

## EVALUATION DF SECTION PROPEATIES FOR HOLLDW COMFDSITE BEAMS by

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#### Abstract

The paper describes the plan and the first resulta of a doint reeearch project, aiming at the development and the validation of design procedures for composite beam-like structares and structural somponents. Analytical as well as experimental results are reported; one of the most gigeificant of such resuits has been the development of the program Haxba (Holiow Anisotropic Beam Analysis) which, based on an original displacement metiod approach, aliows the eveluation of section stiffinesses and stresses. The program works on a finite elament idealization of jine beam section and computes the stresses from the resultant forces and zoments acting on the particular section, as the usual engineer's beam theory. Ezperimental results are mainly concerming the identification of elastic pro perties for composite laminates, at relativeveiv low sureas levels, and the Falidation or the results of the program Laત3A. The latter has been planned through the evaluation of a large number of jenditag and torgion tasts to be performed on blade apars and on compiate bladea. inalytical respits compare very well with axperimental ones, the largest differences in section stilfenesaes so far evaiuatad belng of the order of lat. The plan is still going on and future activitias gre also outlined in the paper.

\section*{1. INHRODUCTION}

The continuous evclution of composites materials is bringing them among the most interesting and the most atudied materials for the manufacturing of aircreft, end of course also of hellcopter. Among their most significant features, besides having, quite obviously, a favourable strength to weigt Fatio, composites promise a hygh fatigue zesistance and a good damage tolerance. AGOSTA, following this trend, has undertairen a plan for composite application development, which aill lead, in the near future, to the use of composites in the manulacturing of primary structaral componenta, as rotor hubs and blades, and in that of secondary components, as doors, fairings etc.. Rigure 1 shows the potential of composite application in the structare of a modern Agusta helicopter. Such developenent plan brought up the need of reliable analysis procedures for primary structural subsssemblies made from composites. Besides being sufficiently reliable such procedures mast also be: - sufficiently efficient and simple in use to be employed from the eariy design stages, in. Interactiva modes by the designers as well as integrated in optimizing routines; - sufficiently sophisticated to take into account the presence of different materials in the same section, non-orthotropic materials and non-linear cou pling between shear and normal stresses, as it may be needed in the more ad venced designs; - sufiticiently flexible to be able to compute the inertia and stifinese properties of rotor blades, to be uged in dynamics and dynamic instability


aralysis of rotors.
For such reasons a cooperation has been undertaken between AGOSTA and IASP (Iatituto di Ingegneria Aerospaziale del Politecnico di Milano) for the development and the validation of experimental and computational procedures pertaining the analysis and design of composite beams, with particular empiasis on rotor blades.
The cooperation has been a complete success, partly due to good planning and effective management; undoubtedly it hes been a new evidence of the tremendous potential of the cooperation of different ad complementary competencies, as the ones of Dniversity and Industry.
It may be enough to note that, the first contract having been signed the 27th sept. 1978, now a significant amount of experimental work has been completed, and the first version of the program HavBa is being used for deaign purposes, Such version, which in fact is a medium size code, has already been validated trough the evaluation of a certain number of theoretical tast cases, i.e. cases where exact solutions based on monocoquer or semi-monocoque schemes mere available.
Besides, the first results of EANBA compare very well also with the resulta of ad-hoc measurements cade on blade spers in bending and torsion.

## 2. THE PESEARCE TLAN

The main activities of the plan are liated below, grouped according to metho dologies.
4. BASIC THEOREMTCAL DETEFORNGNS

Al Eormulation of a displacement method procedure for the analysis of a bean section

A2 Finite element formulation of the same procedore for linear elaatic me terials; element formulation
a3 Critical statement of the problem of identipying non-elastic behaviour of composite materials at high strass levela
14 Development of non-elastic matertal models and of failurs criterita
A5 Formulation of Pinite element procedures employing the models developed by activity 44 , based on the general formulation stated in AI

B EXPERTMENTAL ACIIVITIES
B1 Braluation of techniques for the direct measurement of elastic proper ties of composite laminates, in uniadal stresa gtates, at low stress levela
B2 Development and evaluation of techniques for the diract measurement of elastic properties of composite laminates in plariaxial strese states
B3 Observation of failure conditions and failure modes for composite laminates in pluriaxial stress states
B4 Deplections and stiffness measurements on blade spars and complete blade sections in various bending and toraion load conditions, in the elastic range
B5 Observation of atrength properties and of fallure characteristics of longerons and complete blade sections in Various combined load conditions

C1 Drafting of the general specifications of the program
C2 Detail specifications of the Innear version
C3 Coding, teat and validetion of the linear veraion
C4 Detail specifications for an extended version of the progrem Including non-elastic material bohaviours and strength evaluations
C5 Coding, test and validation of the extended version specified by actiFity 64.
The logical flow of the activities of the plan, showing also dependence connections, is reported in figure 2.
At the present time the activities A1, A2, B1, B3, C1, C2 and C3 have been completed.

