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Abstract

The Ffowcs Williams and Hawkings equation is analyzed with the intent of making aerodynamic applications. The flow is assumed inviscid, incompressible, and the quadrupole term is neglected. The result is transformed into a linear integral equation of the Fredholm type using Green's functions. A new interpretation of the monopole term is given, which accounts for the motion of the body with respect to a frame of reference fixed to the fluid at rest and for velocities induced by this motion. A solution to steady and two-dimensional problems is developed and applied to families of elliptic cylinders and symmetric airfoils. An iterative procedure is established between velocity and pressure fields using Bernoulli's equation. Improved correlation is obtained with results from potential theory for non-lifting bodies of appreciable thickness. The new results suggest that the quadrupole term is important in correcting overstagnation pressures near the leading edge of airfoils, in improving the overall solution for bluff bodies, and in representing circulation effects. Another conclusion is that in inviscid flow the monopole term should be considered unknown whereas in viscous flow it is known for a given motion of the body. However, in the latter case the importance of the quadrupole term should not be underestimated.

Nomenclature

a	non-dimensionalized minor axis of elliptic cylinder
a_j	coefficient of the pressure series expansion on the upper surface of the body
b_j	coefficient of the pressure series expansion on the lower surface of the body
$\vec{b}_i^B$	unit vector of reference frame fixed to moving body
$\vec{b}_i^F$	unit vector of reference frame fixed to undisturbed fluid at rest
c	speed of sound in undisturbed medium
c_l	lift coefficient
c_p	pressure coefficient
f	function that describes the body surface when equal to zero
h	airfoil thickness to chord ratio
i	index referring to ith observer station
j	index referring to jth pressure mode
$\mathbf{K}_m$	integrand of the motion integral
$\mathbf{K}_p$	integrand of the pressure integral
l	index referring to the lower surface of the body
le	subscript indicating leading edge
m_i	forcing vector of two-dimensional problem
M	Mach number of body motion seen by fluid particle at rest
M_n	Mach number in the direction normal to the body surface
M_r	Mach number in the direction source-observer
$\hat{n}$	unit vector normal to body surface
p	perturbation pressure
P_{ij}	compressive stress tensor or system matrix
q	function describing the non-dimensionalized velocity tangent to the body surface
$\hat{r}$	unit vector in the direction source-observer
r	$\lvert\vec{x}(t) - \vec{y}(\tau)\rvert = (\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2)^{\frac{1}{2}}$
ret	subscript indicating expression evaluated at retarded time
S	body surface
t	time at the observer point

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$\hat{t}$	unit vector tangent to body surface
te	subscript indicating trailing edge
T_{ij}	Lighthill's stress tensor $P_{ij} + \rho u_i u_j - c^2(\rho - \rho_o)\delta_{ij}$
u	index referring to the upper surface of the body
u_i	fluid particle velocity vector
$\vec{u}_m$	velocity vector of the absolute motion of the body
V	airfoil velocity in two-dimensional steady problem
v_n	body velocity in the direction normal to its surface
v_r	body velocity in the direction source-observer as seen by fluid particle at rest
$\vec{x}$	position vector of observer point
$\vec{y}$	position vector of source point
α	airfoil angle of attack
δ	Dirac's delta function
δ_{ij}	Kronecker delta
θ	$\cos^{-1}(\hat{n} \cdot \hat{r})$
κ	parameter to assess the convergence of the iterative scheme
ρ	density of fluid
ρ_o	density of undisturbed fluid
τ	source point retarded time $t - \frac{r}{c}$
ϕ	pressure mode shape function
Ω	scalar quantity related to rotations of the body
Δx_i	difference of spatial coordinates
∇	gradient operator
$\bar{\nabla}^2$	generalized Laplacian operator
$\bar{\Box}^2$	generalized wave operator $\frac{1}{c^2}\frac{\bar{\partial}^2}{\partial t^2} - \frac{\bar{\partial}^2}{\partial x_i^2}$

1 - Introduction

Analysis of the aerodynamics of rotating blades is a challenge to any skilled aerodynamicist. The analysis is complex due to the nature of the flow, which can be described as essentially three-dimensional, compressible, unsteady, non-linear and viscous. Other characteristics such as wake effects, blade-vortex interaction, dynamic stall, reversed flow, and unsteady free stream add to the complexity of the problem. The lack of appropriate solution techniques usually forces the aerodynamicist to oversimplify the analysis. The majority of current aeroelastic problems are solved using two-dimensional, incompressible, quasisteady, linear, and inviscid aerodynamic theories. If one attempts to go beyond that, the sophistication of the analysis and the accompanying computing cost increase dramatically.

Here we consider a solution technique which has powerful analytical capabilities, yet requires modest computational effort compared to more complex methods of Computational Fluid Dynamics. The approach is based on the application of the acoustic analogy of Lighthill[1] to aerodynamics. The Ffowcs Williams and Hawkings (FW-H) equation[2] may be written as follows:

$$\bar{\Box}^2 \left[c^2(\rho - \rho_o)\right] = \frac{\bar{\partial}}{\partial t}\left[\rho_o v_n |\nabla f| \delta(f)\right] - \frac{\bar{\partial}}{\partial x_i}\left[P_{ij}\frac{\partial f}{\partial x_j}\delta(f)\right] + \frac{\bar{\partial}^2 T_{ij}}{\partial x_i \partial x_j} \tag{1}$$

where $\bar{\Box}^2$ is the generalized wave operator. The bars over the derivative signs indicate that the functions operated on should be considered as generalized functions[3,4]. Recognizing the product $c^2(\rho - \rho_o)$ as the linear approximation for the perturbation pressure p in isentropic flow, equation (1) can be regarded as an inhomogeneous wave equation in the perturbation pressure. The three forcing terms are known in the literature respectively as monopole, dipole and quadrupole terms. The equation gives p at time t and position vector $\vec{x}$ due to point sources at retarded time τ and position vector $\vec{y}$. These sources are everywhere on the flow field surrounding a body of surface S which moves through a fluid of undisturbed density ρ_o. The body surface is described by the equation $f = 0$. The quantity v_n denotes the absolute body velocity normal to its surface, P_{ij} is the compressive stress tensor, T_{ij} is the stress tensor of Lighthill and $\delta(f)$ is the Dirac delta function of the body surface.

