EIGHTH EUROPEAN ROTORCRAFT FORUM

Paper No. 6.2

# NUMERICAL ANALYSIS AND EXPERIMENTAL

# VERIFICATION OF ELASTOMERIC BEARINGS

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August 31 through September 3, 1982

AIX-EN-PROVENCE, FRANCE

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Abstract:

In comparison with metallic materials, elastomers show incompressibility or near-incompressibility, leading to analysis problems when using finite elements based on the conventional displacement method. This paper describes methods of overcoming these difficulties by using finite elements based on HERRMANN's principle for linear analysis and on MOONEY-RIVLIN's approach for the nonlinear case. The computer programme MARC offers various finite elements based on these variational principles.
Results are presented for various elastomeric parts obtained by using MARC and are compared with respectively; exact solutions; results taken from literature; and test results.

For a thick-walled cylinder subjected to internal pressure, the nonlinear approach approximated the exact nonlinear solution quite well, even with load increments of 10% of maximum load.

For the plain strain state of a rubber layer, bonded between steel plates and subjected to compression load, the linear analysis for the vertical displacement corresponds well with the measurement found in literature up to a compression strain of 5%. With HERRMANN's constitutive equation the bulging fits the measurement well even for compression strains higher than 10%, whereas the results with MOONEY-RIVLIN's constitutive equation differ significantly from the test results.

For a spherical/conical thrust bearing, the stiffness in various modes as well as the hoop shim and housing strains were calculated and compared with test results. A fairly good agreement was found.

## 1. Introduction

In modern helicopter design, engineers have constantly to attempt to improve reliability and maintainability whilst striving to reduce weigth and production as well as life cycle costs for all components.

The main rotor head plays a fundamental role among the helicopter mechanical components. It supports the rotor blades at rest and in rotation, transmits the drive torque to the blades, as well as the control inputs in terms of blade pitch change, and transmits to the fuselage the blade lift loads and rotor moments generated by the rotating blades.

The fulfillment of these primary functions historically resulted
in the well known rotor head configurations with flap, lag and feathering
hinges and different dampers.

To overcome the disadvantages of hinges viz. weight, manufacturing
and maintenance costs, much progress has been made, from this beginning,
by the use of flexible elements instead of hinges and/or by use, of
elastomeric bearings, leading to the following rotor systems:

- <u>rigid rotor</u>  with feathering hinge, where the flapping and lagging
  hinges are replaced by an elastic element in the rotor head or in
  the blade root. The feathering hinge uses conventional bearings
  (Fig. 1).
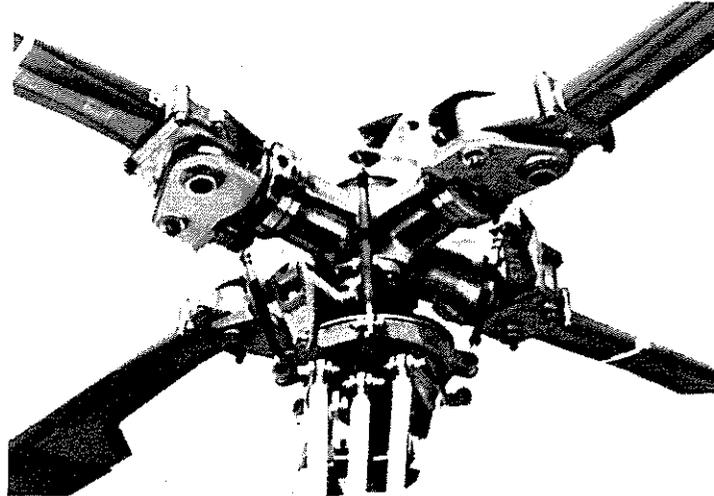


Fig. 1:  Rigid Rotor Head of BO 105 and BK 117

- <u>bearingless rotor</u>, where the flapping, lagging and feathering hinges are
  replaced by a flexible, torsionally weak component (Fig. 2).
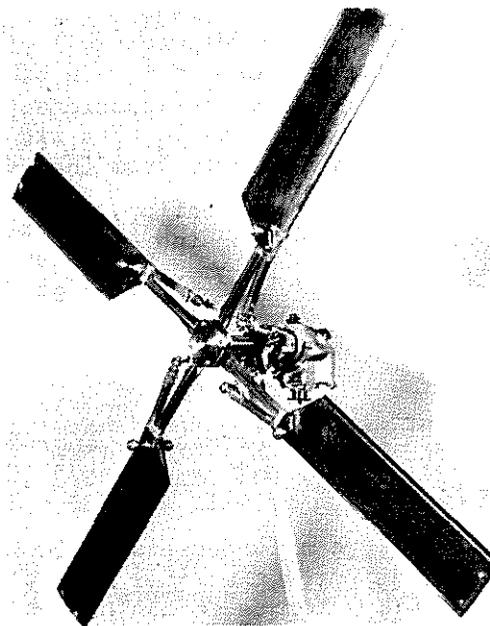


