NUMERICAL INVESTIGATION OF HELICOPTER DYNAMICS IN EXTREME FLIGHT CONDITIONS

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Abstract

This paper presents a review of the most important phenomena associated with flying in extreme, limited conditions. These investigation is characterised by set of common features related to a course near limits, estimated return possibilities and includes causes of transgression such as: pilot errors, helicopter failures, purposeful or forced by situation contravention of regulations and causes of uncontrolled transgression during exploitation of a helicopter in extreme situations. A short note is given on problems encountered by high angle of attack aerodynamics. Problems associated with deep stall phenomena are considered. Dynamics of spatial motion of a helicopter has been considered. The complete set of non-linear equations of motion has been applied. Some results of numerical analysis of aircraft motion are presented.

1.Introduction

Investigations of controlled flight of a helicopter during extreme flight conditions and breaking through various limits of usage are of great cognitive and practical importance. Such investigations consist of transformations of the rotorcraft through its functional limits. This produces the unique set of information about the rotorcraft behaviour, effects correlation, mutual limits configuration, enabling the rotorcraft to improve safety and widens designed range of usage.

These investigations are characterised by a set of common features related to course near limits, estimation of return possibilities and include causes of limit transgression such as: pilot errors, rotorcraft failures, purposeful or forced by situation in contravention of regulations and reasons for uncontrolled limit transgressions during exploitation of the helicopter in extreme situations.

In flight battle areas, military rotocraft fly close to the ground to utilise the surrounding terrain, vegetation, or manmade objects. If is referred to such low altitude flight tactics as terrain flight. There are three common modes of terrain flight: low-level, contour, and nap-of the-Earth (NOE), providing increasing levels of concealment (Fig.1). The most effective and complex of these is nap-of-the-Earth (NOE) flight, during which the rotorcraft operates below treetop level. The obstacle-avoidance manoeuvres are repeatedly realise in extreme, limiting flight conditions. Such manoeuvres are jointed with a number of singularities, including unexpected rotorcraft motion. As result of them it is possible faulty pilot's action. Therefore it is necessary to investigate rotorcraft flight phenomena in extreme conditions.

In the present paper a non-linear dynamic model of a rotorcraft is considered which enables to determine the helicopter's motion. Manoeuvres used to demonstrate the flight simulation model involve rapid, large amplitude control inputs are classified as aggressive manoeuvres. It is shown (e. g. Ref. 1 - Ref. 5) that a individual blade main rotor model, including a correct representation of the rotor-engine drive train, is required to adequately predict engine and drive system response for aggressive manoeuvres. It is assumed that the helicopter fuselage is a rigid body and the motion of rigid blades about flap hinges, lead-lag hinges and axial hinges is considered, while the tail rotor is a linear model using strip/momentum theory with a uniformly distributed inflow. The blade dynamics is modelled to teetering rotor with pitch-flap coupling.

The induced velocity has been determined making use of the Biot-Savart law. Simplified model of vortex field has been applied and spatial structure of tip vortex trajectories being taken into consideration (e. g. Ref. 2).

In the present paper unsteady aerodynamics for prediction of rotor blades loads is included. The method of Tarzanin type stall model for calculating aerodynamic loads on airfoils is used (e. g. Ref. 5).

2.Formulation of the Problem

On the basis of physical model presented above, a set of non-linear differential equations has been obtained, which describes the dynamics of translatory and rotating motion of the helicopter and the blade motions around flap, lead-lag and axial hinges. The kinematic relations are also included in this set.

Several systems of co-ordinates have been applied in the problem under study. They are presented in Fig.2. Mathematical model describing the helicopter's flight can be defined in several ways i.e.:

- From classical mechanics (e. g. Ref. 2, 4, 5, 8, 9, 10) applying
  - –principle of conservation of momentum for the system, or
  - –principle of conservation of angular momentum
- From analytical mechanics, applying:
  - –Lagrange's equations, or
  - –Boltzmann-Hammel equations for non-holonomic systems (e. g. Ref. 10, 11, 12, 13).

Methods of analytical mechanics require twice differentiation of kinetic energy. This gives a large number of elements of equations and increases possibility commission of mistake. Difficulties with correctness verification of receiving equations appears.

Eduction of equations of rotorcraft's motion by application of methods of classical mechanics has two approaches:

-applying the principle of conservation of momentum and angular momentum refers to all rigid bodies forming a framework of investigated material system. The weak point of this approach is, that number of obtained equations is greater than number of degrees of freedom. Equations educed using this method contains inner reactions, which usually are not interesting in solving the problem;

- applying the principle of conservation of momentum and angular momentum refers to all investigated material system. In this case principle of conservation of angular momentum may be used in relation to any pole, not necessarily to the centre of gravity.

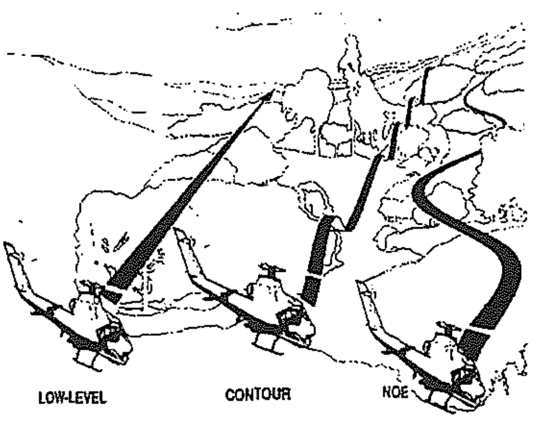

Fig. 1 Three modes of terrain flight

It is applied the first approach of eduction of equations of rotorcraft's motion by application of methods of classical mechanics in this work; i.e. method of the principle of conservation of momentum and angular momentum refers to all rigid bodies forming the helicopter's framework.