The FW-H equation results from the conservation laws of mass and momentum, coupled with boundary conditions introduced via the theory of distributions. Derived from first principles, the equation is quite general and one should expect to obtain accurate results from its application. However, this has not been the case. Simplified versions of this equation have failed to correlate with experimental results[5]. These failures have resulted in controversial discussions among aeroacousticians[6].

The generality of the equation also makes it applicable to aerodynamics. Studies of this subject were given by Long[7] and by Brandão[8]. However, the same difficulties experienced in aeroacoustic applications appear in aerodynamic problems. Without the use of the quadrupole term, the method fails in large disturbance and lifting problems. The reasons for this failure have not been clarified yet, but have prompted researchers to examine alternate solution techniques which are equally valid for aeroacoustic and aerodynamic applications. One of these formulations was given by Farassat[9], discussed by Long and Watts[10], and explored more extensively by Farassat and Myers[11]. After some simplifications, this approach leads to a Volterra type integral equation which is difficult to solve. The method seems to be successful in describing circulation effects, but so far, applications have been restricted to the case of zero thickness wings.

Here we explore the idea of using the original FW-H equation to improve the solution to simple problems of two-dimensional and incompressible flow. The improvement results from a new interpretation of constituents of the equation. It is shown that a linear formulation, with the quadrupole term neglected, leads to better correlation with results of potential theory. Although the numerical applications are elementary, they provide a basis on which to perform more advanced applications.

2 - The Simplified Problem

As pointed out earlier, the first approximation is to consider the product $c^2(\rho - \rho_o)$ equal to the perturbation pressure p. The second approximation consists of taking the idealized case of inviscid flow. These two assumptions reduce the compressive stress tensor P_{ij} to its diagonal form and the Lighthill stress tensor T_{ij} to $\rho u_i u_j$. The third approximation is to assume that the spatial gradients of T_{ij} only make a small contribution to the inhomogeneous part of the equation. This last assumption is not true, for example, near the leading edge of airfoils, for problems involving bluff bodies or for flows with vortices and shock waves. The combined effect of these simplifications is equivalent to neglecting the quadrupole term of equation (1). The FW-H equation then reduces to

$$\bar{\square}^2\, p = \frac{\bar{\partial}}{\partial t}\left[\rho_o v_n |\nabla f| \delta(f)\right] - \frac{\bar{\partial}}{\partial x_i}\left[p n_i |\nabla f| \delta(f)\right] \tag{2}$$

The result above presents only surface source terms, which somewhat simplifies the solution approach. Farassat[9] used full domain Green's functions to transform this differential equation into an integral equation of the Fredholm type with the following structure:

$$p - \frac{1}{4\pi}\int_{f=0} \mathbf{K}_p \, dS = \frac{1}{4\pi}\int_{f=0} \mathbf{K}_m \, dS \tag{3}$$

The integral on the left hand side results from the dipole term and will be called *pressure integral* whereas the integral on the right hand side results from the monopole term and will be called *motion integral*. For the subsonic compressible case the integrands are given by

$$\mathbf{K}_p = \left[\frac{p\Omega_r + \dot{p}\cos\theta}{cr(1-M_r)^2} + \frac{p\dot{M}_r \cos\theta}{cr(1-M_r)^3} + \frac{p\left[\cos\theta(1-M^2) - M_n(1-M_r)\right]}{r^2(1-M_r)^3}\right]_{ret} \tag{4}$$

$$\mathbf{K}_m = \left[\frac{\rho_o c(\Omega_M + \dot{M}_n)}{r(1-M_r)^2} + \frac{\rho_o c M_n \dot{M}_r}{r(1-M_r)^3} + \frac{\rho_o c^2 M_n (M_r - M^2)}{r^2(1-M_r)^3}\right]_{ret} \tag{5}$$

where the definition of all quantities is given in reference [8].

The above integrands bear an inherent complexity because they must be evaluated at retarded time. In other words, information arriving at time t at the observer position $\vec{x}$ has been emitted by sources at position vectors $\vec{y}$ and past times τ, and has propagated towards $\vec{x}$ with the speed of sound c. To bypass this complexity and at the same time allow a basic investigation of the present technique, we introduce one more simplification, namely, the assumption that the flow is incompressible. Due to the infinite speed of sound, the concept of retarded time disappears, and source information arrives instantaneously at the observer position. With this assumption, equation (2) becomes

$$\bar{\nabla}^2 p = -\frac{\bar{\partial}}{\partial t}\left[\rho_o v_n |\nabla f| \delta(f)\right] + \frac{\bar{\partial}}{\partial x_i}\left[p n_i |\nabla f| \delta(f)\right] \tag{6}$$

where $\bar{\nabla}^2$ is the generalized Laplacian operator.

Applying the Green's function technique to equation (6), we obtain an integral equation with the same structure of equation (3), but with integrands now simplified to

$$\mathrm{K}_p = \frac{p\cos\theta}{r^2} \qquad \mathrm{K}_m = \frac{\rho_o(\Omega_v + \dot{v}_n)}{r} + \frac{\rho_o v_r v_n}{r^2} \tag{7}$$

This formulation can be applied to three-dimensional and unsteady problems of fixed or rotating wings. It can easily be adapted for rotorcraft applications because we consider the body motion from a frame of reference fixed to the undisturbed fluid at rest, a common concept for aeroacousticians, but a not so common idea for aerodynamicists. With this perspective, there is no restriction to the body being perfectly rigid. Therefore, the field of aeroelasticity may also benefit from its use.