Fig. 2:  MBB Bearingless Tail Rotor

- rigid rotor with elastomeric bearing to form the feathering hinge, where the flapping and lagging hinges are replaced by an elastic element in the rotor head or in the blade root (Fig. 3).
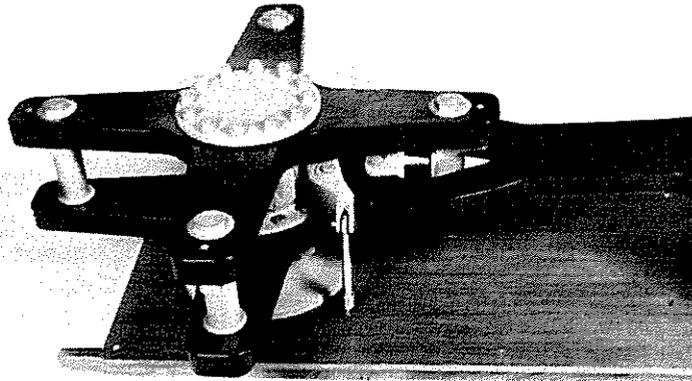


Fig. 3: MBB FEL Rotor

- articulated rotor with elastomeric bearings to replace the flapping, lagging and feathering hinges (Fig. 4).
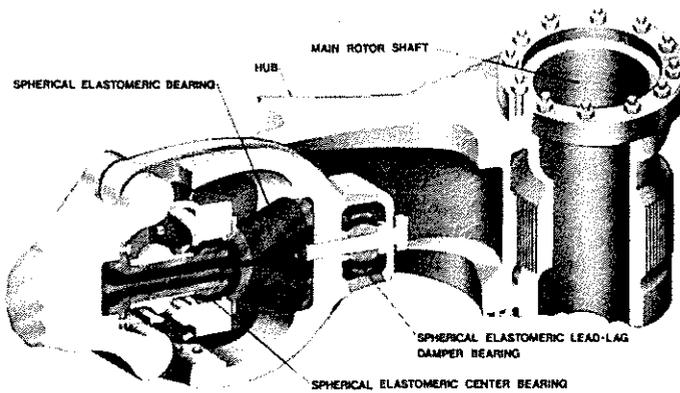


Fig. 4: Sikorsky Spherical Elastomeric Bearing Concept

Due to their low polymerised chain molecules, elastomeric materials are able to undergo large but reversible deformations. This capability, in combination with the nearly incompressible material behaviour is used in designing elastomeric bearings. In the special case of a low stiffness requirement in the direction of a possible shear deformation, together with a high stiffness requirement normal to that axis, the elastomeric bearing is constructed from alternating rubber and metal layers (shims). This generates the neccessary augmentation in the bearing stiffness perpendicular to the laminates (i.e. in comparison to the compression stiffness of the elastomeric bearing without shims). The use of shims has only a secondary effect on the shear stiffness.

The construction of the elastomer shim composite is mainly orientated by the load cases. The demands of stiffness and strength may be accomodated by the designer using shims in the forms of disks, cylindrical, conical or spherical shells.

In designing elastomeric bearings, questions with regard to stiffness, static and fatigue strength as well as damping arise for different environmental conditions.

In engineering practice, elastomeric bearings are designed according to simple closed-form solutions with respect to stiffness and strength requirements for relatively simple and regular part geometries. These solutions are based on the uniaxial linear constitutive equation of HOOKE with kinematic relations and shape factors [1]. The emphasis will continue to be on the reduced weight and size of elastomeric bearings for the aircraft industry without sacrificing performance while improving cost effectiveness. To minimise development costs and risks, sophisticated analytical techniques have to be developed.

Numerical methods to analyse elastomeric bearings have to consider the peculiarities of the elastomeric materials and must be able to give a real geometric approximation. Finite element techniques have proven their efficiency in analysing complex and irregular part geometries under multiaxial loading and may also be used for numerical analysis of elastomeric bearings. Due to the near or complete incompressibility of elastomeric materials, problems may arise with finite elements, based on the conventional displacement method. As most finite element programme systems use these types of elements, this paper describes one possibility of overcoming these problems. To show the capacity of the proposed method, some examples have been analysed and are compared with the test results.

