IMPROVEMENT OF HELICOPTER ROBUSTNESS AND PERFORMANCE CONTROL LAW USING EIGENSTRUCTURE TECHNIQUES AND H_∞ SYNTHESIS

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Abstract

This paper deals with the design of a robust helicopter control law. A two-loop structured feedback is proposed. The first loop is static and computed using eigenstructure assignment. Its objective is to provide some decoupling between the different axes. The second loop is designed using H_∞ synthesis, and tends to zero as frequencies tend to infinity. The objective of the outer loop is to improve performance in terms of minimizing the error between the reference and output signals and some robustness against additive perturbations due to plant uncertainties. This procedure allows the compensator order to be reduced with respect to more classically derived H_∞ solutions.

1 Problem presentation

We are concerned with the design of a robust control law for improving the handling qualities of a helicopter using a combination of eigenstructure assignment and H_∞ synthesis. The proposed controller has to achieve some performance specification in terms of input / output response properties and some robustness requirements because the aircraft eight state linearized model we consider is highly uncertain. High frequency dynamic effects arising, for example, from the rotor dynamics and flutter modes have been neglected in the nominal linear plant as they are difficult to modelize. The nominal rigid body linear model description takes the following state space form :

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (1)$$

The state variables, inputs and outputs are described in tables 1 and 2

Table 1 State description

State	Description
u	Forward velocity
vz	Vertical velocity
q	Pitch rate
θ	Pitch angle
v	Lateral velocity
p	Roll rate
r	Yaw rate
ϕ	Roll angle

Table 2 Input output description

Output	Input	Description
q	θ_2	Longitudinal cyclic
θ	θ_1	Lateral cyclic
p	θ_r	Tail rotor collective
r		
ϕ		

The purpose of the H_∞ synthesis design is to find a dynamic controller which internally stabilizes the system and minimizes the H_∞ norm of a weighted transfer function matrix denoted by $F_l(P(s), K(s))$ (s is the Laplace transform). The optimization problem can be written as :

$$\min_{K_{stabilizing}} \quad \|F_l(P(s), K(s))\|_\infty \quad (2)$$

$F_l(P(s), K(s))$ characterizes the desired performance and/or robustness specifications. Its inputs are the (considered) errors and its outputs are the signals to be minimized. $P(s)$ is the plant and $K(s)$ the dynamic controller to be designed. Unfortunately, despite all the studies done on the subject of H_∞ synthesis, the achievable compensator order is generally two or three times the plant order. This is a major drawback of the H_∞ approach, especially for physical implementation. In order to cope with such a disadvantage, we introduce an inner constant gain loop to produce some decoupling between the three elementary axes which correspond to forward, lateral and yaw motions. Considered then as sub-systems, these motions are further simplified by taking into account only the dominant state variables (see figure 1)

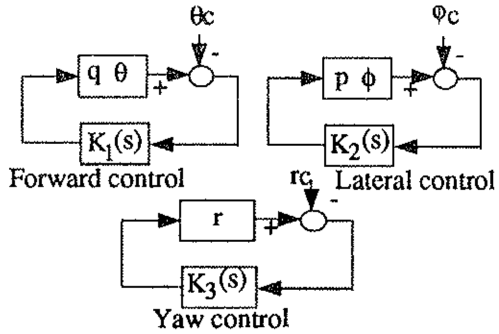


Figure 1 H_∞ decoupled single input single output control schemes

Note that the state vz is not considered here in order to simplify the design procedure and can be assumed to be controlled by the collective pitch input θ_0. An H_∞ controller is synthesized for each sub-system by considering all the desired specifications with the additional constraint that the H_∞ controller direct transmission equals zero to not disturb the inner static loop. Consequently, the outer loop compensator has the following diagonal structure:

$$K(s) = \begin{pmatrix} K_1(s) & 0 & 0 \\ 0 & K_2(s) & 0 \\ 0 & 0 & K_3(s) \end{pmatrix} \quad (3)$$

Therefore, the proposed compensated system has the structure shown in figure 2 :

