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HELICOPTER SYSTEM IDENTIFICATION IN THE FREQUENCY DOMAIN

K.-H. FU

and

M. MARCHAND

Deutsche Forschungs- und Versuchsanstalt
für Luft- und Raumfahrt e.V.,
Institut für Flugmechanik
Braunschweig, Germany

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K.-H. Fu
M. Marchand

Deutsche Forschungs- und Versuchsanstalt
für Luft- und Raumfahrt e.V. (DFVLR)
Institut für Flugmechanik, Braunschweig

Abstract

Accurate mathematical models are a prerequisite for a reliable description of aircraft dynamic behavior. These models containing the stability and control derivatives of the actual aircraft can be extracted from flight test data by system identification techniques. For this, a new identification method operating in the frequency domain has been developed. In comparison to existing time domain methods it enables a reduction in the number of data to be evaluated and a concentration on specific frequency ranges. Therefore, this technique is particularly suitable for the determination of higher order and more complex helicopter mathematical models.

The paper presents first the frequency domain technique and differences from other existing methods. Then, application apsects are emphasised. Results obtained from both computer simulated and measured Bo 105 flight data are compared with results obtained from a Maximum Likelihood time domain technique.

Notation

a_x, a_y, a_z	longitudinal, lateral, vertical acceleration, m/sec^2
L, M, N	normalized rolling, pitching, yawing moment, rad/sec^2
L_u, L_v, ...	moment derivatives (= $\partial L/\partial u$, $\partial L/\partial v$, ...)
p, q, r	roll rate, pitch rate, yaw rate, rad/sec
u	input vector
u, v, w	longitudinal, lateral, vertical velocity
V	velocity
x	state vector
X, Y, Z	normalized forces, m/sec^2
X_u, X_v, ...	force derivatives (= $\partial X/\partial u$, $\partial X/\partial v$, ...)
δ_o, δ_y, δ_{TR}	collective, lateral, tail rotor control
ϕ	bank angle
Θ	pitch angle
ω	frequency, rad/sec
ε	equation error

1. Introduction

For the investigation of helicopter stability and control an accurate mathematical model describing the helicopter dynamics is required. When this model is derived only from theoretical calculations and from wind tunnel test data it is not possible to sufficiently include all influences acting on the helicopter. Consequently, flight tests are needed to validate or correct the model with respect to actual flight conditions. Therefore, specific flight tests are conducted to provide adequate flight test data for the extraction of system parameters using system identification techniques (see fig. 1).

System identification techniques are widely used for the evaluation of flight test data from fixed wing aircraft. The time domain methods have been shown to be particularly effective. However, the application of system identification techniques to rotorcraft is a very difficult task (ref. 1). The results published until now are very limited in comparison with the number of results published for fixed wing aircraft and can be judged as being only partly satisfying though different evaluation techniques have been applied. For example, Regression Analysis and Extended Kalman Filter have been applied to the identification of CH-53-D rotorcraft (ref. 2), Regression Analysis to Bell 205 (ref. 3), Stepwise Regression both in time and frequency domain to RSRA Rotor Systems Research Aircraft (ref. 4), Least-Squares, Instrumental-Variable and Maximum-Likelihood-Methods to Bo-105 (ref. 5, 6).

The authors of references 4, 5, 6 showed the need of combining several manoeuvres with excitation of each of the controls (Multi-Run-Evaluation).

In most cases, a reduced mathematical model representing 6 degrees of freedom for the rigid body motion was used. For various applications, however, it is necessary to extend this model and to explicitely describe rotor degrees of freedom. In this case the evaluation in the time domain may reach its limits of applicability due to the following reasons:

- high system order
- large spread of smallest and largest eigenvalue
- large number of data due to long data record, high sampling rate and large number of recorded input/output variables
- large number of parameters to be identified (flight mechanical derivatives and additive constants within the equations)

An approach to alleviate the numerical difficulties of time domain evaluation is the frequency domain evaluation. This approach enables a reduction of the number of data to be evaluated by applying the Fourier-Transformation and the subsequent elimination of all data not included in the frequency range of interest. In addition, the number of parameters to be identified is reduced since only derivatives have to be identified. Additive constants that have to be estimated in the time domain techniques (e.g. for taking into account measurement zero shifts and nonzero steady states) are not needed in the frequency domain model, as data for the frequency $\omega = 0$ can be excluded from the evaluation.

Frequency domain identification methods have been used since about 1950 (ref. 7, 8). The approach in the first methods was to minimize either the transfer function errors or the equation errors (ref. 9 through 12). During the last few years, the theoretical background for the implementation of more advanced methods has been developed and Output-Error-Methods as well as Maximum-Likelihood-Methods have been proposed (ref. 13, 14). But the practical application of these methods to fixed wing aircraft or rotorcraft identification did not lead to fully satisfying results.

Therefore, in this paper, a modified version of one of these proposed methods is presented. To test this method, both computer simulations as well as flight tests using the Research Helicopter Bo 105 of DFVLR were performed and evaluated. The paper presents some results from these evaluations and a comparison with time domain identification results.

2. The DFVLR-Frequency-Response-Method

2.1. Basic approach

The basic approach of the DFVLR-method is shown in fig. 2. This approach corresponds to the Output-Error-Technique published by V. Klein (ref. 13, 14). In this method, the output errors are calculated as the differences beetween the frequency domain model outputs and the Fourier-Transform of the measured test data.

The DFVLR-Method differs from the methods published up to now with respect to the following items:

- Modification for nonperiodic signals:
  This modification was necessary to ensure the applicability to arbitrary flight test data records.

- Multi-Run Evaluation:
  Data from different manoeuvres can be combined for one evaluation

- Two-Step-Identification:
  The advantages of two identification methods are combined. First, a robust Least-Squares-Equation-Error-Method is applied, which quickly provides preliminary results without requiring a-priori-values of the parameters. Then, an Output-Error-Method is used to further improve these results and to obtain unbiased estimates (without systematic errors).

