WATER IMPACT OF A FILLED TANK

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Abstract

As statistics show, the water impact of a helicopter in emergency is likely to have tragic consequences. In that, the behaviour of the subfloor is fundamental for accident survivability – especially when in the subfloor structure is integrated a tank. In this work, the impact with the ground and with the water of a filled tank integrated in the helicopter subfloor has been numerically studied. Initially, the numerical model has been developed and validated referring to the experimental data collected during a ground impact of the filled tank. Hence, the impact with the water in the same condition has been investigated. Two different approaches have been adopted to model the fluid: the Lagrangian Finite Element and the Arbitrary Lagrangian Eulerian approaches. Advantages and disadvantages of these approaches have been highlighted and the results show that the Arbitrary Lagrangian Eulerian approach gives an effective advantage in describing the events considered. Finally, on the base of the obtained results, the differences between ground and water impact have been discussed.

Keywords

Tank Crashworthiness, Ground Impact, Water impact, Fluid-Structure Interaction, Explicit Finite Element codes, Arbitrary Lagrangian Eulerian approach

Acronyms

FEM	*Finite Elements Method*
ALE	*Arbitrary Lagrangian Eulerian*
EOS	*Equation of State*
FSI	*Fluid-Structure Interaction*

Introduction

As statistics show (Ref 1), the water impact of an aircraft in emergency is likely to have tragic consequences for the occupants. Remarkable losses not only economical but also in human lives justify a new interest in the study of the event.

Up to few years ago, due to the lack of relevant knowledge necessary to develop adequate simulation tools, the approach to the problem was mainly based on experimental tests. These tests are both difficult to perform and expensive – whence the need for numerical tools which support the development of safer structure. Several efforts have been provided and, nowadays, explicit codes based on Finite Element (FE) Method are successfully used to analyse water impact (Ref 4-5). Nevertheless, though the use of these codes have made possible to reduce the number of tests to be performed, the experimental tests are still fundamental to understand the complicate dynamic of a water impact and to validate the numerical models, as well.

During the impact with the water, the structure undergoes a complicated system of forces not easy to be reproduced because these arise from the mutual influence between hydrodynamic loads and response of the structure. Nevertheless, as a significant number of helicopter accidents occur on water, there is an actual need to improve design methodologies to improve helicopter crashworthiness during emergency water impact.

Most of the investigations made in last years with regard to helicopter crashworthiness (Ref 2, 3) have brought as results the publication of crash survival design guides and the development of crashworthiness requirements. Nevertheless, the most of improvements have been done considering ground impact, even if water impact has been shown to be relevant both for civil and military operation.

The dynamic of a water impact causes more severe crash conditions than that of a ground impact at equivalent velocity. When impacting *solid surfaces*, loads distribute through high stiffness structural elements of subfloor designed to absorb impact energy. These elements crush progressively, and lower skin is not involved. When impacting *water surface*, loads distribute differently and lower skin panels are loaded. Their failure prevents stiff elements from crushing and absorbing energy. Often, also internal structures such as fuel tank are exposed to the water and the inrush of that is likely to cause the collapse of these components.

Tank crashworthiness is fundamental for accident survivability. Indeed, the collapse of the fuel-system, as a consequence of the impact with the ground of a helicopter in emergency, is the main cause of death among the cabin crew and, therefore, the design of fuel-system undergoes severe rules, military (MIL) and civil (such as FAR or JAR/CE). Accordingly, in this work, the impact with the *ground* and the *water* of a filled tank integrated in the helicopter subfloor has been numerically investigated with particular regard to the interaction between the structure and the fluid, using LSTC LS-Dyna, release 960 (Ref 6, 7).

At first, a FE numerical model of the tank was developed and validated referring to the experimental data collected during ground impact tests (Ref 8). These tests were carried out with the simulacra of the tank integrated in the subfloor of a helicopter that is currently produced and commercialised. Then, the validated tank model has been used to simulate the impact with the water.

Two different approaches were used to model the fuel inside the tank and the impacted water surface: the Lagrangian FE and the Arbitrary Lagrangian Eulerian (ALE) approaches.

The results obtained were evaluated comparing them among themselves and advantages and disadvantages of the two approaches have been discussed. Furthermore, differences between a ground and a water impact have been highlighted with particular regard to the behaviour of the structure in the two different cases.

Experimental tests

Ground impact tests were performed in 1998 at the *Dipartimento di Ingegneria Aerospaziale* (Department of Aerospace Engineering), of the *Politecnico di Milano* (Ref 8).