2. Basics on the Description of Elastomeric Behaviour

In comparison with metallic materials, elastomers show the following peculiarities which must be taken into consideration for finite elements:

- near-incompressibility

- viscoelastic behaviour

- large deformations at low loads

- nonlinear stress-strain characteristic

- temperature dependent properties

- properties' change by ageing

6.2-4

To analyse laminated elastomeric bearings, both by the constitutive equations and by the variational principle used to formulate finite elements, the above mentioned material properties should be considered or at least it must be seen which simplifications have been introduced.


## 2.1 Constitutive Equations

Properties of rubber-like materials can be described by constitutive equations using either a linear or a nonlinear approximation.

The l i n e a r approximation of a stress-strain relation is by a modified HOOKE's law, known as HERRMANN's approach [2]. Another degree of freedom, the mean pressure variable

$$h = S_1 / [2G(1 + \nu)] \tag{1}$$

is introduced, leading to the following constitutive equation:

$$\underline{S} = 2G(\underline{D} + \nu h \underline{1}) \tag{2}$$

where

$\underline{S}$ = stress tensor

$S_1$ = first invariant of $\underline{S}$

$\underline{D}$ = strain tensor

$\underline{1}$ = unit tensor

$G$ = shear modulus

$\nu$ = POISSON's ratio


For n o n l i n e a r approximation of a stress-strain relation, the MOONEY-RIVLIN constitutive equation seems to be most suitable, with respect to computational expense and exactness, for the most interesting field of practical applications [3]. This equation reads:

$$\underline{T} = \hat{p} \, \underline{C}^{-1} + 2 \, [c_1 \, \underline{1} + c_2 \, (C_1 \, \underline{1} - \underline{C})] \tag{3}$$

where

$\underline{T}$ = second PIOLA-KIRCHHOFF stress tensor

$\underline{C}$ = right CAUCHY-GREEN strain tensor

$\underline{C}^{-1}$ = inverse of $\underline{C}$

$C_1$ = first invariant of $\underline{C}$

$\hat{p}$ = arbitrary hydrostatic pressure function

$\left.\begin{array}{c} c_1 \\ c_2 \end{array}\right\}$ = MOONEY-RIVLIN material constants

Using the GREEN strain tensor $\underline{G}$, as is the case in most existing finite element programme systems, the MOONEY-RIVLIN constitutive equation reads:

$$\underline{T} = a_1 \underline{1} + a_2(G_1 \underline{1} - G) + 2 \hat{p} [\underline{1} + 2(G_1 \underline{1} - \underline{G}) + 4 \det\underline{G} \; \underline{G}^{-1}] \qquad (4)$$

where

$$a_1 = 2 (c_1 + 2c_2)$$
$$a_2 = 4c_2$$

A complete derivation of equations (3) and (4) as well as a proposition on how to establish the MOONEY-RIVLIN material constants $c_1$ and $c_2$ can be found in [4].

Influences of ageing, working temperature etc. on material behaviour may be considered both with HERRMANN's and MOONEY-RIVLIN's constitutive equation by using empirical derived changes of the material properties but without changing the constitutive equations themselves.

Time dependent phenomena like creep, stress relaxation, and damping could be tackled satisfactorily by a viscoelastic theory, but this is not within the scope of this paper.


## 2.2 Variational Principles

To overcome analysis problems caused by the incompressibility of the elastomer, a mean pressure h or a hydrostatic pressure function $\hat{p}$ has been introduced in the constitutive equation as another unknown besides the displacements $\underline{u}$.

The unknown h or $\hat{p}$ is not a material constant, but depends on the boundary condition of the structure to be analysed. For this reason it must be incorporated in addition to the displacements in the variational equation.

A convenient approach was produced by HERRMANN [2] for the linear case by introducing into the variational principle, valid for compressible materials, an incompressibility constraint as follows:

$$D_1 - (1 - 2\nu) \cdot h = 0 \qquad (5)$$

The variational principle for linear elastic incompressible or nearly incompressible materials then becomes

$$\int_{(V)} \underline{S} \, .. \, \delta\underline{D} \, dV + \int_{(V)} [D_1 - (1 - 2\nu) \, h] \, \delta\lambda \, dV - \int_{(A)} \underline{t} \, . \, \delta\underline{u} \, dA - \int_{(V)} \underline{b} \, . \, \delta\underline{u} \, dV = 0 \qquad (6)$$

where the variation of the mean pressure h results from LAGRANGE multiplier

$$\delta\lambda = 2\nu G \, \delta h . \qquad (7)$$

The first term of equation (6) represents the strain energy, the second contains the incompressibility constraint, whereas term three and four represent the energy from boundary and body forces respectively. The operators (_) .. (_) and (~) . (~) symbolise the double scalar product of two tensors and the scalar product of two vectors respectively.