Flight simulation model description

A blade element model was used to determine the main rotor's motion and loads. The blade element rotor has provisions for the rotor rotational degree of freedom and n-number of blades to dynamically couple with the drive train to body degree of freedom. The primary degree of freedom involved in the rotor-body-drive train dynamic coupling are the main rotor lead-lag collective mode, main rotor rotation, tail rotor rotation, drive train rotation, and body roll, yaw and pitch. The coupled equations of motion which involve the main rotor and body degrees of freedom are solved simultaneously, as is presented in details in Ref. 4 and Ref. 5. The blade element rotor model, in addition to providing the means to represent non-linear, unsteady aerodynamics, enables correct representation of the main rotor-body-drive train dynamics coupling.

The main rotor inflow in presented work is based on the Biot-Savart law and simplified model of vortex field (e. g. Ref. 3, 14). The spatial structure of tip vortex trajectories is taken into consideration (e. g. Ref. 15). Equations of dynamic equilibrium of forces and moments have been determined in the system of co-ordinates $Ox_k y_k z_k$ fixed with the fuselage, and the blades motions have been considerated in the systems of co-ordinates $P_H x^i y^i z^i$ and $P_V x^i y^i z^i$ fixed with the hinges; (i=1,2,3,..n), n-number of rotor blades.

Detailed way of determining these equations can be found in Ref. 4, 5, 8, 9.

Finally it is obtained a set of 10+2n (n - number of main rotor blades) non-linear differential equations with periodic coefficients which can be presented in the form:

$$\mathbf{A}(\mathbf{X},t)\cdot\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X},\mathbf{S},t) \qquad (1)$$

Where $\mathbf{X}$ is the state vector:

$$\mathbf{X} = \big[u, v, w, p, q, r, \dot{\beta}_1,..\dot{\beta}_n, \zeta_1,..\dot{\zeta}_n, \Omega, \\ \psi_1,..\psi_n, \beta_1,..\beta_n, \zeta_1,..,\zeta_n, \Theta_e, \Phi_e, \Psi_e\ \big]^T \qquad (2)$$

and u, v, w are linear velocities of the centre of fuselage mass in the co-ordinate system $Ox_k y_k z_k$ fixed with the fuselage; p, q, r are angular velocities of the fuselage in the same co-ordinate system; Θ_e, Φ_e, Ψ_e are pitch, roll and yaw angles of the fuselage, β_i -i-th blade flap motion about flap hinge P_H, ζ_i-i-th blade lead-lag motion about lead-lag hinge P_V.

Vector S is the control vector:

$$\mathbf{S} = \mathrm{co}\big[\theta_0, \kappa_s, \eta_s, \varphi_T\big] \text{ or } \mathbf{S} = \mathrm{co}\big[\theta_0, \theta_1, \theta_2, \varphi_T\big] \qquad (3)$$

where: θ_0 is angle of collective pitch of the main rotor, κ_s is control angle in the longitudinal motion, η_s is control angle in the lateral motion and φ_T is angle of collective pitch of the tail rotor; θ_1, θ_2 - are angles of cyclic pitches of the main rotor:

$$\theta_1 = \kappa_s \sin\psi_0 + \eta_s \cos\psi_0 \qquad (4)$$
$$\theta_2 = \kappa_s \cos\psi_0 - \eta_s \sin\psi_0$$

where ψ_0 is a retardation angle of cyclic pitch control.

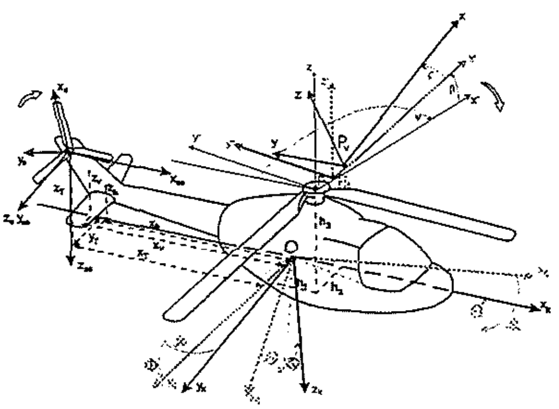

Fig. 2. Systems of co-ordinates

3. Aerodynamic Loads

Aerodynamic loads modelling is a difficult task in rotary wings flight simulation. The requirements for method of aerodynamic loads calculations stem both from flow environment and from algorithms used in analysis of helicopter flight. The airframe model consists of the fuselage, horizontal tail, vertical tail, landing gear and wing (if applicable). The fuselage model is based on wind tunnel test data (as function of angle of attack α and slip angle β). For high angles of attack and slip, the fuselage longitudinal and lateral forces and moments are calculated using method presented in Ref. 16. The horizontal tail and vertical tail are treated as aerodynamic lifting surfaces with lift and drag coefficients computed from data tables as functions of angle of attack α and slip angle β. The tail rotor is a linear model using strip-momentum theory with an uniformly distributed inflow. The effects of rotor wash on the airframe is included in the model. The technique used provides the essential effects of increased interference velocity with increased rotor load and decreased interference as the rotor wake deflects reward with increased forward speed.

Blade Aerodynamics

Aerodynamic data is for a NACA 23012 airfoil in the range +/-23° and the compressibility effects have been included. The data have been blended with suitable low speed data for the remainder of the 360° range to model the reversed flow region and fully stalled retreating blades. Dynamic stall effects have been included.