Simulacra of the tank integrated in the subfloor of a (small-size) helicopter actually produced and commercialised were made following the design specifications of the manufacturer (Fig 1).

The tank was 670 *mm* in width, 638 *mm* in length, 300 *mm* in height and was able to contain up to 80 *kg* of fuel. The structure was realised with Aluminium-Alloy 2024 T42 panels jointed one with each other with blind rivets. The panels had different thickness (Table 1) depending on their position in the structure of the helicopter subfloor. On each side of the tank, were also riveted three vertical L-stiffeners 8 *mm* thickness made in the same aluminium-alloy of the tank.

A drop-tower was built on purpose to perform these tests and the tanks were dropped from a height of 7.5 *m* in order to obtain an impact velocity of about 12 *m/s*.

A massive *lifting system* (the weight of lifting system was about 17 *kg*) was manufactured to facilitate lifting during the tests by welding four steel C-bars one with another. The weight of the lifting system was not representative of the weight of the part of helicopter above the test article. Nevertheless, its high stiffness and weight made necessary to model the lifting system in detail when simulating the event.

A number of eight tests were performed using two different levels of the fluid inside the tank: 50% and 80% of the maximum fuel level.

The data collected during the tests consisted of:

- the accelerations in correspondence of the four upper corners of the tank,
- the photographic documentation of deformations after the impact, and
- the photos of the tank taken just before the impact from two opposite side using two high-speed cameras.

The accelerations on the corner that for first impacts the ground, in particular, were considered to evaluate *quantitatively* the results obtained after numerical simulations. The photos of the tank after the impact were used to have a further *qualitative* comparison between experimental tests and numerical simulations.

As a remark on tests, it is worth noticing that the simulacra during the fall were not restrained. The oscillations during the fall conditioned the impact angles, and hence the severity of the impact and the motion of the fluid inside the tank. Accordingly, the measured accelerations are slightly different in peaks and in the time profiles. The numerical model was developed referring to all the different impact condition documented, though, here, are presented only the results obtained for one of those.

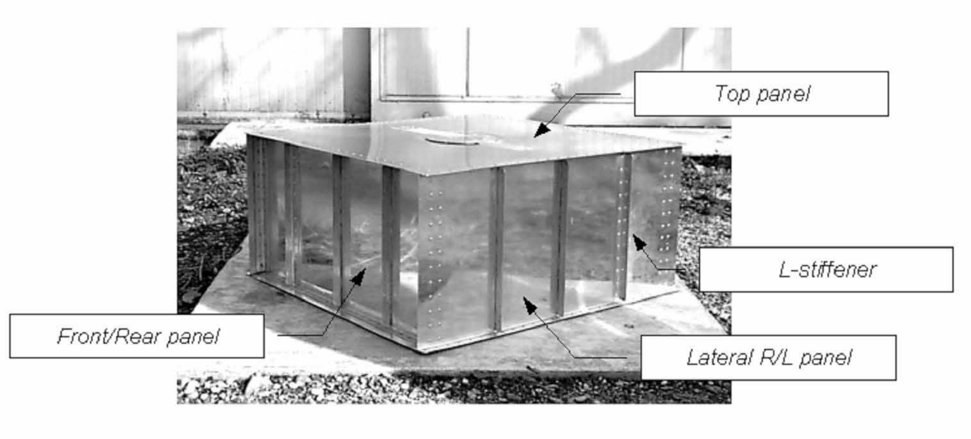


Figure 1 – A photo of the tank used in the tests.

Table 1 – Features of the tank simulacra. Measures are in millimetres (Ref 8).

Part of the tank	Length	Width	Thickness
Top panel and plug	670	638	0.51
Bottom panel	638	670	1.02
Front and rear panels	336	595	1.02
Lateral panels	336	670	0.81
Front and rear panels L-stiffeners	291	38	1.60
Lateral panels L-stiffeners	193	40	0.80

Numerical model

Referring to the experimental data, a numerical model of tank filled with water was developed and validated.

Two different approaches were adopted to model the fluid inside the tank: Lagrangian FE and ALE.

