"THE ANALYTICAL DEFINITION OF ROTOR STREAMTUBE TO STEP IN 3D NAVIER-STOKES FLOW NUMERICAL SOLUTION FOR HELICOPTER IN HOVER"

DR.I.Tugrul Karamisir
ATI Aviation Technologies Industries
Ucaksavar sit. Pamir Apt. 32/5 ETILER-34337 ISTANBUL/TURKEY
E- MAIL:ibra2rul@karamisir.com

Abstract

This study is aimed to develope some analitical solutions of Actuator Disc Theory, for hovering case of the rotor as precise as possible. Starting with Navier-Stokes Equations, by making least amount of assumptions to enable to reach real time physical results, progressing towards Eulerian Equation and Viscous Energy consideration, and finally an analitically solveable Bernoulli Equation, including some physical considerations are taken into account at the scope of this work. After explaining problem in 1D space, the first exact analytical solution of the 3D, viscous Navier-Stokes laminar flow by Von Karman is adopted to our case.
Actuator Disc Theory is defined in Three Dimensional Space and the care is focused to Induced Velocity which is essential to calculate Thrust produced by rotor, and Thrust Induced Power that dominates during hover. Navier- Stokes Equations tell us that, Induced Velocity varies along the span, in other words it is not axial on each incremental location of the blade due to tip effects, loading effects, swirl effect and ground effect. Cylindrical Components of the Navier-Stokes Equations from "Streeter,V.1961(Ref. 1)" are preferred due to nature of the problem. Finally, Von Karman's solution technique is intruduced.

Key words

Navier-Stokes Equations, Bernoulli Equation, Induced Velocity, Thrust, Power.

Introduction

Accurate numerical simulation of the flow field of helicopter rotors is one of the most complicated and challenging problems in the field of aerodynamics. For performance prediction of helicopter rotors, the numerical method must have a capability of accurately capturing the flow not only on the blade but also in the vortical wake generated from the blade tip, which significantly affects the overall rotor performance, vibration and noise.
Although many researchers have been successfully performed for hovering rotors using inviscid methods, the capability of handling viscosity is still a very desireable feature for simulation of realistic flow mechanism such as tip vortex formation. It is particularly true to simulate accurately viscous-inviscid interactions involving shock-induced seperation at transonic tip Mach numbers with a relatively high collective pitch setting. Several viscous simulations have been performed to predict the rotor performance using direct wake capturing methods on structured grids. "Srinivasan et al. 1992 (Ref.2)" descibed one of the earliest

viscous simulations using a single-block grid with a number of grid points close to one million for the complete rotor and wake system. Their results showed good agreement with the experimental pressure distrubution and the tip vortex trajectory. "Wake and Baeder.1996 (Ref.3)" presented comparisons of their viscous prediction to the UH-60A rotor performance data. The results included spanwise loading distribution, owerall thrust, torque, and figure of merit over a range of collective pitch angles. "Ahmad and Strawn,1999 (Ref.4)" used an overset grid viscous method and investigated the dependency of three different wake grid resolutions. They successfully tracked the tip vortex for 630 degrees of vortex age using up to 17.1 million grid points at the finest level. Recently " Kang and Kwon, 2002 (Ref.5)" made an attempt to predict numerically the performance of a hovering rotor using an unstructured mesh Navier-Stokes flow solver. The tip vortex trajectory is traced through a series of spatial mesh adaptation starting from a very coarse initial mesh.

So many works have been successfully done to solve numerically the Navier-Stokes flow,after a broad analytical solution and Navier-Stokes interpretation has been achived by Von Carman at the begining of the 20th century. In the present study, an attempt is made to reduce Navier-Stokes flow to a Bernoulli Equation to attack the problem from 1D to 2D as the nature of it is two dimensional for each and every possible assumptions. After explaining physical sense of the problem in 1D. Von Carman's challenge to solve 3D Navier-Stokes for this particular problem is introduced.