By introducing equation (2) into equation (6), the stress tensor can be deleted and under consideration of equation (7) one obtains

$$\int_{(V)} [2G\underline{D} .. \delta\underline{D} + 2G\nu h\delta D_1 + 2G\nu D_1 \delta h - 2G(1-2\nu)\nu h\delta h] \, dV$$

$$- \int_{(A)} \underline{t} . \delta\underline{u} \, dA - \int_{(V)} \underline{b} . \delta\underline{u} \, dV = 0. \tag{8}$$

Equation (8) is used to formulate finite elements to analyse rubber-like products with linear elastic incompressible and nearly incompressible material behaviour.

As the strain tensor $\underline{D}$ represents the derivations of the displacement vector $\underline{u}$, equation (8) contains only the displacements $\underline{u}$ and the mean pressure h as unknowns.

The formulation of a variational principle for the nonlinear case is somewhat different from the way shown before. Here must be exactly defined which stress and strain tensors have to be connected. For computer analysis it was found advantageous to use the second PIOLA-KIRCHHOFF stress tensor in combination with the GREEN strain tensor.

The incompressibility constraint according to equation (5) cannot be used in the nonlinear case. Because of det $\underline{C}$ = 1 at incompressibility, the relation

$$C_3 - 1 = 0 \tag{9}$$

is offered as incompressibility constraint.

The variational principle for nonlinear elastic incompressible materials then becomes

$$\int_{(V)} \underline{T} .. \delta\underline{G} \, dV + \int_{(V)} (C_3 - 1) \, \delta\lambda \, dV - \int_{(A)} \underline{t} . \delta\underline{u} \, dA - \int_{(V)} \underline{b} . \delta\underline{u} \, dV = 0 \tag{10}$$

The LAGRANGE multiplier can be chosen to be

$$\delta\lambda = \delta\hat{p}/2. \tag{11}$$

Based on equation (10), the nonlinear problem is reduced by an incremental procedure to a stepwise solution of a finite number of linearized subproblems (step-by-step algorithm). This method means a superposition of finite quantities with infinitesimal ones, symbolised by Δ ( ) in the following equations.

An incremental form of the variational principle for nonlinear elastic incompressible materials then becomes

$$\int\limits_{(V)} (\underline{T} + \Delta\underline{T}) \; .. \; \delta\Delta\underline{G} \; dV + \int\limits_{(V)} (C_3 + \Delta C_3 - 1) \; \delta\Delta\lambda \; dV$$

$$- \int\limits_{(A)} (\underline{t} + \Delta\underline{t}) \; . \; \delta\Delta\underline{u} \; dA - \int\limits_{(V)} (\underline{b} + \Delta\underline{b}) \; . \; \delta\Delta\underline{u} \; dV = 0 \; . \tag{12}$$

By introducing equation (4) into equation (12), the second PIOLA-KIRCHHOFF stress tensor can be deleted, one gets after some transpositions and under consideration that the strain tensor $\underline{G}$ can be divided into a linear portion $\underline{G}^l$ and a nonlinear one $\underline{G}^n$

$$\int\limits_{(V)} [4c_2 \; (\Delta G_1^l \; \underline{1} - \Delta\underline{G}^l) - 2\hat{p} \; \underline{C}^{-1}\Delta\underline{G}^l \; \underline{C}^{-1} + \Delta\hat{p}\underline{C}^{-1}] \; .. \; \delta\Delta\underline{G}^l \; dV$$

$$+ \int\limits_{(V)} \{\hat{p}\underline{C}^{-1} + 2 \; [c_1\underline{1} + c_2 \; (C_1\underline{1} - \underline{C})]\} \; .. \; \delta\Delta\underline{G}^n \; dV$$

$$+ \int\limits_{(V)} \underline{C}^{-1} \; .. \; \Delta\underline{G}^l \; \delta\Delta\hat{p} \; dV - \int\limits_{(A)} \Delta\underline{t} \; . \; \delta\Delta\underline{u} \; dA - \int\limits_{(V)} \Delta\underline{b} \; . \; \delta\Delta\underline{u} \; dV \tag{13}$$

$$+ \int\limits_{(V)} \{\hat{p}\underline{C}^{-1} + 2 \; [c_1\underline{1} + c_2 \; (C_1\underline{1} - \underline{C})]\} \; .. \; \delta\Delta\underline{G}^l \; dV$$

$$- \int\limits_{(V)} \underline{b} \; . \; \delta\Delta\underline{u} \; dV - \int\limits_{(A)} \underline{t} \; . \; \delta\Delta\underline{u} \; dA + \int\limits_{(V)} \frac{1}{2} \; (1-C_3^{-1}) \; \delta\Delta\hat{p} \; dV = 0 \; .$$

Equation (13) is used for finite element formulation of nonlinear elastic and incompressible material behaviour described by the MOONEY-RIVLIN constitutive equation.
The dashed terms in equation (13) represent the virtual work of the inequality of inner forces against outer ones and the last term characterizes the violation of the incompressibility averaged over the integral region. These terms are suitable as the correction terms in an iterative procedure.

