A MACHINE LEARNING APPROACH FOR DATA-DRIVEN ENHANCEMENT OF ROTORCRAFT DEVELOPMENT EMULATION TOOLS

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Abstract

First principles models, based on prior physical knowledge of the system of interest, are widely adopted in the aerospace industry, as they provide valuable emulation tools for core development and certification activities, such as requirements validation and verification, performance analyses and flight simulations. Incomplete representation of underlying physics, as well as simplifications driven by computational constraints, could lead to errors in model's prediction. Oftentimes, real-world measurement data of the system of interest are available, particularly for projects in an advanced stage of development. This data availability possibly represents a valuable resource to fill the gap between model and real-world data, thanks to dedicated machine learning techniques. This paper presents an autoregressive grey-box approach based on Gaussian Process regression, that exploits both prior knowledge and available data for the enhancement of existing first principles aircraft actuation models.

1. NOTATION

Acronyms:

EHV	Electro-Hydraulic Servo-valve
ERF	European Rotorcraft Forum
GB	Grey Box
GP	Gaussian Process
ITFV	Integrated Three Function Valve
RAM	Power Stage
RMSE	Root Mean Squared Error
VFE	Variational Free Energy
VFS	Vertical Flight Society
WB	White Box

2. INTRODUCTION

Aircraft development and certification often requires the use of emulation tools. Emulation can significantly reduce testing costs, if compared with equivalent testing activities perpetrated on the corresponding real system. However, these tools are often subject to computational constraints related to the necessity to run them in real-time. Moreover, first principles models used to power said emulations may presents some limits in describing complex behavior of the system of interest, such as transient dynamics or aging of the components.

During aircraft development, necessity for real-world testing usually arises, alongside emulation activities discussed above. In these instances, real world data obtained from the real system are naturally collected and stored, during bench testing of the component or flight tests. In this availability of data lies a valuable opportunity to fill the gap between emulation models and the actual behavior of the system.

Available data could be employed to derive fully data-driven models of the object to be emulated, or to enhance existing first principle models in a *grey-box* fashion. However, the grey-box approach is often preferred, as it allows to exploit prior knowledge of the system and typically leads to accurate models in a more data-efficient way. Use of grey-box approaches found wide application in recent years in the aerospace industry and in affine fields. Bidarvatan et al. [1] showed the potential of simple feedforward neural networks architectures for grey-box modeling of thermal engines, with application in performance analysis and control design for the engine of interest. Kosova et al. [2] adopted a random forest algorithm for the development of a grey-box model for failure detection of a hydraulic pump, while Li et al. [3] used a support vector machine to improve the performance of a model for prediction of a pump pressure signal. Rahideh et al. [4] developed a grey box model of the dynamics of an experimental vehicle, based on a genetic algorithm approach for parameters identification. Jackson et al. [5] used a gaussian process approach to create a grey-box model for the prediction of pitch, roll and yaw rate of a helicopter. Zilletti et al. [6] compared a neural network approach and a gaussian process approach to develop a grey-box model of a helicopter main rotor lead-lag damper.

Among the different machine learning techniques discussed above, Gaussian Processes offer an interpretable measure of the accuracy of the prediction, along with a powerful generalization capability [7]. The adoption of a Gaussian Process regression approach naturally comes at the cost of a higher computational burden, but effective and well consolidated approximation techniques for sparse Gaussian Processes are available [9] [10] to deal with this issue. Moreover, Gaussian Processes have been proven to be well suited for application to regression of dynamical systems [11] [12], which is often the category in which items replicated by emulation tools of interest for the aerospace industry lies (whether they are components, whole sub-systems of the aircraft or the aircraft dynamics itself). Schrunner et al. [13] used a sliding-window approach to provide a hydrological model starting from precipitation data. Even though the sliding-window approach was proven to be effective for some tasks, it fails to represent long term time dependencies. A popular option for time-series regression is given by the category of the Gaussian Process Dynamical Models [14], that allows to perform dynamics regression on a low dimensional latent space, which is subsequently mapped to an observable one with a second regression model. Based on a somehow similar concept, autoregressive models perform time-series regression by exploiting previous inference results to perform regression on correlated outputs of interest [15] [16].