Problem Formulation

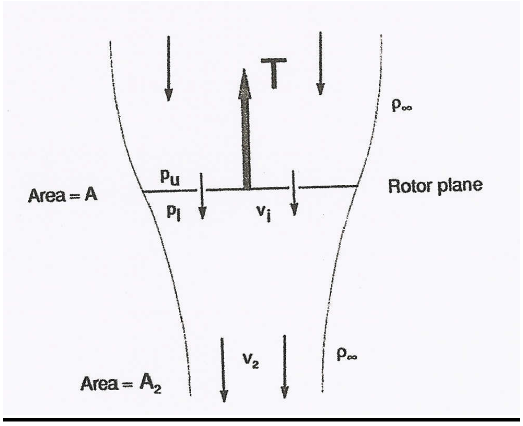

Figure 1 Actuator disc streamtube (Ref.6)

In order to make a clear explanation, we should start by selecting the most broad set of Navier-Stokes flow as far as the physical nature of the wake concerned. "Streeter,V. 1961(Ref.1)" gives the most convenient form of Navier-Stokes equations for incompresssible flows with constant viscosity, The dynamical equation:

$$\frac{DV}{Dt} = F_{body} - \tfrac{1}{\rho}\nabla p + \vartheta\nabla^2 V \qquad (1)$$

The cylindrical components of "Eqn.(1)" are as follows:

$$\frac{\partial \upsilon_r}{\partial t} + \upsilon_r \frac{\partial \upsilon_r}{\partial r} + \frac{\upsilon_\psi}{r}\frac{\partial \upsilon_r}{\partial \psi} + \upsilon_i \frac{\partial \upsilon_r}{\partial z} - \frac{\upsilon_\psi^2}{r} =$$

$$F_r - \frac{1}{\rho}\frac{\partial p}{\partial r} + \vartheta\left(\nabla^2 \upsilon_r - \frac{2}{r^2}\frac{\partial \upsilon_\psi}{\partial \psi} - \frac{\upsilon_r}{r^2}\right)(2)$$

$$\frac{\partial \upsilon_\psi}{\partial t} + \upsilon_r \frac{\partial \upsilon_\psi}{\partial r} + \frac{\upsilon_\psi}{r}\frac{\partial \upsilon_\psi}{\partial \psi} + \upsilon_i \frac{\partial \upsilon_\psi}{\partial z} + \frac{\upsilon_r \upsilon_\psi}{r} =$$

$$F_\psi - \frac{1}{\rho}\frac{\partial p}{r\partial \psi} + \vartheta\left(\nabla^2 \upsilon_\psi + \frac{2}{r^2}\frac{\partial \upsilon_r}{\partial \psi} - \frac{\upsilon_\psi}{r^2}\right)(3)$$

$$\frac{\partial \upsilon_i}{\partial t} + \upsilon_r \frac{\partial \upsilon_i}{\partial r} + \frac{\upsilon_\psi}{r}\frac{\partial \upsilon_i}{\partial \psi} + \upsilon_i \frac{\partial \upsilon_i}{\partial z} =$$

$$T - \frac{1}{\rho}\frac{\partial p}{\partial z} + \vartheta\nabla^2 \upsilon_i (4)$$

First terms on above equations are due to unsteady characther of the flow,

following terms up to equal signs are lifting terms, first terms after equal signs are body Forces and finally , the terms after υ are the viscous terms. Notice that, main difficulty on above aquations are from lifting terms due to nonlineeraty.
Now, we shall proceed to make some assumtions:

1. Rotor has infinetly many blades, so it can be assumed to be uniform.
2. Flow is steady, so first terms can be omitted.
3. Once we define the actuator disk, "Equation (4)" will solely satisfiy our needs.
4. For the begining let us assume constant viscosity for the flow.

Now, Let us analyze the "Equation(4)" as far as the physical nature of the problem concerned. The term $\frac{\partial \upsilon_i}{\partial t}$ is due to time dependent irregularities of the induced velocity. By assuming Steady flow we shall omit this term. The term $\upsilon_r \frac{\partial \upsilon_i}{\partial r}$ is the component due to span, tip effect and loading. We shall omit this term, and analyze the variation of induced velocity with respect to radius, by superposition in loading distribution (compansate it in the n term of the method). The term $\frac{\upsilon_\psi}{r}\frac{\partial \upsilon_i}{\partial \psi}$ is the component due to swirl effect. As $\upsilon_\psi << \upsilon_i$ we shall also omit this term, but will compansate its effect in n term of the superposition method. The term $\upsilon_i \frac{\partial \upsilon_i}{\partial z}$ is the component which causes Kinetic Energy along the wake in axial direction. The term T is the body force which is Thrust in our problem. The term $\frac{1}{\rho}\frac{\partial p}{\partial z}$ is the component due to pressure variation along the wake which also causes Energy. The term $\vartheta \nabla^2 \upsilon_i$ is the component due to Kinematic viscosity, which is one of the scope of this paper is the proof , why this expression is negligeble analiticly.
So , "Equation(4)" reduces to;

$$\vartheta \frac{d^2 \upsilon_i}{dz^2} - \upsilon_i \frac{d\upsilon_i}{dz} = -T + \frac{1}{\rho}\frac{dp}{dz} \quad (5)$$

