



DYNAMIC RESPONSE OF WINDTURBINE

TO YAWED WIND

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SUMMARY

Dynamic response of a two-bladed windturbine to yawed wind is analyzed by means of the local circulation method. The dynamic system is considered to consist of blade deformation, rotor rotational motion and yawing motion of the windturbine. The amplitude of the 2-P vibration in the bending moment and the rotor torque are more significant in the change of wind direction than in that of wind speed. The exemplified windturbine can follow the change of wind direction with fairly small response time. The inertial forces and moments are much smaller than the aerodynamic components because of the high rigidity of the present rotor.

1. Introduction

As precisely explained in Ref. 1, the aerodynamic forces and moments acting on horizontal-axis or propeller type windturbine are appreciably influenced by the change of wind direction with respect to the rotor axis. Thus the power coefficient based on the power of inflow or the mechanical efficiency of the windturbine is, as shown in Fig. 1a, strongly deteriorated by the yawing angle of the rotor. Similarly, as shown in Fig. 1b, the blade bending moment also fluctuates severely during one revolution.

Usually the speed and direction of wind cannot be controlled artificially and vary from site to site and time to time. Therefore, the rotor plane is adjusted to be normal to the wind velocity by making yawing motion of the rotor shaft around a vertical axis prepared by a swivel and a tail fin. In small windturbines as shown in Fig. 2, when the wind direction does not coincide with the shaft axis, the restoring moment to make the above yawing motion can usually be generated by the aerodynamic force acting on the tail fin which is, like weather vane, installed on an opposite side of the rotor shaft. Then the lateral or side slip angle and the resulted yawing motion will bring various fluctuations on the aerodynamic and inertial forces and moments of the windturbine.

2. Method of Analysis

Aerodynamic Forces and Moments

The airloading of the respective blades of two-bladed windturbine and the resulted aerodynamic forces and moments of the rotor are calculated by the "Local Circulation Method (LCM)," the detailed description of which is given in Ref. 1 ~ 2.

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By referring to Fig. 2 and by assuming that (i) the pre-coning angle β_p , the lead-lag, flapping, and torsional deformations (v , w and ϕ) are small, and (ii) the aerodynamic forces and moments may be given by quasi-steady treatment because of small reduced frequency such as $\kappa=0.03$, the relation between the airload ℓ and the circulation Γ can be given by

$$\ell = \frac{1}{2}\rho U^2 c C_{\ell} = \rho U \Gamma \quad (1)$$

where

$$U = \sqrt{U_T^2 + U_P^2} \quad (2)$$

$$U_T \cong R\Omega\{\mu \sin\psi + x + \dot{v}/R\Omega\}$$

$$U_P \cong R\Omega\{\lambda - \beta_p \mu \cos\psi - \dot{w}/R\Omega\}$$

$$\alpha = \tan^{-1}(U_P/U_T) - (\theta + \phi) \quad (3)$$

$$\mu = \{V \sin(\Psi_w - \Psi) + \ell_r \dot{\Psi}\}/R\Omega \quad (4)$$

$$\lambda = \{V \cos(\Psi_w - \Psi) - v_i\}/R\Omega$$

and where the wind speed V may be a function of height h .

If the deformations and yawing motion are specified, then equation (1) can be solved by the LCM. Usually the effects of the rate of deformation of the blade on the aerodynamic force, which are given by the final term of the expression of U_T and U_P , may be neglected as small quantities.

Then the aerodynamic forces and moment (about the elastic axis) at radius r and azimuth ψ can be given by

$$\begin{aligned} dF_{Ay}/dr &= d(U_T/U) - \ell(U_P/U) \\ dF_{Az}/dr &= \ell(U_T/U) + d(U_P/U) \\ dM_{Ax}/dr &= m_{\theta} + (dF_{Ay}/dr)e_{A,z} + (dF_{Az}/dr)e_{A,y} \end{aligned} \quad (5)$$

Inertial Forces and Moments

The inertial forces and moments of a blade element are given by

$$\begin{aligned} dF_{Ix}/dr &= -m\left\{2\left(\frac{dv}{dt}\right)\Omega - r\Omega^2\right\} + e_y\left\{-\frac{d^2}{dt^2}v + \Omega^2\right\}\left(\frac{dv}{dr}\right) \\ &\quad + e_z\left\{-\frac{d^3v}{dt^2 dr} - 2\Omega\frac{d\phi}{dt} + \Omega^2\frac{dw}{dr} + \frac{d^2\Psi}{dt^2}\cos\psi + \beta_p\Omega^2\right\} \\ dF_{Iy}/dr &= -m\left\{\frac{d^2v}{dt^2} - (v+\delta_y)\Omega^2\right\} + e_y\left\{2\Omega\frac{d^2v}{dt dr} - \Omega^2\right\} \\ &\quad + e_z\left\{-\frac{d^2\phi}{dt^2} + 2\Omega\frac{d^2v}{dt dr} + \frac{d^2\Psi}{dt^2}\sin\psi + \phi\Omega^2\right\} \end{aligned} \quad (6)$$

$$dF_{Iz}/dr = -m \left[\left\{ \frac{d^2 w}{dt^2} - r \frac{d^2 \psi}{dt^2} \cos \psi + 2\Omega r \frac{d\psi}{dt} \sin \psi + r \beta_p \Omega^2 \right\} + e_y \left\{ \frac{d^2 \phi}{dt^2} - \frac{d^2 \psi}{dt^2} \sin \psi - 2\Omega \frac{d\psi}{dt} \cos \psi \right\} \right] \quad (6)$$

and

$$dM_{Ix}/dr = -I_x \left\{ \frac{d^2 \phi}{dt^2} - \frac{d^2 \psi}{dt^2} \sin \psi \right\} - I_z \left\{ -2\Omega \frac{d\psi}{dt} \cos \psi + \phi \Omega^2 \right\} + I_y \left\{ -2\Omega \frac{d^2 w}{dt^2 dr} + \phi \Omega^2 \right\} - I_{yz} \left\{ \Omega^2 - 2\Omega \frac{d^2 v}{dt^2 dr} \right\} - m \left[e_y \left\{ \frac{d^2 w}{dt^2} - r \frac{d^2 \psi}{dt^2} \cos \psi + 2r\Omega \frac{d\psi}{dt} \sin \psi + r \beta_p \Omega^2 \right\} + e_z \left\{ -\frac{d^2 v}{dt^2} + (v + \delta_y) \Omega^2 \right\} \right]$$

