time domain unsteady aerodynamic model is used to obtain the air loads, and the rotor induced inflow is calculated using a free wake model. The vehicle trim is also obtained from a fully coupled trim/rotor aeroelastic analysis. Thus this simulation capability has many common features with Refs. 95-97, except that the aerodynamic model is more refined when compared to that used by Robinson and Friedmann [95-97]. The higher harmonic control algorithm is also very similar to that employed in Refs. 95-97. This aeroelastic simulation capability was validated by comparing it with the wind tunnel tests conducted on a one-sixth dynamically scaled three-bladed articulated rotor model tested by Boeing Helicopter Co. [82] up to high advance ratios, $\mu = 0.40$, and fairly good correlation with the experimental data was obtained.

The conclusions obtained from this study are in agreement with the majority of the conclusions noted in Refs. 95-97. Thus the performance of the global controller was superior to the local controller. When applying HHC on hingeless rotors larger control angles were required than for articulated rotor and a substantial increase in the HHC power requirements was also noted. Furthermore HHC effects on rotor performance were found to be small. An interesting effect noted was the increase in stall areas over the rotor disk, when the local HHC controller was at the edge of the flight boundary.

Fairly extensive tests on a four bladed hingeless rotor, with a minimum variance controller were also described in detail in a recent paper by Lehmann and Kube [103]. An adaptive local controller was found to perform quite well over the whole envelope tested. Considerable detail on the actual hardware, and digital control implementation of HHC for the test configuration is presented in the paper. Effective vibration reduction for the hingeless rotor was obtained with a gain adjustment (adaptation) algorithm. The power requirement for HHC during the test was not discussed in this study.

A very comprehensive wind tunnel test program on a three bladed articulated model rotor (CH-47D) was also conducted by Shaw et al. [82]. It was found that HHC is highly effective on articulated three bladed rotors. Harmonic hub load response to HHC was found to be linear up to 3 degrees and it was insensitive to flight condition. Furthermore a fixed-parameter control law was found to be fully effective for vibration reduction. Figure 16, taken from Ref. 82, shows that closed loop HHC was extremely effective in suppressing vibratory hub forces. The fixed-gain controller, when configured to regulate 3/rev vertical force and 2/rev and 4/rev inplane shears, suppressed all three of these components simultaneously by 90 percent in almost all of the trimmed and quasi-steady maneuvering test envelope.

From the discussion presented in this section it is evident that comprehensive aeroelastic simulations of HHC have a very useful role in this field. In addition to being much more cost effective than wind tunnel tests and flight tests, they provide one with an ideal basis for planning such tests. Furthermore experience with flight tests [67] indicates that scale model tests do not always correlate well with wind tunnel tests. Thus correlation between aeroelastic simulation programs of HHC which correlate well with wind tunnel tests provide one with a tool suitable for simulating flight tests.

The evidence available from recent studies seems to imply that the practical implementation of HHC on hingeless rotor systems could be more difficult, and less effective, than similar implementation of HHC on articulated rotors. Control angles needed for HHC on hingeless rotors are significantly larger and power requirements could be also between 2-5 times larger.

Simultaneous reduction of both hub shears and moments on hingeless rotors is also more difficult, and rotating blade root loads can be also substantially larger. One possible explanation for these observations is the strong physical coupling between the bending and torsional degrees of freedom which exists in most hingeless rotor blades. This can also cause the increase in the power required for implementing HHC on such rotors. Since the higher harmonic movement of the complete blade during its HHC, introduces more complex motion due to the coupling effect and thus requires more power.

Finally, it appears that adaptive controller may not be always required in order to implement an effective HHC for vibration reduction.

6. Concluding Remarks

This paper provides a detailed discussion of four important topics in helicopter rotor dynamics and aeroelasticity. Hopefully this discussion will provide an improved fundamental understanding of the current state of the art. Thus future research on these topics can be focused on problems which remain to be solved instead of producing marginal improvements on problems which are already well understood.

For aeroelastic and aeromechanical stability problems incorporation of geometrically nonlinear terms due to moderate deflections has become fairly routine. Both moderate deflection beam theories and large deflection theories are available, and both can serve as a basis for aeroelastic stability and response analysis. It is noted that improvement of the unsteady aerodynamics in rotary-wing aeroelastic analyses is much more important than the incorporation of higher order geometrically nonlinear terms. Excessive preoccupation with such higher order terms provides only diminishing benefits.