In this paper, an autoregressive Gaussian Process approach is proposed for the enhancement of an existing white-box model of a hydraulic actuator, used for the control of the surface of an aircraft. Firstly, an overview on Gaussian Processes is discussed, with insight on sparse approximation and Gaussian Process autoregressive formulation. Subsequently, the model architecture is presented, starting from some key characteristics of the original white-model and leading to the formulation of the learning model investigated in this research. Then, results of the application of the autoregressive grey-box model are discussed. Experimental scenarios will involve synthetic data, used to assess the performance of the autoregressive model in a controlled environment. Afterwards, results obtained from application of the model to real data will be displayed, to attest whether the proposed method shows promise for further development and integrated test applications. As the problem of interest emerged from needs in the field of certification and system engineering, possible application of the presented technology to said field are listed and discussed. Finally, main results and considerations are collected as a conclusion of this research.

3. GAUSSIAN PROCESS

According to the *function-space view* discussed by Rasmussen and Williams [7], a Gaussian Process can be described as a distribution over functions. More specifically, a Gaussian Process is a collection of random variables, any finite number of which have a joint Gaussian distribution.

A Gaussian Process $f(x)$ is completely defined by its mean function $m(x)$ and its covariance function $k(x,x')$, namely

$$m(x) = \mathbb{E}[f(x)]$$

$$k(x,x') = \mathbb{E}\big[\big(f(x)-m(x)\big)\big(f(x')-m(x')\big)\big]$$

leading to the following notation:

$$f(x) \sim \mathcal{N}\big(m(x), k(x,x')\big)$$

Therefore, a crucial aspect of the definition of a GP model for regression is the selection of a mean function, that will be here assumed to be always zero, and of a kernel function. A popular and versatile choice of kernel function is offered by the Squared Exponential kernel

$$k(x,x') = s_f^2 \, exp - \frac{(x-x')P^{-1}(x-x')}{2}$$

where s_f^2 represents the overall Gaussian Process variance and P is a length-scales diagonal matrix.

The terms s_f^2 and $P_{[i,i]}$ represents the hyperparameters of the Gaussian Process model and can be tuned by means of empirical methods, such as cross validation, or by maximization of the training data marginal likelihood [7]. For Gaussian Process models used in this paper, this last training method has been adopted.

3.1. Gaussian Process Regression

Assume to have access to noisy observation drown from function $f(x)$, namely

$$y = f(x) + \varepsilon$$

with ε representing identically distributed Gaussian noise with variance σ_n^2. The joint distribution representation of new test target values $\boldsymbol{f}^*$ of the function is hence given by

$$\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{f}^* \end{bmatrix} \sim \mathcal{N}(\mathbf{0}, \begin{bmatrix} K(X,X) + \sigma_n^2 I & K(X,X^*) \\ K(X,X^*) & K(X^*,X^*) \end{bmatrix}$$

where X and X^* are respectively inputs of function f that led to observation of $\boldsymbol{y}$ and – supposedly - of $\boldsymbol{f}^*$ and $K(X,X^*)$ is a matrix of covariance evaluated at all pairs of training and test points. By conditioning the joint Gaussian prior information on the test observations, one can obtain the predictive equations of Gaussian Process regression [7]:

$$\boldsymbol{f} \mid X, \boldsymbol{y}, X^* \sim \mathcal{N}\left(\bar{\boldsymbol{f}}^*, cov(\boldsymbol{f}^*)\right)$$

with

$$\bar{\boldsymbol{f}}^* = \mathbb{E}[\boldsymbol{f}^* \mid X, \boldsymbol{y}, X^*] = K(X^*,X)[K(X,X) + \sigma_n^2 I]^{-1}\boldsymbol{y}$$

$$cov(\boldsymbol{f}^*) = K(X^*,X^*) - K(X^*,X)[K(X,X) + \sigma_n^2 I]^{-1}K(X,X^*)$$