3 - A New Interpretation of the Monopole Term

In three-dimensional compressible flow the conservation laws of mass and momentum have five unknowns. If these two laws are combined into a single wave equation, it is expected that these unknowns are included somewhere in its terms. Usually in aeroacoustics the monopole term is assumed known from the absolute motion of the body. Effects resulting from unknown velocities induced by the motion are included in the quadrupole term. However, this is the term that has been neglected in most applications of the FW-H equation.

The key point of the interpretation given here is to recognize that the sources of the wave equation are *in the fluid*. Once this is accepted, if we consider surface sources we should look at the *fluid* near the surface and not at the surface itself. In the case of viscous flow, due to the no-slip condition, the fluid close to the surface has the same absolute motion as the body. This is the case where the monopole term is known. In other words, this is the case where the vector $\vec{v}$ in the monopole term is given by the prescribed motion. However, the quadrupole term becomes more important because it now includes the spatial gradients of the boundary layer velocity field. Therefore, the quadrupole term should not be neglected if the monopole term is assumed known.

So far, research with equation (1) has been conducted under the general assumption of inviscid flow. In this case the velocity of the fluid close to the surface *is not* the same as the absolute velocity of the surface. Therefore, in inviscid flow we should use an expression for the vector $\vec{v}$ which accounts not only for the known absolute motion of the body, but also for the unknown velocities induced by this motion. This interpretation is consistent with the fact that the FW-H equation is a single equation with more than one unknown. It is also consistent with the assumption of neglecting the quadrupole term because in inviscid flow there are no boundary layer velocity gradients and, accordingly, the quadrupole term should contribute less to the overall problem.

To make this informal discussion more exact, let us present the subject considering the two-dimensional steady motion of airfoils with velocity V. In this case equation (3) assumes the form

$$p - \frac{1}{4\pi}\int_{f=0} \frac{p\cos\theta}{r^2}\, dS = \frac{1}{4\pi}\int_{f=0} \frac{\rho_o v_r v_n}{r^2}\, dS \tag{8}$$

Using the geometry defined in Figure 1, the usual way of writing the velocity vector that appears in the motion integral is

$$\vec{v} = -V\,\vec{b}_2^F = -V\left[\sin\alpha\,\vec{b}_1^B + \cos\alpha\,\vec{b}_2^B\right] \tag{9}$$

This definition gives only the absolute motion of the body. It does not provide any hint of the occurrence of induced velocity in the flow surrounding the surface. In contrast, we propose using

$$\vec{v} = \vec{u}_m - qV\,\hat{t} - \left(\vec{u}_m \cdot \hat{t}\,\right)\hat{t} \tag{10}$$

where $\vec{u}_m$ is the pure motion part of the velocity vector, given by equation (9), $q(x_2)$ is an unknown function that includes motion and velocity induced by thickness and lifting effects of the airfoil, and $\hat{t}$ is the unit vector tangent to the airfoil surface. Note that in the above definition of the vector $\vec{v}$ the tangent component of $\vec{u}_m$ is subtracted because it is already included in the function $q(x_2)$.

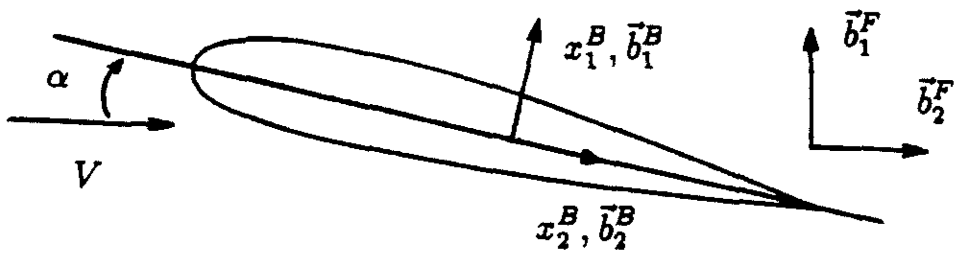

Figure 1: The geometry of the two-dimensional airfoil problem.

Two comments about the above definition are necessary here. First, in establishing this formula it is useful to choose a fluid particle close to the airfoil surface, imagine that it is at rest, and ask how it is seeing the motion of the airfoil. With this reasoning, equation (10) becomes the result. Second, we should note that the two new and last terms introduced in the definition of $\vec{v}$ are *tangent* to the body surface. This means that they do not contribute to the normal component v_n that appears in the formula of the monopole. This part of the problem continues to be given entirely by the absolute motion of the body. Therefore, there is no conflict between the present proposition and the original work of Ffowcs Williams and Hawkings[2]. However, these two terms do contribute to the radiation component v_r of the monopole. Here we try to determine the importance of this contribution.

Using definition (10) for the velocity vector of the monopole term, integrating spanwisely along an infinite wing and non-dimensionalizing equation (8) so that

$$c_p = \frac{p}{\frac{1}{2}\rho_o V^2}$$

we obtain the following:

$$\begin{aligned}
c_p(x_2^o) - \frac{1}{2\pi}\int_{-1}^{1}\left[\frac{c_p(x_2)}{r^2}\left(\Delta x_1\frac{\partial f}{\partial x_1} + \Delta x_2\frac{\partial f}{\partial x_2}\right)\right]_{(u+l)} dx_2 \quad = \\
\frac{1}{\pi}\int_{-1}^{1}\left\{\frac{1}{r^2}\left\{\sin^2\alpha\left\{\frac{\partial f}{\partial x_1}\Delta x_1\left[1-\left(\frac{\partial f}{\partial x_2}\right)^2\frac{1}{|\nabla f|^2}\right] + \frac{\partial f}{\partial x_2}\frac{\Delta x_2}{|\nabla f|^2}\right\}\right.\right. \\
+ \cos^2\alpha\left\{\frac{\partial f}{\partial x_2}\Delta x_2\left[1-\frac{1}{|\nabla f|^2}\right] + \frac{\partial f}{\partial x_1}\Delta x_1\left(\frac{\partial f}{\partial x_2}\right)^2\frac{1}{|\nabla f|^2}\right\} \\
+ \sin\alpha\cos\alpha\left\{\frac{\partial f}{\partial x_1}\Delta x_2\left[1-\frac{1}{|\nabla f|^2}+\left(\frac{\partial f}{\partial x_2}\right)^2\frac{1}{|\nabla f|^2}\right]\right. \\
\left.\left.\left. + \frac{\partial f}{\partial x_2}\Delta x_1\left[1+\frac{1}{|\nabla f|^2}-\left(\frac{\partial f}{\partial x_2}\right)^2\frac{1}{|\nabla f|^2}\right]\right\}\right\}\right\}_{(u+l)} dx_2 \\
+ \frac{1}{\pi}\int_{-1}^{1}\left\{\frac{q(x_2)}{r^2|\nabla f|}\left\{\sin\alpha\left[\frac{\partial f}{\partial x_1}\Delta x_2 - \frac{\partial f}{\partial x_2}\Delta x_1\right]\right.\right. \\
\left.\left. + \cos\alpha\left[\frac{\partial f}{\partial x_2}\Delta x_2 + \frac{\partial f}{\partial x_1}\Delta x_1\left(\frac{\partial f}{\partial x_2}\right)^2\right]\right\}\right\}_{(u+l)} dx_2
\end{aligned} \tag{11}$$

This equation applies to any point on the airfoil surface. The subscript $(u+l)$ indicates that the integrations should be carried out on both upper and lower surfaces of the airfoil. Each integral should be interpreted in the limit as the observer, being initially outside the body, approaches it along the normal line to the airfoil surface. This concept allows continuity in the value of each integral as the observer approaches the body and also generates interesting singularities of the Dirac delta type at the surface, as discussed in reference [8]. When the integration runs through the observer, the integral is formally singular. This happens with half of the integrals in equation (11). In this case, the integrals have been regularized by the process described in reference [12]. Full details of the treatment of these integrals and also of the derivation of the equation above can be found in reference [13].

4 - Some Applications

In the next sections we describe results obtained with equation (11). The unknown pressure coefficient is expanded into an appropriate series. Coefficients a_j and b_j are used respectively to represent the series expansions on the upper and lower surfaces of the body. The exact shape of the surface is used to obtain the function f and all integrals are evaluated using Gauss-Legendre quadrature[14] with an accuracy of the order of 10^{-6}. Using n terms in each series expansion and applying equation (11) to $2n$ collocation points results in the following system of linear equations:

$$\sum_{j=1}^{n} P_{ij}^{uu} a_j + \sum_{j=1}^{n} P_{ij}^{ul} b_j = m_i^{uu} + m_i^{ul}$$

$$\sum_{j=1}^{n} P_{ij}^{lu} a_j + \sum_{j=1}^{n} P_{ij}^{ll} b_j = m_i^{lu} + m_i^{ll} \tag{12}$$

The subscript i refers to the collocation station x_{2_i} and the subscript j to the jth pressure mode. The double superscripts of P_{ij} and m_i indicate, in the order given, on which surface of the body observer and source points are located. For example, ul means observer on the upper surface and sources on the lower surface.

4.1 - Family of Elliptic Cylinders

A family of elliptic cylinders can be described by the following equations in terms of body-fixed spatial coordinates:

$$f_u(x_1, x_2) = x_1 - a\sqrt{1 - x_2^2} \qquad f_l(x_1, x_2) = -\, x_1 - a\sqrt{1 - x_2^2}$$

The governing parameter here is the non-dimensionalized minor axis a, which yields a flat plate when equal to zero and a circular cylinder when equal to one. For the pressure distribution on the upper surface we have assumed a Fourier sine series given by

$$c_p(x_2) = a_o + \sum_{j=1}^{n} a_j \sin\left[j\frac{\pi}{2}(1 + x_2)\right]$$

A similar expansion is assumed for the lower surface. These series have all the attributes necessary to describe the expected solution. The unknown function $q(x_2)$ was taken from Milne-Thomson[15] as the exact result from potential theory. This function may be written as

$$q(x_2) = (1 + a)\sqrt{\frac{1 - x_2^2}{a^2 + (1 - a^2)(1 - x_2^2)}}$$

Results for the circular cylinder are presented in Figure 2. As observed in reference [8], convergence with respect to the series expansion is not a problem because a solution with only 7 pressure modes ($n = 6$) nearly coincides with a solution with 11 modes. Compared to the exact potential result, the answer given by the present approach appears to be shifted by a value of about 0.5 towards the compression side. This difference must be attributed to the absence of the quadrupole term in the formulation. However, this result is qualitatively much better than the one obtained with the previous interpretation of the monopole term, which is given by the dashed line. The latter not only fails by a factor of two at critical points, but also does not present a pressure distribution similar in shape to the solution of potential theory.

The circular cylinder is a difficult test for the present formulation. To assess how the method behaves for thinner bodies, we can repeat the procedure for smaller values of the parameter a. The results are presented in Figure 3. For $a = 0.7$ the two curves are much closer than in the case of the circular cylinder. For $a = 0.5$ the result over half of the chord coincides with the potential solution. For $a = 0.3$ the curves differ basically only near the edges and for $a = 0.1$ the agreement is very good. Note that the thinner the body, the greater the number of pressure modes required for a converged solution. This happens because more sine modes are necessary to describe a shallower curve and to avoid the occurrence of the Gibbs phenomenon[16] near the edges. This suggests that better behaved series or alternative ways of describing the pressure distribution on the body surface should be considered in the future.