By the product of dz operator

$$\vartheta \frac{d\upsilon_i}{dz} - \upsilon_i d\upsilon_i = -Tdz + \frac{dp}{\rho} \quad (6)$$

This is an energy equation for unit mass

$$\frac{d\upsilon_i}{dz} = \frac{d\upsilon_i}{dz}\frac{dz}{dt} / \frac{dz}{dt} = \frac{a_i}{\upsilon_i}$$

unit=1/second (7)

The term $\vartheta \frac{d\upsilon_i}{dz}$ indicates that Kinematic viscous energy is significant only for the part close to rotor disc of the wake. We can see from "Equation(7)" that, Viscous energy is linearly propotional with induced acceleration, and inversly proportional with induceed velocity, and it looses its significance at downwash as the endurance of the wake increases. On the other hand Kinematic viscousity of air is $\vartheta = 1.5x10^{-5}$ m^2/second at 20^{o}C and sea level. That is why we neglect viscous energy term.
Now "Equation (6)" reduces to:

$$Tdz = \upsilon_i d\upsilon_i + \frac{dp}{\rho} \quad (8)$$

Integration yields:

$$\frac{\upsilon_i^2}{2} + \frac{p}{\rho} - Tz = cons\tan t \quad (9)$$

everwhere in the actuator disc streamtube
Tz is the potential and this equation is the Bernoulli's equation.
Mass flow through the rotor disc per unit time is " Newman,S.1994(Ref.6)";

$$M_{flow} = \rho A \upsilon_i \qquad (10)$$

The flow in the wake is assumed to come from air ahead of the rotor at rest with the pressure p_∞ and return to pressure p_∞ far downstream of the rotor.

The rotor thrust is given by:

$$T = A(p_l - p_u) \qquad (11)$$

Applying Bernoulli's equation above the rotor gives

$$p_\infty = p_u + \frac{1}{2}\rho \upsilon_i^2 \qquad (12)$$

Applying Bernoulli's equation below the rotor gives:

$$p_\infty + \frac{1}{2}\rho \upsilon_2^2 = p_l + \frac{1}{2}\rho \upsilon_i^2 \qquad (13)$$

From "Equations (11)-(12)-(13)" we have

$$T = \frac{1}{2}\rho \upsilon_2^2 A \qquad (14)$$

$$T = M_{flow}\upsilon_2 = \rho A \upsilon_i \upsilon_2 \qquad (15)$$

In every second, air of mass M_{flow} enters the streamtube with zero vertical velocity, whilst an equal mass of air leaves the streamtube with vertical velocity v_2 . In this time period the air in the stremtube acquires a vertical thrust of $M_{flow}\ v_2$.

From "Equations (14) and (15)" we find that:

$$\upsilon_2 = 2\upsilon_i \qquad (16)$$

From which substituting into "Equation (14)" and rearrenging gives:

$$\upsilon_i = \left(\frac{T}{2\rho A}\right)^{1/2} \qquad (17)$$

The value for v_i , so obtained, is called the ideal induced velocity and, because it is uniform over the rotor disc, is the minimum value for a given rotor thrust. The importance of this result lies in the predictions it makes for the power required to generate this rotor thrust. In hover, the greatest source of power is that required to maintain the rotor thrust T working against an air inflow of v_i . This is called the thrust induced power an is given by:

$$P_i = T v_i \qquad (18)$$

And is called the ideal induced power. An indication of the hover performance efficiency can be derived from the above analysis using, as a measure, the thrust per unit power required:

$$\frac{T}{P_i} = \frac{1}{\upsilon_i} \qquad (19)$$

Typical values for this parameter, for a selection off thrusting devices, are given in Table 1. (From Newman, S.1994(Ref.6))

Table 1

Aerospace vehicle designation	*Aerospace vehicle type*	*Equivalent disc loading* (N/m^2)	*Ideal induced velocity* (m/s)	*Hover efficiency* (N/kW)
Mil Mi-10k	Crane helicopter	388	12.6	79.4
Westland Lynx	Utility helicopter	364	12.2	82.0
Sikorsky S64A	Crane helicopter	496	14.2	70.3
Sikorsky CH-53E	Heavy helicopter	718	17.1	58.5
Bell V22 Osprey	Tilt rotor	1161	21.8	45.9
Hiller-Ryan XC-142	Tilt wing	2387	31.2	32.1
Ryan XV5A	Fan lift	16823	82.8	12.1
Harrier GR3	Jet lift	114403	216	4.6
Space shuttle	Rocket	465479	436	2.3
Saturn V F1	Rocket	2964912	1087	0.9

Non-ideal behaviour of the rotor

Charecteristics of a real rotor differ from the ideal situation which will be described largely through two effects, namely, viscous effects and induced velocity effects.