A number of recent composite beam theories, suitable for modeling composite blades have been developed however these have yet to be implemented in a comprehensive aeroelastic stability or response analysis. The significant potential for aeroelastic tailoring inherent in composite rotor blades remains to be exploited.

The body of research available on coupled rotor/fuselage aeromechanical problems has reached a remarkable level of maturity in a fairly short period of time. Reliable models for this class of problems should contain coupled flap-lag-torsional blade models combined with a fuselage which has pitch and roll as well as two translational degrees of freedom. Simpler models can easily lead to inaccurate results. Air resonance in hingeless rotors can be actively controlled throughout the entire flight envelope using a simple control system designed using modern control system synthesis techniques.

The state of the art in applying HHC to articulated rotors appears to be quite promising. The application of HHC to hingeless rotors could have practical implementation problem. Aeroelastic simulations of HHC have a very useful role in providing an improved fundamental understanding on the implementation of HHC on different types of rotor systems. A substantial amount of additional research in this topic is required so as to guarantee its implementation on the next generation of rotorcraft.

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Table 1

Frequency Increase due to Ω			**Frequency Decrease due to Timoshenko Beam Effects**		
Inplane Modes			*Inplane Modes*		
Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
3.5%	2.6%	1.0%	-1.6%	-9.7%	-19.2%
Out of Plane Modes			*Out of Plane Modes*		
71.2%	12.8%	4.6%	-0.4%	-2.5%	-5.5%

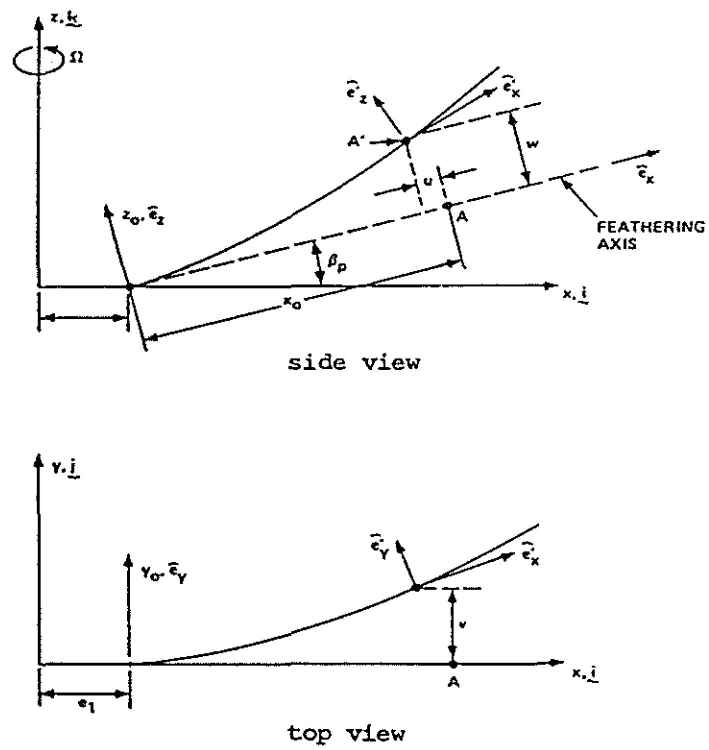

Figure 1a: Geometry of the Blade Elastic Axis Before and After Deformation.