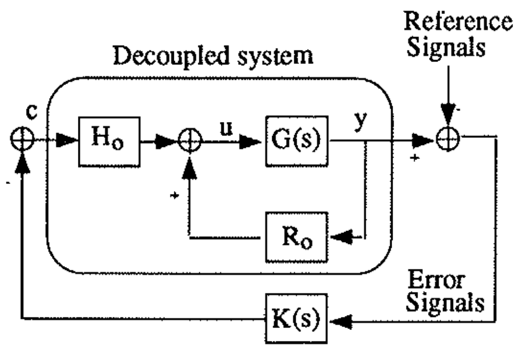


Figure 2
The two-feedback loop controller structure
$G(s)$: Helicopter linear model
H_0, R_0: Static controllers computed with eigenstructure technique
$K(s)$: H_∞ compensator

The paper is organized as follows. In section 2, we present the eigenstructure assignment method used to obtain some decoupling. To assess the decoupled structure in terms of diagonal dominance, the Gershgorin bands are introduced. Section 3 deals with the H_∞ synthesis applied to each sub-system. In section 4 we present the significant results of the overall closed-loop system design. Gershgorin bands are plotted and singular values of each sub-system are drawn in order to justify the simplifications made. The comparison of singular values plots by considering first, the inner loop with additive perturbation on the helicopter model and secondly the overall system with the same type of perturbation but considered on the inner loop, highlights the effect of introducing the H_∞ loop. To assess robustness against plant parameter variations, simulations in the time domain with the twelve states model are illustrated. All the results obtained show that the simplifications made are valid for an application. This permits the compensator order to be "only" eleven. Note that the idea of the two loop structured compensator is essentially based on physical insight.

2 Eigenstructure assignment

This section is concerned with the application of eigenstructure assignment in order to achieve diagonal dominance between the different inputs and ouputs. The objective is to design a static output feedback control law (see figure 2) of the form :

$$u = H_0 c + R_0 y \quad (4)$$

which achieves partial internal stabilization and decoupling between c and y. The helicopter's outputs are then given by :

$$y = C\left(sI_d - (A + BR_0C)\right)^{-1} BH_0 c \quad (5)$$

Now consider the right eigenvector matrix V associated with the corresponding spectral matrix Λ of $A + BR_0C$. We have $(A + BR_0C)\, v_i = \lambda_i v_i$ which can be written as

$$\begin{cases} (A - \lambda_i I_d)\, v_i + Bw_i = 0 \\ w_i = R_0 C v_i \end{cases} \quad (6)$$

It follows from these equations that, in order to assign a pair (λ_i, v_i) , the vector $[v_i^T \; w_i^T]^T$ must satisfy

$$\begin{bmatrix} A - \lambda_i I_d & B \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} = 0 \quad (7)$$

Under controllability and observability assumptions, only $max(dim(y)dim(c))$ eigenvalues and eigenvectors can be assigned because an output feedback is considered [1] .

If $dim(y) \geq dim(c)$, the algorithm to design the feedback is the following [2]

1. Choose dim(y) eigenvalues $\{\lambda_1, \lambda_2 ... \lambda_p\}$

2. Choose dim(y) eigenvectors $\{v_1, v_2 ... v_p\}$ and $\{w_1, w_2, ... w_p\}$ satisfying equation (7).
3. Find R_0 given by

$$R_0 = [w_1, w_2 ... w_p] (C [v_1, v_2 ... v_p])^{-1} \quad (8)$$

The static output feedback R_0 must be designed in such a way as to achieve some decoupling of the three elementary axes (see table 3). This can be done by a suitable choice of right eigenvectors [3], [4]

Relation (5) can be expanded in modal form as :

$$y = \left(\frac{P_1}{(s - \lambda_1)} + ... \frac{P_n}{(s - \lambda_n)} \right) c \quad (9)$$

where n is the number of states. The residue matrix P_k is given by

$$P_k = C v_k u_k^* B H_0 \quad (10)$$

Hence, one can eliminate the mode λ_k from a transfer function (j^{th} input, i^{th} output) by setting $(P_k)_{i,j} = 0$, or similarly

$$(U B H_0)_{k,j} = 0 \quad (11)$$

This relation permits the computation of the static matrix H_0.