2.2. Modification for nonperiodic signals

For the basic method, as described in ref. 13, it is assumed that the signals can be regarded as periodic with period T, so that

(1) $$x(T) = x(0)$$

and the Fourier-Transform of $\dot{x}(t)$ can be written as

(2) $$\dot{x}(\omega) = j\omega\, x(\omega) \quad .$$

In this case, the equations of motion can be transformed from time domain into frequency domain as follows

(3) $$\dot{x}(t) = A\, x(t) + B\, u(t) \qquad \text{(time domain)}$$

(4) $$j\omega\, x(\omega) = A\, x(\omega) + B\, u(\omega) \qquad \text{(frequency domain)}$$

Unfortunately, in general, real flight test data do not meet the condition of equation (1). As shown in the Appendix, the differences between the state variables at the limits of the transformation interval

(5) $$\Delta x = \left[x(T - \Delta t/2) - x(- \Delta t/2)\right]/T$$

have to be taken into account. Consequently the equations of motion of the frequency domain model have to be modified to

(6) $$j\omega\, x(\omega) = A\, x(\omega) + B\, u(\omega) + \Delta x\, e^{j\omega\Delta t/2} \quad .$$

The additional term in equation (6) can be treated in a simple manner by adding a fictitious control variable and setting the value of this variable to $e^{j\omega\Delta t/2}$ in the frequency domain. The corresponding additional column of the control matrix B has to be filled with the elements of the vector Δx or with unknown parameters to be identified. The equations of motion then are

(7) $$j\omega\, x(\omega) = A\, x(\omega) + \hat{B}\, \hat{u}(\omega)$$

where

$$\hat{B} = \left[B,\ \Delta x\right], \qquad \hat{u}(\omega) = \left[u(\omega),\ e^{j\omega\Delta t/2}\right]^T \quad .$$

During testing of the modified method, both the unmodified equation (4) and the modified equation (7) were used for identification from computer simulated helicopter flight data. To evaluate the accuracy of the obtained results two different approaches were used:

1. The maximum and the mean errors of the identified derivatives were calculated. They are shown in fig. 3. It clearly demonstrates the improvement that is obtained when the modified technique is applied.

2. For each derivative, its contribution to the total aerodynamic force or moment is calculated and compared with the equation error of the identified model. For example, the normalized pitching moment is

(8) $$\dot{q} = M_u\, u + M_v\, v + M_w\, w + M_p\, p + M_q\, q + \ldots$$

and the derivative contributions are

$$M_u\, u,\ M_v\, v,\ M_w\, w,\ M_p\, p,\ M_q\, q,\ \ldots$$

For a reliable identification of the pitch derivatives, it is necessary that the pitch equation error

(9) $$\varepsilon_{\dot{q}} = \dot{q} - M_u\, u - M_v\, v - M_w\, w - M_p\, p - M_q\, q - \ldots$$

be small in comparison with the magnitudes of the contributions $M_u\, u$, $M_v\, v$, ... In fig. 4 it can be seen that this requirement is not met by the unmodified method. The equation error level is even higher than most of the derivative contributions. Only when the modified technique was applied could the error be reduced to a negligible level. This equation error results mainly from numerical inaccuracies in the simulated data (digital integration) and the Fourier Transformation.

2.3. Multi-Run-Evaluation

For the identification of complex systems it is necessary to use data which contain information about each of the eigenmotions and about the effectiveness of each of the controls (ref. 1). In general, it is not possible to obtain all this information from one single manoeuvre, because the test pilot is not able to excite several controls simultaneously in a prescribed manner. Apart from this it is also often impossible to sequentially perform the input signals due to helicopter instabilities. Therefore, the only practical approach is to use an evaluation method that is able to use the information from different runs that were flown independently. The time records suitable for identification are selected and their data are combined as shown in fig. 5. Thus, the "equivalent run" is the time history formed by the sum of the n manoeuvers. For example, from three manoeuvres with excitations of lateral control, collective pitch control, and tail rotor control an "equivalent run" is generated, which corresponds to a manoeuvre with simultaneous excitations of the three controls. In this way, the number of data to be evaluated is reduced whereas the information content is concentrated into a shorter data record.

2.4. Selection of Frequency Ranges

After calculation of the Fourier-Transform of the data set corresponding to the "equivalent run", the frequency range of interest (or several ranges) can be selected. As a help for defining the range of interest, the magnitude of the Fourier coefficients or of the "coherence functions" give an indication whether the system has been sufficiently excited at a specified frequency or not. By eliminating the frequencies outside the range of interest, the number of data to be evaluated can be further reduced.

2.5. Identification from Simulated Data

In order to test the entire procedure, the rigid body motion of a Bo 105 helicopter was identified from computer simulated data. Data preprocessing included the calculation of the "equivalent run" data from three different simulation runs and the selection of data from only the first eight frequencies. The identified parameters agreed very well with the values used in the simulation. As shown in fig. 6, the outputs of the identified model and the simulated model are identical.

3. Identification from Flight Test Data

3.1. Flight Tests for System Identification

For System Identification, a flight test program with the MBB Bo-105 research helicopter of the DFVLR was conducted (ref. 15). To help the pilot implement the optimized input signal an additional cockpit display was developed, which shows both the desired and the actually performed signal (fig.7). For the evaluation presented in this paper, the data from three manoeuvres flown at a trim speed of V = 150 km/h are used. These manoeuvres consist of different inputs into lateral control, collective control and tail rotor control, as shown in fig. 8. For the frequency domain identification, these data are combined in one "equivalent run" with a length of 20 sec, which is shown in fig. 8 on the right.

3.2. Mathematical Model

The mathematical model to be identified consists of the equations of motion for the six rigid body degrees of freedom and the equations defining the eleven measured output signals.