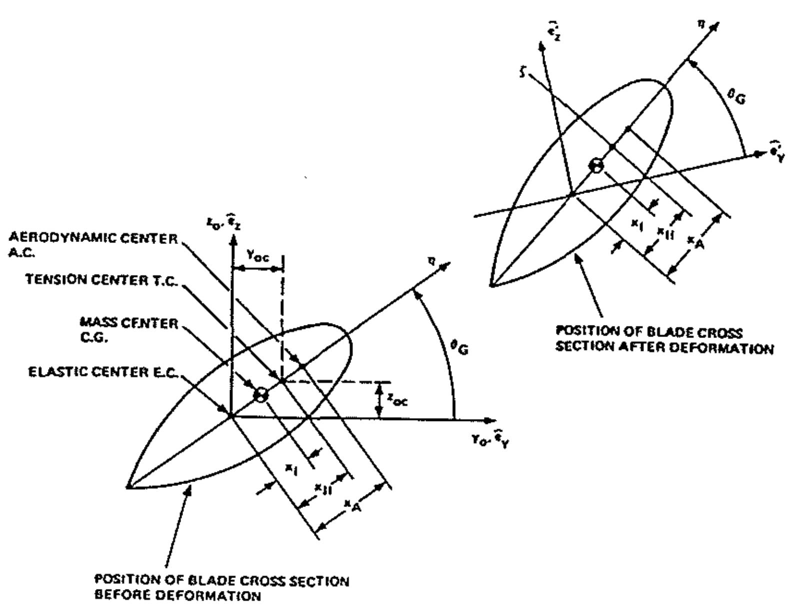

Figure 1b: Blade Cross Sectional Geometry.

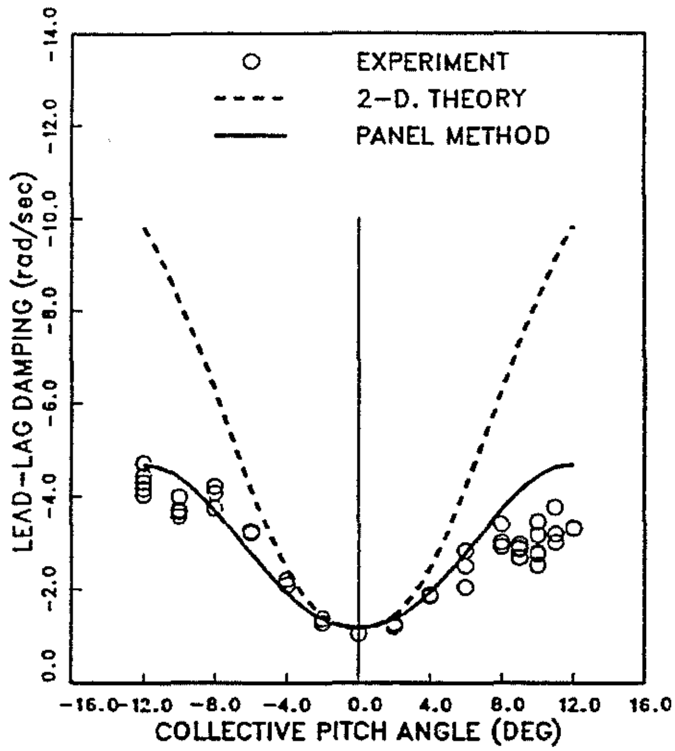

Figure 2: Lead-lag Damping Versus Blade Pitch Angle, Soft Pitch Flexure, Zero Droop, Zero Precone (Ref. 30).

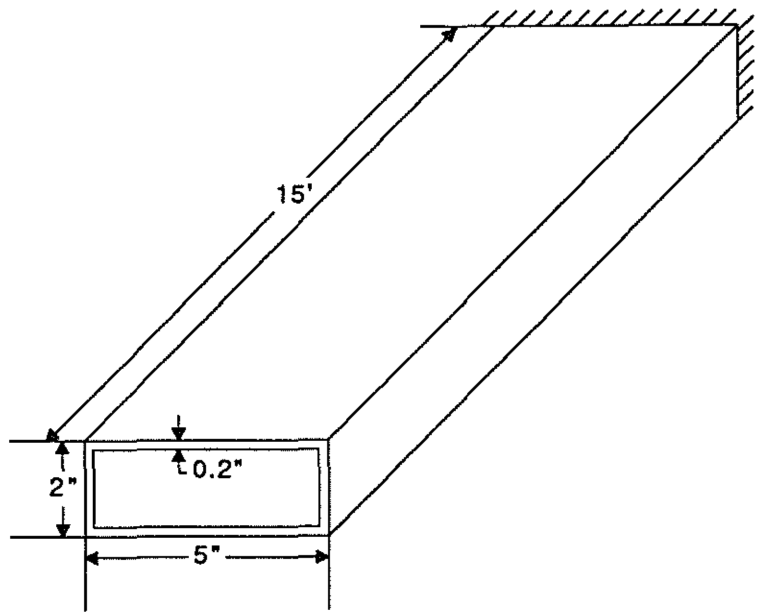

Figure 3: Box Beam for Study of Timoshenko Beam Effects.