The three elementary axes to be decoupled are given by the following distribution of states

Table 3

Forward axis	u q θ
Lateral axis	v p φ
Yaw axis	r p φ

In order to obtain low feedback gains and robust dominance properties with respect to the flight condition, the five closed-loop modes have been selected to stabilize and improve the handling qualities (eg damping) of the open-loop system [5].

To synthetize the three H_∞ controllers, only the dominant variables have been considered (see figure 1). The velocities u and v have been left out because they have a very low frequency effect which can be neglected with regard to the second order sub-systems described by (q, θ) for forward flight and (p, ϕ) for lateral flight, respectively. Moreover, p and ϕ have been left out from the yaw axis because their effects are negligible (see figure 10). Since θ, ϕ and r are considered as the outputs of each sub-system corresponding respectively to forward, lateral and yaw motions, the number of inputs is the same as the number of outputs. We can also examine the diagonal dominance properties of the system using the approach described by Rosenbrock [6].

The rational $m \times m$ transfer function matrix $Z(s)$ is said to be *diagonally row dominant* on the contour D if z_{ii} has no pole on D, for $i = 1, 2, ... m$ and

$$\forall s \in D, \ |z_{ii}(s)| - \sum_{j=1/j \neq i}^{m} |z_{ij}(s)| > 0 \quad (12)$$

for $i = 1, 2, ..., m$

With the same assumptions, $Z(s)$ is said to be *column diagonally dominant* on the contour D if $z_{ii}(s)$ has no pole on D and

$$\forall s \in D, \ |z_{ii}(s)| - \sum_{j=1/j \neq i}^{m} |z_{ji}(s)| > 0 \quad (13)$$

$Z(s)$ is *diagonally dominant* if it is row and column diagonally dominant.

A graphical method can be used to determine whether or not diagonal dominance is achieved. Consider a transfer function matrix which is row dominant for instance, and plot the Nyquist array of a dominant transfer function. For a specified frequency, a circle can be centered on this Nyquist array with a radius equal to the sum of the moduli of the corresponding row off diagonal terms. If a large set of frequencies is considered, then many corresponding circles are obtained. They define the Gershgorin band. Row diagonal dominance is then achieved if and only if none of the circles include the origin (see figure 3)

To verify diagonal dominance, Gershgorin bands corresponding to off diagonal terms in row and in column have to be drawn separately. For instance, for a three input three output system, nine curves are required.

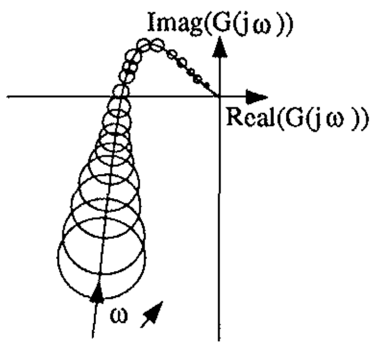


Figure 3 The Gershgorin band
$G(s)$: dominant transfer function

3 The H_∞ design

During the last few years, H_∞ synthesis techniques have received vast attention. Studies were initiated by the work of Zames [7]. Then, several approaches followed. The early methods [8] require intricate computations because they involve operator theory. Despite the use of Hankel singular values model reduction method, the compensator order is prohibitive for physical implementation. The later approaches involve the resolution of two Riccati equations. These new theories have been derived from two approaches. One is directly connected with the first technique which solves the *general distance problem* [9] and the other decomposes the problem into an H_∞ full state feedback problem and its estimation dual [10] [11]. The compensator order obtained with these new techniques is the same order as the order of the plant $P(s)$ which characterizes all the desired specifications (see figure 4).

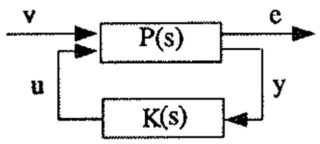


Figure 4 Standard configuration in H_∞ synthesis
v all external inputs , $dim(v) = m_1$
e error signals which are to be regulated, $dim(e) = p_1$
u control inputs, $dim(u) = m_2$
y measurements, $dim(y) = p_2$

The H_∞ technique tested here is the one which makes use of the dual Riccati equation approach. The compensator obtained is sub-optimal in the sense that, if $p_1 > m_2$ and/or $p_2 < m_1$ an iterative algorithm is needed in order to reach the minimal H_∞ norm of $F_l(P(s), K(s))$ with respect to internal stability :

$$\|F_l\,(P\,(s)\,,K\,(s))\|_\infty < \gamma \qquad (14)$$

The "γ iteration" is continued until one of the conditions derived from the Riccati equations becomes invalid.