Viscous effects on a wing give rise to profile drag and hence , to overcome this, an extra demand is made, known as profile power. It is present irrespective of whether the rotor is producing thrust or not and its magnitude is normally below the other power drains on the rotor. For instence, in hover induced power dominates, however, when the rotor blades penetrate the stall boundry, the profile drag forces on the rotor blades increase substantially with a consequent increase in profile power. We have explained why we have omitted Viscous energy term in "Equation(6)" under normal conditions.

The momentum theory discussed above assumes uniformity over the entire rotor disk particularly the air flow velocity through it. However, in reality, this induced velocity varies across the disc and these differences can arise from various sources: suh as, tip effects, loading effects, swirl effects, and the ground effect.The tip loss factor is usually denoted by B, making the effective rotor radius as BR. Since the satisfactory various models are developed for tip loss factor by "Newman, S. 1994(ref.6)" we shall investigate Loading effects, and Swirl effects all in one n term by assuming the induced velocity expressible as a power of rotor radius. We have the following result:

$$\upsilon_i(r) = \upsilon_i^T \left(\frac{r}{R}\right)^n \qquad (20)$$

where zero downwash flow is assumed at the rotor centre and υ_i^T is the value of the downwash at the blade tip. In order to examine the effect of a non-uniform downwash distrubibution the idea of an annulus theory needs to be described. Rather then the entire rotor disc being used for the momentum analysis, the disc divided into a series of annuli, which are infinitesimally small in width and then apply to each annulus the analysis which has previosly been described. The overall rotor performance is then determined by integrating the results for a general annulus over the complete rotor. The general annulus has the radius r, width dr and a local downwash $\upsilon_i(r)$.

Annulus area is then:

$$dA = 2\pi r dr \qquad (21)$$

The mass flow through the annulus is:

$$2\pi\rho r dr\, \upsilon_i(r) \qquad (22)$$

As before, the vertical velocity far downstream is given by $2\upsilon_i$, therefore the thrust obtained from the annulus is equal to the rate of change of momentum of the flow passing through it. i.e.

$$dT = 2\pi\rho r dr\, \upsilon_i 2\upsilon_i = 4\pi\rho\upsilon_i^2 r dr \; (23)$$

The induced power for the annulus is then:

$$dP_i = dT\upsilon_i = 4\pi\rho\upsilon_i^3 r dr \qquad (24)$$

Using the above results and upon integrating over the whole rotor disc, and by substitution of "Equation (20)" the rotor thrust is given by:

$$T = \frac{4\pi\rho\upsilon_i^{T2}}{R^{2n}} \int_0^R r^{(2n+1)} dr = \frac{4\rho A \upsilon_i^{T2}}{2n+2} \; (25)$$

Similarly induced power is given by:

$$P_i = \frac{4\pi\rho\upsilon_i^{T3}}{R^{3n}} \int_0^R r^{(3n+1)} dr = \frac{4\rho A \upsilon_i^{T3}}{3n+2} \; (26)$$

Now from the thrust expression "Equation(25)", we have

$$\upsilon_i^T = (n+1)\left(\frac{T}{2\rho A}\right)^{1/2} \qquad (27)$$

and noting that the ideal analytical induced power is given by:

$$P_{iAIDEAL} = T\left(\frac{T}{2\rho A}\right)^{1/2} \qquad (28)$$

And expression for induced power in 3D (axial, radial, azimuthal) from "Eqn.(26)"is:

$$P_i = \left(\frac{2n+2}{3n+2}\right) T\upsilon_i^T \qquad (29)$$

whence;

$$\frac{P_i}{P_{iAIDEAL}} = \left(\frac{2n+2}{3n+2}\right)(n+1)^{1/2} = \frac{(n+1)^{3/2}}{1+\frac{3n}{2}} = k_i \qquad (30)$$

where $P_{iAIDEAL}$ is the ideal analytical induced power of the actuator disc given by "Eqn.(18)"

The term k_i , as defined above, is the amount that the induced power varies compared with the analytically ideal case of uniform downwash (n=0), and expressed as a ratio.