3. Finite Element Computer Programme

The finite element method process is characterized by the handling of the integral region of the functionals in equation (8) and (13) as a summation of subregions, the finite elements. Due to the variational equations derived in the chapter before, it is necessary to use both displacements and mean pressure or hydrostatic pressure function as unknowns.

To analyse laminated elastomeric bearings, i.e. bodies consisting of
several layers made of several different materials, types of elements with
and without incompressibility constraints have to be joined. This is possible
only by introducing additional constraints on the boundaries of elements of
a different kind and leading to the fact that each layer must be handled as
a complete substructure built up of one element type. Compared to monolithic
structures therefore the number of unknowns will be very high, causing high
computer costs.

The finite element computer programme MARC [5] has been used for the
analyses presented in the next chapter. This programme offers on the basis
of the theory described before various finite elements both with HERRMANN's
and MOONEY-RIVLIN's approach.

## 4. Applications

In the following, results will be presented for various elastomeric
parts obtained by using MARC and will be compared with exact solutions,
results taken from literature, and test results respectively.

### 4.1 Thick-walled Cylinder Subjected to Internal Pressure

One of few examples, of which an exact solution is known for geometrical
and physical nonlinear behaviour, is the plain strain state of an infinitely
long thick-walled cylinder subjected to internal pressure.
Both the exact results and a finite element approach are presented in [6,7].
We have analysed the problem using axisymmetric 8-node elements, where the
plain strain state was realized by suitable boundary conditions.
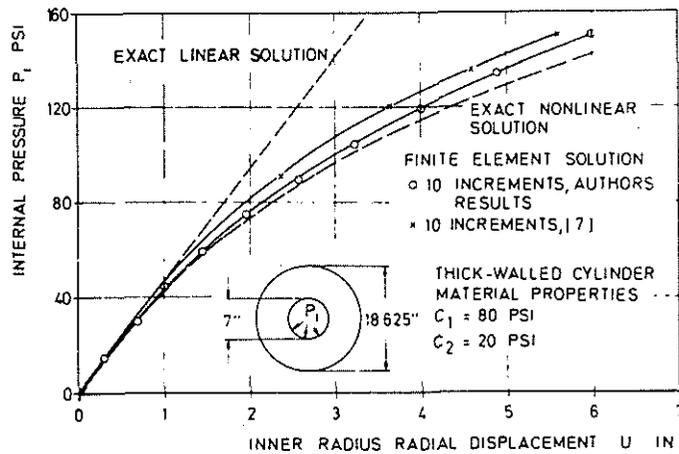


Fig. 5:   Load versus Displacement
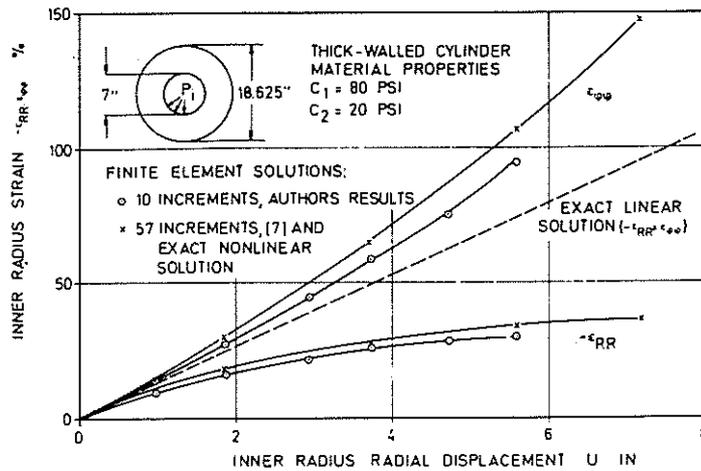
Fig. 6: Strain versus Displacement

From Fig. 5 and Fig. 6 it can be seen that at or below incremental load steps of 10% of maximum load the exact nonlinear solution is approximated quite well. Better agreement can be found using smaller increments, but the exact nonlinear solution can be found only with increments smaller than 2% of maximum load. For strains larger than 10% there are significant differences between linear and nonlinear results justifying the additional work of a nonlinear approach.