$$dM_{Iy}/dr = -I_y \left\{ -\frac{d^3 w}{dt^2 dr} - 2\Omega \frac{d\phi}{dt} + \Omega^2 \frac{dw}{dr} + \frac{d^2 \psi}{dt^2} \cos \psi + \beta_p \Omega^2 \right\} - I_{yz} \left\{ -\frac{d^3 v}{dt^2 dr} - \Omega^2 \frac{dv}{dr} \right\} - m \left[e_y \left\{ -r \phi \Omega^2 \right\} + e_z \left\{ 2\Omega \frac{dv}{dt} - r \Omega^2 \right\} \right] \quad (7)$$

$$dM_{Iz}/dr = -I_z \left\{ \frac{d^2 v}{dt^2} - 2\Omega \frac{dv}{dr} \right\} - I_{yz} \left\{ \frac{d^3 w}{dt^2 dr} + 2\Omega \frac{d\phi}{dr} - 2\Omega^2 \frac{dw}{dr} - \frac{d^2 \psi}{dt^2} \cos \psi - \beta_p \Omega^2 \right\} - m \left[e_y \left\{ -2\Omega \frac{dv}{dt} + r \Omega^2 \right\} - e_z \left\{ r \phi \Omega^2 \right\} \right].$$

Yawing Moment and Torque

By referring to Fig. 1, the external yawing moment about the vertical axis M_ψ , which includes the inertial components of the rotor, is comprised of the hub moment M_Y , the moment caused by the horizontal force H and the moment generated by the tail-fin force L_t as follows:

$$M_\psi = -M_Y - H \ell_r - L_t \ell_t \quad (8)$$

where

$$M_Y = \int_0^R \left\{ (dF_{Az}/dr + dF_{Iz}/dr) r \cos \psi + (dM_{Ax}/dr + dM_{Ix}/dr) \sin \psi \right\} dr$$

$$H = \int_0^R \left\{ (dF_{Ay}/dr + dF_{Iy}/dr) \sin \psi + (dF_{Ix}/dr) \cos \psi \right\} dr \quad (9)$$

$$L_t = \frac{1}{2} \rho n_t v^2 S a \alpha_t$$

$$\alpha_t = \tan^{-1} \left[\frac{l_t \left(\frac{d\psi}{dt} \right) - \sqrt{n_t} V \sin(\psi_w - \psi)}{\sqrt{n_t} V \cos(\psi_w - \psi)} \right],$$

and where Σ specify to take the summation for all b blades.

Similarly, the torque about the rotor shaft Q_Ω is comprised of the rotor torque Q which also includes the inertial torque as well as the aerodynamic torque and the torque Q_m generated by the mechanical torque for driving an installed load.

$$Q_\Omega = Q - Q_m \quad (10)$$

where

$$Q = \Sigma \int_0^R \left\{ (dF_{Ay}/dr + dF_{Iy}/dr) r + dM_{Iz}/dr - (dM_{Ax}/dr + dM_{Ix}/dr) \beta_p \right\} dr \quad (11)$$

Elastic Equations of Motion of the Blade

The elastic deformation of a blade, the pretwist of which is far larger than that of the helicopter-rotor blade, can be written by the generalized force balance for the generalized coordinate \bar{q}_j

$$\bar{M}_j (\ddot{q}_j + \bar{\omega}_j^2 \bar{q}_j) = \bar{Q}_j \quad (12)$$

where the generalized mass \bar{M}_j and the generalized force \bar{Q}_j in non-dimensional form are given by

$$\begin{aligned} \bar{M}_j = \int_0^1 & \left\{ [\bar{I}_x \bar{\phi}_j + \bar{m} (\bar{e}_y \bar{w}_j - \bar{e}_z \bar{v}_j)] \bar{\phi}_j \right. \\ & + \bar{m} (\bar{w}_j + \bar{e}_y \bar{\phi}_j) \bar{w}_j + (\bar{I}_y \bar{w}'_j + \bar{I}_{yz} \bar{v}'_j) \bar{w}'_j \\ & \left. + \bar{m} (\bar{v}_j - \bar{e}_z \bar{\phi}_j) \bar{v}_j + (\bar{I}_z \bar{v}'_j + \bar{I}_{yz} \bar{w}'_j) \bar{v}'_j \right\} dx \\ & - [(\bar{I}_y \bar{w}'_j + \bar{I}_{yz} \bar{v}'_j) \bar{w}_j + (\bar{I}_z \bar{v}'_j + \bar{I}_{yz} \bar{w}'_j) \bar{v}_j]_0^1 \\ \bar{Q}_j = \int_0^1 & \left\{ [d\bar{M}_{Ay}/dx - (\bar{I}_{yz} + \bar{m} \bar{e}_z \bar{\delta}_y) + 2(\bar{I}_y \bar{w}'_j + \bar{I}_{yz} \bar{v}'_j) \right. \\ & + \bar{I}_x \ddot{\psi} \sin \psi + 2\bar{I}_z \ddot{\psi} \cos \psi - \bar{m} \bar{e}_y (-x \ddot{\psi} \cos \psi \\ & \left. + 2x \dot{\psi} \sin \psi + x \beta_p)] \bar{\phi}_j \right\} \end{aligned} \quad (13)$$

$$\begin{aligned}
& + \{d\bar{F}_{Az}/dx - \bar{m}x\beta_p + \bar{m}(x\ddot{\Psi}\cos\psi - 2x\dot{\Psi}\sin\psi) \\
& + \bar{m}\bar{e}_y(\ddot{\Psi}\sin\psi + 2\dot{\Psi}\cos\psi)\}\bar{w}_j - \{\bar{m}\bar{e}_z x + 2\bar{I}_y \dot{\phi} \\
& - 2\bar{m}\bar{e}_z \dot{v} - \bar{I}_y \ddot{\Psi}\cos\psi\}\bar{w}_j' \\
& + \{d\bar{F}_{Ay}/dx + \bar{m}\bar{\delta}_y - \bar{m}\bar{e}_z \ddot{\Psi}\sin\psi\}\bar{v}_j \\
& - \{\bar{m}\bar{e}_y x + 2\bar{I}_{yz} \dot{\phi} - 2\bar{m}\bar{e}_y \dot{v} - \bar{I}_{yz} \ddot{\Psi}\cos\psi\}\bar{v}_j' dx \\
& + \{[\bar{m}\bar{e}_z x + 2\bar{I}_y \dot{\phi} - 2\bar{m}\bar{e}_z \dot{v} - \bar{I}_y \ddot{\Psi}\cos\psi]\bar{w}_j \\
& + [\bar{m}\bar{e}_y x + 2\bar{I}_{yz} \dot{\phi} - 2\bar{m}\bar{e}_y \dot{v} - \bar{I}_{yz} \ddot{\Psi}\cos\psi]\bar{v}_j\} \Big|_0^1
\end{aligned} \tag{14}$$

and where ($\bar{\quad}$)'s are nondimensionalized quantities of () and the subscript j shows jth mode.