## 3. ANALITICAL APPPOACR

The approach is based on the hypothesis that the solid can be considered as a "beam", $1 . e$. that, wth a good approximation, it has the shape of a cJlin der of any crose section, loaded only on its end sections. In other words the basic hypotheais of De Saint Venant are supposed to hold, and we seek a solution which is correct only in a certain distance prom the end sections, where loads and/or constraint may be applied.
In this case it is well bnown that the atress and/or the strain state can be determined on a cross section of the beam, only from the resultant forces and moments acting on the same cross section, fgnoring the situation of the neighboring sections.
Let us consider a solid that can be assumed to be a beam, or shortiy a beam, and let $0 x y z$ be an orthogonal reference set having the $z$ exis paralell to the generatrices of the cylinder; the point 0 is a point of the cross secHon that has been choosen as a reference point.
If the beam is moderately bent the $z$ ajia is only locally parallal to the generatrices, or, more precisely, parallel to the straight lines tangent to the generatrices at the crose section considered.
So $z$ can also be considered an abscissa measured along the iine connecting the points 0 of the cross sections of the beam.
Let us imagine that the set Oxyz movea during the deformation th such a way that $z$ axis remains tangent to the deflected axis of the bean and $x$ and $y$ axis rotates around $z$ axis by and angle $\theta=\theta(z)$ corresponding to the torsion of the section ( $x$ ).
If $\underline{W}=\underline{W}(z)$ is the displacement of the points 0 , in the case of small deplections, the rotation $\beta=\beta(z)$ of the set $O x y z$ has the components:

$$
\hat{B}=\left\{\begin{array}{l}
-w^{\prime} y  \tag{1}\\
w^{\prime} x \\
\theta
\end{array}\right.
$$

(x) In general this definition would require a further specification about torsion: In the case of hollow sections the projection of the section on xy plane can be assumed to zemain undeformed and torsion is uniquely apecifiad.
where the apex means derivation raspect to the abscissa $z$.
The displacement $B$ of any point $P$ of the cross section can then be regarded as the sum of the rigid displacement of the reference set plus the follows:

$$
\begin{equation*}
\underline{s}(P, z)=\underline{w}(z)+\underline{\beta}(z) \wedge(P-0)+\underline{g}(P, z) \tag{2}
\end{equation*}
$$

The third component $g_{z}$ of the relative displacement $g$ is generally lenown es the "warping" of the pointa of the cross section.
The decomposition (2), within the apecification (1) of the rotation $\beta$ gives the following formulations for the significant strain components:

$$
\begin{align*}
\varepsilon_{z} & =w_{z}^{\prime}-x w_{z}^{u}-y_{y}^{u^{n}} \\
\gamma_{n z} & =2 \Omega_{(n)} \theta^{\prime}+\frac{\partial g_{z}}{\partial \eta} \tag{3}
\end{align*}
$$

where:

$$
\begin{equation*}
2 \Omega_{(n)}=(p-0) \wedge \text { nax }=x \ln -\operatorname{yn} x \tag{4}
\end{equation*}
$$

Is tine double of the area dashed in 21gure $4[1],[2]$. It may be noted that the signleicant componenta of strain, from (3), depend on the 4 global deflection parameter of the cross section $w_{z}^{\prime}, w_{x}^{\prime \prime}$, $W_{y}^{n}$ and $\theta^{\prime}$, gnd on the waroing $g_{z}$.
If the constitutive laws of the materials are specifiled, stresses, Virtual work of deformation, and anything else usable for atudying equilibrium, can be erpressed as sunctions of the 4 global deflecticn parameters and of the fanction $\xi_{3}$.
In this case the desplacement method implies that the above global deflection paramaters and the marpiag function are assumed as the urknown of the equili brium equations.
An efficient solution procedure can be obtained by means of finite element techniques, i.e. by dividing the cross section surface in a certain number of finite elements, each element being characterized by a certain number of nodes or grid potnts; the unknown, discrete in number, are then assumed to be the 4 global desplacement parameters, plus the values $\Gamma_{i}(1=1,2, \ldots, m)$ of the warping function $g_{z}$ in the $I f$ srid points of the cross section. The basic element types that can be useful for idealizing the section of as ronautical beams and beamlike structures seem to be the following:
a) panel elements, i.e. elements thin enough to be representable on the cross section by a suitable mean line;
b) Plange elements that on the cross section mast be considered as auriace elemente;
o) joint elements.

The material model can be any linear material model, and generally anysotrom pic model (i.e. non-orthotropic) must be allowed. So also semi-mono schemes can be reproduced just specififing for panol elementa a material ham Fing only the shear modulus $G$ non-gero, and for flange elements a material capable only of normal stress ( $G=0$ ). For the panela it has been convenient to develop isoparametric elementa haVing 2,3 and 4 nodes each.
The joint elements are used to represent a longitudinal joint in the beam: on the crose section a joint element is always connecting 2 grid points and has a stifiness that can be specified independently of the shape and the dimen-
sions of the joint itaele.
Figure 5 shows all these types of elements with some possible way of using them.
To deacribe briefly the ifnite-element approach it is convenient to develop, as an example, the contribution, to the equilibrium of the whole cross sec tion, of an isoparametric panal having 3 nodes idealizing a piace of laminate of thicimess $t$.