The frequency domain equations are

(10) $$j\omega x = A\ x(\omega) + B\ u(\omega) + \Delta\dot{x}\ e^{j\omega\Delta t/2}$$

$$+ \text{ nonlinear gravity and inertia terms}$$

(11) $$y(\omega) = C\ x(\omega) + D\ u(\omega)$$

with

state vector	$x = [u,\ v,\ w,\ p,\ q,\ r,\ \phi,\ \Theta]^T$
control vector	$u = [\delta_o,\ \delta_y,\ \delta_{TR}]^T$
measurement vector	$y = [a_x,\ a_y,\ a_z,\ u,\ v,\ w,\ p,\ q,\ r,\ \phi,\ \Theta]^T$
system matrices	A, B, C, D

The matrices A, B, C, D contain 35 state and control derivatives to be identified.

3.3 Identification Results

Identification results obtained from both frequency and time domain methods are presented. For the evaluation in the time domain, a well established Maximum Likelihood technique commonly used in the DFVLR Institute for Flight Mechanics was applied. Results are shown in the form of output spectra and time history plots. The main identified derivatives are also given.

Fig. 9 presents the output spectra from the measured data and identified model. It can be seen that both frequency and time domain methods yield satisfactory results. There are major differences only in the spectra of the yaw rate r.

In fig. 10 time histories of the measured data and the identified models are presented. A good agreement could be obtained for both estimation methods. Again there are some discrepancies in the yaw rate fit.

Extensive evaluations of different data runs using the time domain Maximum Likelihood Method showed similar discrepancies. From this, it can be assumed that measurement errors or a gyro malfunction have deteriorated the yaw rate data. Therefore, depending on the cost functions applied, different methods can lead to different results for the model outputs and the identified derivatives.

Fig. 11 gives the main identified Bo 105 derivatives. In general there is a satisfactory agreement between time and frequency domain results. As discussed before, the differences in the identified yaw derivatives can be explained by the poor yaw rate measurement. Since yaw rate forms one of the most important terms of the side force equation, the side force derivative Y_v may also be influenced by measurement errors.

Summarizing, it can be stated that the frequency domain identification from flight test data yields satisfactory fits in both output spectra and time histories. The accuracy of the identified derivatives is comparable with results obtained from time domain techniques.

4. Concluding Remarks

A system identification technique to extract aircraft stability and control derivatives from flight test data was presented. The method operates in the frequency domain and provides an alternative in rotorcraft identification to existing time domain techniques that reach their limits of applicability with increasing number of unknowns and large amounts of data. To evaluate the efficiency of the method, the frequency domain technique and a time domain method were applied to both computer simulated and measured Bo 105 flight data. Output spectra and time history fits as well as identified derivatives were presented. The comparison of the results demonstrated that the frequency domain method provides a reliable and accurate identification of dynamic systems.

Future helicopter identification will be extended from 6 degree of freedom rigid body models to higher order equation systems that also include rotor degrees of freedom. This will not only lead to a larger number of unknown parameters and, due to high sampling rates, to a high number of data points but also to data from two significantly different frequency ranges (rigid body, rotor dynamics). Since the frequency domain technique enables both data reduction and concentration on selected frequency ranges it can be expected that the frequency domain technique will be more suitable and powerful than other existing techniques in helicopter system identification.

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6. Appendix: Fourier Transform of $\dot{x}(\omega)$

The discrete Fourier Transform of a time series $x_k = x(k\Delta t)$, $k = 0, 1, 2, \ldots N-1$ is defined as

$$\text{(A1)} \qquad x(\omega) = \frac{1}{N} \sum_{k=o}^{N-1} x_k \; e^{-j\omega k\Delta t}$$

This can be approximated by the integral

$$\text{(A2)} \qquad x(\omega) = \frac{1}{T} \int_{-\Delta t/2}^{T-\Delta t/2} x(t) \; e^{-j\omega t} \, dt \quad .$$

From (A2), the Fourier Transform of $\dot{x}(t)$ can be derived

$$\text{(A3)} \qquad \dot{x}(\omega) = \frac{1}{T} \int_{-\Delta t/2}^{T-\Delta t/2} \dot{x}(t) \; e^{-j\omega t} \, dt$$

$$= \frac{j\omega}{T} \int_{-\Delta t/2}^{T-\Delta t/2} x(t) \; e^{-j\omega t} \, dt$$

$$+ \left[x(T-\Delta t/2) e^{-j\omega(T-\Delta t/2)} - x(-\Delta t/2) e^{-j\omega(-\Delta t/2)} \right] / T \quad .$$

Since $e^{j\omega T} = 1$ for all frequencies used, and with equ.(A1), the Fourier Transform of $\dot{x}(t)$ is

$$\text{(A4)} \qquad \dot{x}(\omega) = j\omega \, x(\omega) + \Delta x \; e^{j\omega\Delta t/2}$$

where

$$\text{(A5)} \qquad \Delta x = \left[x(T-\Delta t/2) - x(-\Delta t/2) \right] / T \quad .$$

Δx can be calculated from the sampled data by the linear interpolation

$$\text{(A6)} \qquad \Delta x \approx (x_{N-1} + x_N - x_{-1} - x_o)/2T \quad .$$

Note, that this interpolation requires two extra data points, x_{-1} and x_n, not used in the Fourier Transformation.