## 4.2 Rubber Bonded between Steel Plates Subjected to Compression Load
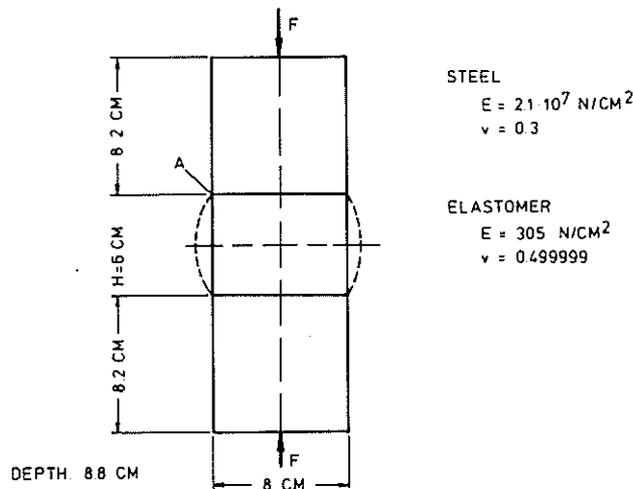


Fig. 7: Rubber/Metal Element

For the plain strain state of a rubber layer, bonded between steel plates and subjected to compression load, no exact analytical solution is known. As this structure is also interesting as a practical application, some numerical solutions can be found in literature [7,8,9].

We have conducted a numerical analysis based on HERRMANN's consti-
tutive equation for the nearly incompressible elastomeric material and
HOOKE's law for the steel plates.
Due to double symmetry of the structure only one quarter needed to be ana-
lysed, consequently the number of unknowns was reduced drastically.



Fig. 8: Deformed Plot of a Rubber/Metal Element

The deformed plot in Fig. 8 clearly shows the effect of nearly in-
compressible material behaviour, where the elastomeric material bulges in
the unrestrained area. The higher the shape factors, i.e. load area divided
by bulge area, the larger the bulging in compression.
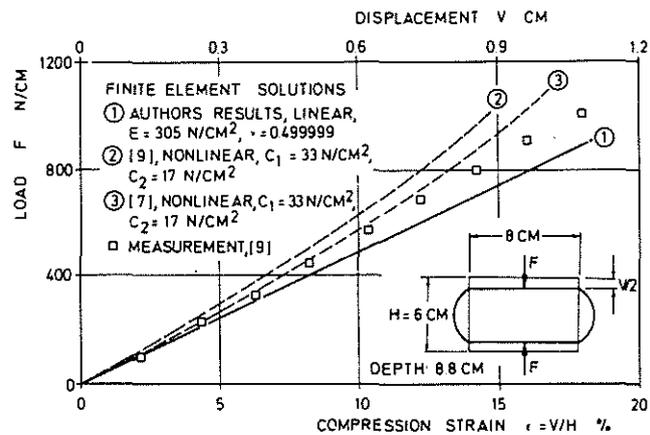


Fig. 9: Load versus Compression Strain

From Fig. 9 it can be seen that the vertical displacement from linear
analysis corresponds well with the measurement from [9] only up to a com-
pression strain of 5%. For higher compression strains a better approximation
can be found by nonlinear analysis.
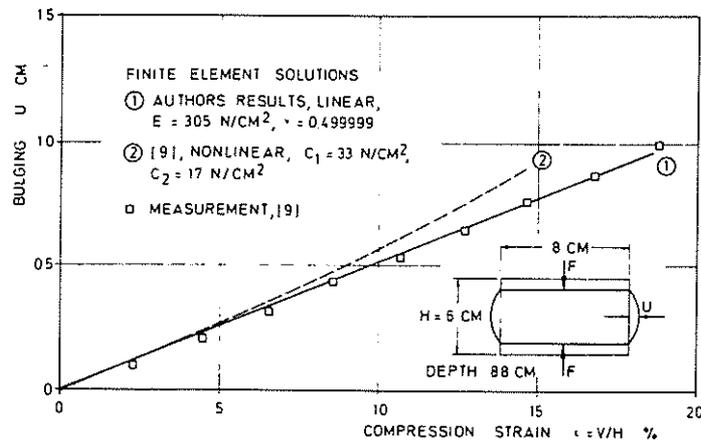
Fig. 10: Bulging versus Compression Strain

From Fig. 10 it can be seen that with HERRMANN's constitutive equation the bulging fits the measurement from [9] well even for compression strains higher than 10%, whereas the results with MOONEY-RIVLIN's constitutive equation differ significantly from the test results.