Fith reference to figure 6 we define

$$
\begin{equation*}
p=\int_{t} \sigma_{z} d \bar{y} \quad ; q=\int_{t} \tau \bar{x} x d \bar{y} \tag{5}
\end{equation*}
$$

and In accordance with the usual approach for hollow beam we can asaume that:

$$
\begin{equation*}
\int_{t} \sigma \bar{x} d \bar{y}=0 \quad \text { and } \quad \int_{t} \tau_{\bar{y} z} d \bar{y}=0 \tag{6}
\end{equation*}
$$

To shorten the notation it ia convenient to denote the stress ilowa with the column $\{P\}$ and the significant strains in the column $\{\varepsilon\}$, i.e.:

$$
\{\underline{p}\}=\left\{\begin{array}{l}
p  \tag{7}\\
q
\end{array}\right\} ; \quad\{\varepsilon\}=\left\{\begin{array}{l}
\varepsilon_{z} \\
\gamma_{\bar{z} z}
\end{array}\right\}
$$

For linear elastic materials, taking into account (5) and (6), the constitution relation can be put in the following symbolic form:

$$
\begin{equation*}
\{p\}-\angle \bar{c}]\{\varepsilon\} \tag{8}
\end{equation*}
$$

where $\bar{C} \overline{7}$ is a symuetrical matrix sumarizing the signipicant elastic propartices of the laminate.
[C] will be a diagonal matrix only if the unidirectional laminee are so orien ted that the resultant laminate is orthotropic respect to the direction of the zaxis, or in other words has an orthogonal aymetry of the elastic properties respect to a direction perallel to $z$ arls.
The sigrilitcant deflection parameters, i.e. the 4 global parameters pins the 3 values $\{\Gamma\}$ of the marping at the panel nodes can be denoted by the column:

$$
\left\{\begin{array}{l}
u
\end{array}\right\}=\left\{\begin{array}{c}
\Gamma_{1}  \tag{9}\\
\Gamma 2 \\
\Gamma 3 \\
\theta^{2} \\
w_{7}^{\prime \prime} \\
w_{z}^{\prime \prime} \\
w_{z}^{\prime}
\end{array}\right\}
$$

Denoting by $\{I\}$ and $\{Y\}$ the colamn of the $I$ and $Y$ coordinates of panel nodes, the idea of isoparametric element comes off into the fact that the coordinate $x y$ and the warping function $g_{z}$ of any point of the parel mean line ars expressed in the same way:

From (3), taking into account (4), (9), and (10), the strains can be expressed as linear functions of displacement parameters, as follows:

$$
\begin{equation*}
\{\varepsilon\}=[B]:\{\bar{n}\} \tag{11}
\end{equation*}
$$

$\lceil B]=\left[\begin{array}{ccccccc}0 & 0 & 0 & 0 & -x & -y & 1 \\ \frac{d N_{1}}{d \bar{x}} & \frac{d x_{2}}{d \bar{x}} & \frac{d \mathbb{N}_{3}}{d \bar{x}} & \mathrm{xn}_{y}-\mathrm{yn}_{x} & 0 & 0 & 0\end{array}\right]$
To derive equilibrium equations we now start writing the virtual work pertaining to a slice of the bean bound by two cross sections at the vanishing distance dz .
The contribution $\delta I_{e}$ of the-isoparametric panel that we are considering to the virtual work of "external" forces (external for the elementary beam slice), with the notations of figure 7 is:

$$
\begin{equation*}
\delta^{*} I_{0}=d z \int_{1}^{3}\left(p \delta s_{z / z}+p / z \delta s_{z}+q \underline{n} \cdot \delta_{s / z}\right) d \bar{x} ; \tag{13}
\end{equation*}
$$

whence, for (1) and (2)

$$
\begin{equation*}
\frac{\delta^{*} I_{e}}{d z}=\left(\bar{T}_{z}+\bar{M}_{y}^{\prime}\right) \delta w_{x}^{\prime}+\left(\bar{T}_{y}-\bar{M}_{z}^{\prime}\right) \delta w_{Y}^{\prime}+T_{z}^{\prime} \delta w_{z}+\{\delta \bar{u}\}^{T}\{\bar{p}\} \tag{14}
\end{equation*}
$$

where $\bar{T}_{x}, \bar{T}_{J}$ and $\bar{T}_{z}, \bar{W}_{X}, \bar{M}_{Y}$ and $\bar{M}_{z}$ are, respectively, the contribution of the panel to the resultant forces and moments acting on the cross section, and:

$$
\{\vec{p}\}=\left\{\begin{array}{c}
\bar{\phi}_{1}  \tag{15}\\
\dot{\Phi}_{2} \\
\bar{\phi}_{3} \\
\vec{M}_{z} \\
\vec{M}_{y} \\
-\bar{M}_{z} \\
\bar{T}_{z}
\end{array}\right\} ; \vec{\phi}_{k}=\int_{1}^{3}-\bar{I}_{k} \quad \bar{p} / z d \bar{x}
$$