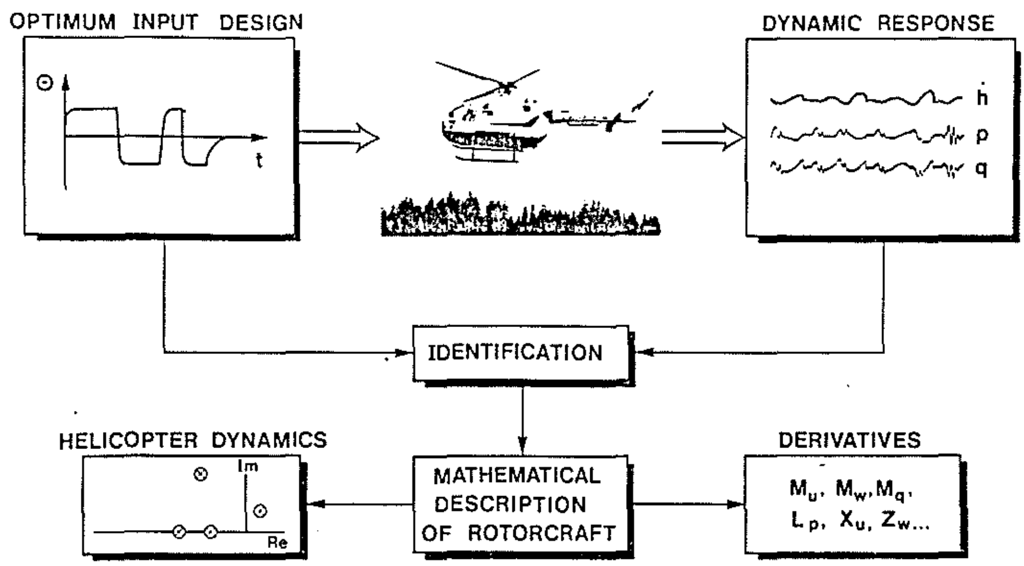

Fig. 1 Helicopter identification procedure

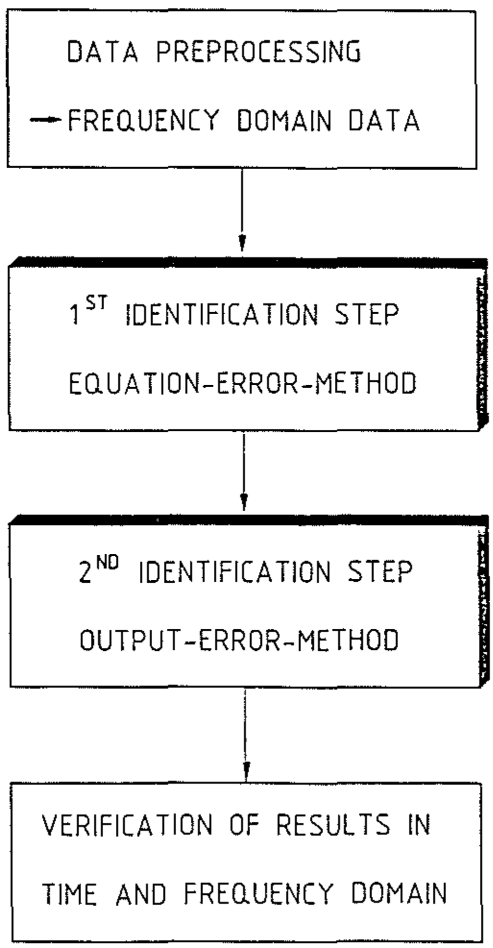

Fig. 2 Concept of the DFVLR-Frequency Domain Identification Method

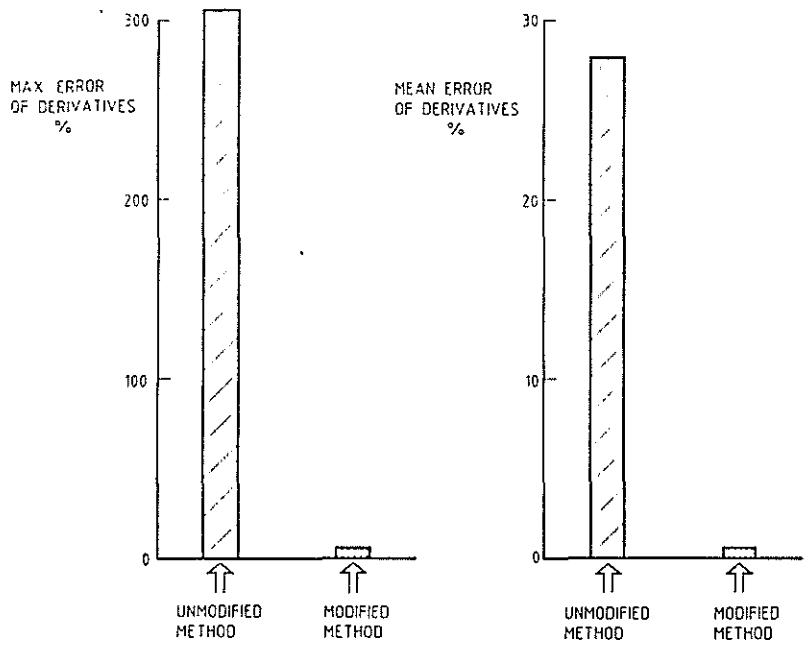

Fig. 3 Maximum and mean error of 21 identified derivatives for two identification methods (from computer simulated Bo 105 flight data)

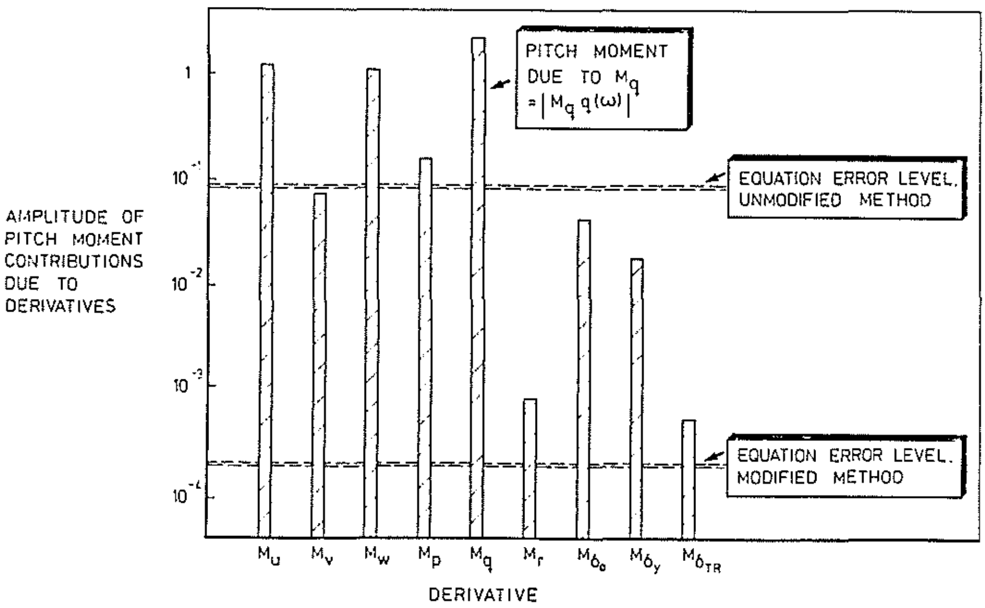

Fig. 4 Comparison of pitching moment equation errors with aerodynamic pitching moment contributions due to stability and control derivatives (from computer simulated Bo 105 flight data)