If $p_1 = m_2$ and $p_2 = m_1$ the general distance problem can be reduced to a best approximation problem. The optimal value equals the norm of the Hankel operator associated with an unstable transfer function.

Choice of P

The choice of the criterion optimized with the H_∞ technique is here justified. Applications on helicopters of H_∞ synthesis already exist, [12], [13]. The same criterion is used but on the unsimplified multi input multi output linear model.

First objective Consider a transfer function matrix $G(s)$ such as

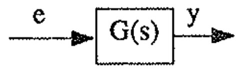


Figure 5

The H_∞ norm of $G(s)$ is then defined as the output maximum energy, as the input energy is less than equal to unity. Since our first objective is to minimize disturbances that could produce output signal deviations, the H_∞ norm of the transfer function between the references and the error signals has to be minimized as illustrated in figure 6.

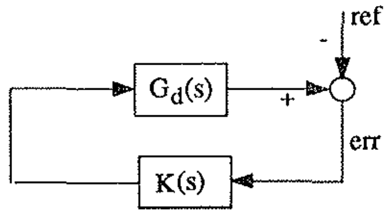


Figure 6 Sensitivity objective
$err = (I_d + G_d(s)K(s))^{-1}ref$
$G_d(s)$: decoupled system
I_d : Identity matrix
ref : reference signals
err : error signals

A weighting function $W_1(s)$ is introduced to normalize and select a frequency range. Hence, the first optimization problem is the following

$$\begin{array}{l} min \left\| W_1\,(s)\,(I_d + G_d\,(s)\,K\,(s))^{-1} \right\|_\infty \\ K\,(s)\;\; stabilizing \end{array} \qquad (15)$$

$(I_d + G_d(s)K(s))^{-1}$ is the *sensitivity transfer function.*

Second objective The H_∞ norm is the maximum singular value of $G(j\omega)$ over all frequencies (see figure 7 for the single input single output case)

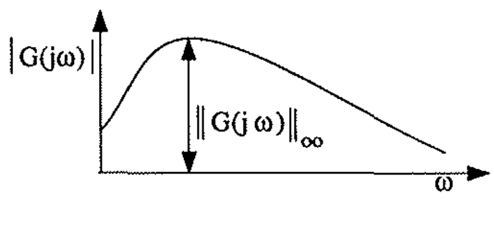


Figure 7 H_∞ norm for the single input single output case

Consider now the feedback configuration figure 8, where $\Delta(s)$ is an additive perturbation which characterizes the nominal plant uncertainties. Suppose that $\Delta(s)$ is stable with

$$\|\Delta(s) W_\delta(s)\|_\infty \leq 1 \quad (16)$$

where $W_\delta(s)$ is a weighting transfer function. The *small gain theorem* then states that the closed loop stability is ensured if the nominal feedback system is stable and if

$$\left\| W_\delta^{-1}(s) K(s) (I_d + G_d(s) K(s))^{-1} \right\|_\infty \leq 1 \quad (17)$$

Hence, our second objective is to improve robustness in the sense of the above weighted transfer function.

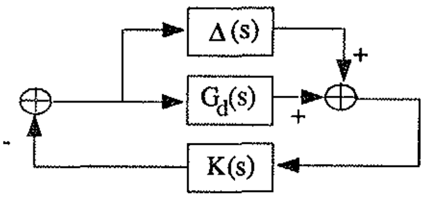


Figure 8 Additive perturbation

The criterion

Consider now

$$Z(s) = \begin{pmatrix} X(s) \\ Y(s) \end{pmatrix} \quad (18)$$

where $X(s)$ and $Y(s)$ are transfer function matrices. It is well known that $\|Z(s)\|_\infty$ satisfies the following properties [14] :

$$\|Z(s)\|_\infty \geq max\{\|X(s)\|_\infty, \|Y(s)\|_\infty\} \quad (19)$$

Moreover, by considering this result, our two objectives can be expressed as

$$\min_{K(s)\ stabilizing} \left\| \begin{pmatrix} W_1(s)(I + G(s)K(s))^{-1} \\ W_2(s)K(s)(I + G(s)K(s))^{-1} \end{pmatrix} \right\|_\infty \quad (20)$$