The effect of various values of n on k_i are shown in table 2, "Reference(6) NEWMAN,S 1994"

Table 2

n	0	0.5	1	2	3	5
k_i	1.0	1.05	1.13	1.30	1.45	1.73

It can thus be seen that as the downwash becomes more peaky towards the tip, increasing the value of n, so the induced power factor k_i increases.

A Solution Technique by Von Karman

After a trip to recent years, let us have a journey to the history where father of the Fluid Dynamics, Sir Von Carman came up with the exact solutions of the Navier-Stokes equations for three-dimensional steady flow. Case is the laminar flow due to an infinite rotating plane disc of constant angular velocity.

Navier-Stokes equations of an incopressible fluid in cylindrical coordinates are:

From "Streeter, V.1961(ref.1)"

$$\frac{1}{r}\frac{\partial \upsilon_r r}{\partial r}+\frac{1}{r}\frac{\partial \upsilon_\psi}{\partial \psi}+\frac{\partial \upsilon_i}{\partial z}=0 \qquad (31)$$

$$\frac{D\upsilon_r}{Dt}-\frac{\upsilon_\psi^2}{r}= -\frac{1}{\rho}\frac{\partial p}{\partial r}+\vartheta\left(\Delta\upsilon_i-\frac{\upsilon_r}{r^2}-\frac{2}{r^2}\frac{\partial \upsilon_\psi}{\partial \psi}\right) \qquad (32)$$

$$\frac{D\upsilon_\psi}{Dt}+\frac{\upsilon_r\upsilon_\psi}{r}= -\frac{1}{\rho r}\frac{\partial p}{\partial \psi}+\vartheta\left(\Delta\upsilon_\psi+\frac{2}{r^2}\frac{\partial \upsilon_r}{\partial \psi}-\frac{\upsilon_\psi}{r^2}\right) \qquad (33)$$

$$\frac{D\upsilon_i}{Dt}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\Delta\upsilon_i \qquad (34)$$

inwhich

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+\upsilon_r\frac{\partial}{\partial r}+\frac{\upsilon_\psi}{r}\frac{\partial}{\partial \psi}+\upsilon_i\frac{\partial}{\partial z}$$

and

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \psi^2}+\frac{\partial^2}{\partial z^2}$$

By assuming that the angular velocity of the disc is Ω, the boundary conditions of the present problem are:

$\upsilon_r(r,\psi,0)=0$

$\upsilon_r(r,\psi,\infty)=0$

$\upsilon_\psi(r,\psi,0)=r\Omega$ (35)

$\upsilon_\psi(r,\psi,\infty)=0$

$\upsilon_i(r,\psi,0)=0$

where disc is situated at $z = 0$

In the present problem , all the variables are independent of angular coordinate ψ.

In order to satisfy the boundry conditions, equations (35) , the velocity components and the pressure are placed in the following forms:

$\upsilon_r = r\,f(z)$, $\upsilon_\psi = r\,g(z)$, $\upsilon_i = h(z)$,
$p = p(z)$ (36)

After substituting Equations (36) in to Equations (31) to (34) a system of ordinary differential equations for f , g , h , and p are obtained, as follows:

$2f + h' = 0$

$f^2 - g^2 + h\,f' = \nu f''$

$2\,f\,g + h\,g' = \nu g''$ (37)

$h\,h' = -(p'/\rho) + \nu h''$

Equations (37) and the corresponding boundry conditions may be transformed in to non dimensional form by writing:

$f = \Omega F$, $g = \Omega G$, $h = (\nu\Omega)^{1/2} H$,
$p = \rho\nu\Omega P$, $r = (\nu/\Omega)^{1/2} R$,
$z = (\nu/\Omega)^{1/2} Z$ (38)

In terms of the nondimensional parameters of Equations (38), Equations (37) become:

$$H' + 2F = 0$$
$$F'' - H F' - F^2 = - G^2$$
$$G'' - H G' + 2 F G = 0 \qquad (39)$$
$$P' - H'' + H H' = 0$$

In which the primes now mean differentiation with respect to Z
The boundary conditions for F, H ,G, and P are:

$$F(0) = 0 \qquad F(\infty) = 0$$
$$G(0) = 1 \qquad G(\infty) = 0 \qquad (40)$$
$$H(0) = 0 \qquad P(0) = P_o$$

Equations (39) may be solved by series expansion method or by numerical integration. Numerical values of F, G, and H are calculated by Cochran and the functions are given in Figure 3. of ref (1)