Equations of Yawing Motion and Driving Motion

In the present example, the windturbine has a degree of freedom around the vertical axis. Then the torque about the rotor shaft is affected by the yawing motion of the windturbine as well as the bending deformations of the blade. The following nondimensional equations are established:

$$\bar{I}_\Psi \ddot{\Psi} = \bar{M}_\Psi \tag{15}$$

$$2\bar{I}_\Omega \dot{\bar{\Omega}} = \bar{Q} - \bar{Q}_m \tag{16}$$

3. Method of Computation

For solving the above nonlinear equations of motion, (11), (15) and (16) of the rotor dynamic system in yawing motion, the calculus of finite differences has been applied. The external forces and moments were calculated at the time of one step before. The time step was 0.031 second, which was equivalent to time of blade passing over 10 degrees of azimuth angle, for the calculation of the airloading, and was 0.0031 second for the calculation of the blade deformation and yawing motion.

The loading torque was assumed empirically to operate in proportion to the square power of the rotor rotational speed,

$$\bar{Q}_m = k_m \bar{\Omega}^2 \tag{17}$$

4. Results of Computation

Geometrical dimensions of an exemplified windturbine and the elastic characteristics of the blade in nondimensional form are given in Table I and II. The windturbine is under development and is used to be a heat generator for agricultural purpose.

Modes of Deformations

As shown in Fig. 3, since the eigenvalues of the blade deformations are almost insensitive for the change of the rotational speed of the rotor, the eigenvalues and the modes of deformation are treated as constant values specified at the normal operation state.

Effects of Wind Shear

As shown in Fig. 4, the windturbine is considered to operate in the wind speed of 8.0 m/s at the rotor hub, for the case of (i) uniform flow or (ii) sheared flow of $V = V_{10}(h/10)^{1/6}$. Shown in Fig. 5 are the torque variation, $100\{Q-Q(\psi=0)\}/Q(\psi=0)$, and the bending moment variation, $\{M_B-M_B(\psi=0)\}/M_B(\psi=0)$, in comparison between the above two cases. It can be seen that the effect of wind shear is obvious specifically in the bending moment variation.

Effects of Yawed Wind

Here let us assume for simplicity that the yawing motion of the rotor system and the rotor speed are constrained or fixed to their initial values. Shown in Fig. 6 is the torque change caused by the yawed wind ($\Psi_w=45^\circ$) in comparison with that of normal wind ($\Psi_w=0^\circ$). It must be notified that the level of effective torque or torque output is appreciably reduced (about 35 percent) by the diagonal angle of the wind and the torque variation of twice per revolution (2P) is observed.

The torque variation and the bending moment variation are shown in Fig. 7. They have peak values of 7 and 25 percent respectively compared with those of the no yawed angle ($\Psi_w=0^\circ$).

Effects of Yawing Motion

Let us consider here two examples such that (i) the wind speed is abruptly increased in a step form, $\Delta V = 0.15V$, and (ii) the wind direction is suddenly changed from $\Psi_w=0^\circ$ to $\Psi_w=30^\circ$. Here the yawing motion of the rotor system about the vertical axis and the rotor speed are considered free.

(i) The results of the former case are shown in Fig. 8 for the variations of rotor speed $100\{\Omega-\Omega(\psi=0)\}/\Omega(\psi=0)$, the torque variation, the yaw angle, Ψ , the variation of normal force, $100\{N-N(\psi=0)\}/N(\psi=0)$, and the variation of bending moment. As the wind speed increases, every quantity increases gradually and approaches to each final value. It is interesting to find that a small yawing motion is induced by the change of aerodynamic forces and moments. The period of the yawing

motion is, in the present example, about five second or 5 revolutions of the rotor.

(ii) The results of the latter case are shown in Fig. 9. At the initial stage the effective torque is reduced by 0.5 percent and is, then, recovered to the initial value because the yawing motion reduces the diagonal angle of the rotor with respect to the wind, $\Psi \rightarrow \Psi_w$. During this period the 2P variations of aerodynamic and inertial forces and moments are predominant. Their peak-to-peak values are 10 percent in the torque, 5 percent in the normal force and 8 percent in the bending moment. This fact is important for the design of structural configuration and of material selection of the blade, drive shaft, gear trains and tower, all of which are under influence of the above exciting forces and moments.

Although the angular rate and acceleration of the yawing motion are predominant, the inertial effects on the rotor dynamics are not so significant that the inertial forces and moments acting on the blades are much smaller than the aerodynamic components. This is because the blade of the exemplified rotor have high rigidity and thus the blade deformation is very small.

The time constant of the damped yawing motion is about three second in the present example. This enables the exemplified wind-turbine to follow the change of wind direction, the predominant frequency of which is more than the above time constant.

5. Conclusion

By applying the local circulation method (LCM) to the aerodynamic analysis of the rotor of a two-bladed windturbine, the dynamic response of the system, which was comprised of the blade elastic deformations, the rotor driving motion and the yawing motion of the windturbine, was analyzed. The following facts were drawn: (i) The yawed angle of the wind reduces the mean values of the aerodynamic forces and moments, but it induces the vibratory change in the above every quantity. (ii) The change of wind speed including the wind shear has almost no effect on the vibratory change in the forces and moments, but it induces a yawing motion slightly. (iii) The change of wind direction affects strongly the yawing motion and the vibratory change in the aerodynamic and inertial forces and moments. (iv) The vibratory change has 2P variation at the early stage of the motion and is attenuated by the yawing motion.