Inikewise, the contribution of the same panel to the virtual work of deforamtion:

$$
\begin{equation*}
\delta_{d}=d z \int_{1}^{3}\{\delta \varepsilon\}^{T}\{p\} \quad d \bar{x} \tag{16}
\end{equation*}
$$

for (8) and (11) aan be expressed as :

$$
\begin{equation*}
\frac{\delta^{4} I_{d}}{d x}=\{\delta \bar{u}\}^{T}\left[\frac{x}{T} \backslash \bar{u}\right\} \tag{17}
\end{equation*}
$$

where:

$$
\begin{equation*}
[x]=\int_{1}^{3}[B][C][B] d x \tag{18}
\end{equation*}
$$

Elements of other type will give contributions to external and internal Firtaal work formally identical to (14) and (17), even if the number of nodal union $\{\Gamma\}$, the shape function and the matrix [B] may be different. In any case the total virtual work of the beam since comes from the $s u_{\text {m }}$ of the contribution of all the elements, after having arranged all the unknown parameters in the column:

$$
\{u\}=\left\{\begin{array}{l}
\Gamma_{1}  \tag{19}\\
\Gamma_{2} \\
\vdots \\
\Gamma_{m} \\
\theta^{\prime} \\
w_{x}^{n} \\
w_{y}^{\prime \prime} \\
w_{z}^{\prime}
\end{array}\right\}
$$

an being the number of nodes of the whole section.
The total external and internal virtual works come of as follows:

$$
\begin{align*}
& \frac{\delta^{\prime} I_{e}}{d z}=\left(I_{x}+M_{y}^{\prime}\right) \delta w_{z}^{\prime}+\left(I_{y}-w_{x}^{\prime}\right) \delta w_{y}^{\prime}-T_{z}^{\prime} \delta w_{z}+\{\delta u\}^{T}\{P\} \\
& \frac{\delta^{\prime} I_{d}}{d z}=\{\delta u\}^{T}\langle z]\{u\} \tag{20}
\end{align*}
$$

where the forces $T$, the moments $M,\{P\}$ and $[K]$ come Prom the summing, or assembling of all element contributions.
The principle of Virtual works states that at equilibrium, for any choice of the virtual variation of the parameters, it mat be:

$$
\begin{equation*}
\delta^{x} I_{d}=\delta^{*} I_{e} \tag{21}
\end{equation*}
$$

whence, for ( 20 ), it must be:

$$
\begin{array}{ll}
u_{Y}^{\prime}=-I_{x} & ; M_{I}^{\prime}=I_{J} \quad ; I_{z}^{\prime}=0 \\
\text { and }: & {[X]\{u\}=\{P\}} \tag{23}
\end{array}
$$

The equations (22) are the well known equilibrium equations of the resultant forces acting on the section. The set of linear equations (23), in the unknown displacement parameters $\{n\}$
are the actual equilibrium equations of the finite element section scheme. Yet it must be noticed that not all the terms of the column $\{P\}$ are actualIy known. As a matter of fact the first a terma $\varnothing$ are depending on $\mathrm{p} / \mathrm{z}$ (15), which at the moment cannot be known.
Hevertheless we may note tnat according to the basic hypothesis, streasea and atrains can only be constant with $z$ or linear function of $z$, while the stifness matrix $\langle\mathrm{K}\rceil$ must be constant with $z$. Then deriving both menbers of (23) with regard to $z$, the following aet of equations is obtained:

$$
\begin{equation*}
[\pi]\left\{a^{\prime}\right\}=\left\{P^{\prime}\right\} \tag{24}
\end{equation*}
$$

where

$$
\left\{P^{*}\right\}=\left\{\frac{d P}{d z}\right\}=\left\{\begin{array}{c}
0  \tag{25}\\
0 \\
\vdots \\
0 \\
0 \\
-T_{x} \\
-T_{J} \\
0
\end{array}\right\}
$$

So the known terms being actally known, the (25) can be solved for $u$. Eventaally dariving equations (8) and (11) with regard to $z$ :

$$
\begin{equation*}
\{p / z\}=[C]\{\varepsilon / z\} ;\{\varepsilon / z\}=\lceil\bar{B}]\{u \cdot\} \tag{26}
\end{equation*}
$$

the terms $\Phi$. of the known term of (23) can be evaluatad by means of (15). It it is worth noting that the set of equations (23) and (25) have the same coefficient matrix, go the latter can be factorized once for both solutions. Once the matrix $[\mathbb{K}]$ has been factorized it will be convenient to compute geparately the 6 solutions corresponding to unit values of any one of the 6 components of resuitant forces and moments.
If all the elements have orthotrofic anterials normal susasses and shear stresses remain unconpled. In tiois stmpler case it is possible to define, in the usual way, the centroid of normal stresees, the primeipal ares for bending and the corresponding bending stiffnessea, the shear center and the cor responding torsional stiffiness.
In the more general case where normal and shear stresses are coupled the above usual defintions no longer hold, and the stiffiess properties of the cross section must be expressed by a matrix.
Hevertheless such general cases and the possible ways of presenting the cor responding etiffnesses deserve a further study, due to the interest inherent in the possibility of using non-orthotropic laminates to improve section pro perties.