FE model of the tank

The original geometry of the tank was already simple (Fig 1). However, it was further simplified in order to built a rather regular FE mesh (Fig 2) and hence focus the attention on the interaction between the structure and the fluid (FSI). Indeed, the characteristic length of the element was a compromise between the need to properly reproduce the buckling of the lateral panels of the tank and the need to have a regular but relative coarse mesh to reduce the required CPU-time. Eventually, the model consisted of 13464 four-nodes shell elements, having a reference length of 10.0 *mm*.

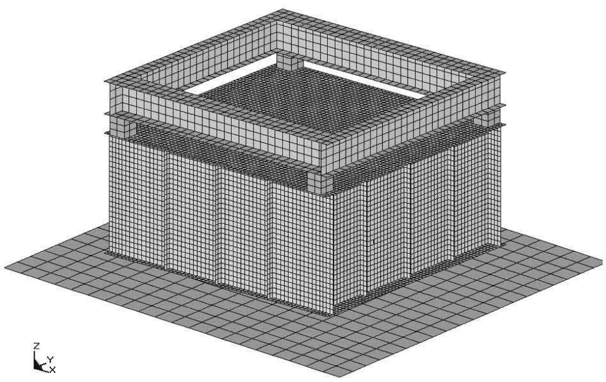


Figure 2 – Finite Element model of tank and lifting-system.

The piecewise linear plasticity material model (*MAT_24 in Ref 6, 7) was used to reproduce the mechanical behaviour of the Aluminium alloy used in manufacturing the tank.

As a further simplification, the presence of the rivets was ignored. Indeed, the benefits of modelling in detail the junction were considered not sufficient to justify the increase in model complexity and in required CPU time. Furthermore, as observed in the tests, the number of collapsed rivets after the impact was negligible.

Particular attention was paid to initial and boundary conditions such as impact velocities and tank incidence (i.e. impact angles). These showed to be fundamental in order to find a good agreement between experimental data and numerical results.

Furthermore, as the lifting system was observed to have a deep influence on the dynamic of the impact because of its stiffness and weight, a detailed FE model of that was eventually realized.

Model of the water inside the tank

For the water inside the tank two different approaches were considered: the Lagrangian FE and the Eulerian/ALE approach.

Lagrangian FE model. The Lagrangian FE approach is customary for the Continuum Mechanic (Ref 9).

The Lagrangian approach is extremely efficient when considering nonlinear problems though it has its weak point in the excessive mesh distortions, which are usual in events featuring soft-bodies or fluid-like materials. In fact, adopting the Lagrangian FE approach, the FE mesh is constructed on the material. Hence, when the material undergoes large distortions, also the mesh undergoes the same large distortions, which are likely to cause an unacceptable loss in accuracy, a considerable increase in required CPU time, and sometimes, a premature analysis termination.

The FE model of the water used in the simulation consisted of 6300 eight-nodes solid elements, which had characteristic length of 20 *mm*.
Furthermore, an hourglass control was introduced and properly calibrated.
A particular attention was paid to the choice of the material model. In order to accurately reproduce the main features of the fluid inside the tank, several simulations were performed considering different Constitutive Laws. In particular, the following material model were considered:

- the *elastic-fluid material model* (*MAT_1 in Ref 6, 7)
- the *elastic plastic hydrodynamic material model* (*MAT_10 in Ref 6, 7)
- the *null material model* (*MAT_9 in Ref 6, 7), i.e. a material characterised by the absence of deviatoric stresses featured with the customary polynomial Equation of State (*EOS_LINEAR_POLYNOMIAL in Ref 6, 7) and the Mie-Grüneisen Equation of State (*EOS_GRUNEINSEN in Ref 6, 7)