Nomenclature

AC	:	aerodynamic center
a_t	:	lift slope of tail fin
b	:	number of blades
C_ℓ	:	lift coefficient
c	:	blade chord
CG	:	center of gravity
d	:	section drag
EI_y, EI_z, EI_{yz}	:	bending rigidity
(e_y, e_z)	:	CG position from EA in (x, y, z) coordinate
(\bar{e}_y, \bar{e}_z)	=	$(e_y, e_z)/R_b$
$(e_{A,y}, e_{A,z})$:	AC position from EA in (x, y, z) coordinate
(e_η, e_ζ)	:	CG position from EA in (ξ, η, ζ) coordinate
$(e_{A,\eta}, e_{A,\zeta})$:	AC position from EA in (ξ, η, ζ) coordinate
EA	:	elastic axis
FA	:	feathering axis
(F_{Ax}, F_{Ay}, F_{Az})	:	aerodynamic forces in (x, y, z) coordinate
$(\bar{F}_{Ax}, \bar{F}_{Ay}, \bar{F}_{Az})$	=	$(F_{Ax}, F_{Ay}, F_{Az})/M_b \Omega_0^2$
(F_{Ix}, F_{Iy}, F_{Iz})	:	inertial forces in (x, y, z) coordinate
$(\bar{F}_{Ix}, \bar{F}_{Iy}, \bar{F}_{Iz})$	=	$(F_{Ix}, F_{Iy}, F_{Iz})/M_b \Omega_0^2$
GJ	:	torsional rigidity
H	:	horizontal force in the X-axis
h	:	height
(I_x, I_y, I_z, I_{yz})	:	inertial moments in (x, y, z) coordinate
$(\bar{I}_x, \bar{I}_y, \bar{I}_z, \bar{I}_{yz})$	=	$(I_x, I_y, I_z, I_{yz})/M_b R^2$
I_ψ	:	inertial moment of windturbine without rotor about Y-axis
\bar{I}_ψ	=	I_ψ/MR^2
I_Ω	:	inertial moment of windturbine without rotor about Z-axis
\bar{I}_Ω	=	I_Ω/MR^2
k_A	:	radius of gyration
k_m	:	coefficient of loaded torque in Eq. (16)
L_t	:	lift of tail fin
ℓ	:	section lift
ℓ_r	:	distance between Z-axis to rotor hub

l_t	:	distance between Z-axis to tail fin
M	:	total mass of windturbine
(M_{Ax}, M_{Ay}, M_{Az})	:	aerodynamic moments in (x, y, z) coordinate
$(\bar{M}_{Ax}, \bar{M}_{Ay}, \bar{M}_{Az})$	=	$(M_{Ax}, M_{Ay}, M_{Az})/M_b R \Omega_0^2$
M_B	:	flatwise bending moment
M_b	:	mass of a blade
(M_{Ix}, M_{Iy}, M_{Iz})	:	inertial mementis in (x, y, z) coordinate
$(\bar{M}_{Ix}, \bar{M}_{Iy}, \bar{M}_{Iz})$	=	$(M_{Ix}, M_{Iy}, M_{Iz})/M_b R \Omega_0^2$
\bar{M}_j	:	nondimensionalized j-th generalized mass
(M_X, M_Y, M_Z)	:	total moments in (X, Y, Z) coordinate
M_Ψ	:	total yawing moment about Y-axis
m	:	section mass
\bar{m}	=	mR/M_b
m_θ	:	aerodynamic pitching moment (positive for head-up)
N	:	normal force
Q	:	torque
Q	:	$Q/M(R\Omega_0)^2$
\bar{Q}_j	:	nondimensional generalized force
Q_m	:	loaded torque
\bar{Q}_m	=	$Q_m/M(R\Omega_0)^2$
Q_Ω	:	torque about Z-axis, see Eq. (10)
\bar{q}_j	:	nondimensional generalized coordinate
R	:	rotor radius
r	:	spanwise position of a blade
S	:	rotor disc area
S_t	:	fin area
t	:	time
U	:	resultant velocity $= \sqrt{U_T^2 + U_P^2}$
U_P	:	perpendicular velocity against wing section
U_T	:	tangential velocity against wing section
V	:	wind velocity
V_h	:	wind velocity specified at heigh h (h = 10 m, 21 m)
v	:	blade deformation along y axis
v_i	:	induced velocity
\bar{v}_j	:	j-th lead-lag-wise mode

w : blade deformation along z axis
 \bar{w}_j : j th flapwise mode
 x = r/R
 (X, Y, Z) : coordinate fixed in the windturbine
 (x, y, z) : coordinate fixed in and rotating with the rotor hub
 α : angle of attack
 α_t : angle of attack of tail fin
 β : flapping angle of blade
 β_p : precone angle
 Γ : circulation on a blade
 δ_y, δ_z : distances between EA and FA along y and z axes
 $\bar{\delta}_y, \bar{\delta}_z$ = $\delta_y/R, \delta_z/R$
 (ξ, η, ζ) : local coordinate fixed on a blade
 η : efficiency or local axis fixed on the blade
 η_t : decrement of dynamic pressure at tail fin
 θ : pitch angle positive for pitch down
 λ : inflow ratio positive for downflow
 μ : advance ratio
 ρ : air density
 Σ : summation
 ϕ : torsional blade deformation
 $\bar{\phi}_j$: j th torsional mode
 Ψ : yaw angle
 Ψ_w : wind direction about Z-axis
 ψ : azimuth angle of a blade
 Ω_0 : initial rotor rotational speed
 Ω : rotor rotational speed
 $\bar{\Omega}$ = Ω/Ω_0
 ω_j : j th natural frequency
 $\bar{\omega}_j$ = ω_j/Ω_0

Superscripts:

$$\dot{(\quad)} = \frac{d(\quad)}{dt} / \Omega_0$$

$$\prime(\quad) = \frac{d(\quad)}{dx} = R \frac{d(\quad)}{dr}$$

$\bar{(\quad)}$: nondimensionalized value of ()