## 4. THE PROGRAM HANBA

The finite element procedare outlined above has been implemented in the program HaNBA (Eollon Anisotropie Beam Analyisis).
Before the first statement of the program had been written, the deaign of the - program has been developed, and detailed to the description of the subrouti-
nes, of data base and test cases.
This allowed different parts of the program to be developed independentiy by 8 persons, belonging to AGUSTA and IASP, in a pretty short time, and integra tion to be pertormed without any particular problem.
In particular $\operatorname{FANBA}$ has been designed to have an open modular structure, and with the possibility of stop and reatart at intermediate pointa. The first level structure of the program is done by the following modules:

- Input and Preprocessing
- Anslysis
- Stresa computation and output.

The input and preprocessing module reads the data given or specified by the user.
It makes use of a library of matarials, laminae and laminates, fith different modes of operation, at user's choice.
Por instance the user may specily a laminate by choosing one of the laminates of the librery, or by selecting a certain number and spectffige a certain arrangement of unidirectional laminae.
Modules are also provided to unprove the library.
In addition the preprocesaing module performs checks and emits diagnostic messages about input data, prepares the printed and graphic output ior data check, and the input for the subseguent analysis. The analysis module computes the inertia and stipeness properties of the beam section, and the solutions needed by the posaible spbeequent computations. The atresa module computes the atresses in the laminae forming the laminates, in ilange elements and in joint elements, for given load conditions. The gross flow diagram of program \#ANBA is depicted in ilgure 8. at present the following elaments have been implemented:

- panel elements,isoparametric, 2, 3 and 4 nodes;

Gauss numertcal integrative of order 2, 3 or 4;

- flange elements capable of morikiag also in normal stresses;
- joint elements.

Besides the element nodes may be displaced from the grid points by a certain displacement or offset. So the rrid points can be fixed on the external profille and the actual element noder are automatically placed, in positions depending on the laminate.
Clearly the Program FANSA can be also a flexible and oificient instrument for the analysis of hollow beams of any aaterial, and then also wings, tailplanes ote.
In particular, as it computes the beam section by section, it is much more ver satile and manageable than the usual fintte element codes, especially in the sarly design stages.
As auch it is already being. used at AGJSTA.

## 5. EXAMPTE OF AN IDRATIZAITON

To give an impresaion of the idealization that can be used with the program EAKBA, the gcheme adopted for analyzing a rotor blade is reported in figare 9. Figure $9 A$ shows the actual cross section of the blade, while ilgure 9 B shows the idealization prepared by the user, with the grid pointa located on the outer proidle.
Pigure 9C reports the element nodes, computed by the program, whose actual position defenda on element ththess.

The scheme has a total of 59 degrees of freedom, 6 cubic panel elements ( 4 nodes), 11 parabolic ( 3 nodes) and 7 linear ( 2 nodes), plus 5 flange elements and 12 joint elements.
Despite its semplicity and relatively low cost this model is capable of representing the crosa section with a very good accuracy.

## 6. ELASTIC PROPERTIES OP LAMTNATES

The elastic moduli of laminates have been measured directly, employing a techni que based on uniarial tension tests of specimens cut from the laminate at different angles. Figure 10 shows the three types of specimens, denoted as I, II and III: the spectmen longitudinal axes $x$ form with the laminate reference direction $X$ angles of $0^{\circ}$, $90^{\circ}$ and $45^{\circ}$, respectively. For non-orthotropic laminates the strain $y_{\text {of }}$ of specimens type III may be non negligible, and then it may be important that the end clamps allow this strain without interferences. For that reason the clamps were hinged in the specimen centerline, at the stations where the apecimen emerges from the clampe, as shown in ingure 11.
Each specimen have been equipped with two strain rosettes, one on each face, to eliminate possible bending effects from the measurements. So, for each stress level, three gtrain aeasures can be extracted from one specimens, and then 9 for one zroup of 3 specimens of the 3 types. For an orthotropic laminate these 9 messured values are obviously redundant to determine the 4 independent elastic constanta.
In the first teats these 9 strains where used to deteraine the 9 teras of the elastie matrix, Whout making any assuption on orthotropy, nor even on symuetry. The finst tests were ran with 12 specimens, coming from the same sheet, 4 for each of the types in ingure 10.
The sheet was done by three unidirectinal layers, material SP250 S2920,placed at $0^{\circ}-90^{\circ}-0^{\circ}$.
AII the teata have been run with a Data Acquisition System, making use of a saall computer, programed to evalnate the elastic moduli from atrain measurements, on line.
The first testa ghowed essential symmetry and orthotropy of the elastic matrix, and a rather strong dependence of the tangent modulus $G$ on stress level. To make easier the observation of possible stress-dependence of the moduli, a simpler procesaigeg of meassured data was programmed, accepting a priori ortho tropy and symmetry. Particularly 5 moduli were directly extracted, as follows:

| $E_{\bar{L}}$ | and | $\nu_{I}$ | from specimens tjpe I |
| :---: | :---: | :---: | :---: |
| 3 | and | $\nu_{7}$ | Prom specimens type II |
| $G_{\text {ITI }}$ |  |  | from specimens type III |