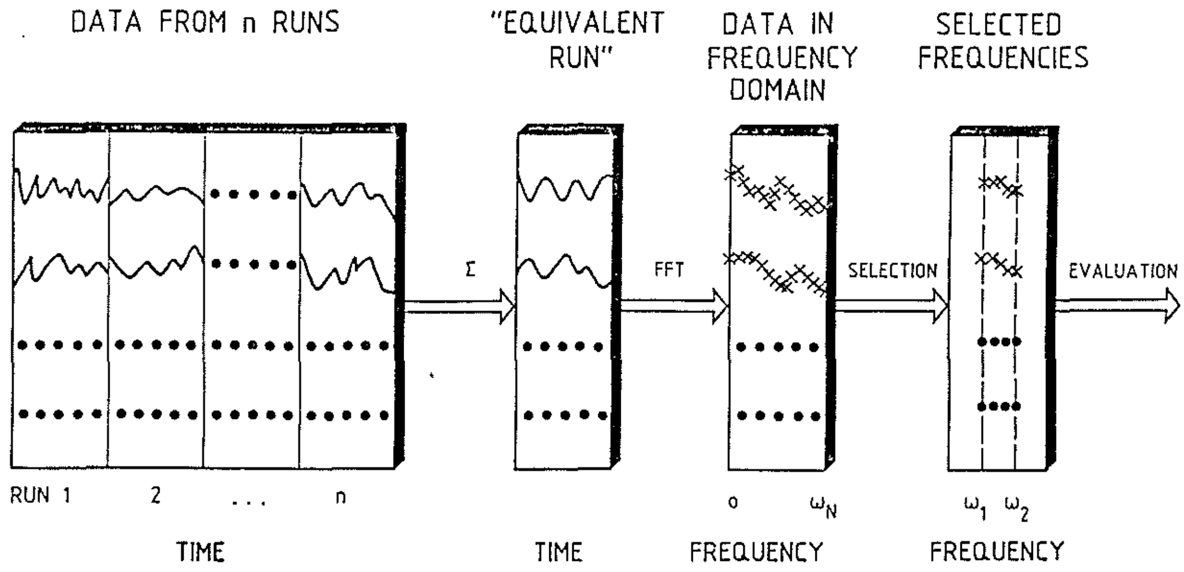

Fig. 5 Data preprocessing for multi-run evaluation in frequency domain:

- Generation of an "equivalent run"
- Fast Fourier Transformation
- Selection of the frequency range of interest