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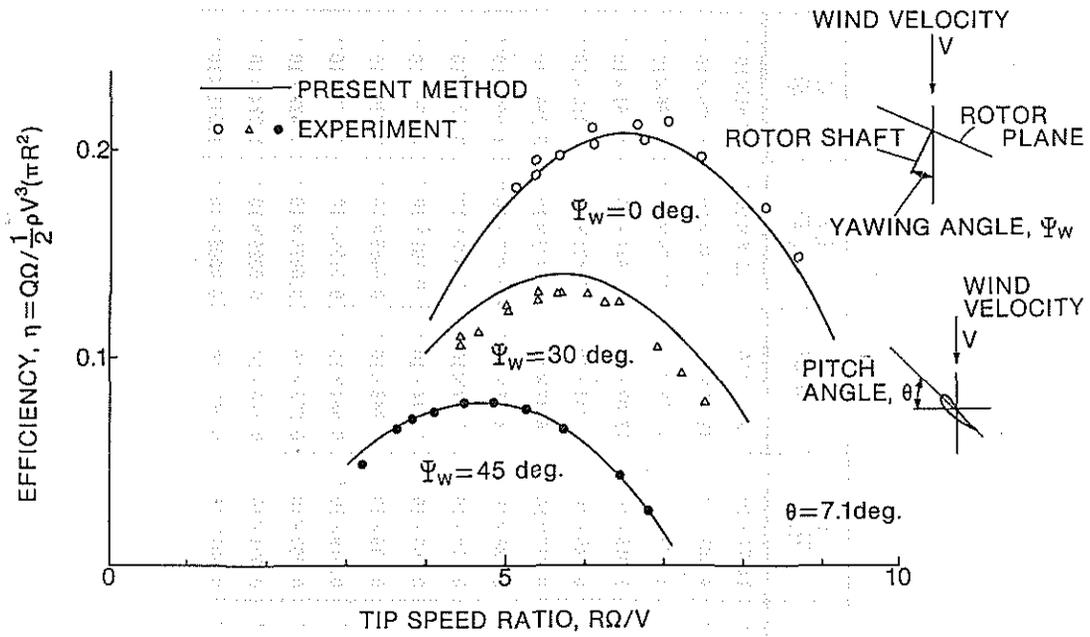
- 1) Nasu, Ken-ichi and Azuma, Akira: An Experimental Verification of the Local Circulation Method for a Horizontal Axis Wind Turbine. The 18th Intersociety Energy Conversion Engineering Conference, Orlando, Florida, August 21-26, 1983, pp. 245-252.
- 2) Azuma, A., Nasu, K. and Hayashi, T.: An Extension of the Local Momentum Theory to Rotors Operating in a Twisted Flow Field. *Vertica*, Vol. 7, No. 1, 1983, pp. 45-59.

Table I Dimensions of a windturbine blade

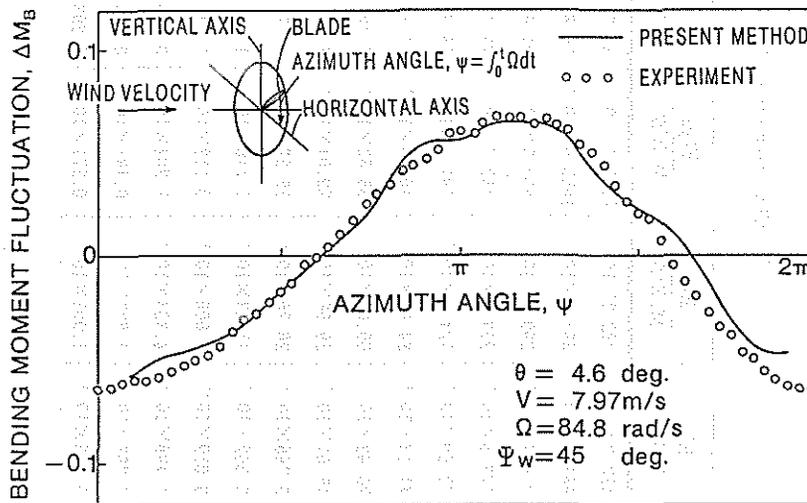
Items	Dimensions
Rotor radius, R	7m
Blade mass, M_b	122kg
No. of blades, b	2
Preconing angle, β_r	0.0deg
Collective pitch angle, θ_0	1.6deg
Wing section	NACA 4418
Coefficient of loaded torque, k_m	6.23
Reynolds number, R_e	8×10^5
Tail fin area, S_t	15.39m ²
Distance, l_t	7.0m
Distance, l_r	3.5m