Modelling of Deep Stall Phenomenon

Term „deep stall" means phenomenon of increasing of lift coefficient C_L over the value C_{Lmax} achieved in static air-flow conditions. This phenomenon has been discovered by dint of helicopters. Designers observe that helicopters may fly with higher than resulted from restrictions due to transgression of critical angles of attack on recurring rotor blades speeds. One of the first work describing this phenomenon was published by Harris and Pruyn (Ref. 17). This question has been investigated in many works. It has been discovered similar phenomena occurring on turbo-compressors and aeroplane wings. Modelling air-flow on dynamic stall conditions belongs to very involved problems. It is not always possible or profitable to use CFD methods. Therefore dynamic stall phenomenon was a subject of many experimental works. As result of them factors affecting this phenomenon has been identificated. Taking as basis manner of utilisation of experimental data three groups of methods describing the deep stall phenomenon can be classified (e. g. Ref. 18).

I. Group of approximation methods. In this group results of experimental researches help either to calculate airfoil loads or to estimate parameters of mathematical description (for example factors of approximation polynomial). In this group will be three methods can be distinguished.
   a) method of aerodynamic factors. Airfoil loads are calculated directly from experimentally assigned (in wind tunnels) aerodynamic factors;
   b) method of generalised airfoil data. Airfoils loads are calculated by help of mathematical expressions received on basis of approximation of aerodynamics factors measured experimentally;
   c) analytical methods. Aerodynamic loads are calculated on basis of analytical expressions which are chosen in such manner that they describe measured experimentally process of deep stall phenomenon. An example is a procedure of calculation of aerodynamic loads of helicopter rotor blade airfoil in deep stall conditions suggested by Tarzanin (e. g. Ref.19), or methods described by Wayne (e. g. Ref. 20).
II. Semi-empirical methods which use differential equations for prediction of unsteady aerodynamic loads. The form and coefficients of this equations are determined by techniques of parameter identification. The basic model was developed by ONERA for loads at rotor blade section in stall conditions (Dat et al. (e. g. Ref. 21), Tran and Petot (e. g. Ref. 22), McAlister et al. (e. g. Ref. 23), Narkiewicz et al. (e. g. Ref. 24)). Also model of deep stall phenomenon suggested by Leischmann and Beddoes (e. g. Ref. 25, 26) belongs to this method. The ONERA model is a semi-empirical, unsteady, non-linear model which uses experimental data to predict aerodynamic forces on an oscillating airfoil which experiences dynamic stall.
III. Analytical methods, based either on the unsteady vortex lattice method (ULV) (cf. Konstadinopoulos et al. (e. g. Ref. 27)), or Euler and Navier-Stokes models (cf. Rausch et al. (e. g. Ref. 28), Guruswamy (e. g. Ref. 29), Schuster et al. (e. g. Ref. 30), Risk and Gee (e. g. Ref. 31)).

Some efficient models developed in computational fluid dynamics are difficult to be adopted in algorithms for solving a flight simulation problems. For instance application of a panel method leads to a large number of states. Also efficiency of some numerical perturbation methods and differential equation solvers could be questioned when such models are utilised. The requirements which stem from restrictions mentioned above concern:

- expressing the flow motion in state variables,
- describing the loads or state changes by ordinary differential equations,
- covering the possibility of feed-back loops, which occur in control problems.

State variable formulation of aerodynamic loads allows to use existing codes for rotorcraft flight simulation. Differential equations account for arbitrary airfoil motion and model the history of motion which is important in unsteady case. The Tarzanin deep stall model was chosen for adaptation to rotorcraft flight analysis in extreme flight conditions.

4. Engine model

The engine model is adapted from the code, data, and flow charts provided by the engine manufacturer. Usually the engine control system consists of two parts: an electronic control unit which acts as a trimming device to provide isochronous governing and various transient compensations, and the hydromechanical unit which performs the main task of pumping fuel to the engine in quantities grossly matched to compressor inlet temperature, and compressor discharge pressure. Other tasks of the hydromechanical unit include acceleration /deceleration scheduling to protect against engine stall and flameout, respectively. Inputs to the hydromechanical unit are the power available spindle, load demand spindle, and a trimming signal from the electronic control.

5. Steady Flight Conditions

The first step of solution process is the calculation of the trim state of helicopter. The flight conditions are defined by airspeed, turn rate, climb angle. The trim procedure is described in detail in Ref. 32 and Ref.33. The unknowns of the trim problem are: the steady state values of main rotor and tail rotor pitch controls, angles of attack and slip of the fuselage, average inflow over the main and tail rotor discs, fuselage pitch and roll angles (Θ and Φ), and roll, pitch, and yaw rates p, q, and r. The trim solution also provides the steady state periodic motion of the blades in flap and lag in the form of a truncated Fourier series for the quantities in flap-lag equations of motion. Usually it has been assumed that the helicopter is performing a steady horizontal trimmed flight. The terms of this flight are:

- linear and angular accelerations are equal to zero:

$$\dot{u} = \dot{v} = \dot{w} = \dot{p} = \dot{q} = \dot{r} = 0 \qquad (5)$$

- angular velocities are:

$$p = q = r = 0 \qquad (6)$$

- and the velocity component parallel to the Oy_k axis is:

$$v = 0 \qquad (7)$$

On the basis of Eqs.(5)-(7), making use of Eqs.(1) following vector of flight parameters can be obtained:

$$X_0 = (U_0, 0, W_0, 0,0,0, \dot{\beta}_{i0}, \beta_{i0}, \dot{\zeta}_{i0}, \zeta_{i0}, \Theta_0, \Phi_0, \Psi_0)^T \qquad (8)$$

(i=1,2,3,4) and the vector of control parameters:

$$S_0 = (\theta_0, \kappa_S, \eta_S, \varphi_{SO})_0^T \qquad (9)$$

Vectors (8) and (9) define the initial conditions for further calculations of flight simulation.