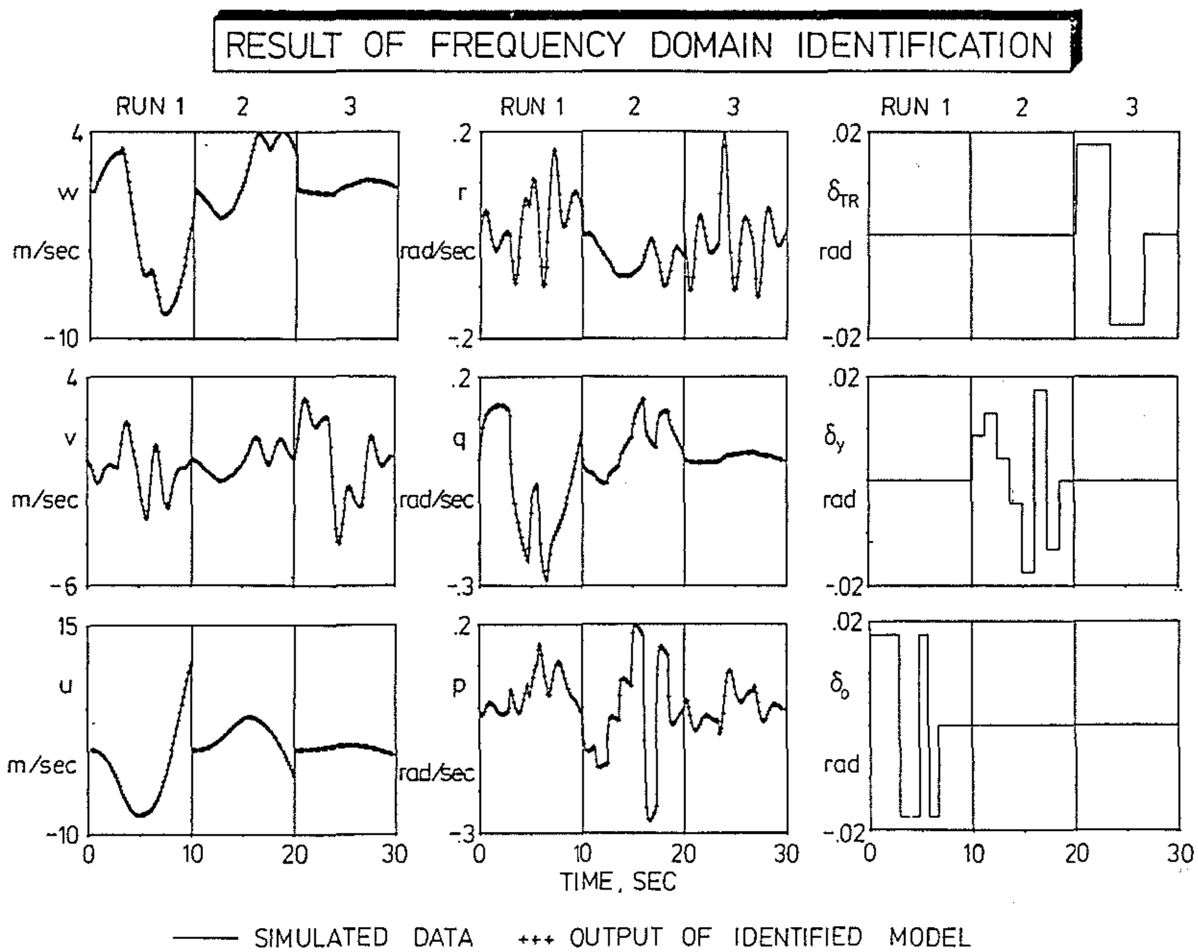

Fig. 6 Test of the DFVLR-Frequency Domain Identification Method

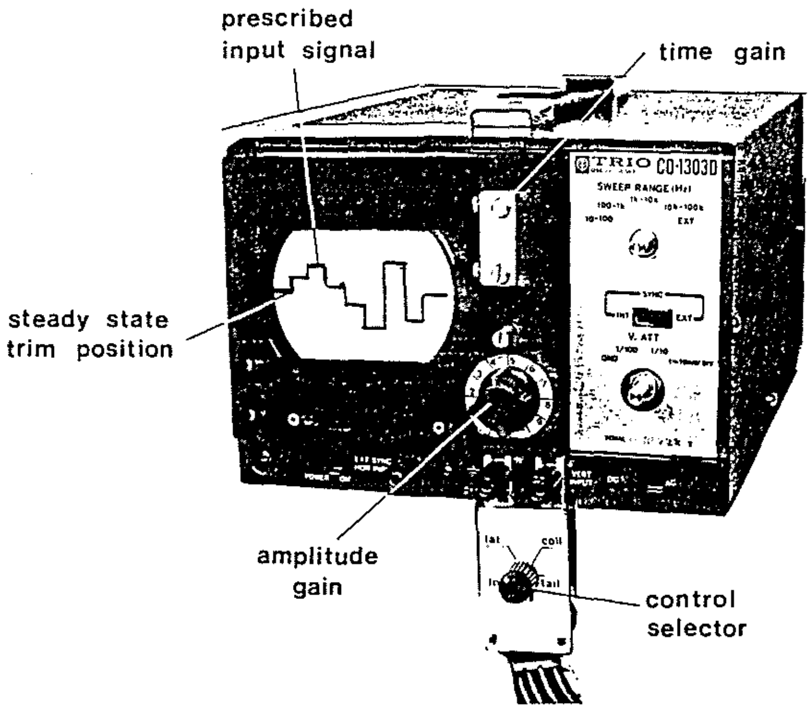

Fig. 7 Cockpit display for input signal implementation

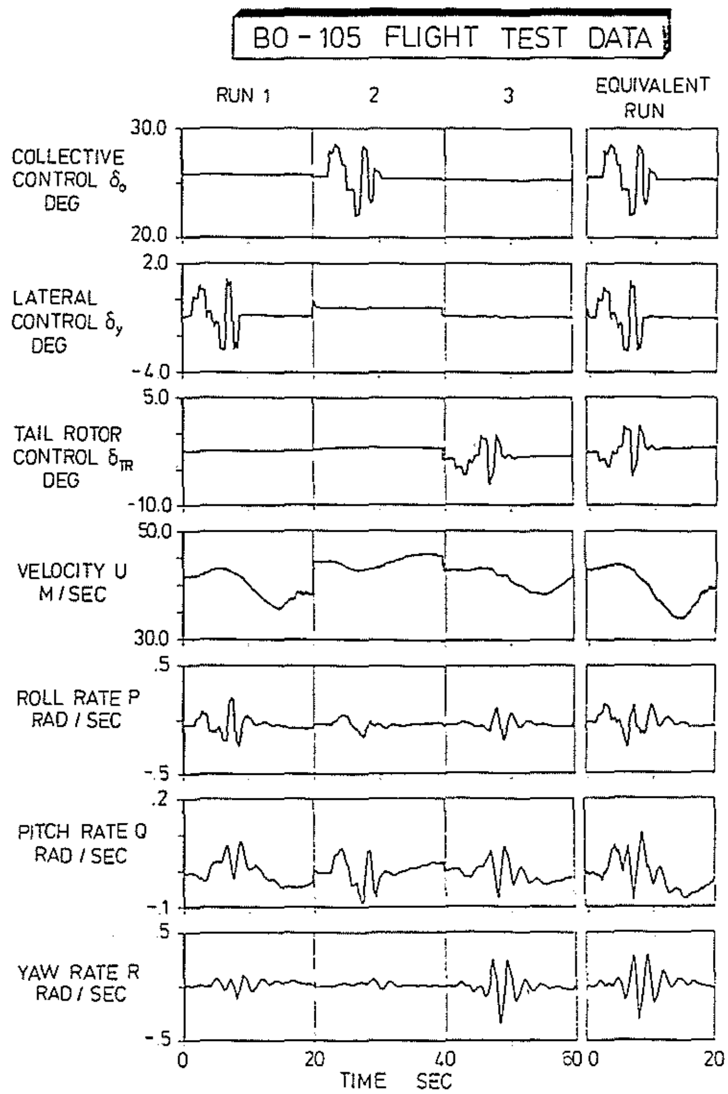

Fig. 8 Three flight test manoeuvres and corresponding "equivalent run" used for system identification (Bo 105, V = 150 km/h)