Table II Characteristics of a windturbine blade

No.	Station r/R	Chord c/R	Pretwist θ_r (deg)	Mass per unit length m/M_b	$\frac{EI_y}{M_b R^3 \Omega^2}$	$\frac{EI_z}{M_b R^3 \Omega^2}$	$\frac{EI_{yz}}{M_b R^3 \Omega^2}$	$\frac{GJ}{M_b R^3 \Omega^2}$	$\frac{I_x}{M_b R^2}$	$\frac{I_y}{M_b R^2}$	$\frac{I_z}{M_b R^2}$	$\frac{I_{yz}}{M_b R^2}$	$\frac{k_A^2}{R^2}$
1	0.10	0.043	27.9	0.210	1422.0×10^{-3}	382.2×10^{-2}	2137.4×10^{-3}	33.72×10^{-3}	5.447×10^{-3}	14.77×10^{-4}	39.7×10^{-4}	22.2×10^{-4}	1.061×10^{-4}
2	0.14	0.051	21.2	0.180	881.6×10^{-3}	393.7×10^{-2}	1655.0×10^{-3}	16.86×10^{-3}	5.186×10^{-3}	9.488×10^{-4}	42.37×10^{-4}	17.81×10^{-4}	1.490×10^{-4}
3	0.21	0.109	14.0	0.183	360.2×10^{-3}	299.1×10^{-2}	850.5×10^{-3}	21.62×10^{-3}	4.630×10^{-3}	4.976×10^{-4}	41.32×10^{-4}	11.75×10^{-4}	6.796×10^{-4}
4	0.29	0.135	9.4	0.174	173.5×10^{-3}	223.7×10^{-2}	452.7×10^{-3}	27.71×10^{-3}	1.410×10^{-3}	1.015×10^{-4}	13.09×10^{-4}	2.649×10^{-4}	10.51×10^{-4}
5	0.36	0.113	6.6	0.177	73.60×10^{-3}	124.8×10^{-2}	191.8×10^{-3}	18.32×10^{-3}	0.979×10^{-3}	0.545×10^{-4}	9.244×10^{-4}	1.421×10^{-4}	7.310×10^{-4}
6	0.43	0.096	4.6	0.186	41.13×10^{-3}	84.07×10^{-2}	100.3×10^{-3}	11.80×10^{-3}	0.679×10^{-3}	0.317×10^{-4}	6.47×10^{-4}	0.772×10^{-4}	5.286×10^{-4}
7	0.50	0.083	3.0	0.180	25.93×10^{-3}	60.65×10^{-2}	55.83×10^{-3}	8.429×10^{-3}	0.453×10^{-3}	0.186×10^{-4}	4.35×10^{-4}	0.400×10^{-4}	4.000×10^{-4}
8	0.57	0.074	1.9	0.162	15.61×10^{-3}	39.56×10^{-2}	29.04×10^{-3}	5.057×10^{-3}	0.290×10^{-3}	0.110×10^{-4}	2.79×10^{-4}	0.205×10^{-4}	3.122×10^{-4}
9	0.64	0.066	1.0	0.137	9.798×10^{-3}	26.26×10^{-2}	15.27×10^{-3}	3.078×10^{-3}	0.181×10^{-3}	0.065×10^{-4}	1.744×10^{-4}	0.101×10^{-4}	2.490×10^{-4}
10	0.71	0.059	0.3	0.104	5.552×10^{-3}	15.43×10^{-2}	7.146×10^{-3}	1.979×10^{-3}	0.102×10^{-3}	0.036×10^{-4}	0.988×10^{-4}	0.0458×10^{-4}	2.020×10^{-4}
11	0.79	0.054	-0.3	0.071	3.019×10^{-3}	8.606×10^{-2}	3.113×10^{-3}	1.246×10^{-3}	0.0567×10^{-3}	0.019×10^{-4}	0.584×10^{-4}	0.0199×10^{-4}	1.673×10^{-4}
12	0.86	0.050	-0.8	0.042	1.496×10^{-3}	4.334×10^{-2}	1.202×10^{-3}	0.660×10^{-3}	0.0283×10^{-3}	0.009×10^{-4}	0.273×10^{-4}	0.0075×10^{-4}	1.429×10^{-4}
13	0.93	0.046	-1.2	0.031	0.783×10^{-3}	2.293×10^{-2}	0.481×10^{-3}	0.513×10^{-3}	0.0124×10^{-3}	0.004×10^{-4}	0.120×10^{-4}	0.0025×10^{-4}	1.224×10^{-4}
14	1.0	0.043	-1.6	0.030	0.645×10^{-3}	1.901×10^{-2}	0.271×10^{-3}	0.293×10^{-3}	0.006×10^{-3}	0.002×10^{-4}	0.057×10^{-4}	0.0008×10^{-4}	1.041×10^{-4}

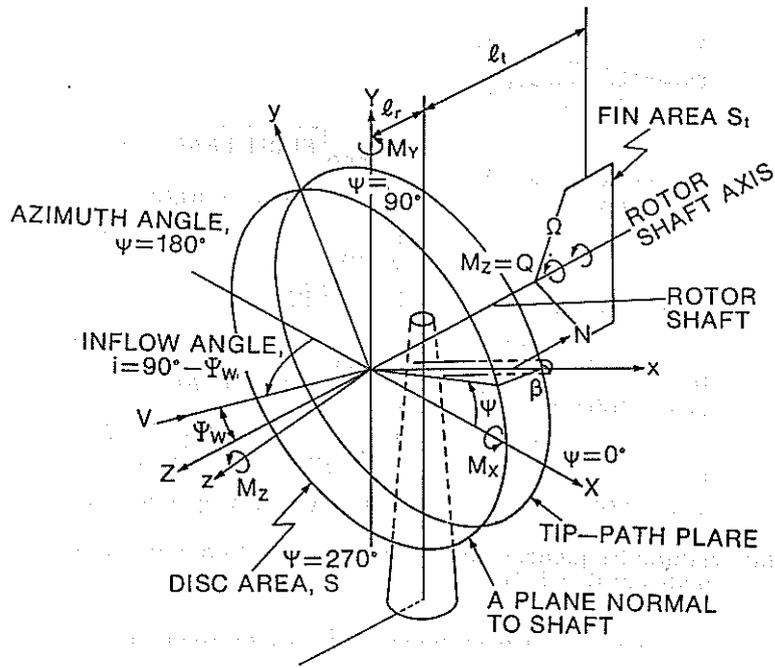


(a) On the efficiency

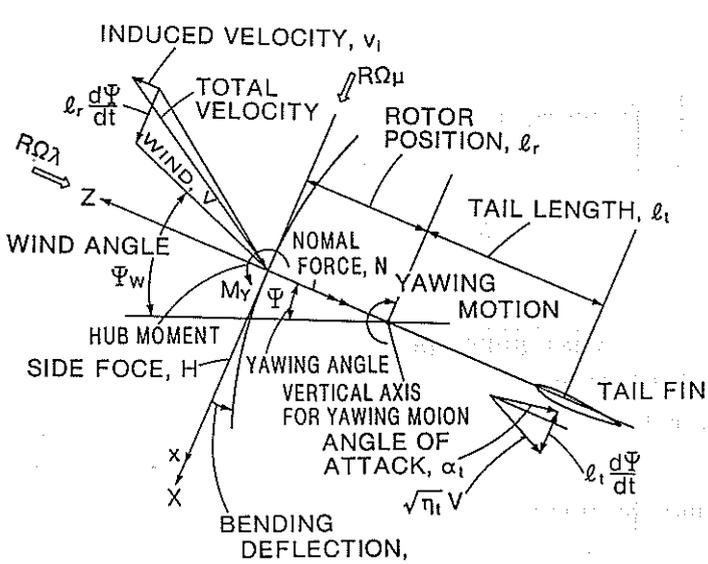


(b) On the bending moment fluctuation

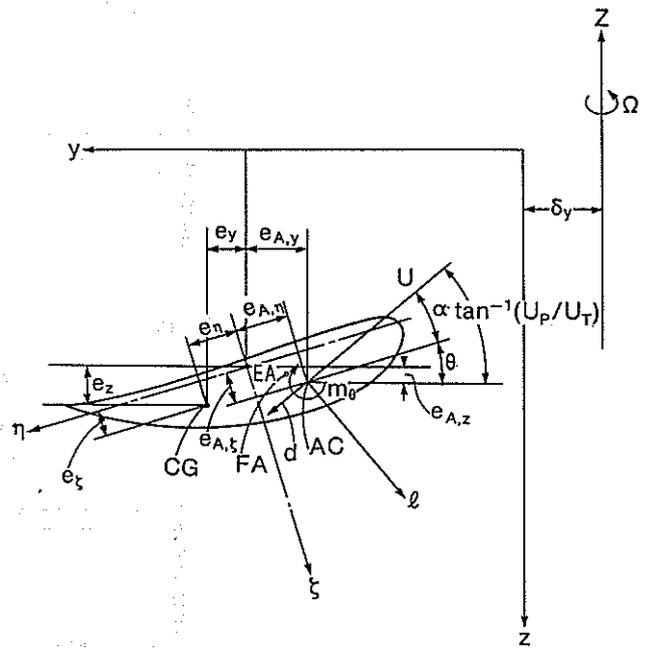
Figure 1: Effects of yawing angle of a windmill¹⁾