6. Results

All the results presented in this section refer to a PZL „Sokol" helicopter in forward flight and a gross weight 6500 kg, with the control system turned of (bare airframe configuration). Some results of computation are presented in this paper.

The rotor blade stall affects the limiting condition of operation of a helicopter. Stall on a helicopter blade limits the high speed possibilities of the helicopter. This is understandable, when one considers that the retreating blade of the helicopter rotor encounters lower velocities as the forward speed is increased. The retreating blade must produce its portion of the lift, therefore as the velocity decreases with forward speed, the blade angle of attack must be increased. It follows that at some forward speeds the retreating blade will stall. In forward flight the angle of attack distribution along the blade is far from uniform, so that it must be expected that some portion of the blade will stall before rest.

The hump of a helicopter realised with high entry velocity is characterised by some singularities. Usually, when the pilot pulls the stick, the helicopter's angle of attack and normal load factor will increase. The normal load factor should decrease after pushing the stick. This is expected helicopter's behaviour. However, it is observed unexpected motion of the helicopter. When a helicopter has realised the hump with high velocity, the normal load factor can increase, after pushing the stick. This phenomenon can be called as helicopter's hump with „the snatch up". Figures 3-23 show the results of the numerical simulation of such unexpected motion of a helicopter. The swash-plate longitudinal deflection is shown in Fig. 11. As it is shown in this figure, the swash-plate is deflected backward during the first second of control process. Next, it is deflected forward during approximately 1.5 sec., and then remained unchanged during approx. 2 sec. Fig. 12 shows variation of normal load factor corresponding to such control. The normal load factor increases during the first second of motion. This is typical helicopter's reaction. When the swash-plate has deflected backwards, the normal load factor doesn't decrease (as during expected motion). It was increasing at this time. This is significant singularity of motion of a helicopter. This phenomenon can be called as „the snatch up" of the helicopter. Figs 13-21 show, how the angle-of-attack distribution around a rotor disk was changing during this motion. This figures explain physical meaning of this phenomenon. It is explained, that the direct cause of „the snatch up" of a helicopter is loss of effectiveness of control. It is connected with transgression throw critical angles-of-attack on significant rotor disk area. It is observed, that at initial phase of the flight, the stall area and the reverse flow area occurs at 200^0-345^0 angles of a blade azimuth positions (recurring blade position) (Figs. 13, 14). The following turns of the main rotor are shown in Figs. 15-21. Figs. 17-19 shows, that from 10th to 16th turn of the main rotor (i.e. from 2.11 sec. to 3.76 sec. of the simulation) stall area has been increasing very fast. The right and rear side of the main rotor disk area are in stall conditions. It follows that the retreating blade will stall, and appears a strong nose up pitching moment. This phenomenon can explained unexpected behaviour of a helicopter. During this time the swash-plate is deflected forward maximally