Thus, one can adopt $\bar{\boldsymbol{f}}^*$ as a statistical estimator of function f on test inputs X^*, while $cov(\boldsymbol{f}^*)$ provides insightful information on the accuracy of the prediction. Intuitively, $cov(\boldsymbol{f}^*)$ can be decomposed as the difference between two terms: the prior covariance to which is subtracted a term representing the information given by the available data on the target function. Notice that to compute this posterior prediction, a matrix inversion must be performed, leading to a cubic computational complexity $\mathcal{O}(N^3)$ where N is the number of prior observations.

3.2. Gaussian Sparse Approximation

In order to tackle the computational complexity issue mentioned above for Gaussian Process regression on large datasets, one can rely on sparse Gaussian Process approximations. Specifically, the Gaussian Process posterior can be approximately represented by selecting a set of *inducing points*, namely a small group of points in the input space that are used as support for approximation of the process covariance and hence for speeding up both training and inference. This kind of approximation grants inference computational complexity of $\mathcal{O}(M\,N^2)$, with M being the number of inducing points.

Inducing points can be selected manually or even algorithmically, as several available off-the-shelf approximation methods are suitable for gradient based selection of the sparse dataset [8][9][10]. However, for time-series regression induing points are often preferred to be equally spaced over the original training data [15]. For the model presented in this paper, inducing points are selected as equally spaced, and a Variational Free Energy approximation of the covariance function is adopted [8], following both observations on preliminary trials of this research and characteristics outlined in comparative study of sparse approximations performed by Bauer et al. [9].

3.3. Autoregressive Gaussian Process

Gaussian Process autoregressive models can be used to perform regression for multi-output systems, while capturing and exploiting dependencies between outputs. Consider the problem of modelling Q functions $y_q(x), q \in \{1, 2, \dots Q\}$ given a relationship of the type

$$y_1(x) = f_1(x)$$

$$y_2(x) = f_2\big(x, y_1(x)\big)$$

$$\dots$$

$$y_Q(x) = f_Q\left(x, y_{1:Q-1}(x)\right)$$

By applying the general product rule of probability over $p(y_{1:q-1}(x))$, the relationship over such functions can be modeled by a set of Gaussian Processes, according to Requeima et al. [15]

$$y_q \mid y_{1:q-1} \sim \mathcal{N}\left(0, \left(\left(x, y_{1:q-1}(x)\right), \left(x', y_{1:q-1}(x')\right)\right)\right)$$

Furthermore, the posterior predictive density also decomposes as a product of Gaussian conditionals, and the same goes for computation of the marginal likelihood for training and inference. Hence, given a natural order of the outputs, inference and learning of Q outputs can be decoupled into Q single-output Gaussian Process regression problems. This autoregressive approach is well suited for regression over complex multi-output systems, as well as for regression of time-series, where previous regression outputs can be adopted as new inputs for future inference [11].

4. MODEL ARCHITECTURE

4.1. White-Box Model

In this paper, a Gaussian Process autoregressive model is applied to predict the behavior of an aircraft control surface actuator for emulation purposes. The actuator is composed of a power stage (RAM) and a control stage manifold. The actuator converts an electrical command signal into mechanical output motion of the surface. The control stage is composed of an Electro-Hydraulic Servo-valve (EHV) and of an Integrated Three Function Valve (ITFV) for engagement control, pressure monitoring and emergency pressure relief.

The relationship between stages of the actuator can be displayed as follows, recalling the notation used previously for the definition of autoregressive models:

$$y_1(t) = EHV(t, u_{EHV})$$
$$y_2(t) = ITFV(t, u_{ITFV}, y_1, y_3)$$
$$y_3(t) = RAM(t, u_{RAM}, y_2)$$

where u_{EHV} accounts for the current control of the EHV stage, $while\, u_{ITFV}$ and u_{RAM} represents the effects of the external loads applied to the actuator; all $u_{EHV,ITFV,RAM}$ inputs are here assumed to have available measures. This structure is adopted by the white-box model of the actuator that will lay the foundations of the grey-box model developed during this research and is depicted in Figure 1.