(a) Coordinate systems



(b) Flow velocities, forces and moments in yawing motion



(c) Sectional configurations and the sectional lift, drag and moment. (Before deformation and $\beta_p \approx 0$)

Figure 2. Schematic view of a windmill in yawing motion.

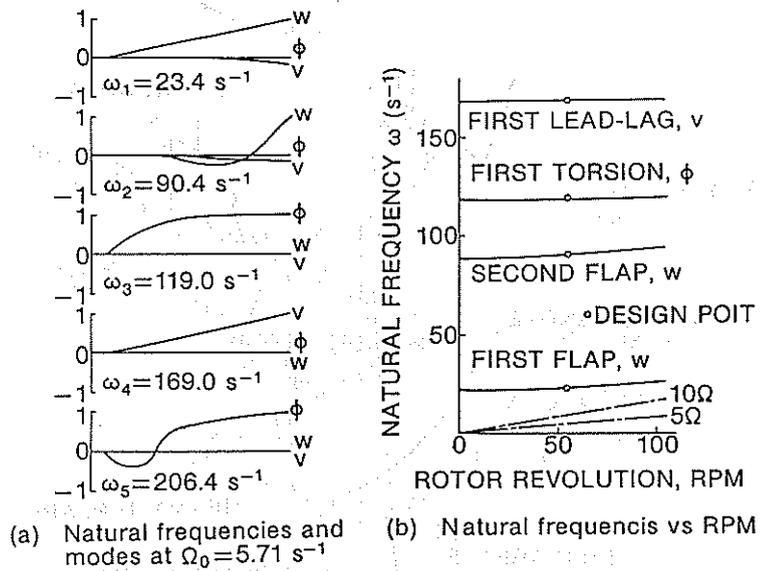


Figure 3. Characteristics of a windturbine blade.

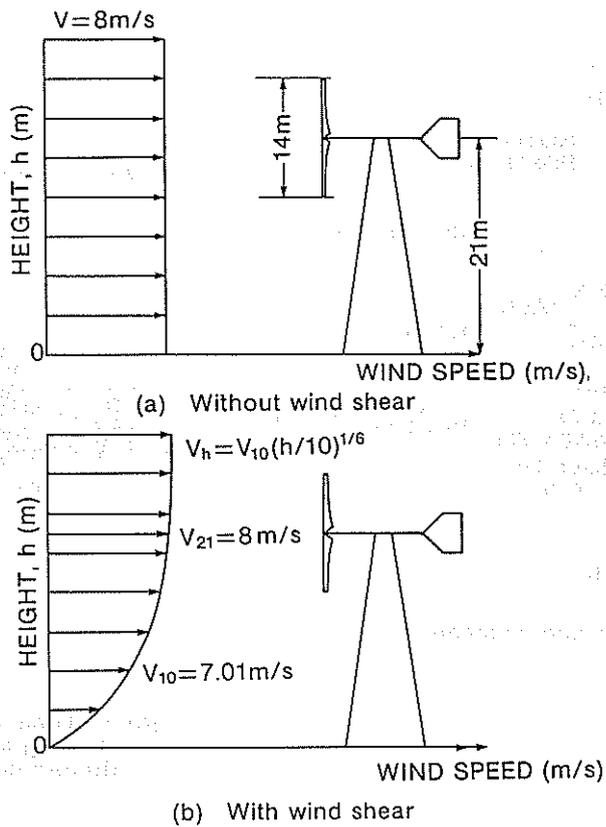
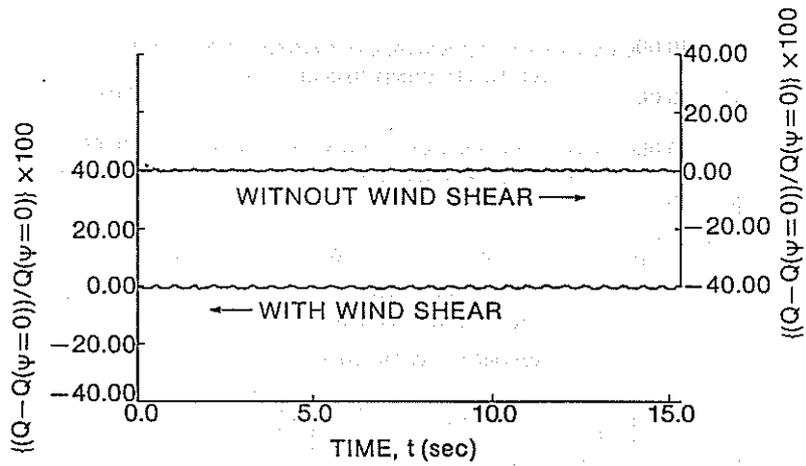
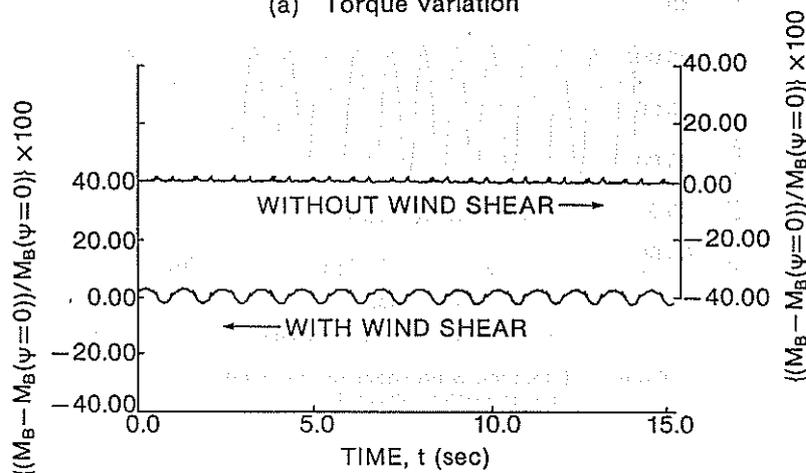