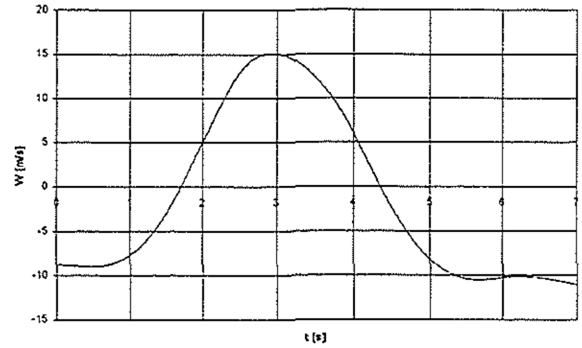

Fig. 3 A hump with „the snatch up" of a helicopter - variation of rate of climb

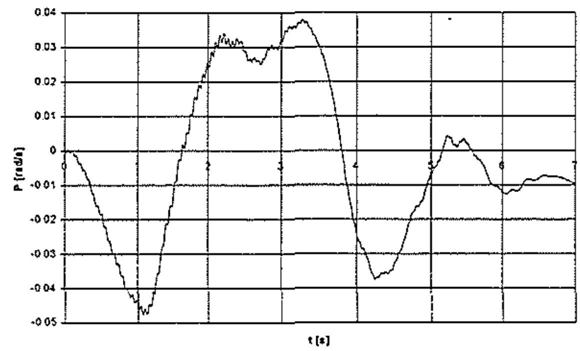

Fig. 4 A hump with „the snatch up" of a helicopter - variation of roll rate

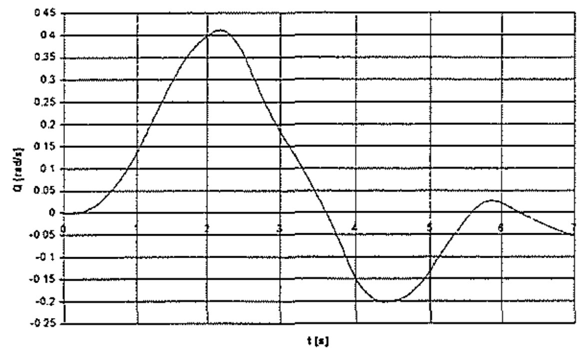

Fig. 5 A hump with „the snatch up" of a helicopter - variation of pitch rate

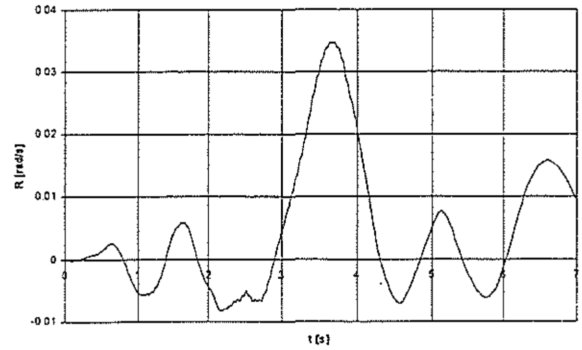

Fig. 6 A hump with „the snatch up" of a helicopter - variation of yaw rate

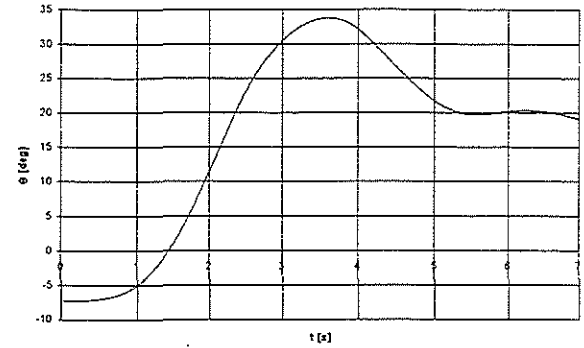

Fig. 7 A hump with „the snatch up" of a helicopter - variation of pitch angle

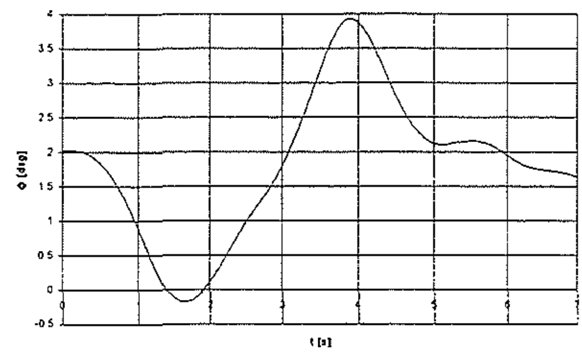

Fig. 8 A hump with „the snatch up" of a helicopter - variation of roll angle

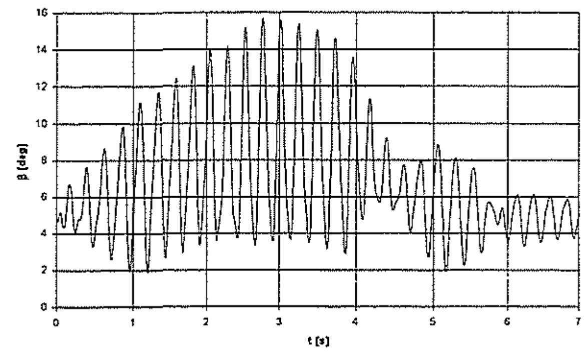

Fig. 9 A hump with „the snatch up" of a helicopter - variation of flap angle

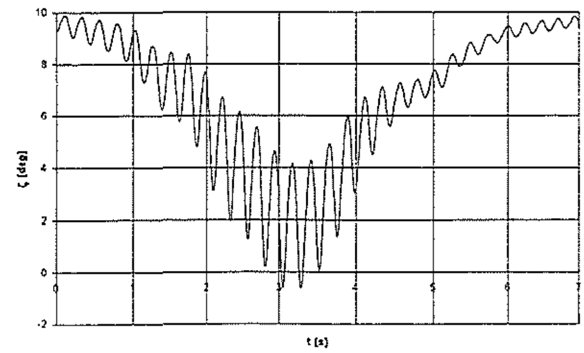

Fig. 10 A hump with „the snatch up" of a helicopter - variation of lag angle

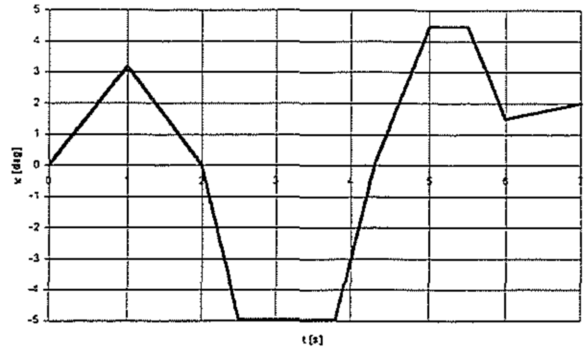

Fig. 11 A hump with „the snatch up" of a helicopter - variation of longitudinal of the swash-plate

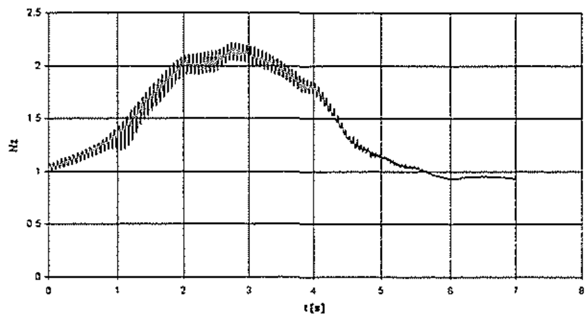

Fig. 12 A hump with „the snatch up" of a helicopter - variation of normal load factor