This leads to the following matrix $P(s)$

$$P(s) = \left(\begin{array}{c|c} W_1(s) & -W_1(s)G(s) \\ 0 & W_2(s) \\ \hline I_d & -G(s) \end{array} \right) \quad (21)$$

The design procedure

For each single input single output sub-system, the optimization problem described by equation (20) is solved using the H_∞ technique.

An important step towards the solution is the selection of the weighting functions. They not only define the frequency range, where performance and robustness specifications must be verified, but they also produce a normalization of the optimization problem.

In order to avoid to increase the controller order, we choose a first order weight for each sub-system. Such weighting filters are generally sufficient to reflect the control requirements [12].

Weight $W_1(s)$ is a high-gain low-pass filter ensuring integral action and thus small tracking error. Moreover, control activity is not necessary at the frequencies which are not included in the sensor bandwidth. This permits the gain at low frequencies to be constant.

$W_2(s)$ is a high-pass filter cancelling the controller activity at the very high frequencies whilst also handling uncertainties due to the neglected high frequency dynamics of the rotor.

This yields the following weighting functions

Table 4

	Forward	Lateral
W_1	$80\frac{0.04s+0.12}{3s+0.04}$	$80\frac{0.015s+0.045}{3s+0.045}$
W_2	$0.2\frac{0.04s+0.04}{s+0.04}$	$0.2\frac{s+0.015}{0.03s+0.015}$

Table 5

	Yaw
W_1	$80\frac{0.03s+0.09}{3s+0.03}$
W_2	$0.2\frac{0.4s+0.012}{0.03s+0.012}$

4 The significant results

The two methods are applied consecutively. This yields an eleven order compensator.

The decoupled structure obtained with eigenvector assignment is shown in figure 9. The Gershgorin bands are plotted in order to highlight the diagonal dominance in row and in column. From these results, we decide to take into account only the dominant transfer functions and to consider the off diagonal functions as uncertainties. The next results validate the idea.

Figure 10 illustrates the simplifications corresponding to each sub-system. The cancellation of the low variables (as forward or lateral velocities) leads to a static error at the low frequency range.

In order to assess the second loop contribution for improving robustness and performance, singular values of $S(s)$ and $K(s)S(s)$ transfer functions are plotted by considering first, only the static loop designed with eigenvector assignment, and additionally by considering the whole compensated plant (see figures 11 and 12). From these figures, it can be shown that our goals are achieved.

The compensator has been generated with the linearized eight states model. However the twelve states model has been used as the helicopter in the simulation. It is considered the nearest available model to the non linear case. For a reference pitch angle input we want this variable to follow it with the effects of the other axes negligible. The same objectives have to be verified for the reference yaw rate and roll angle. The simulations in the time domain (see figure 13) show that the objectives are achieved. Note that when the yaw rate reference input is applied, there is a non negligible effect from the roll angle. This is a natural consequence of the helicopter behaviour.

5 Conclusion

In this paper a two-feedback-loop strategy has been introduced for the design of a robust helicopter control law. The design procedure, based mainly on physical insight is proved to be valid when applied to a simulated linear helicopter model. The compensator direct transmission is designed with eigenvector assignment and its dynamic using the H_∞ synthesis technique. This approach permits the controller order to be eleven without the need of any model reduction. This order is relatively low compared to the classical H_∞ (direct) approach. Moreover, all performance and robustness specifications are achieved.