Figure 4. Wind shear



(a) Torque variation



(b) Bending moment variation

Figure 5. Effect the wind shear on the windturbine characteristics. ($\Psi_w=0^\circ$)

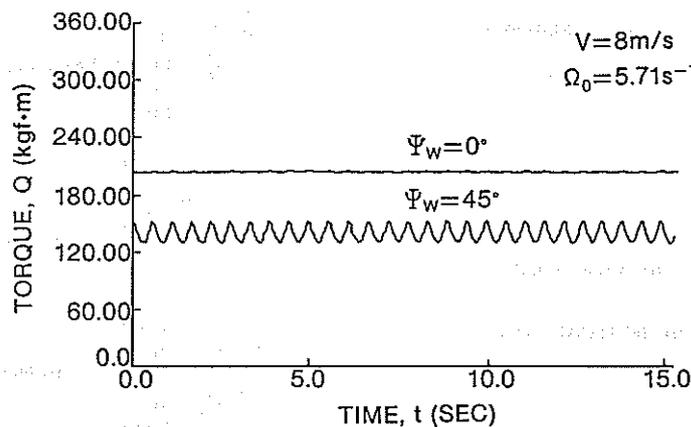


Figure 6. Torque variation to yawed wind.

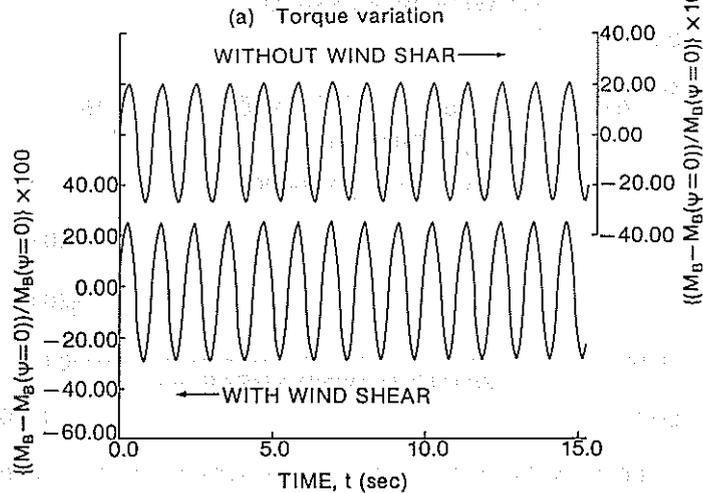
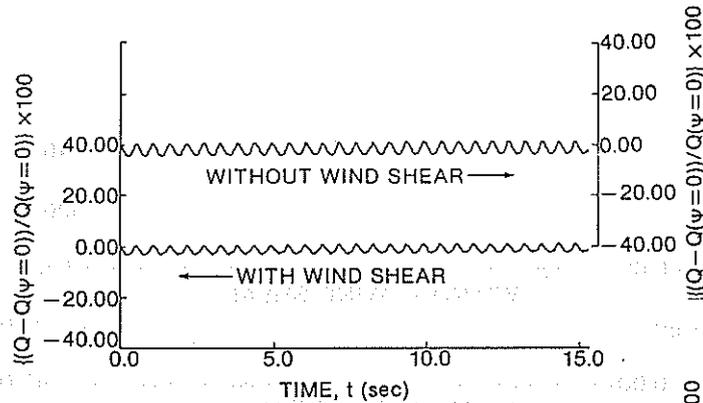


Figure 7. Effect the wind shear on the windturbine characteristics ($\Psi_w=45^\circ$)

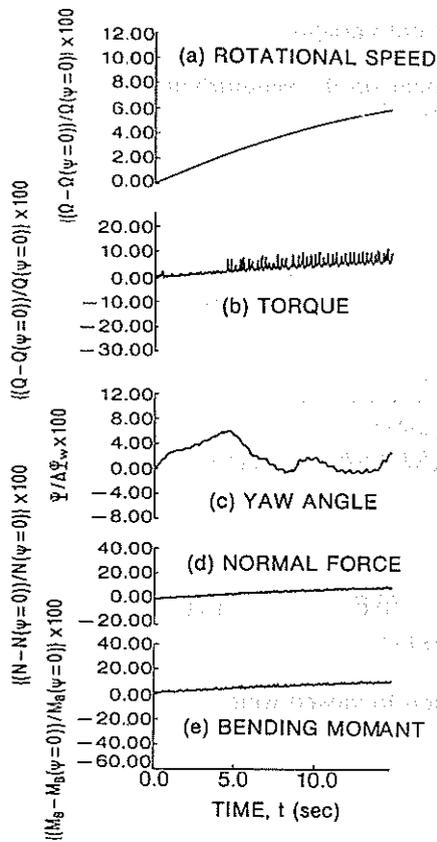


Figure 8. Step response of windturbine to a sudden change of wind speed. ($\Delta V/V=0.15$)

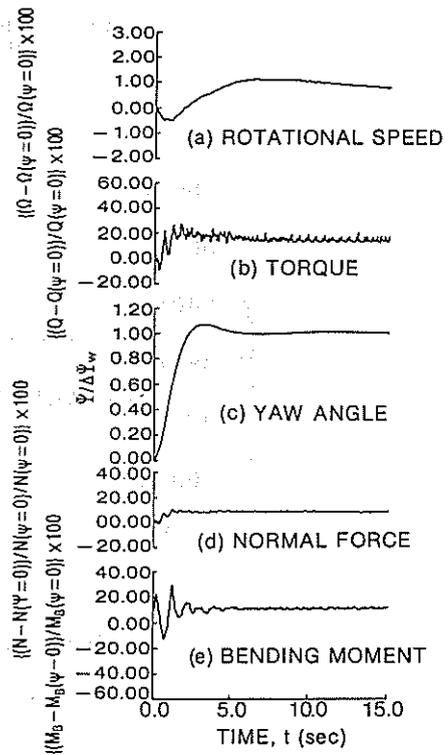


Figure 9. Step response of windturbine to a sudden change of wind direction. ($\Delta \Psi_w=30^\circ$)