For each one of these models, different values of the parameters were tried and the results compared with the experimental data. Eventually, it was concluded that the null material associated with the polynomial Equation of State is the paramount compromise between the total CPU-time required for the simulation and the accuracy of the solution.
The FSI was defined via Contact Algorithm (Ref 6, 7). In particular, it was defined a bidirectional contact based on the penalty method approach, which consists of placing normal interface springs between all penetrating nodes and the contact surface. Springs stiffness is defined on the nodal mass of the part in contact and on the time step. This algorithm is particularly suitable to treat the contact between parts characterised by materials with very different mechanical properties.
ALE model. The ALE approach is originally meant to combine the advantages of Lagrangian and Eulerian approaches (Ref 9). Following a Eulerian approach, the material flows through a mesh fixed in the space. The ALE approach differs from this in that the Eulerian mesh can move arbitrarily versus the material. Consequently, it has an advantage over the pure Eulerian approach when the motion of the material covers a wide region of the space. In this case, the number of elements using the Eulerian approach would have to be so large to maintain a reasonable accuracy in the calculation that the CPU time required for the analysis would be unacceptable. Moreover, despite the FE approach, the ALE approach has the capability of treating problems characterized by high deformations, being the elements distortion limited by the mesh motion versus the material.
With regard to the considered problem, not only the water at the initial instant, but also a small surrounding region was modelled to avoid flowing of the water material out the Eulerian mesh when the velocity of the material were bigger than the *expansion* and *translation* velocity of the Eulerian mesh.
The Eulerian mesh of fluid region consisted of 12716 hexahedral elements: the mesh was then imposed to move following the mass weighted average velocity (Ref 6, 7).
In order to characterise the water behaviour, the same material model and EOS used for the Lagrangian FE model were adopted.
The FSI was defined via Coupling Algorithm (Ref 6, 7). In particular, the nodes of the tank Lagrangian mesh were imposed to have the velocity and acceleration equal to those of the points of the water surface with which are in

contact, only in direction normal to the surface itself.

Ground impact

Using the described numerical model, ground impact simulations were performed referring to a test in which the tank was half filled up with the fluid.
The results obtained using different models for the fluid inside the tank were initially compared with the data collected during the drop test and hence among themselves to highlight advantages and disadvantages of the different approach to the fluid modelisation.

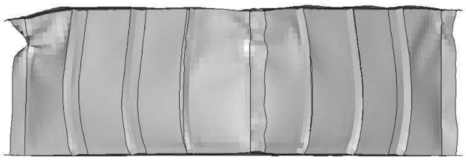
Numerical-experimental correlation

The comparison with the experimental data guided the development of the numerical model in the definition of the initial and boundary conditions (impact velocity and incidence of the tank) and, above all, in the definition of the interaction between the fluid and the tank structure.
A first comparison between numerical results and test data was made referring the tank deformation after the impact (Fig 3, 4a and 4b).
Three parameters were defined:

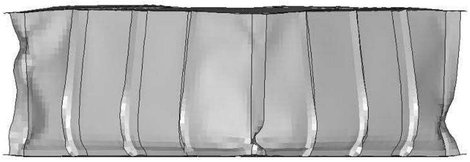
- the *maximum longitudinal displacement* measured between the front and the rear panel,
- the *maximum lateral displacement* measured between the two lateral panels, and
- the maximum *vertical crushing* of the corner that first impacts with the ground.

As reported in Table 2, the vertical crushing obtained with the ALE model is very close to the one measured after the drop test, while the value obtained with the FE model is somewhat higher. On the contrary, with regard to the longitudinal and lateral displacements, the FE model showed a good agreement with the experimental data, while the ALE model led to somewhat smaller values.

Figure 3 – Photographical documentation after impact.



a – Lagrangian Finite Element model



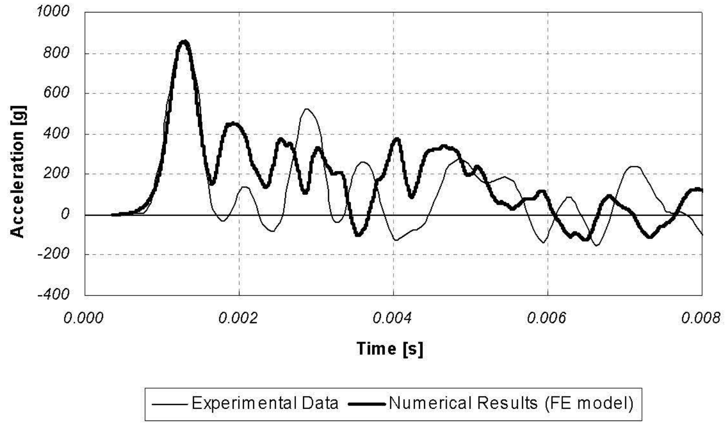
b – Arbitrary Lagrangian Eulerian model

Figure 4 – Numerical results using two different models for the fluid inside the tank.