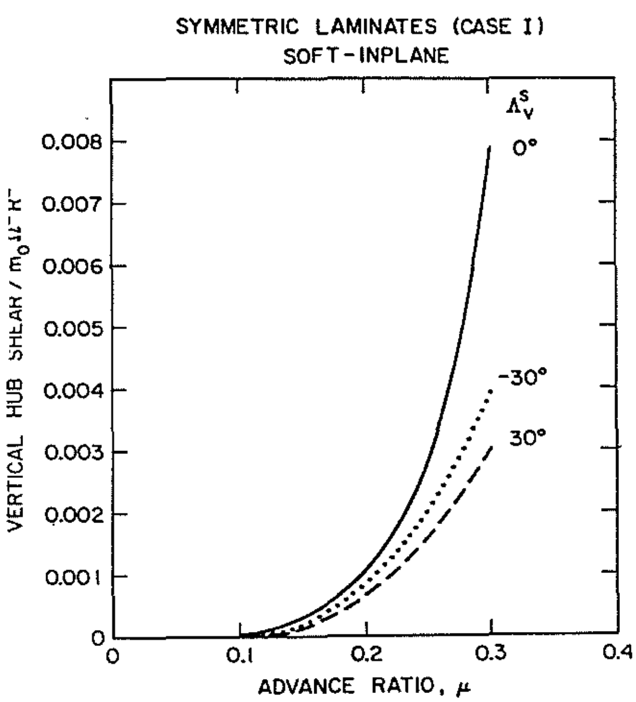

Figure 4: Peak-to-Peak Vertical Hub Shear, Hingeless Composite Blade (Ref. 48).

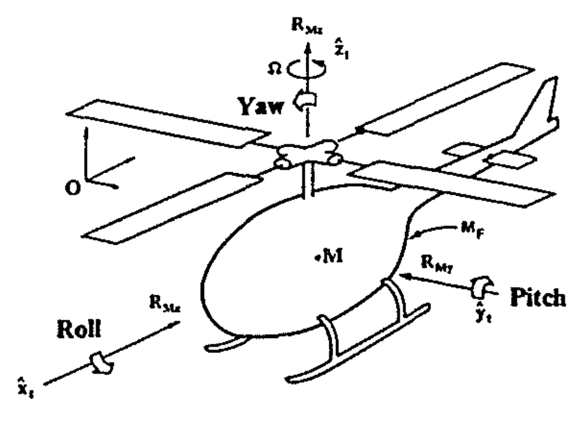

Figure 5: Coupled Rotor/Fuselage Model.

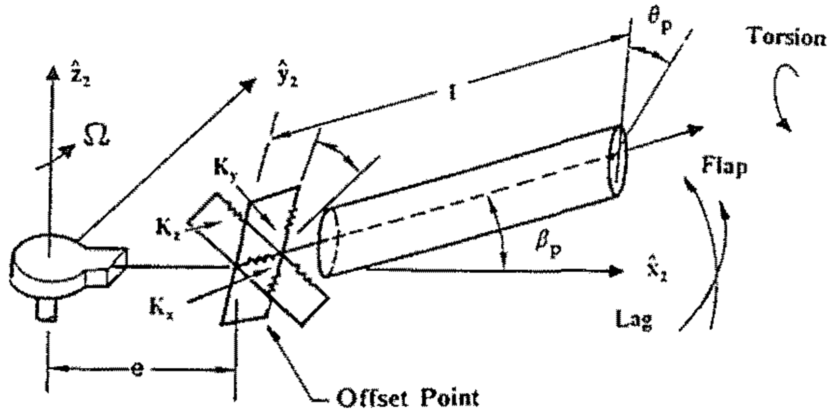

Figure 6: Offset Hinged Spring Restrained Model for Hingeless Blade.