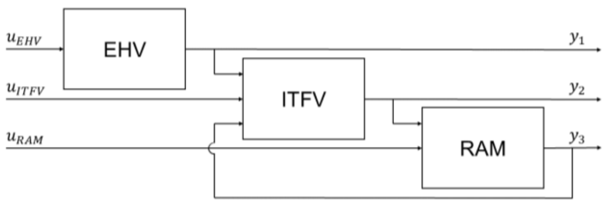

Figure 1: actuator white-box model representation.

4.2. Grey-Box Model

Given some data captured on the real aircraft actuator during bench or flight testing, or in presence of some synthetic target data of interest, the white-box actuator model can be improved thanks to an autoregressive Gaussian Process approach, in order to track the target behavior exploiting both available data and prior knowledge. The stage structure of the actuator suggests to employ autoregression to outline relationship between outputs. However, the structure described by the original white-box model presents a feedback loop between ITFV and RAM dynamics that does not allow to outline a clear order of the y_q outputs. To tackle this issue, the model can be reorganized by accounting for both ITFV and RAM behavior within a single Gaussian Process, while using a second process to map the ITFV output, namely

$$y_1(t) = EHV(t, u_{EHV})$$
$$y_2(t) = RAM_*(t, u_{ITFV}, u_{RAM}, y_1)$$
$$y_3(t) = ITFV(t, u_{RAM}, y_1, y_2)$$

This model structure now well suited for an autoregressive approach. Moreover, to take into account both the knowledge embedded in the white-box model and the evolution in time of the actuator, each Gaussian Process autoregressive unit is fed with inputs augmented with the corresponding prediction performed by the white-box model and the previous posterior prediction of the grey-box model, as suggested also by the results presented by Zilletti et al. for a similar task [6]. The autoregressive architecture of the grey-box model is captured in Figure 2.

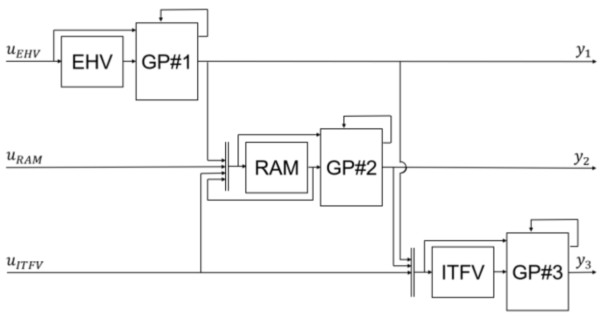

Figure 2: actuator grey-box autoregressive model representation.