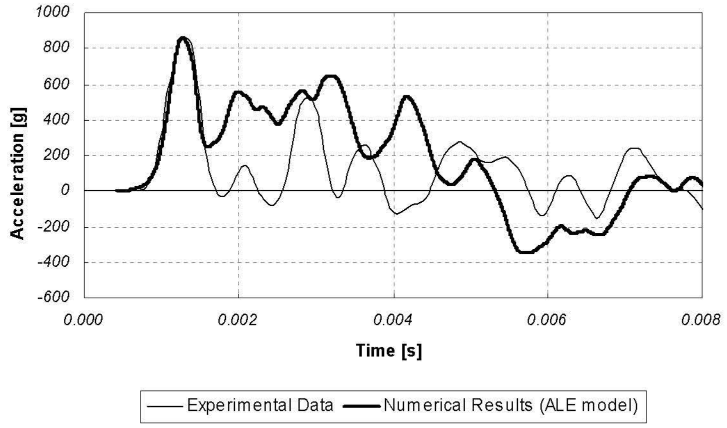
These occurrences have a justification in the definition of the FSI. When performing a hybrid Lagrangian/Eulerian analyses, the coupling between different solvers, the Lagrangian and the Eulerian solver, leads to underestimate the entity of the interaction between fluid and structure. As a results the loads are smaller with respect to those calculated using a fully Lagrangian approach. Since the crushing of the tank basically depends on the shove of the water on the lateral panel, the results reported in Table 3 are not surprising.

As a further comparison, the vertical acceleration in correspondence of the upper corner that first impacted with the ground was considered. Indeed, the vertical deceleration is the most important parameter to evaluate the crashworthiness of the subfloor in which the tank is integrated.

As shown in Fig 3, independently from the model of fluid, the maximum values of the vertical acceleration are close to the first peak measured during the tests. After the first peak, the experimental and numerical curves follow slightly different paths. These disagreements, which are imputable to the sloshing of the fluid inside the tank, are of the order of the approximation and, therefore, definitively acceptable.



a – Lagrangian Finite Element model



b – Arbitrary Lagrangian Eulerian model

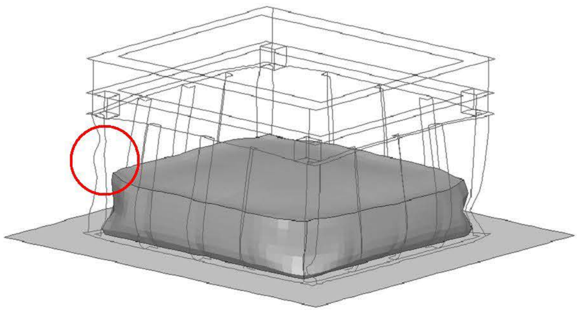
Figure 5 – Numerical-experimental correlation.

Table 2 – Deformation of the tank after ground impact (in millimetres).

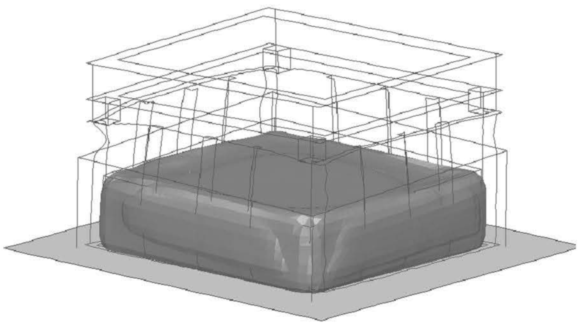
	Longitudinal displacement	Lateral displacement	Vertical crushing
Measured	62	59	14
Lagrangian Finite Element	54	52	19
Arbitrary Lagrangian Eulerian	35	34	13

Remarks on numerical results

Considering the graphical output (Fig 6), it is somewhat evident that the FE mesh of the fluid underwent large distortions (Fig 6a), which caused a drop in the time-step – thought all the simulations performed reached a normal termination. For this reason, it is not wrong to conclude that this model is suitable to reproduce the flow of the fluid only in the early stages of the impact. On the opposite, the ALE model (Fig 6b) led to a *qualitatively* accurate description of the sloshing of the fluid, which could be further improved by defining a larger number of elements. Nevertheless, due to features of the ALE solver implemented in the code used in the analyses, the flow of the fluid was in some respects closer to that of a jelly body than to that of a fluid.
The main difference observed between the two models was in the deformations of the tank: rather smaller when using the ALE model.



a – Lagrangian Finite Element model



b – Arbitrary Lagrangian Eulerian model

Figure 6 – Ground impact at t = $8 \cdot 10^{-3}$ *s*.

The CPU time required for the different models was also considered. Using a Pentium 4 1700 MHz CPU and 256 RAM PC, the CPU-time for 9 *ms* real-time simulation ranged from about 12 *min*, when the FE fluid model was adopted, to a maximum of about 23 *min* for the ALE fluid model.