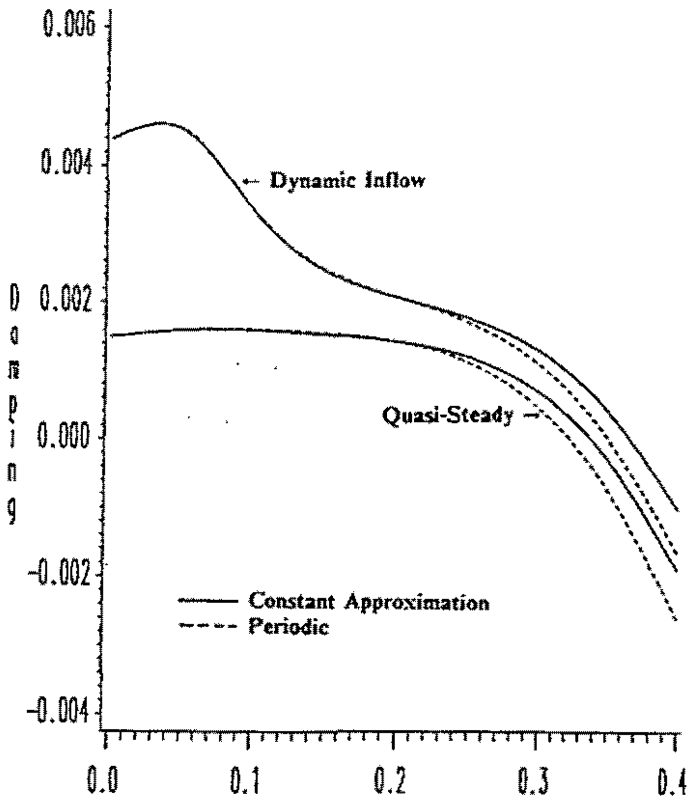

Figure 7: Open Loop Lead-lag Regressing Damping of the Nominal Configuration With and Without Dynamic Inflow.

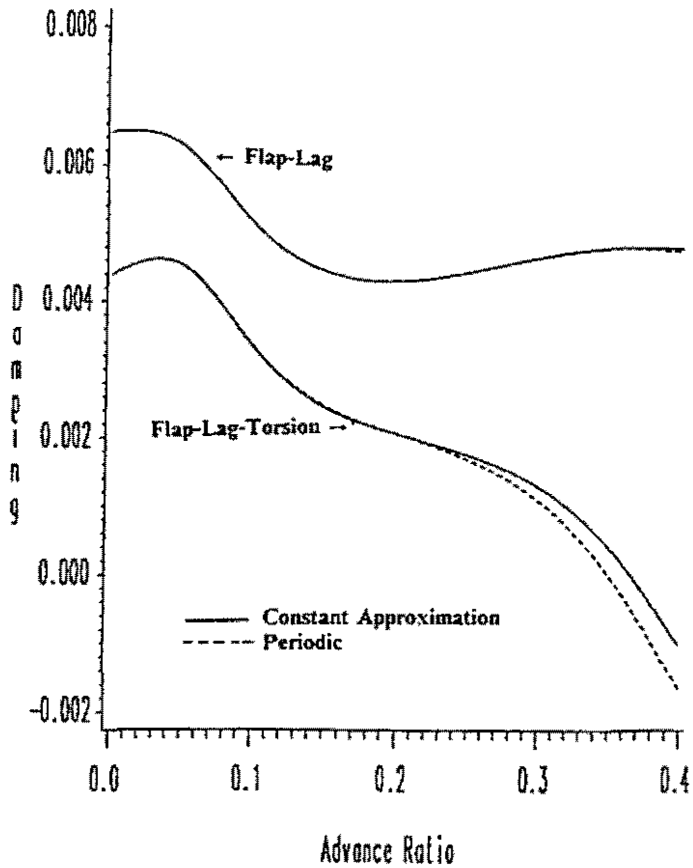

Figure 8: Open Loop Lead-lag Regressing Damping of the Nominal Configuration With and Without Blade Torsional Flexibility.