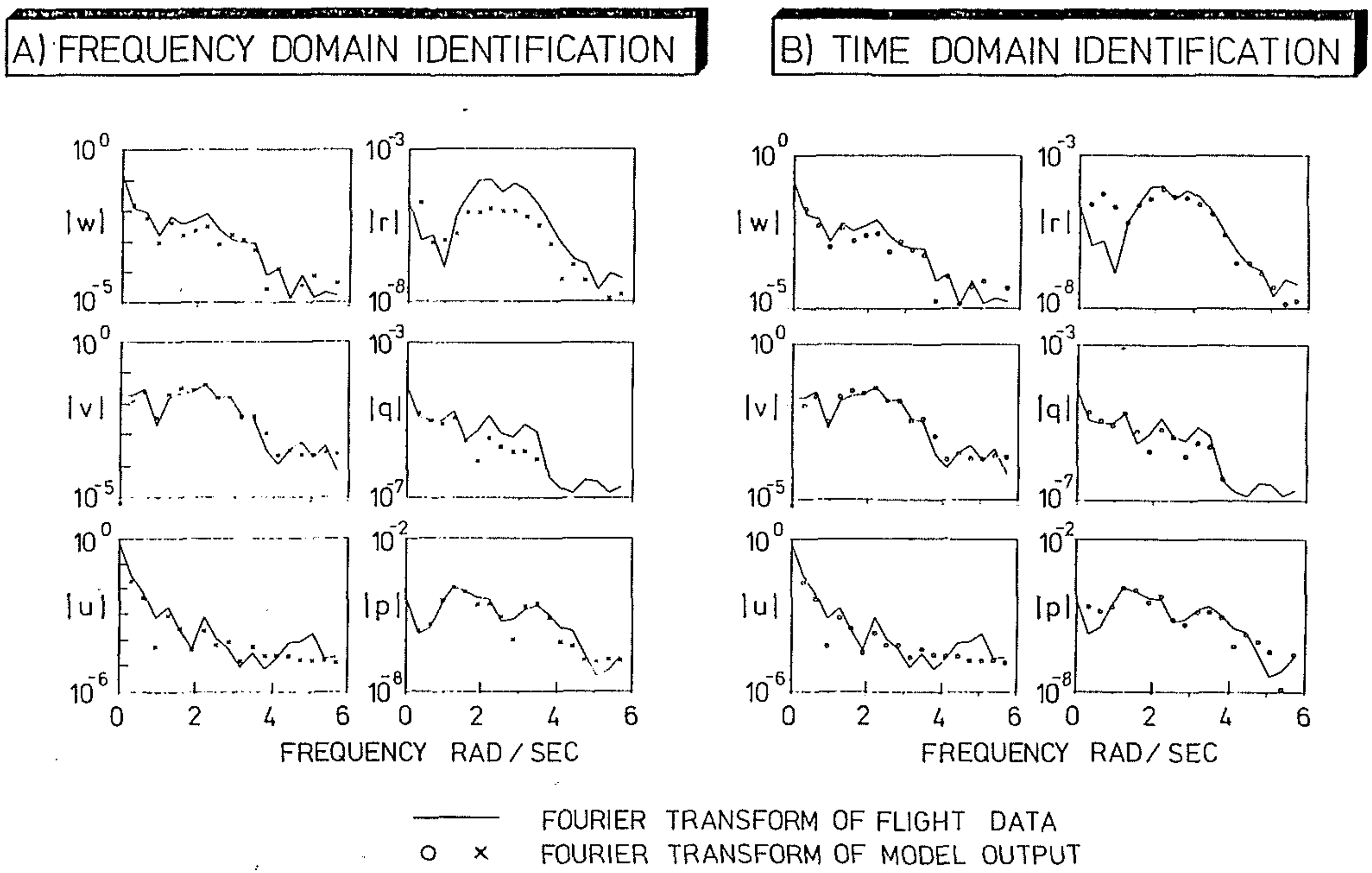

Fig. 9 Fit of output spectra for frequency domain and time domain identification
(Evaluation of the manoeuvres shown in fig. 8)