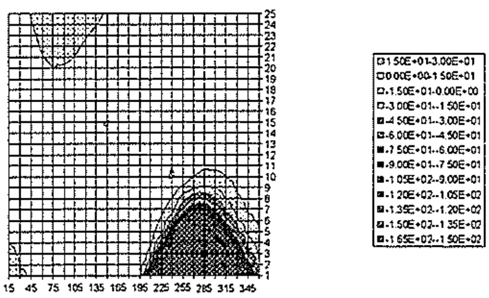

Fig. 13 The angle-of-attack distribution around a rotor disk, first turn of the rotor t=(0-0.235s)

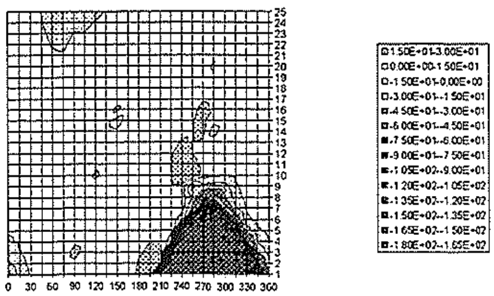

Fig. 14 The angle-of-attack distribution around a rotor disk 4^{th} turn of the rotor t=(o.7-0.94s)

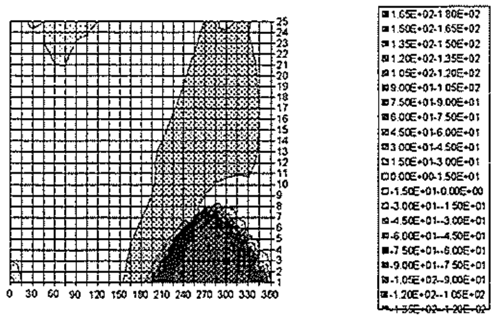

Fig. 15 The angle-of-attack distribution around a rotor disk 6^{th} turn of the rotor t=(1.174-1.41s)

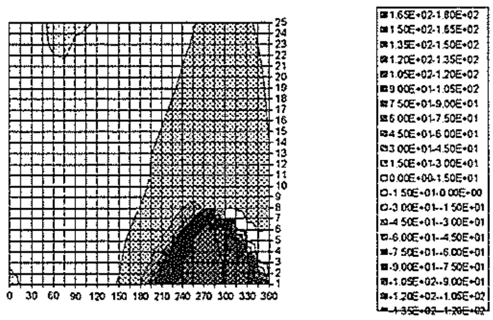

Fig. 16 The angle-of-attack distribution around a rotor disk 8^{th} turn of rotor t=(1.644-1.88s)

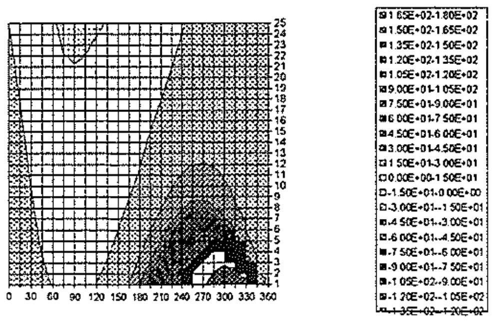

Fig. 17 The angle-of-attack distribution around a rotor disk 10^{th} turn of rotor t=(2.11-2.35s)

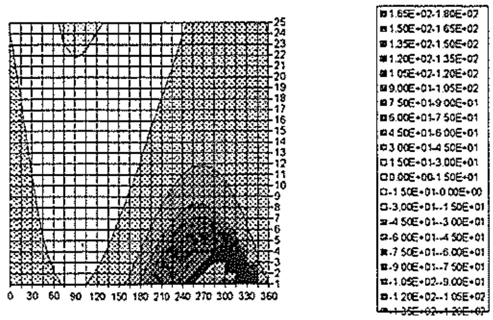

Fig. 18 The angle-of-attack distribution around a rotor disk 14^{th} turn of the rotor t=(3.05-3.29s)

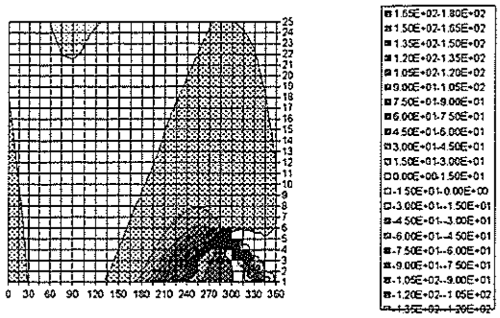

Fig. 19 The angle-of-attack distribution around a rotor disk 16th turn of the rotor t=(3.52-3.76s)

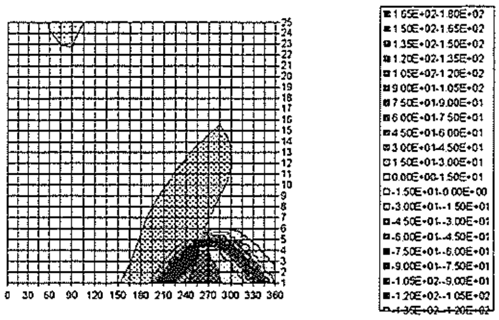

Fig. 20 The angle-of-attack distribution around a rotor disk 18th turn of the rotor t=(4.0-4.235s)

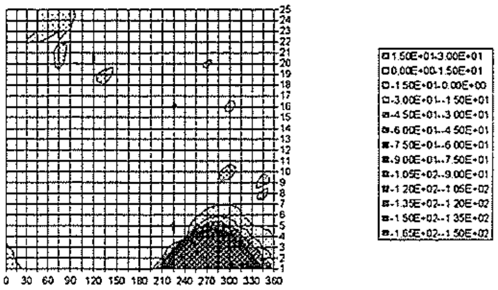

Fig. 21 The angle-of-attack distribution around a rotor disk 20th turn of the rotor t=(4.46-4.7s)