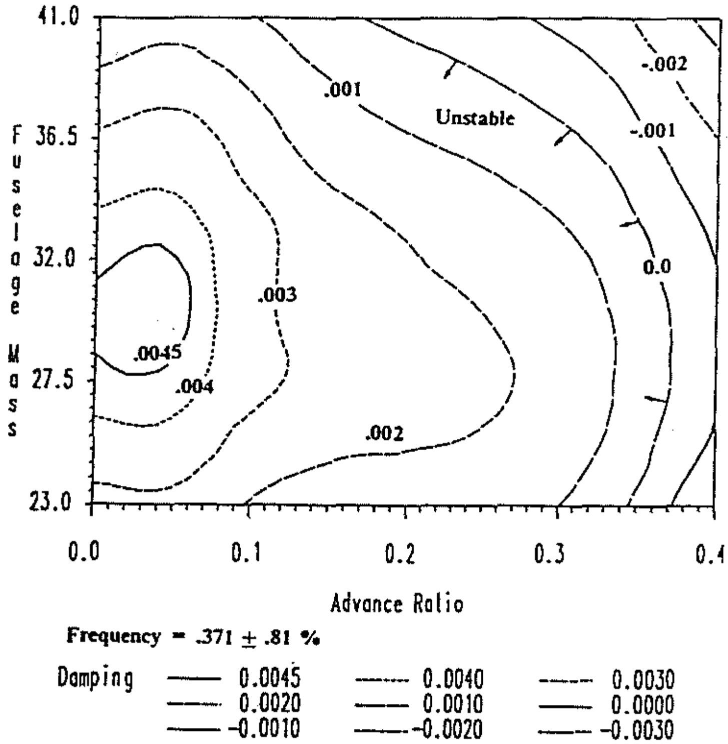

Figure 9: Open Loop Lead-Lag Regressing Damping at Various Fuselage Weights and Advance Ratios.

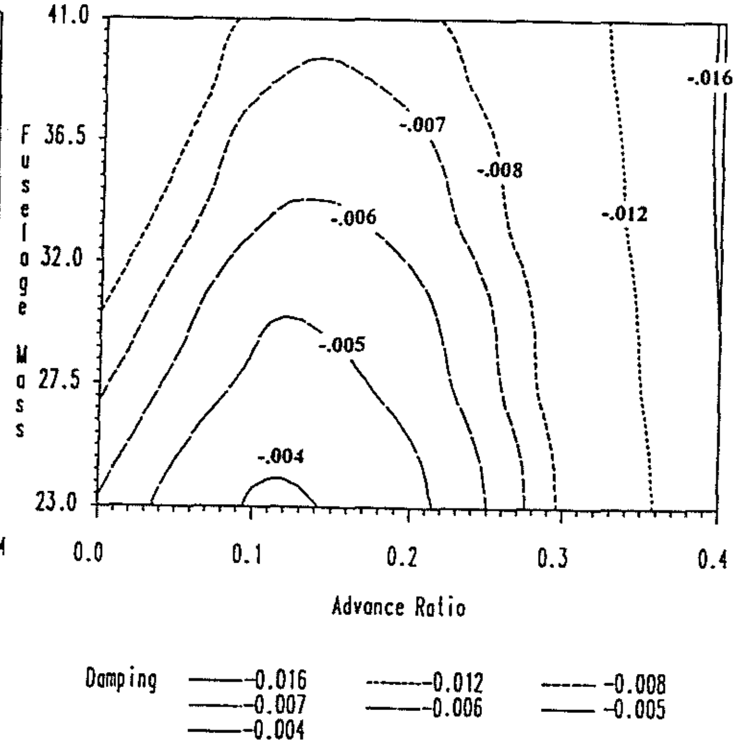

Figure 10: Closed Loop Lead-Lag Regressing Mode Damping Using the Active Controller.

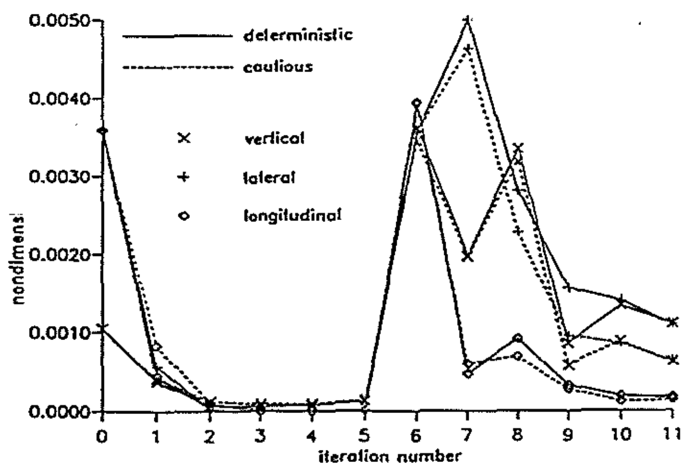

Figure 11: Iteration History of Hub Shears for Deterministic versus Cautious Local Control, Optimization at $\mu = 0.30$, then Step from $\mu = 0.30$ to $\mu = 0.35$.