If P_o is the value of P at the disk,

$$P - P_o = (½)H^2 - H' = (½)H^2 + 2F (41)$$

Strictly speaking, the above results apply only to an infinite disk, by neglecting the edge effect one may find the Frictional moment on a rotating disc of radius R.
The shearing stress at the disc is

$$\tau_{z\psi} = \rho \nu \frac{\partial v_\psi}{\partial z} = \rho(\nu\Omega^3)^{1/2}\, r\, G'(0) \quad (42)$$

so that the moment is

$$M_o = -2\int_0^R 2\pi r^2 \tau_{z\psi} dr = -\pi R^4 \rho \left(\vartheta\Omega^3\right)^{1/2}$$

$$G'(0) = -\frac{\rho\Omega^2 R^5 \pi G'(0)}{R_e^{1/2}}$$

(43)

in which the Reynolds number $R_e = \dfrac{R^2\Omega}{\vartheta}$

The moment coefficient is

$$C_M = \frac{M_o}{(1/2)R^3\Omega^2\pi R^2} = -2\frac{G'(0)}{R_e^{1/2}} = \frac{1.232}{R_e^{1/2}}$$

(44)

Conclusions and Remarks

Although numerical solution of the problem is beyond the scope of this paper, it is an attempt to give an analytical understanding of rotor streamtube, for those who want to step to numerical solution of the 3D Navier-Stokes equations for hovering case of helicopter.

Navier-Stokes flow solvers are available at the top universities and the related industries of the globe. This paper hopes togive ideas, assumptions and boundry conditions of this particular problem and to help better understandingof Streeter,V.1961(ref.1)" and "Newman,S. 1994 (ref.6)".

Von KARMAN's solution is an nostalgic attempt to study the swirl effect of rotor wake downstream. Sedney and Gerber "1985 AUGUST,AIAA Journal Vol.23, Number.8 Page1179 (ref.7)" have used this solution to identify the numerical study of the Critical layer in an rotating fluid. Case was an spining projectail with fluid fuel. Numerical solution of the spin-up eigen value problem is discussed. Each spin perturbation is considered an eigen value which is called critical layer. Time histories of eigenvalues and critical levels are presented along with the typical effects of the critical layer on eigenfunctions and phase of the velocity. If we adopt these studies to our concern Rotor swirl in downstream, we can realize that case is damped, i.e spin-up eigen value problem down to far wake where induced velocity is $2\upsilon_i$. Following this point spin-down flow is the case where we are not concerned with. What we can do is to produce a useful estimate of hover performance of a rotor for no head , tail, and side winds which in turn we can analyze eigen values, i.e, critical layers of the swirl effects independent from eigenfunctions. Results has to be verified by experimental work. This study can be conducted with those who can finance it.

REFERENCES

[1] STREETER. L. Victor "Handbook of Fluid Dynamics, Mc GRAW-HILL Book Co, Inc. 1961"

[2] SRINIVASAN,G.R, BAEDER,J.D, OBAYASHI,S., and McCROSKEY,W.J., "Flow Field of a Lifting Rotor in Hover: A Navier-Stokes Simulation" AIAA Journal Vol.30, (10) 1992, pp. 2371-2378.

[3]WAKE, B.E., and BAEDER,J.D.," Evaluation of a Navier-Stokes Analysis Method for Hover performance Prediction," Journal of AHS , Vol.41, (1), 1996, pp. 7-17.

[4] AHMAD,J.U., and STRAWN,R.C., " Hovering Rotor and Wake Calculations with an Overset-Grid Navier-Stokes Solver," AHS 55th Annual Forum Proceedings, Montreal, Canada May 25-27, 1999,pp. 1949-1959.

[5] KANG and KWON., "Unstructured Mesh Navier-Stokes Calculatiuns of the Flow Fıeld of a Helicopter Rotor in Hover" Journal of AHS April 2002, Vol.47, Num.2

[6] NEWMAN, S., " Foundations of Helicopter Flight" 1994. LONDON

[7] SEDNEY,R., and GERBER,N., "Numerical Study of the Critical Layer in a Rotating Fluid" AIAA Journal August 1985 , Vol.23. Num.8, pp, 1179

[8] HARIHARAN, N.S., and SANKAR,L.N., " First Principles Based High Order Methodologies for Rotorcraft Flow Field Studies," AHS 55th Annual Forum Proceedings, Montreal, Canada, May 25-27, 1999. pp. 1921-1933.