Analyses with various numbers of elements within the elastomeric material have shown, that the vertical displacement and thus the compression stiffness of the rubber/metal element can be established with sufficient accuracy by only one 8-node isoparametric finite element.

However, a more precise establishment of strains and stresses needs a finer mesh, as shown in Fig. 8. For the highest shear strain within the elastomeric material we have found differences of up to 150% relative to the result with one element.
Point A (see Fig. 7) was found to be the high strain area. For a detailed stress analysis the mesh must be refined in the vicinity of point A to consider the high gradients in stress and strain.


4.3 Spherical/Conical Thrust Bearing
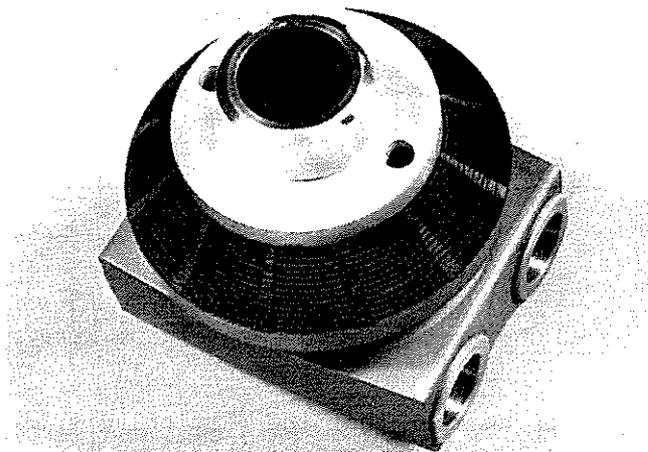


Fig. 11: Spherical/Conical Thrust Bearing

Thrust bearings of the type presented in Fig. 11 may be used in either rigid rotor or articulated rotor systems with elastomeric bearings as shown in chapter 1 of this paper.
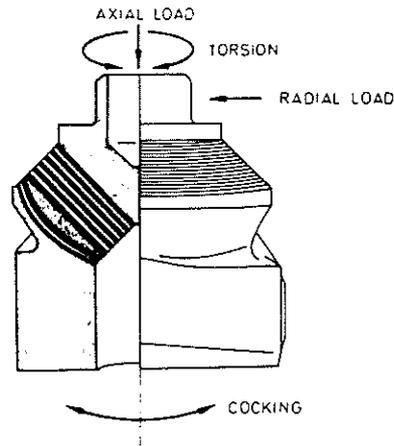


Fig. 12: Loadings and Motions of a Thrust Bearing


This thrust bearing has to support the following loadings:

- axial compression, caused by centrifugal forces;

- radial shear, caused by drag forces;

- radial shear, caused by thrust forces;

while accomodating the following motions:

- torsional shear, due to pitch motion;

- cocking shear, due to flap motion;

- cocking shear, due to lag motion.

These loads and motions are shown in Fig. 12 and are applied at various phases, relative to each other.



Fig. 13: Cross Section of Thrust Bearing

As shown in Fig. 13, thrust bearings are generally built up of several conical and/or spherical shells, consisting of alternating elastomeric and metallic layers and attached to inner and outer support members.
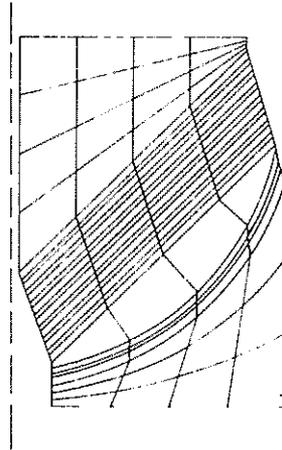


Fig. 14: Computer Plot of a Thrust Bearing

For numerical analysis of the thrust bearing a finite element model as shown in Fig. 14 was used, with major emphasis placed on a realistic representation of the elastomer layers and the shims.

As the loads to be reacted result in a general threedimensional state of stresses and strains, threedimensional finite elements should be used. This, however, would lead to high computational costs, due to a high number of unknowns.

For axisymmetric structures but not axisymmetric loading the computational expense can be reduced drastically for linear analysis by a FOURIER development of the variables and the loads. This method has been used to analyse the thrust bearing.

The finite element analysis was checked by establishing the stiffness of the bearing in various modes and comparing it to the test results. A fairly good agreement was found as shown in Table 1.

| STIFFNESS / ERROR | AXIAL | RADIAL | TORSIONAL | COCKING |
|---|---|---|---|---|
| $\dfrac{C_{TEST} - C_{FEM}}{C_{TEST}}$ | - 5.8 % | + 9.6 % | + 5.7 % | + 4.2 % |

Table 1: Stiffnesses - Analysis versus Test

Strain gauges were installed in hoop direction at different positions on the outer end of conical and spherical shims as well as on the housing.