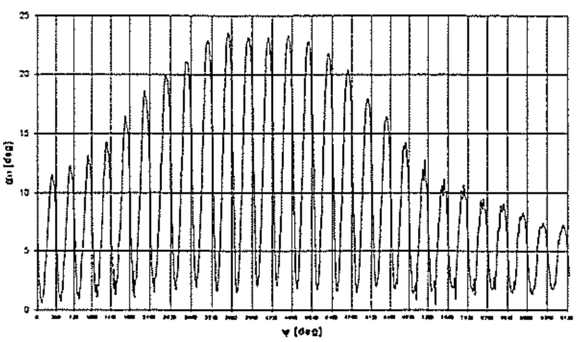

Fig. 22 A hump with „the snatch up" of a helicopter - variation of angle-of -attack of representative rotorblade cross-section

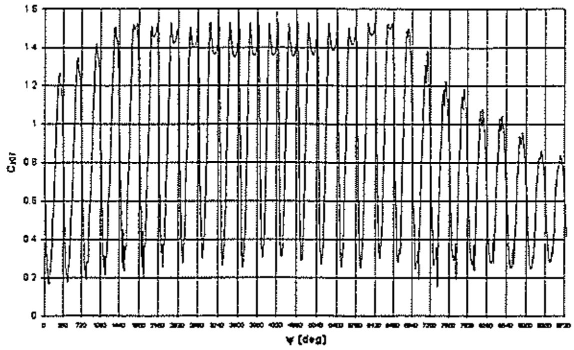

Fig. 23 A hump with „the snatch up" of a helicopter - variation of lift force coefficient C_L on representative rotorblade cross-section

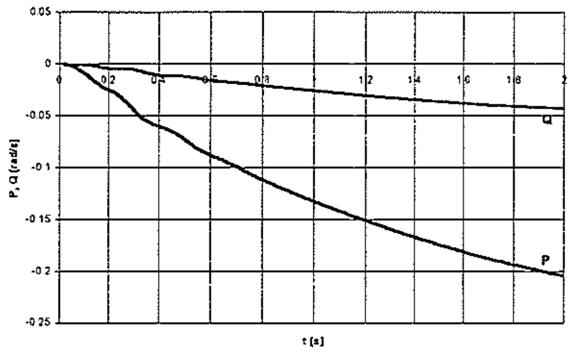

Fig. 24 Port roll, V=28 m/sec, variation of roll and pitch rates

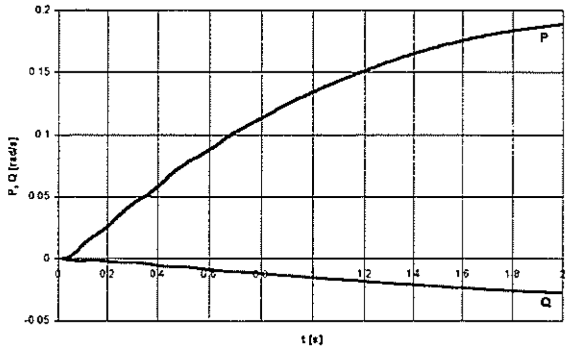

Fig. 25 Starboard roll, V=28 m/sec, variation of roll and pitch rates

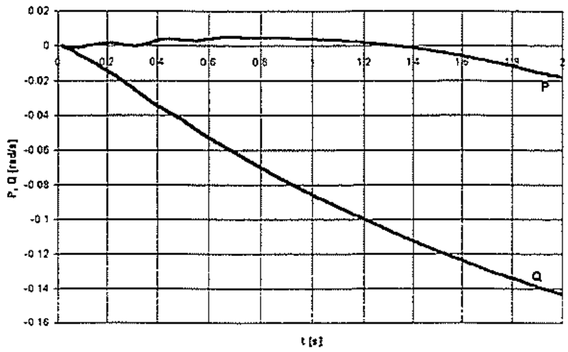

Fig. 26 Pith of the helicopter (nose down), V=28 m/sec, variation of roll and pitch rates

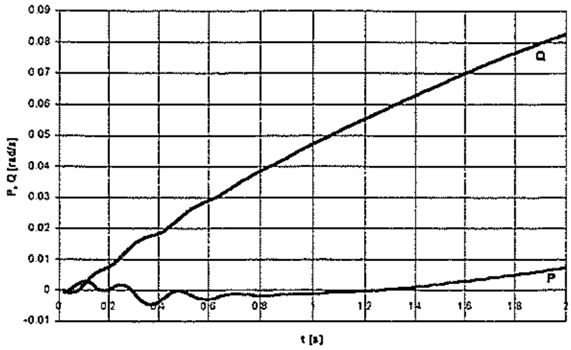

Fig. 27 Pitch of the helicopter (nose up), V=28 m/sec, variation of roll and pitch rates