This type of test was done_on the same 12 specimens used before, with stress levels up to about $0 \mathrm{~kg} \mathrm{~mm}^{-2}$.
Specimens type I and II showed linear stress-strain relations, while spocimens type III manifested remarkable non-linearities, notably creap and hysteresis. Figure 12 showa the decay in time of the gecant modulus $G_{\text {gy }}$, for specimens type III at constant stress: it mas also apparent that sucin modulus is strongiy dependent on specimen temperature.
Figure 13 shows typical stress strain curves for the three types of speczmens; the curve reported ior specimen III was obtained with a fixud load an ucload ve locity of $+.02 \mathrm{~kg} \mathrm{~mm}^{-2} \mathrm{~g}^{-1}$.
Table I gives a summary of the most gignificant features and op the moduli mea sured with the 12 specimens; the aoduli $G$ reported for specimens III are secant moduli, measured with a stress of $5.53 \mathrm{~kg} \mathrm{~mm}^{-2}$, applied for 5.5 minutas.

| Specimen $\neq$ | Type | $\underset{\text { Adhesive }}{\substack{\text { a }}}$ | $\left[\mathrm{E}_{8}^{\mathrm{Ex}} \mathrm{mm}{ }^{-2}\right]$ | $\nu_{x}$ | $E_{y}$ | $\nu_{y}$ | $\left[\begin{array}{c} G x y \\ {\left[x_{8} \mathrm{~mm}^{-2}\right]} \end{array}\right.$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 245 | I | 2 | 3426 | . 1440 |  |  |  |
| 246 | I | 1 | 3397 | . 1645 |  |  |  |
| 247 | I | 1 | 3462 | . 1647 |  |  |  |
| 248 | I | 1 | 3473 | . 1649 |  |  |  |
| 253 | II | 1 |  |  | 2444 | . 1076 |  |
| 254 | II | 1 |  |  | 2347 | . 0989 |  |
| 255 | II | 2 |  |  | 2372 | . 0883 |  |
| 256 | II | 2 |  |  | 2387 | . 0847 |  |
| 249 | III |  |  |  |  |  | 379 |
| 252 | III | 1 |  |  |  |  | 382 |
| 250 | III | 2 |  |  |  |  | 427 |
| 251 | III | 2 |  |  |  |  | 394 |


From table 1 the followng observation can be drama:
-a) Moduly $E_{X}$ and $E_{Y}$ measured from specimens I and II show a very low scatter, below 2\%.
-b) Systematic difierences appear between the measurements coming from strain gages applied wht the two difierent adhesives, at least for $\nu$, and even more for $G_{\mathrm{TY}}$, which are essentially dependent on resin stiffness. In particalar it appears that adhesive $\neq 2$ canees an excesaive stipfening of the resin.
-c) The values of $B_{Y} \nu_{X}$ are systematically lower than the falues of $E_{X} D_{Y}$, even comparing onl $\frac{y}{y}$ the measurementa taken Prom strain gagea applied $\frac{X}{W}$ th the same adhesiva. Thys may appear as a lack of symmetry of the elastic matrix, but most probably it is simply due to the fact that specimen width is not enough to transier transverge contraction from the inner layer to the outer ones, for specimens II.
\#evertheless the authors think that the methodology so far testad has proved to posses a high potential accuracy; in the future larger specimens whil be used, and the effect of adhesive will be carefoully investigated.

## 7. TEST ON BLADES SECTIONS

Several experimental tests wh different beam cross section and construction have been planned. Figurs 14 shows typical cross sections employed in tests. The test articles have been made from different composite materials, different number of pre-preg layers, and difierent fiber angle.
Also the material emplojed come from different manufacturera.
Tests are run with a frame allowing the loads to be applied in a well defined plane, without undesired effects on other planes, and able to give good evaluations of shear center location.
Loads are applied by meights, and deflection have been meagured with dials in the first tasta (figure 15), and subsequently with a set of IVDI transducers (figure 16).

Strain gages have also been fit, to detect the strains of the outer layers. Figure 17 shows some typical bending and twiat measurement obtained from the teats.
Figure 17b contains also results from a metal blade tested for the first evaluation of the equipent and of the results of HANBA. In torsion results it appears that the length of the specimen is sufficient to obtain a significant part of the diagram with a linear trend of the twist angle, i.e. outaide the influence of and restreint.

## 8. EVALUATTON OF HANBA RSSULTS

Four types of beam sections have been so far tested, shown in figure 18. Section $A$ was tested only in bending, in both principal planes; section $D$, corresponding to a typical metal blade mas tested only in torsion, while the remaining aections mere teated both in bending and in torgion.
In each test bending deflection and twist angles were measured at 9 stations, in different load levels.
All these measurements were then used, in an optimizing procedure, to compute the value of the st土fenesses and the values of the slope at the first atation (one for each load), giving the best fit of experimental points.
Such experimental values of the stiffnesses were then compered with the corre sponding values obtatned analytically Irom the program HaNBA, with the ideali zations showed in figure 19.
The tables in $\operatorname{Fig} .20$ report some of these comparison. Further tests are needed to draw conclusive evaluations, and probably a deeper insigit into reain creep may give a bettar interpretation of some or torsional reaults. Nevertineless the agreement between analytical and experimental results seems to be extremely good.