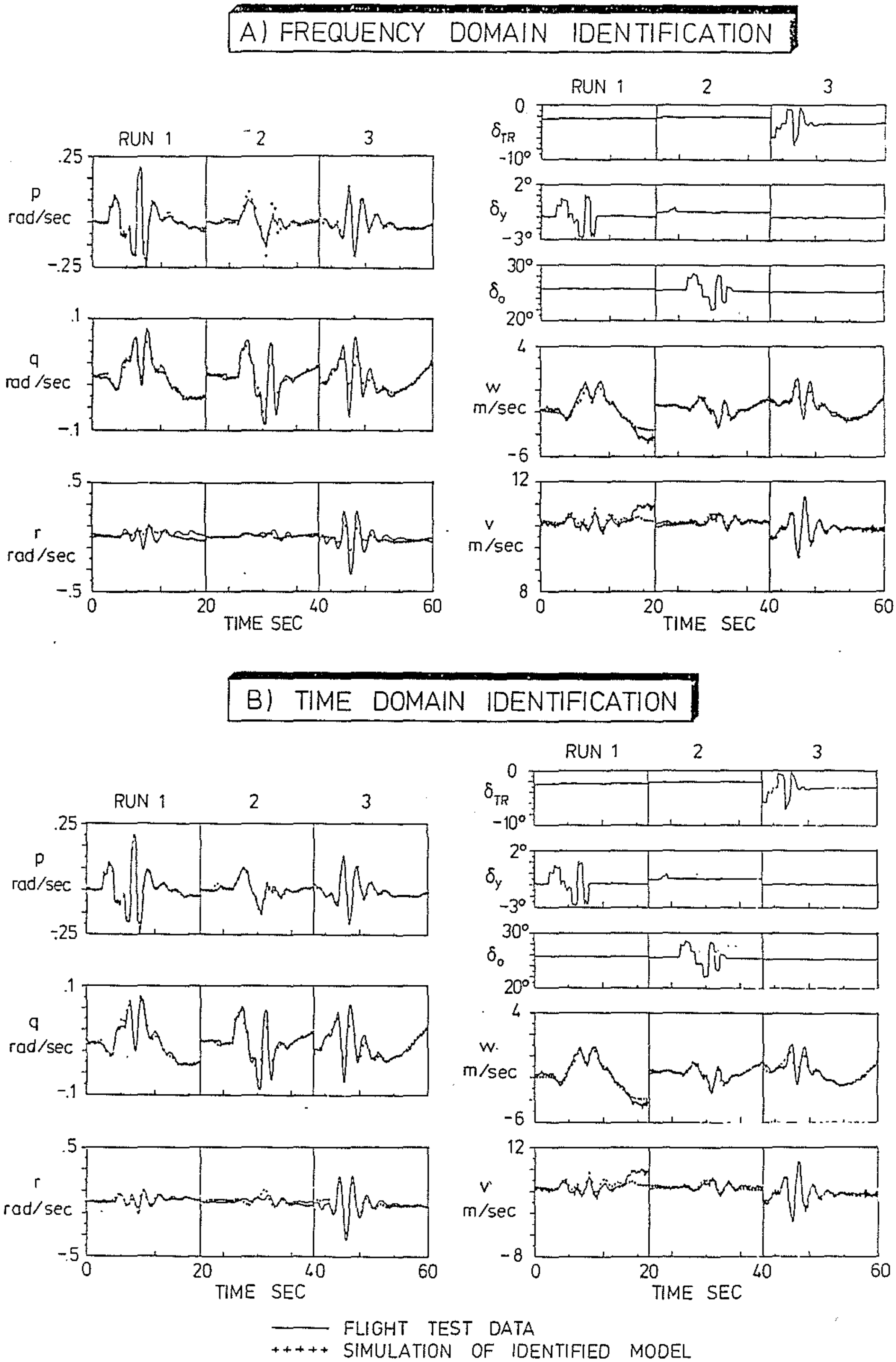

Fig. 10 Fit of time histories for frequency domain and time domain identification (Bo 105, V = 150 km/h)

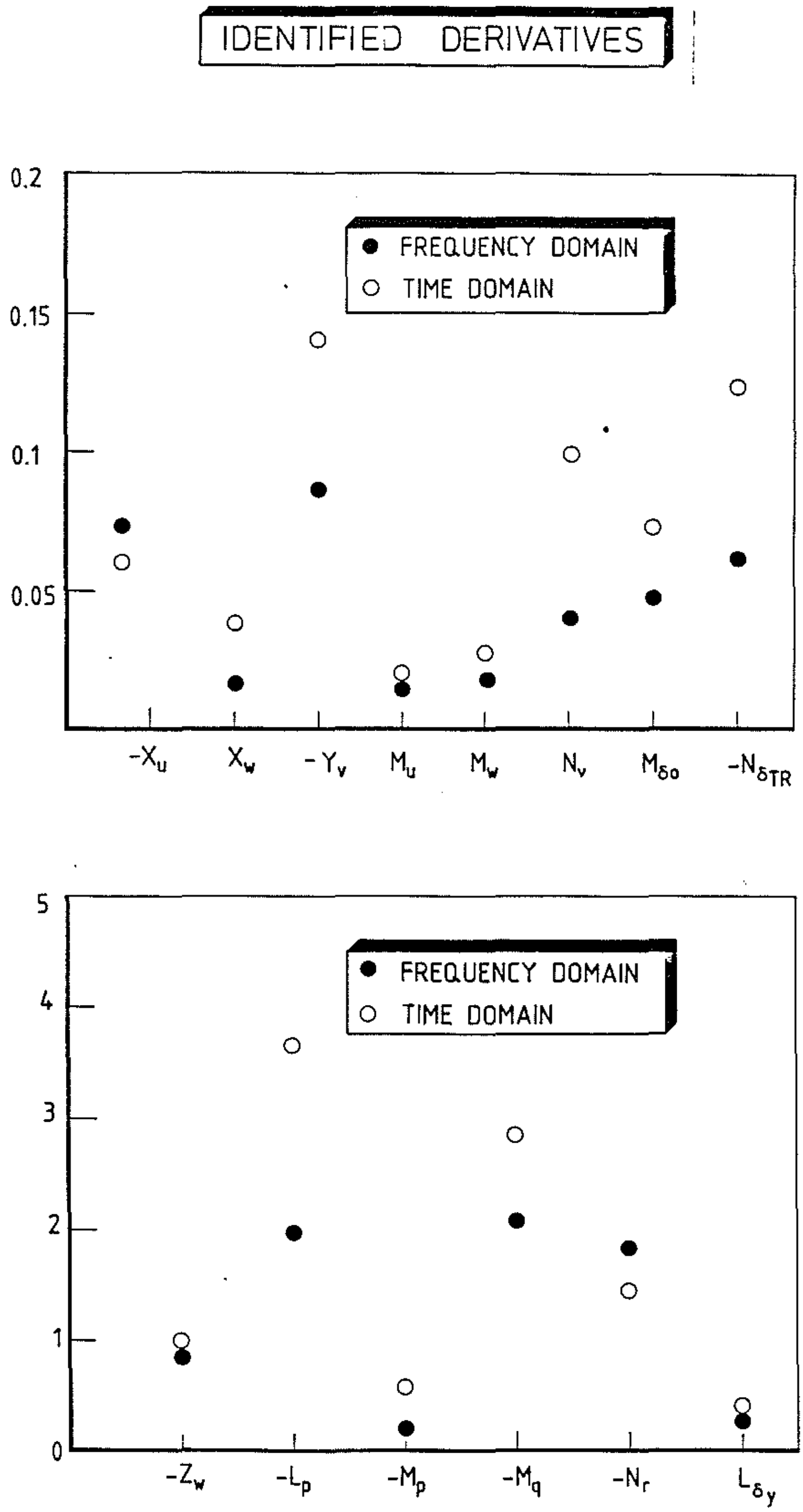

Fig. 11 Comparison of identified stability and control derivatives (Bo 105, V = 150 km/h)