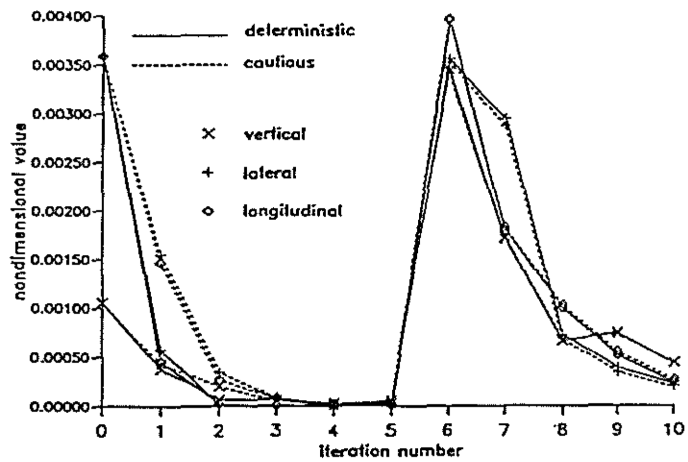

Figure 12: Iteration History of Hub Shears for Deterministic versus Cautious Global Control, Optimization at $\mu = 0.30$, then Step from $\mu = 0.30$ to $\mu = 0.35$.

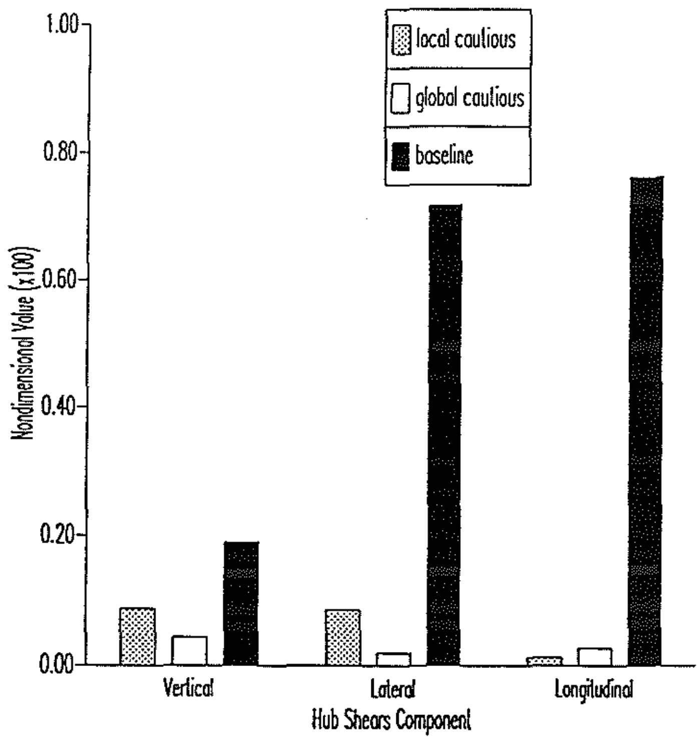

Figure 13: Baseline Shears for $\mu = 0.35$ and $\mu = 0.30$ to $\mu = 0.35$, Local and Global Shears Five Iterations After Step Change from Controllers.

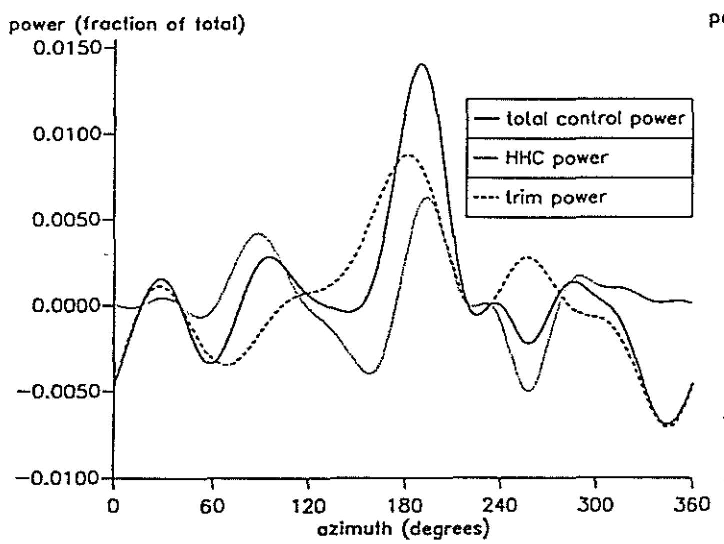

Figure 14: Variation of Trim Power, HHC Power and Total Control Power with Azimuth Articulated Blade, Advance Ratio $\mu = 0.30$; 5% Blade Root Offset, Cautious Global Control.