Water impact

As a significant number of helicopter accidents occur on water, there is an actual need to develop methodologies to improve helicopter design with regard to water impact. Thus, one of the aims of this work has been to numerically investigate the severity of a water impact: whether the onset rate of the acceleration pulse is greater in water than onto a hard surface, and whether a structure designed for hard surface impact behaves differently for water impact.
The feasibility of analysing the water impact with nonlinear FE codes was considered. In particular, simulations reproducing the impact of a filled integrated tank onto a water surface were performed. In these simulations, it was used the previously validated models of the tank and, for the impacted water surface, a numerical model was developed taking advantage from the experience collected in modelling the fluid inside the tank.
The impact scenario (impact velocity and tank incidence) was the same.

Numerical model of the impacted water surface

Two different approaches were adopted to model the water surface impacted by the tank: the Lagrangian FE and the ALE approach.
The model of the water impacted by the tank was basically obtained considering the three following aspects.

- It was used the smallest volume of water that allowed to correctly reproduce the event without requiring excessive computational efforts.
- Proper outflow boundary conditions were adopted to avoid that the reflected shock waves interfering with the impact caused misleading results.
- The dimensions of the fluid region was fixed so that the mass of the fluid were such to avoid rigid displacements of the mesh of the water and, hence, *unrealistic* transfer of energy from the tank to the water.

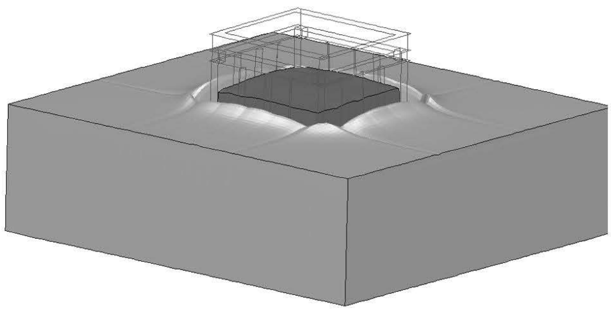
Accordingly, the geometry of the water consists of a box (a parallelepiped) 1914 *mm* in width and length, 600 *mm* in height – to which corresponds a volume of about 2200 *litres*.
In both the cases, the same material model already used for the fluid inside the tank was adopted.

FE model. The Lagrangian FE mesh of water consisted of 219389 hexahedral solid elements.
The mesh was locally made finer in the centre to obtain the necessary accuracy avoiding excessive increases computational efforts. In particular, in the impact region the characteristic length of the elements was chosen as a compromise between accuracy in fluid-structure interaction and required computational effort.
As previously, the FSI was defined via Contact Algorithm.

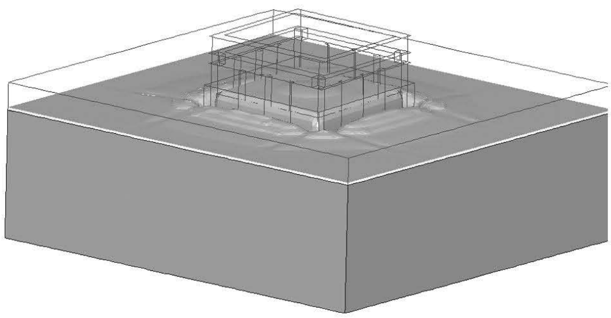
ALE model. As previously mentioned, following an ALE approach, the material flows through a mesh fixed in the space. Consequently, it was necessary to model not only the water domain at the initial instant, but also a region surrounding.
The mesh of the impacted water surface consisted of 279909 hexahedral solid elements. Furthermore, for that, a fixed mesh was defined adopting an Eulerian reference system type (Ref 6, 7), which force the ALE mesh to stay fixed in the space.
As previously, the FSI was defined via Coupling Algorithm.