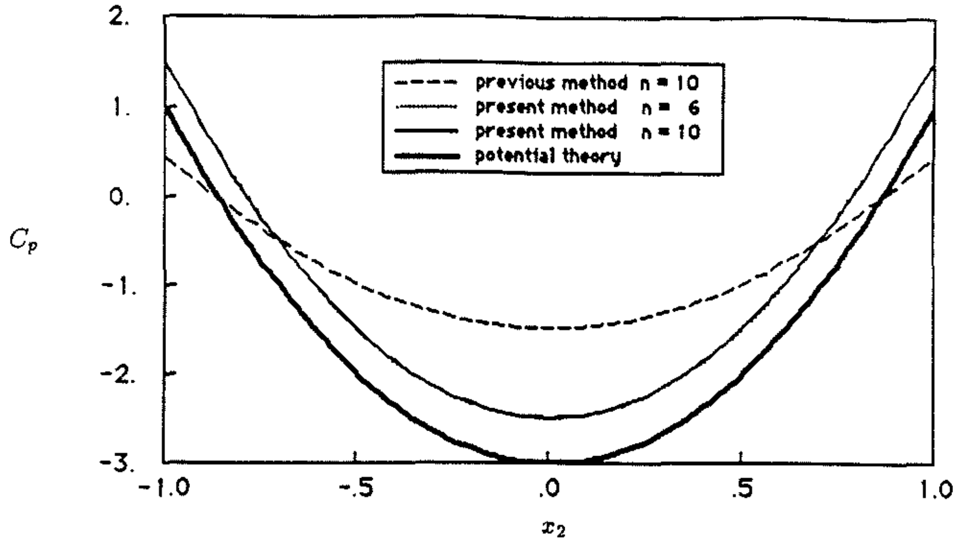

Figure 2: Pressure distribution on the surface of a circular cylinder.

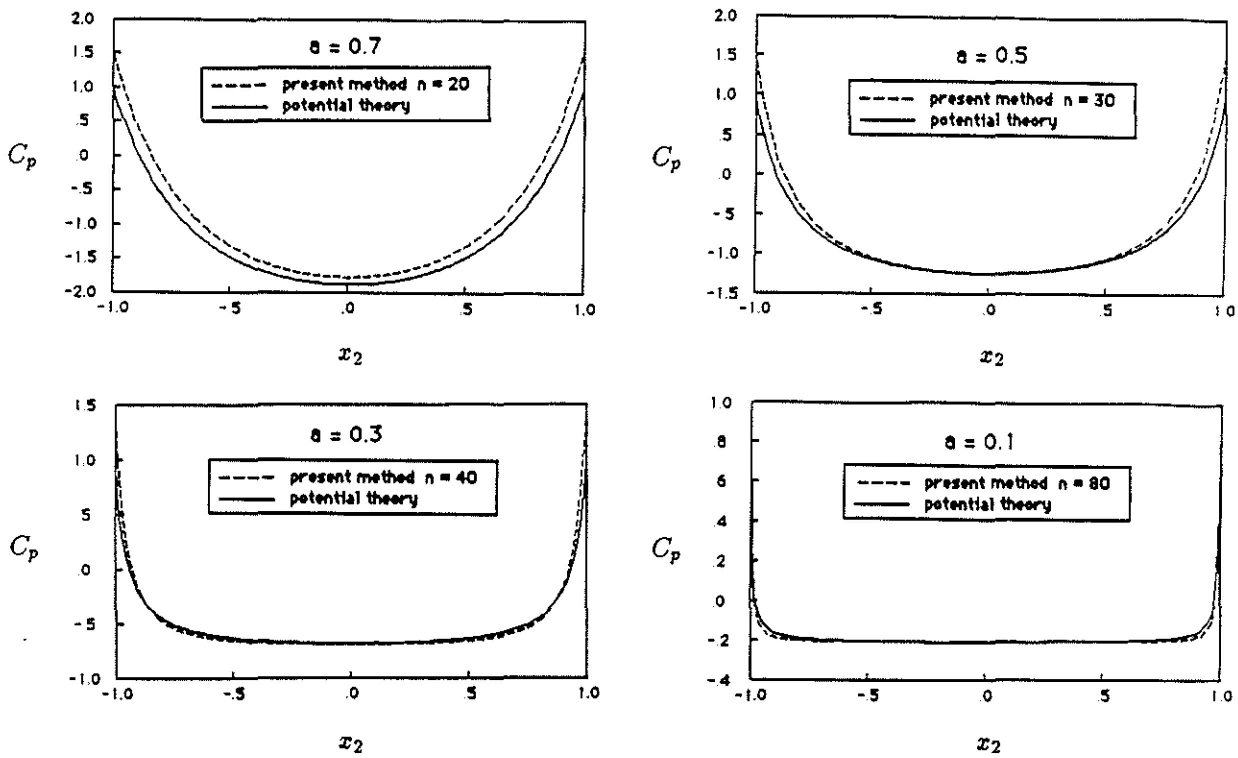

Figure 3: Pressure distribution on the surface of elliptic cylinders in symmetrical flow.

Three-dimensional bodies are less "bluff" than their two-dimensional counterparts of identical maximum thickness. Long[17] presents some results on prolate ellipsoids in three-dimensional and nearly incompressible flow. The results show a deterioration of the solution as the body maximum thickness goes from 5% to 25%. In Long's compressible formulation the monopole term is assumed known from the motion of the body, which does not correspond to the interpretation given in this paper. Thus, it is not surprising why Long's results are not good for thicker bodies. In contrast, results presented in Figure 3 indicate an improved correlation for large disturbance problems. Therefore, the present interpretation of the monopole term seems useful in making this aeroacoustic approach more robust.