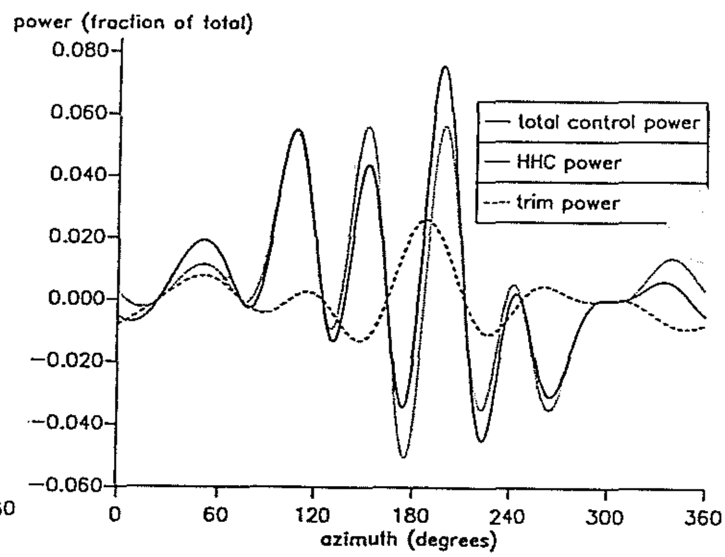

Figure 15: Variation of Trim Power, HHC Power and Total Control Power with Azimuth, Hingeless Blade, Advance Ratio $\mu = 0.30$; 5% Blade Root Offset, Cautious Global Control.

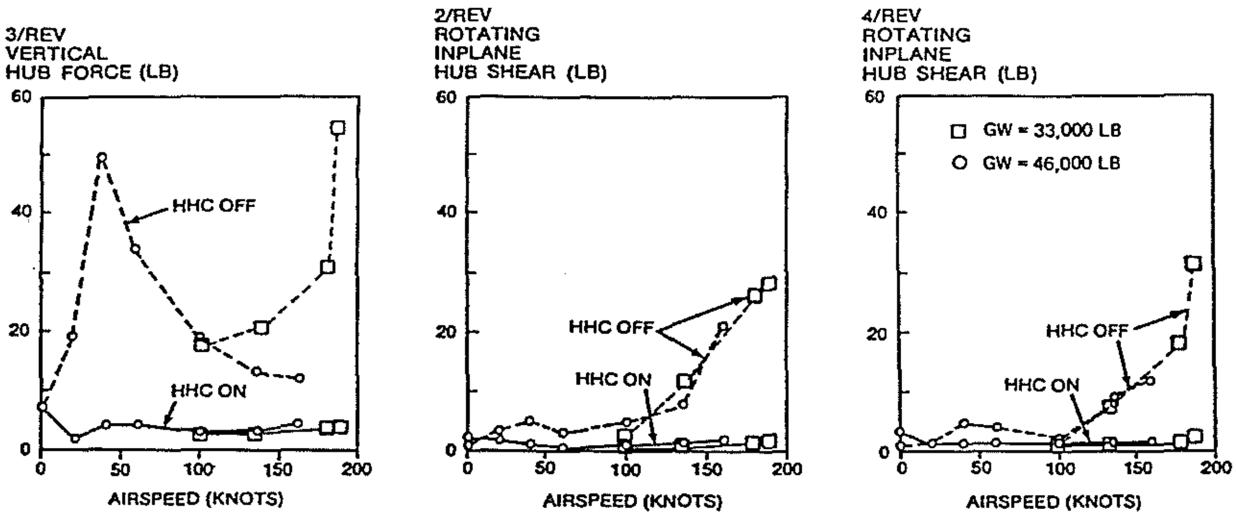

Figure 16: Measured HHC Effectiveness in Trimmed Level Flight, (Ref. 82).