## 9. FUTURE DEVEIOTHCNTS

The main future developments, related to the completion of the research plan, are:

- completion of the linear version of the program HaNBA, trough the development of isoparametric flange elements, capable of developing both normal and shear stresses. Completa development of the stress output module, and of the interfoces for rotor dynamics programs.
- Development of a non linear version of HANBA, allowing for general non linear behaviour of materials, and including also strength evaluations. As it has been outlined above, thits will require some further theoretical work and much experimental actitity.

In the mean time another research project is now being planned as a joint van ture between IASP and AGUSTA, concerning the development and validation of new codes for the analysis of rotor dyamics and instabilities.

## 10. COMCLUDIIS REMARKS

The resulta so far obtained strengthen the opinion of the authors that the ap plication of finite element techniques to the classical problem of De Saint Vénant, can give, for many actual problems, an answer which may be more effec tive, more fleatble and more usable then the one of usual F.E. programs.

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Fig. 1 -POTENTIAL USE OF COMPOSITES IN THE STRUCTURE OF A MODERN HELICOPTER.


Fig. 2 - THE RESEARCH PLAN


Fig. 3


Fig. 4

| SYMBOL | ELEMENT | EXAMPLE OF USE |
| :---: | :---: | :---: |
| $\cdots$ | parel. <br> 2-NODE ISOPARAMETRIC |  |
|  | PANEL <br> 3-NODE ISOPARAIETRIC |  |
|  | PANEL <br> 4-NODE ISOPARAMETRIC |  |
| - | flange <br> concentrated area |  |
|  | FLANGE <br> non concentrated |  |
| $i$ | JOINT |  |

Fig. 5 - elements in program hanba


Fig. 6


Fig. 7


Fig. 8 - GROSS FLOF OF PROGRAM HANBA


$$
\begin{array}{ll}
x=E N D \\
\odot=\text { INTERNAL NODES OF A PANEL ELEMENT } & \Delta \text { FFLANGES HAVING MASS AND STIFFNESS } \\
\text { ELEMENT } & \diamond \text { =FLANGES HAVING ONLY MASS }
\end{array}
$$



Fig. $9 \mathrm{~B} \rightarrow \mathrm{GRID}$ POINTS


Fig. 9C. - ELEMENT NODES


Fig. 10 - SPECIMENS FOR ELASTIC TESTS.


Fig. 1I- ELASTIC TESTHNG


Fig. 12 - CREEP BEHAVIOUR OF SPECIMENS TYPE III.


Fig. 13A - TYPICAL STRESS-STRAIN CURVES FOR THE THREE TYPES OF SPECIMEN.

$$
\text { STRESS } \div\left[\mathrm{kg} \mathrm{~mm}^{-2}\right]
$$



Fig. 13 E - TYPICAS STRESS-STREIN CURVES FOR TEE THBEK TYPES OF SDEGIMEN


Fig. 14 A -"C" SPAR test section


Fig, 14 B-"D" SPAR TEST SECTION


Fig. 14 C - COMPLETE SPAR TEST SECTION


Fig. 14D- COSPLETE BLADE TEST SECTION


Fig. 15 - TEST ON A BLADE SPAR


Fig. 16 - TEST ON BLADE SPAR - GIFASUREMENTS WITH LVDT TRANSDUCERS.




Fig. $18 \mathrm{C}-{ }^{\prime \prime}{ }^{\prime \prime}$ SPAR TESTED


FIg. 18 - BLADE TESTED


Fig. 19 A- "C" SECTION GRID POFNTS.


Fig. 19. B-C ${ }^{\prime 2}$ 'D' SECTION GRID POINTS.


Fig. IG D. $\because G O M P L E A E B L A D E$ SECTION GRID POINTS.

Fig. 20 A TORSIONAL TEST. $\mathrm{N}^{\circ} 1$-BLADE SECTION Fig. 18D

| $\begin{aligned} & \therefore P P L I E D \\ & \text { TORQUE } \\ & \text { Kgmi } \end{aligned}$ | I- BEAM ROTATION [RAD $10^{-3}$ ] |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | STA 1900 | STA 1800 | STA 2600 | STA 3000 ] | ST303400 |
| 391.5 | 0.06966nmen | \%0923: | 0.3826 | 0.0 .3217 - | 0.585 |
| 783 | 0.1449 | - 0.4869 | 0.8111 | $1.0434^{-}$ | 1.200 |
| 1174.5 | 0.2261 | $0.71$ | 1.231\% | 1.4980 | 1.794 |


| $\begin{aligned} & \text { TORQUE } \\ & {[\mathrm{Kgm}]} \end{aligned}$ | 391.5 | 783.0 | 1174.3 |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { EXPERIMENTAI } \\ \text { STIFFYESS } \end{gathered}$ | 0.1813 E 10 | O. 1731.310 | 0.1798 E 10 |
| $\%$ DISPL. | 2. $\mathrm{t}^{\text {a }}$ | \% 0.9 | 2. 1.9 |

$G J$ HAYBA $=0.1765 E 10$

Fig. 20 B BENDING TEST $n^{\circ} 1$ - "D" SPAR Fig. 18C.