4.2 - Family of NACA Symmetric Airfoils

As we did for the elliptic cylinders, a family of four-digit NACA symmetric airfoils can be described by the following equations[18] based on the geometry of Figure 1:

$$f_u(x_1, x_2) = x_1 - h\, g(x_2) \qquad f_l(x_1, x_2) = -\, x_1 - h\, g(x_2)$$

with

$$g(x_2) = 2.0994\,\sqrt{1 + x_2} - 1.2170625 - 1.575625\, x_2 - 0.1935\, x_2^2 + 0.101625\, x_2^3 - 0.0634375\, x_2^4$$

The governing parameter now is the airfoil thickness ratio h, which is equal to 0.12 for the NACA 0012, for example. For the pressure distribution on the upper surface we have used the following series expansion:

$$c_p(x_2) = a_{le}\phi_{le}(x_2) + a_{te}\phi_{te}(x_2) + \sum_{j=1}^{n} a_j \sin\left[j\frac{\pi}{2}(1 + x_2)\right]$$

with

$$\phi_{le}(x_2) = 1 - \sqrt{1 - \frac{(x_2 - 1)^2}{4}} \qquad \phi_{te}(x_2) = 1 - \sqrt{1 - \frac{(x_2 + 1)^2}{4}}$$

A similar expansion is assumed for the lower surface. In both series ϕ_{le} and ϕ_{te} are elliptic modes used to describe the stagnation pressures at the leading and trailing edges respectively.

For the non-lifting problem the function $q(x_2)$ was taken from cubic spline interpolation between points given by Abbott and von Doenhoff[18]. These points were obtained using Theodorsen's potential theory and are known to correlate well with experimental results. The formulation provides good agreement with airfoils of thicknesses ranging from 6% to 24%. For the sake of brevity, however, Figure 4 presents only the results for the NACA 0012 airfoil. The difference between the results obtained using the two distinct interpretations of the monopole term is small because this is a thin body. However, the new interpretation does a better job in reproducing the region of expansion. The behavior of this solution near the trailing edge is not very good due to a failure of the splines in describing the stiff changes of tangent velocity in that region.

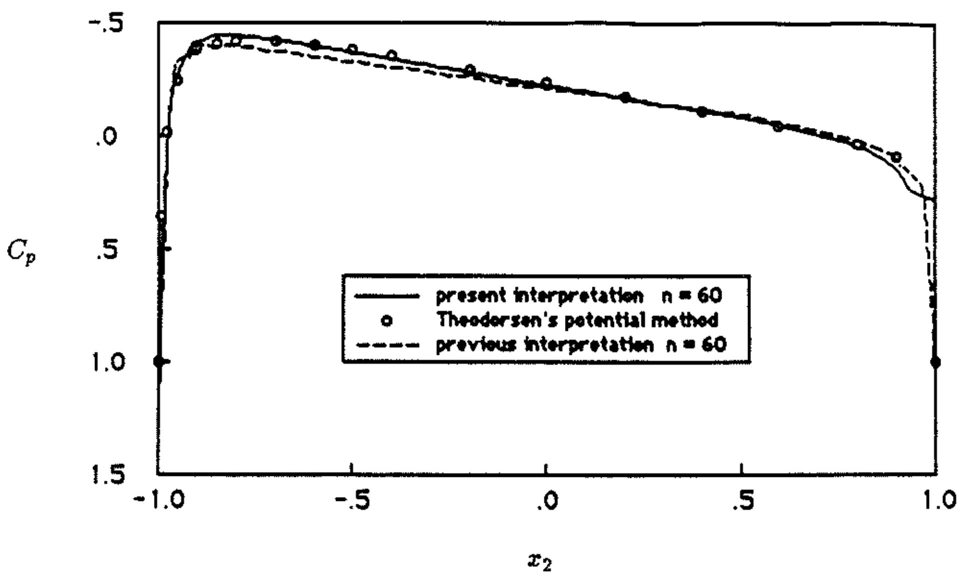

Figure 4: Pressure distribution on the surfaces of the NACA 0012 airfoil at $\alpha = 0$ degrees.

In two-dimensional flows the FW-H equation has three unknowns, i.e., c_p, u_1 and u_2. However, the equation assumes intrinsically that the surface flow is attached, which provides an extra relation between the velocity components. To make the problem well posed, so far we have assumed a tangent velocity distribution $q(x_2)$ in order to solve for $c_p(x_2)$. However, we can introduce another relation to allow a simultaneous solution for both distributions. One relation that is particularly suited to this case is Bernoulli's equation, which may be written as

$$c_p(x_2) = 1 - [q(x_2)]^2 \tag{13}$$

Although simple, this is a non-linear relation. Therefore, the solution calls for an iterative scheme which is described as follows:

1. A tangent velocity distribution $q(x_2)$ is assumed and equation (11) is solved for the perturbation pressure distribution $c_p(x_2)$.
2. With $c_p(x_2)$ available, equation (13) is used to compute a new velocity distribution $q(x_2)$.
3. Convergence is assessed by computing the following parameter:

$$\kappa = \frac{1}{2}\int_{-1}^{1} \mid q_k(x_2) - q_{k-1}(x_2) \mid dx_2$$

   where the index k refers to the kth iteration and so on. If κ is less than a certain tolerance, the process is stopped. Otherwise, the computations resume at step 1.

Occasionally, due to the absence of the quadrupole term in the formulation, values of $c_p(x_2)$ near the leading edge may become greater than one, indicating overstagnation. In computing $q(x_2)$, these values have been reset to one. This procedure improves the stability of the iterative scheme. Experience with this scheme shows that for airfoils of thickness up to 12% the convergence is monotonic to values of κ of the order of 10^{-4}. For thicker airfoils the convergence is oscillatory and never goes below a certain level of κ. The thicker the airfoil, the greater should be the acceptable value of κ. These oscillations occur because although most of the distribution of $c_p(x_2)$ has settled down, values near the leading edge keep jumping between overstagnation and understagnation. This means that the inclusion of the quadrupole term in the formulation may contribute to stabilize the scheme for thicker bodies.