The measured strains are compared to the numerical results for various loading conditions in Fig. 15.
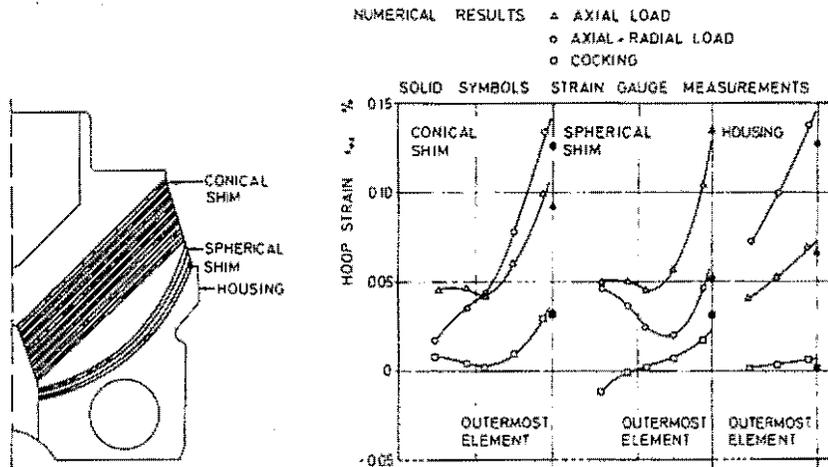
Fig. 15:  Hoop Strains - Analysis versus Test


The test results and the finite element results are found to be in good agreement.

Deformed plots as shown in Fig. 16 are very helpful to find out the high strain areas, so that in a second run, with a refined mesh, high stress and strain gradients can be accomodated satisfactorily.



AXIAL LOAD          RADIAL LOAD          COCKING
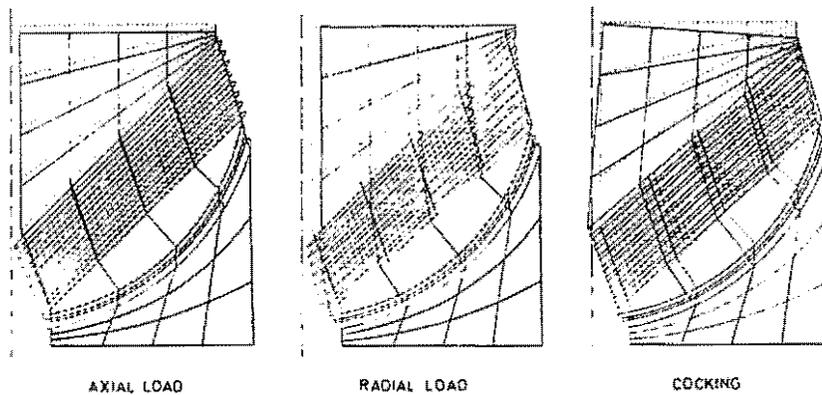
Fig. 16:  Deformed Plots of a Spherical/Conical Thrust Bearing

## 5. Conclusions

In addition to composite materials, elastomeric bearings offer high potential benefits for modern rotor systems. That is mainly due to the very large bulk modulus in comparison to the shear modulus, resulting from the fact that the elastomeric material is incompressible or nearly incompressible Because of this incompressibility, analysis problems occur for complex and irregular part geometries for which closed form solutions are not known. The well known finite element programmes with elements based on the conventional displacement method cannot be used.

This paper shows that there are finite element computer programmes available which can analyse rubber-like structures. These computer programmes use finite elements, based on HERRMANN's principle for linear analysis and on MOONEY-RIVLIN's approach for the nonlinear case.
The important points concerning numerical analysis and their comparisons with respectively;exact solutions; results taken from literature; and test results can be summarized as follows:

- the nonlinear MOONEY-RIVLIN's approach is able to approximate the exact non-linear solution;

- computertime is much greater for the nonlinear case in comparison with the linear HERRMANN's approach;

- for strains higher than 10% there are significant differences between linear and nonlinear results,justifying the additional costs of a nonlinear approach;

- the stiffnesses of laminated elastomeric bearings in various modes can be calculated satisfactorily;

- stresses in the shims and in the rubber layers can be analysed with sufficient accuracy;

- deformed plots are very helpful in resolving where the high strain areas occur.

To fully develop the high potentials inherent in elastomeric bearings, future work should be directed into the fields of fatigue strength and viscoelastic behaviour.

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