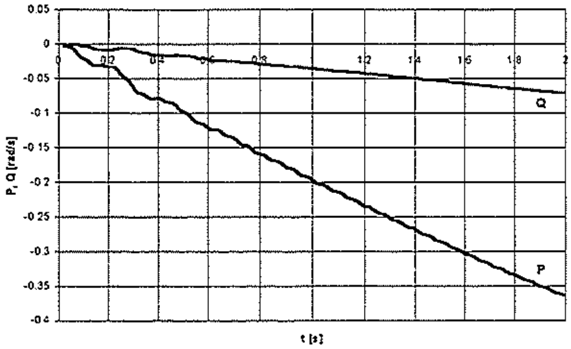

Fig. 28 Port roll, V=84 m/sec, variation of roll and pitch rates

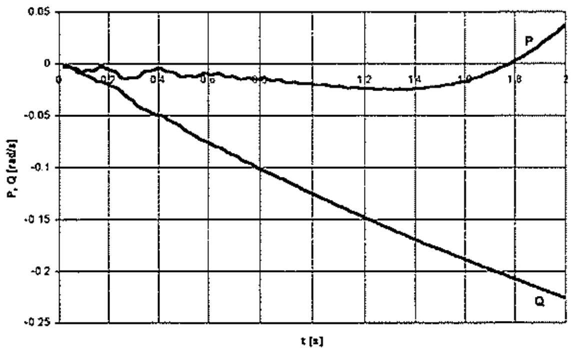

Fig. 29 Pitch of the helicopter (q<0), V=84 m/sec, variation of roll and pitch rates

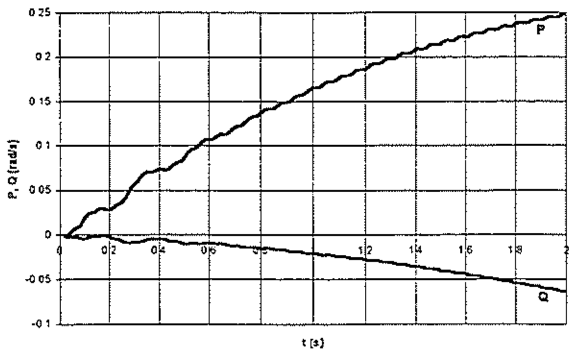

Fig. 30 The reversal roll (starboard displacement of the stick), V=84 m/sec, variation of roll and pitch rates

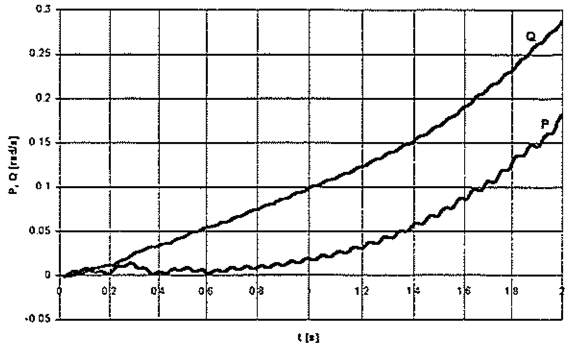

Fig. 31 The reversal roll (port displacement of the stick, V=84 m/sec, variation of roll and pitch rates

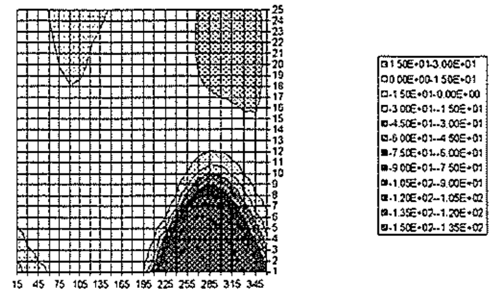

Fig. 32 The angle-of-attack distribution around a rotor disk; the reversal roll(1st turn of rotor)

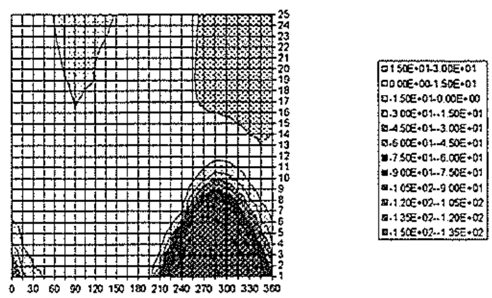

Fig. 33 The angle-of-attack distribution around a rotor disk; the reversal roll (4th turn of rotor)

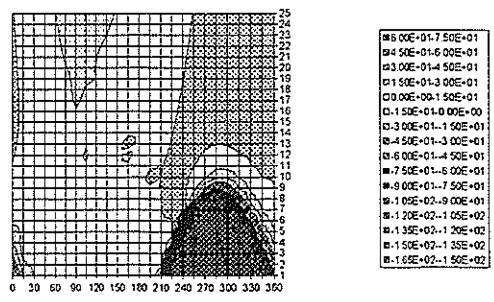

Fig. 34 The angle-of-attack distribution around a rotor disk; the reversal roll (8th turn of rotor)

and collective pitch occurs its maximal value. Figs. 20-21 show 18th and 20th turn of rotor (4-4,7 sec. of simulation). During this time the swash plate is deflected backward. Such control has been decreasing the stall area. The helicopter returns to normal control conditions. Figs 22 and 23 show variation of angle-of-attack, and variation of lift force coefficient on representative rotorblade cross-section as blade azimuth function.

When the airspeed of a helicopter is low, then direction of angular speed of a helicopter will be similar as direction of swash-plate deflection (cf. Figs. 24-27). At high helicopter's speed occurs the phenomenon of unexpected roll of a helicopter on side of retreating rotorblade. It can be called as a „reversal-roll". Figs 32-34 show the physical principle of those phenomenon. If the stick is rolled rapidly, then at high airspeed would have occur the phenomenon of unexpected banking of a helicopter. This banking is resulted in impetuous grow out of the stall and inverse inflow area. Variation of roll and pitch rates and distribution of a angle-of-attack are shown in figs.29, 30, 31, 32. 33, 34.

Conclusions

A comprehensive flight simulation model has been applied to provide numerical investigation of helicopter behaviour at high angles of attack. Piloted simulation was used to evaluate some unexpected helicopter's motion. The results of numerical simulation of helicopter hump witch „snatch up" and "the reversal roll" are presented.

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