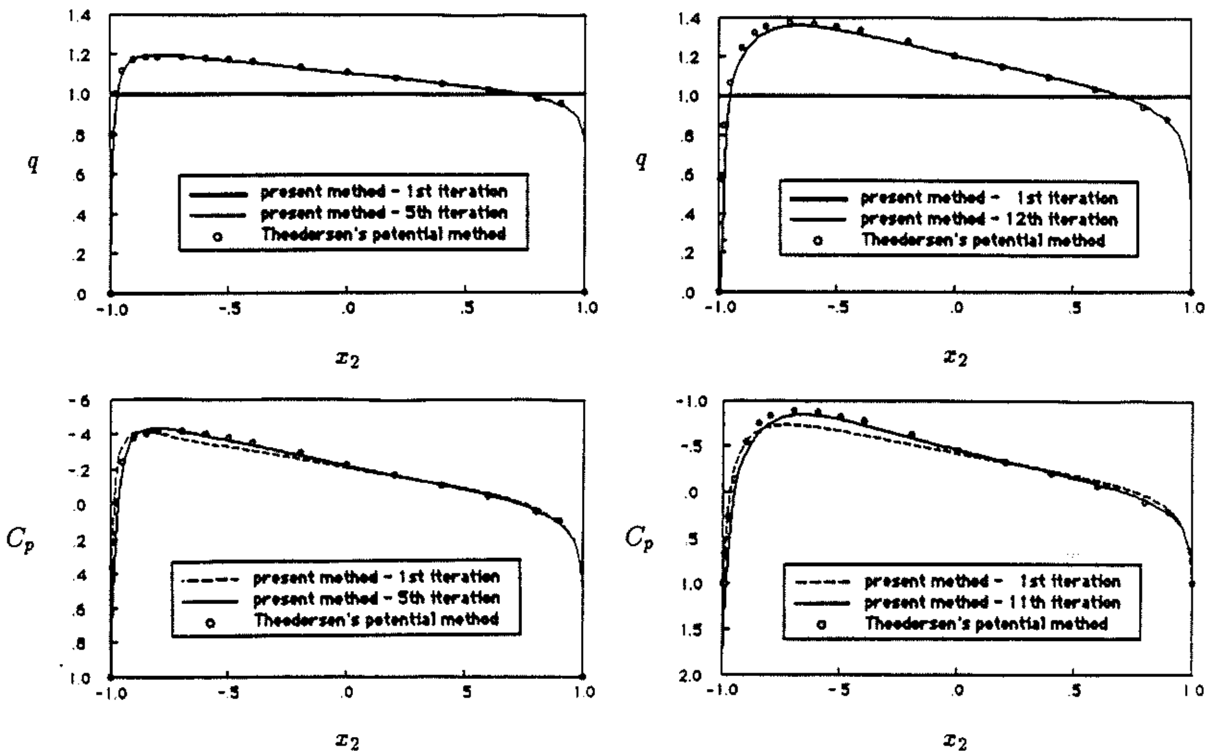

Figure 5: Results of the iterative scheme for the NACA 0012 and 0024 airfoils at $\alpha = 0$ degrees.

Figure 5 presents results obtained using this iterative plan. Due to thickness effects we can start the iterations with the free-stream velocity, which corresponds to assuming that $q_1(x_2)$ equals unity everywhere over the body. For the NACA 0012 airfoil we reach convergence in 5 iterations, with results that agree well with Theodorsen's potential theory. For the NACA 0024 airfoil results of the 11th iteration are more compressive than our standard of comparison, not describing the whole expansion and resulting in overstagnation near the leading edge. However, the results show that the present linear version of the FW-H equation behaves well when applied to a dificult test like this one.

Further discussion is concerned with lifting problems. In reference [8] we showed that in the limiting case of a flat plate airfoil of zero thickness, the previous interpretation of the monopole term leads to the following result:

$$c_{p_u}(x_2^o) + c_{p_l}(x_2^o) = 0 \tag{14}$$

which is independent of the angle of attack. This is the root of the inability of previous works to describe circulation effects with this simplified version of the FW-H equation. In contrast, this time equation (11) yields

$$c_{p_u}(x_2^o) + c_{p_l}(x_2^o) = \frac{2\sin\alpha}{\pi}\int_{-1}^{1}\frac{q_u(x_2) - q_l(x_2)}{x_2^o - x_2}\,dx_2 \tag{15}$$

The integral on the right hand side is similar to the classical downwash integral and depends on the velocity difference between upper and lower surfaces, a concept linked to circulation by definition. However, the equation has too many unknowns to give immediate and useful results.

Lifting results that appear in Figure 6 were obtained using the procedure described herein. To the converged distribution of $c_p(x_2)$ for the non-lifting case we added

$$\Delta c_p(x_2) = -\frac{\partial f}{\partial x_1}\sin\alpha\sqrt{\frac{1-x_2}{1+x_2}}$$

accordingly to both surfaces of the airfoil. Then, Bernoulli's equation was used to compute the input velocity distribution $q(x_2)$. Note that this superposition is allowed because the problem is linear in the perturbation pressure p. Note also that the term added to or subtracted from the non-lifting distribution of $c_p(x_2)$ is a classical result of thin airfoil theory adapted for the present case. With the available distribution for the tangent velocity $q(x_2)$, the system (12) of linear equations was solved for the pressure distribution $c_p(x_2)$.