Numerical result and remarks

The results obtained with the two different approaches were initially compared *qualitatively* and *quantitatively* among themselves.
With regard to the graphical appearance (Fig 7), the two different models provided evidences somewhat similar – though, when using an ALE model of the fluid, the results are closer to common experience.



a – Lagrangian Finite Element model



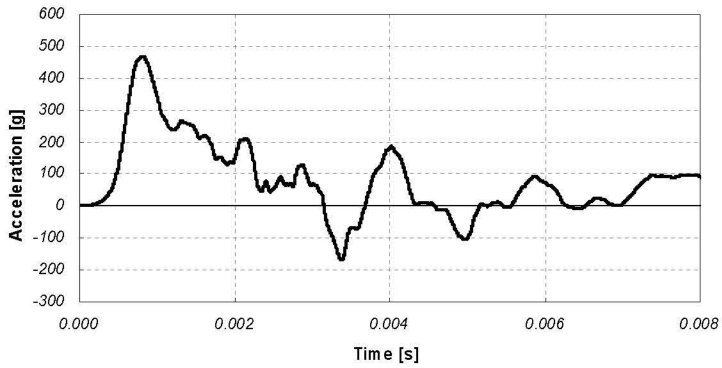
b – Arbitrary Lagrangian Eulerian model

Figure 7 – Water impact using two different fluid models after 15.0 ms real-time simulation.

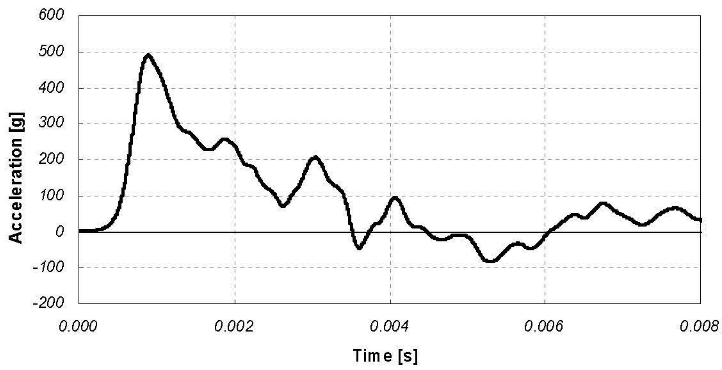
Considering the vertical deceleration in correspondence of the corner that first impacted with the water (Fig 5 and 6), the maximum and the mean values numerically obtained with the two different models are matching. Nevertheless, with regard to these results, it is worth noticing that the simulation performed using the ALE approach for the fluids provided less severe decelerations and that was due to the already mentioned problems arising from coupling between different solvers.

Also when considering the longitudinal and lateral displacement or the vertical crushing obtained with the two different models are close one to the other – as evident in Table 3.

Using a Pentium 4 – 1700 *MHz* CPU, 256 RAM PC, the CPU time required for 9.0 *ms* real-time water impact simulation ranged from two hours when using a Lagrangian FE approach also for the fluid regions, to four hours when using a ALE approach.



a – Lagrangian Finite Element model



b – Arbitrary Lagrangian Eulerian model

Figure 5 – Vertical acceleration during a water impact.

Table 3 – Deformation of the tank after the water impact (in millimetres).

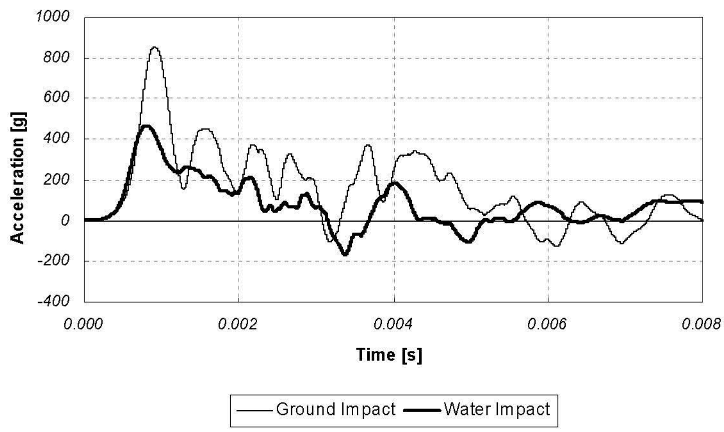
	Longitudinal displacement	Lateral displacement	Vertical crushing
Lagrangian Finite Element	18	16	0.7
Arbitrary Lagrangian Eulerian	15	14	0.6

The difference in required CPU-time was not so relevant because the distortions in the Lagrangian mesh caused a progressive reduction in the stable time-step that partly compensate the higher time-per-cycle of the ALE approach.