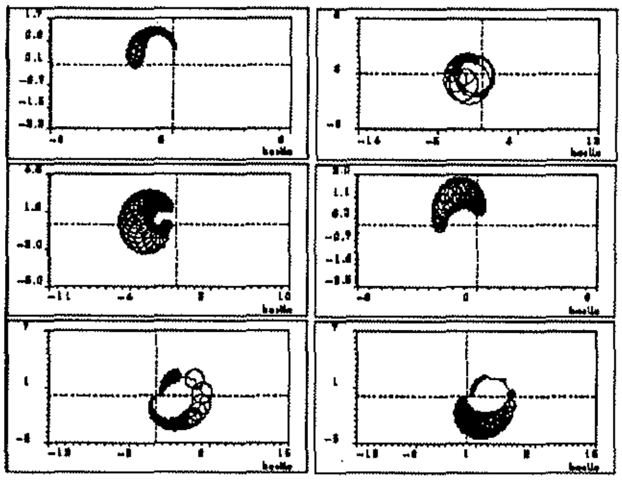


Figure 9 Gershgorin bands
0.01 rad/s ≤ ω ≤ 8 rad/s
First column : in row
Second column : in column

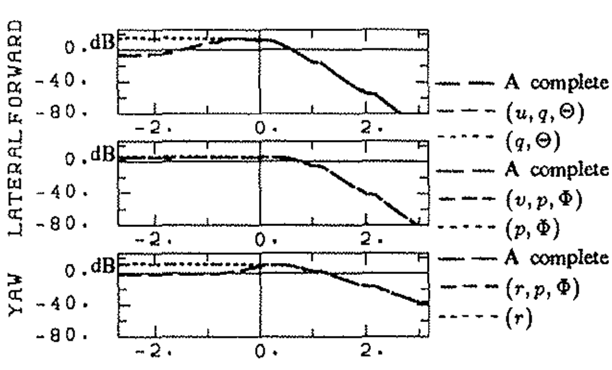
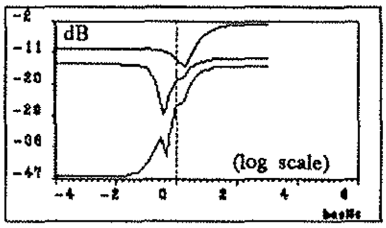
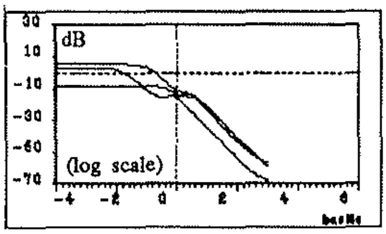


Figure 10 Sub-system singular values

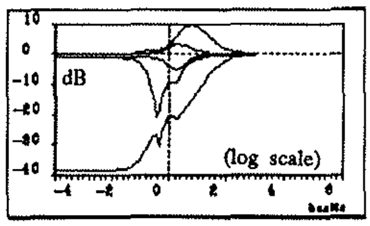


First loop feedback

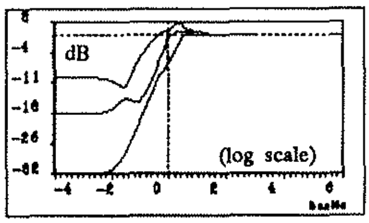


Two loop structured feedback

Figure 11 Robustness test

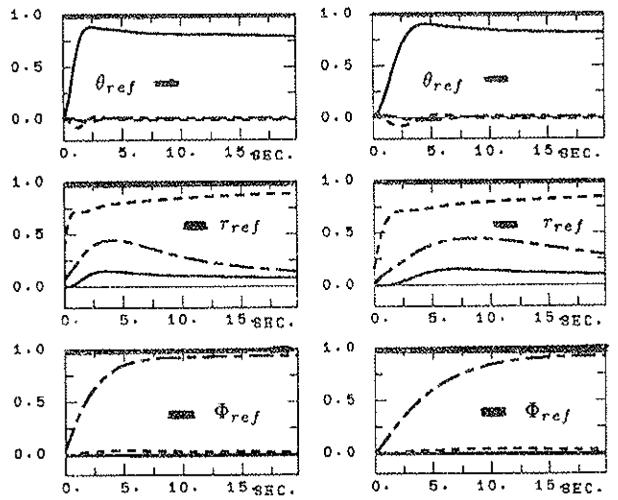


First loop feedback



Two loop structured feedback

Figure 12 Performance test



Eight states model

Twelve states model

Figure 13 Simulation in the time domain
Comparison between the eight and twelve linear models

—— θ
- - - - r
— - — ϕ

Acknowledgment

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