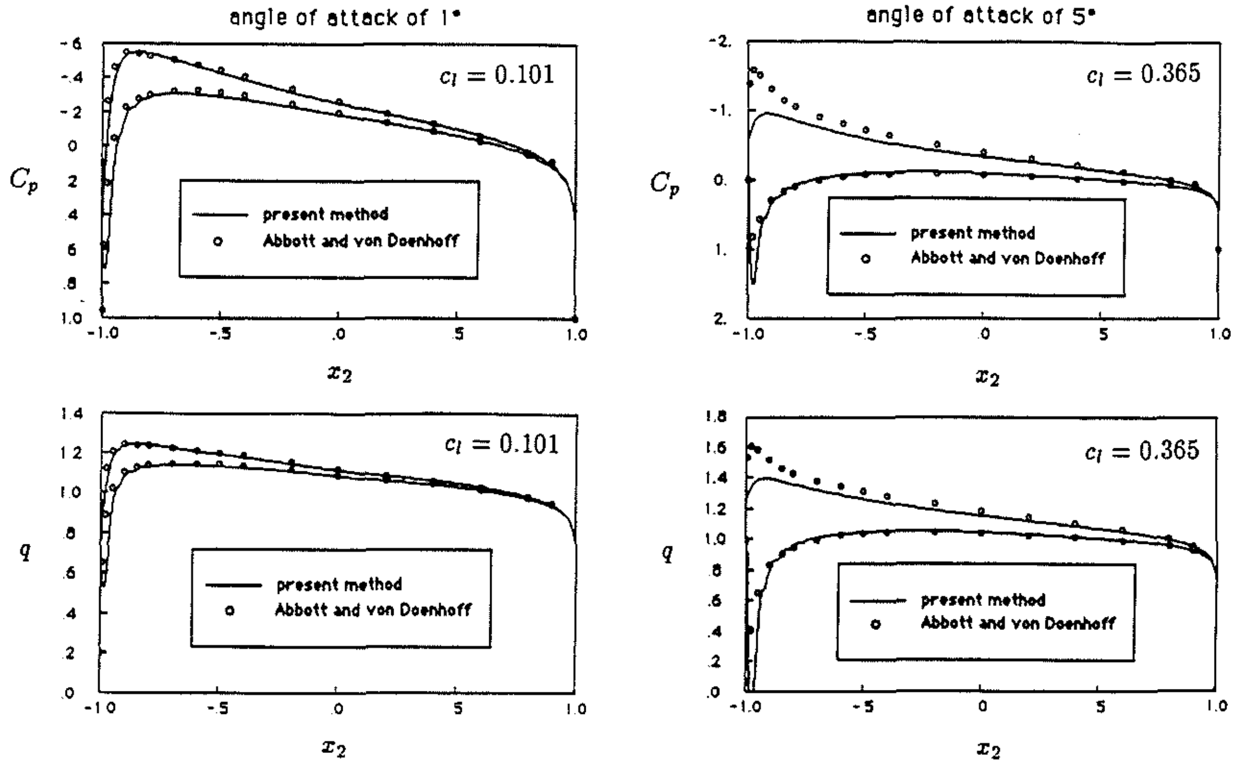

Figure 6: NACA 0012 airfoil at an angle of attack.

The system of equations was prepared in the calculations to enforce zero difference of pressure at both edges of the airfoil, thus ensuring smoothness of velocity distributions and observation of the Kutta condition. At the leading edge this enforcement was accomplished exactly by making the coefficients a_{l_e} and b_{l_e} equal. At the trailing edge this condition was obtained *approximately* by averaging the components of the forcing vector at the collocation points close to the edge.

The reasons for this difference of procedure regarding the edges is twofold. First, as observed in reference [8], the system matrix provides strong coupling for points in the neighborhood of a point being operated on, and very weak coupling for points outside this neighborhood. Second, the integral on the right hand side of equation (15) has a tendency to invert the pressure distributions near the trailing edge. This tendency is obviously a failure of the formulation which does not include the non-linear quadrupole term. To contrapose this inclination of the solution we need a measure which is capable of introducing strong coupling with a larger neighborhood. The process of averaging the components of the forcing vector at certain locations is probably the least arbitrary measure to be considered in the present stage of the formulation. This process was used by Long[16] to "condition" the forcing vector over 70% of the airfoil chord. Here we used the same idea, but applied it over the second half of the airfoil, as suggested by the most important lifting integral. Therefore, at the leading edge we used a weak coupling enforcement of the smoothness conditions, whereas at the trailing edge we used a stronger measure to overcome a weakness of the formulation in applying the Kutta condition.

For an angle of attack of 1^o, agreement between the presently obtained distributions of pressure and tangent velocity and those given by Abbott and von Doenhoff[18] is good. So is also the integrated lift coefficient, which can be obtained using the following simple formula:

$$c_l = \int_{-1}^{1} (q_u - q_l)\, dx_2$$

However, the correlation deteriorates for larger angles of attack. For α equal to 5^o the comparison on the lower surface is still good, but the solution fails to describe the whole expansion on the upper surface. As expected, discrepancies are larger near the leading edge.

The failure of the present development to give an accurate description of lifting effects must again be attributed to the absence of the quadrupole term in the formulation. However, the interpretation given to the monopole term seems to contribute to the understanding of the problem and to reduce the degree of arbitrariness still required to obtain useful results.

5 - Concluding Remarks

A new interpretation has been given to the velocity vector of the monopole term. This idea arises from physical reasoning and is consistent with the original work of Ffowcs Williams and Hawkings[2]. The idea indicates that in inviscid flow the monopole term should be considered unknown and additional relations should be used in order to solve the problem. The assumption that the monopole term is entirely known from the absolute motion of the body is correct for viscous flow. However, for flows with viscosity the quadrupole term should not be neglected because it includes the spatial derivatives of the boundary layer velocity field.

This new interpretation improves the capability of a linearized version of the FW-H equation in predicting pressure distributions in non-lifting, large disturbance problems. The interpretation also helps in addressing the issue of including circulation effects in this aeroacoustic approach. However, a better description of the lifting problem requires the use of the non-linear quadrupole term.

A new equation has been developed for dealing with two-dimensional and steady problems. Several results have been obtained, either by using a known expression for the tangent velocity distribution or by iterating with Bernoulli's equation. The iterative scheme is very robust, in the sense that a very crude guess for the distribution $q(x_2)$ may be used to start the process. These results suggest that the quadrupole term should be used to analyze thick bodies and to correct overstagnation pressures obtained near the airfoil edges.

The major conclusion, however, is that the equation of Ffowcs Williams and Hawkings should be regarded as a condensed form of the Navier-Stokes equation. As such, it is a single equation with more than one unknown. A direct solution, given only the absolute motion of the body, is not possible. Additional relations should be invoked to obtain a well-posed problem.

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