EJ HANBA $=0.9256$ E9 EJ BEST FIT $=0.943 \mathrm{E} 9$

Fig 20 D
BENDING TEST $\mathrm{n}^{\circ} 3-\mathrm{DD}$ SPAR Fig. 18 C .

| APPTIED Co4 [ Kg] |  | BEAM DEETAETION [MM] |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | STA 238 | STA 389 | STA 439 | Sxem-539 | Sma exe | STA 739 | STA 339 | STA 942 |
| $\begin{array}{\|c\|} \hline \\ 82.15 \\ 8 \\ 8 \end{array}$ | Exfrimeray |  | Q5 | -2.9 | - 3.8 | 485 | 6.05 | 7.35 | 8.65 |
|  | Caleuled |  | 300 | $5: 84$ | 3, 8i7 | 4.8 | 6, 068 | 7.309 | 8.641 |
|  |  |  | $-2.5$ | $18$ | 0 | 0.98 | 0. 29 | 0.55 | 0.1 |
| 41.15 | EXPRRIMENTAT | 0.65 | 1. | $\underline{4}$ | $\xrightarrow{3}$ | 2.45 | 3.1 | 3.8 | 4.55 |
|  | CALCUEPD- | 0.65 | 702 | 1.461 | -1. 8.66 | 2.525 | -3,129 | 3.769 | 4.455 |
|  | \% DISPE. | 0. | - $2 \times$ | $-43$ | $4-1.78$ | - | 0.9 | -0.8 | -2. |

EJ EANBA $=0.433 \mathrm{E} 10$ EJ BEST FTH"O.433 E 10

Fig. 20 E BENDING TEST $n^{\circ} 1$ - "C" SPAR Fig. 18 A.

| $\begin{aligned} & \text { APPLIED } \\ & \text { LQAD } \\ & (K G) \end{aligned}$ |  | - |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | STA 238[STA 339] | STA 439 STA -539 | [5TA 639 | STA 739 | STA 839 |  |
|  | EXPERIM. | $.725+1.375$ | $2.15{ }^{1}-05$ | 4.05 | $5.1$ | 6.3 | \% |
| 41.15 | CALC. | .725 1.382 | 2.169 3.071 | 4.071 | 5.15 | $\square \times 4$ -6.307 |  |
| $\therefore \cdots$ | \% DISPL | 0.0 .5 | 0.88 -0.688 | $0.51$ | " 1 | $0.1$ | " |
| - | EXPERIM. | $1 . \overline{1}+2$, | 3.125 4.5 | 5.95 | 7. 7. | 9.35 | +-.... |
| 61:75 | CALC. | 1.1 | $3.216{ }^{4.544}$ | 6.021 | 7. 62 | -9.325 |  |
|  | 9 DISPL. |  | 2.91 .0 .977 | 1.2 | 0.29 | [-0.27 |  |

EJ HANBA $\quad 0.2503 \mathrm{E} 10 \quad \mathrm{EJBEST} F \mathrm{FT}=263 \mathrm{E} 10$

Fig. 20 F BENDING TEST $\mathrm{n}^{\circ} 3$ - "C"SPAR Fig. 18 A

| $\begin{gathered} \text { APPLIED } \\ \substack{\mathrm{GOAD} \\ (\mathrm{Kg}) \\ \hline} \end{gathered}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |
|  | EXPERTM | 1.375 | 32625 | 5379 ${ }^{3} 375$ | +181.5875 | 115.375 | 19.3125 | ¢ |
|  | CaLC.: : | 21.395. | 3.344 | 5.792 .8 | 11.895 | 15.434 | 19.222 |  |
|  | W, DISPIF. | 10. | 2.50 | 2,7402 | 1-1.77 | \%.38 | -0,46 |  |
| $51.3$ | EXPERIM. | 1.6875 | 4.05 | 6.95 10,3125 | 14.5625 | 19.25 | 24.375 |  |
|  | CALC.:- | : 1.688 | 4.167 | 7.243 10.845 | 14.9 | 19.337 | 24.084 |  |
|  | \% DISPI. | 0. res | 2,88 | $4.2 \times 5.16$ | $\mid 2.31$ | 0.45 | -1.2 |  |



Fig. 20 G TORSIONAL TEST $\mathrm{n}^{\circ} 1 \mathrm{Z}$ - D "SPAR Fig. 18 B


| $\begin{aligned} & \text { qraRQUE } \\ & \text { (Kgm): } \end{aligned}$ | $\begin{aligned} & 2-7-7 \\ & : \end{aligned}$ | $1-8.160$ | $13,600$ |
| :---: | :---: | :---: | :---: |
| EXPERIHEMTA STIFENESS | . 310 E 09 | .27 E ${ }^{-109}$ | $-.26 \mathrm{E} 09$ |
| $\mathscr{G}$ DISPL. | 6. $\quad \cdots$ | - | $\cdots \cdots$ |

GJ HANBA $=0.3285 E 09$