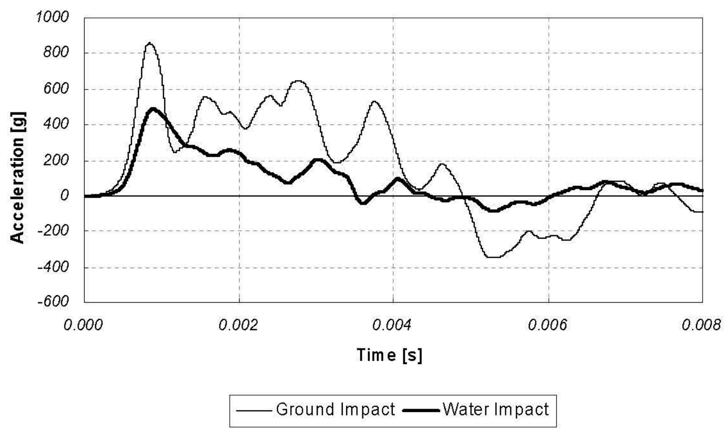
Ground and water impact

Although only referring to the results of numerical simulations, eventually, the basic differences between ground and water impact were highlighted.
In particular, the numerical results have further showed that the decelerations when impacting the ground are higher. Indeed, the water has a relevant role in absorbing the impact energy.

In the first instants of the impact, in fact, the energy transferred to the water is of the same order as the energy absorbed by the structure.
Differently from a rigid impact surface, which does not absorb any of the kinetic energy, the water absorbed the major portion of the kinetic energy of the test article. Furthermore, since water is modelled as an incompressible fluid, the collision involves momentum transfer from the tank to the water that results in a large spray of the water.
As a consequence referring to a water impact the loads on the structure are smaller and therefore the deformations and the energy absorbed by the structure are smaller.



a – Lagrangian model

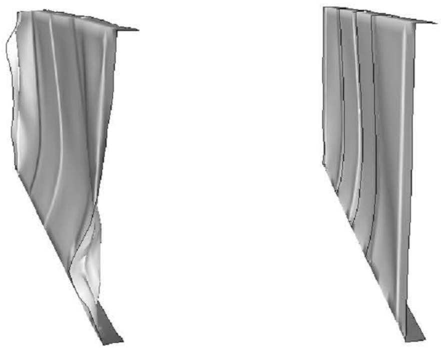


b – Arbitrary Lagrangian Eulerian model

Figure 6 – Vertical acceleration during a *ground* and a *water* impact.

In particular, with regard to the results here obtained, it was observed that the vertical panels and the stiffeners absorbed most of the impact energy progressively collapsing during a ground impact (Fig 6a), but were almost undeformed after a water impact (Fig 6b).

This occurrence is showed in Fig 6 with regard to the results obtained using a Lagrangian FE model of the fluid inside the tank and of impacted water surface.



(a) Ground impact *(b) Water impact*

Figure 6 – After-impact crushing of the lateral panel of the tank (detail).

As a final remark, it is worth noticing that the results shown were obtained considering the simulacrum of a tank and, hence, were not directly applicable to the assembly subfloor/tank/cabin. In fact, as known (Ref 1), the behaviour of the single sub-parts is not representative of the behaviour of the assembly of the sub-parts. Indeed, the study of the assembly is future development of this research.

Conclusions

As a water impact is likely to have tragic consequences, it is important to develop numerical tool that could support the design of structures safer with respect to this event.

An attempt to numerically analyse the complicated sequence of events that characterise a water impact has been made in this research. In particular, the impact behaviour of a filled tank integrated in a helicopter subfloor has been investigated.

The model of the filled tank was initially developed and validated referring to the experimental data collected during a ground impact. Hence, a water impact with the same features was simulated.

Two different numerical models were considered for the water inside and outside the tank: Lagrangian FE and ALE.

As a result, it has been shown that modelling the fluid using an ALE approach allows avoiding those troubles arising from the distortion of the mesh typical of fully Lagrangian approach. Furthermore, the ALE model reproduces the motion of the fluids in a manner closer to common experience. On the other hand, typical limitations of the ALE approach such as the lack of sharp boundary and the diffusivity are not particularly strict in this case and, therefore, the ALE approach seems definitively recommendable for water impact analysis.

With regard to the impact scenario, the numerical results showed that the water has a relevant role in absorbing the kinetic energy of the tank. In fact, in the first instants of the impact the energy transferred to the water is of the same order as the energy absorbed by the structure. As a consequence, the impact behaviour of a structure is completely different when considering a ground or water impact.

Instances of this occurrence, such us the different deformations of the structure and the deceleration profile, have been shown in this research. However, further investigations also supported by specific experimental tests are necessary to deepen the knowledge of the event.

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