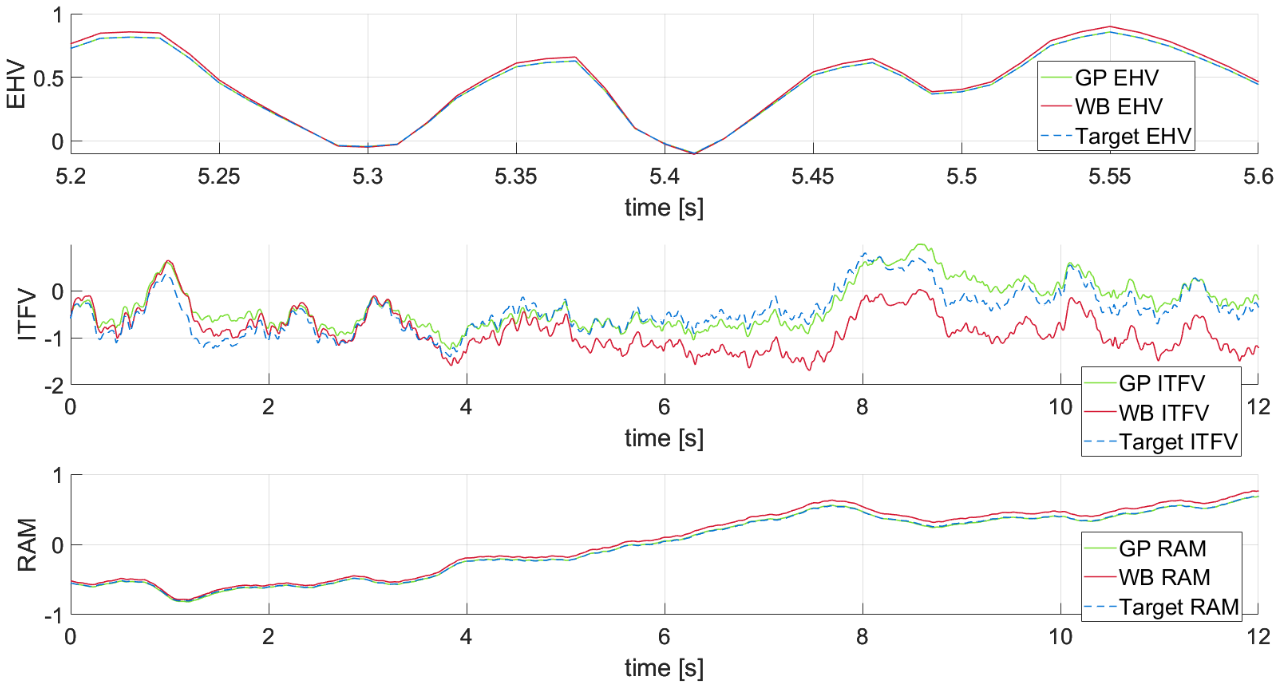

Figure 3: Comparative predictive results on synthetic data, comparing original white-box model (red), modified target model (dotted blue) and autoregressive Gaussian Processes predictions (green). Results for EHV stage are shown on a small portion of the validation sequence, to outline the actual difference between represented signals. Data are normalized over training target values.

5. RESULTS

The autoregressive Gaussian Process models presented above have been tested both on synthetic data and on real world measurements. Tests for both the white-box and the grey-box models have been performed in MATLAB environment. Gaussian Process regression have been implemented with open-source code provided by Rasmussen et al. [7].

The predictive performance of the Gaussian Process models has been evaluated in terms of Root Mean Squared Error (RMSE) over a test data sequence after training, where

$$RMSE = \sqrt{\sum_i \left(\overline{f_i^*} - f_i^*\right)^2}$$

Results obtained with white-box model and grey-box autoregressive GP models have been compared to obtain a measure of the actual improvement offered by the proposed data-drive enhancement technique.

5.1. Synthetic Data

First, the autoregressive Gaussian Process models performance were assessed on synthetic data, obtained by modifying the original white-box model to obtain a test target for model regression. Specifically, a simple proportional mismatch was introduced on the output of the Electro-Hydraulic Servo-valve stage, causing the simulated EHV to reach different positions than what expected by the original white-box model. Although this mismatch appears to be simple, the effects on the whole dynamics of the actuator follow fairly complex behaviors. The actuator models where both fed with an example of current command sampled during a flight test, in order to observe the resulting dynamics in semi-realistic conditions.

Starting from white-box original predictions, the autoregressive brey-box model achieves satisfactory results while tracking the modified dynamics. All three Gaussian Processes, predicting respectively EHV, RAM and ITFV stages, manage to significantly reduce the Root Mean Squared Error with respect to the white-box prediction on a validation dataset, as shown by the numerical results in Table 1. The autoregressive Gaussian Processes were trained on approximately 25000 data samples and subsequently approximated over less than 400 inducing points with a Variational Free Energy approach. Notice that during training the autoregressive inputs had to be replaced with the corresponding target data, as commonly done for autoregressive models training.

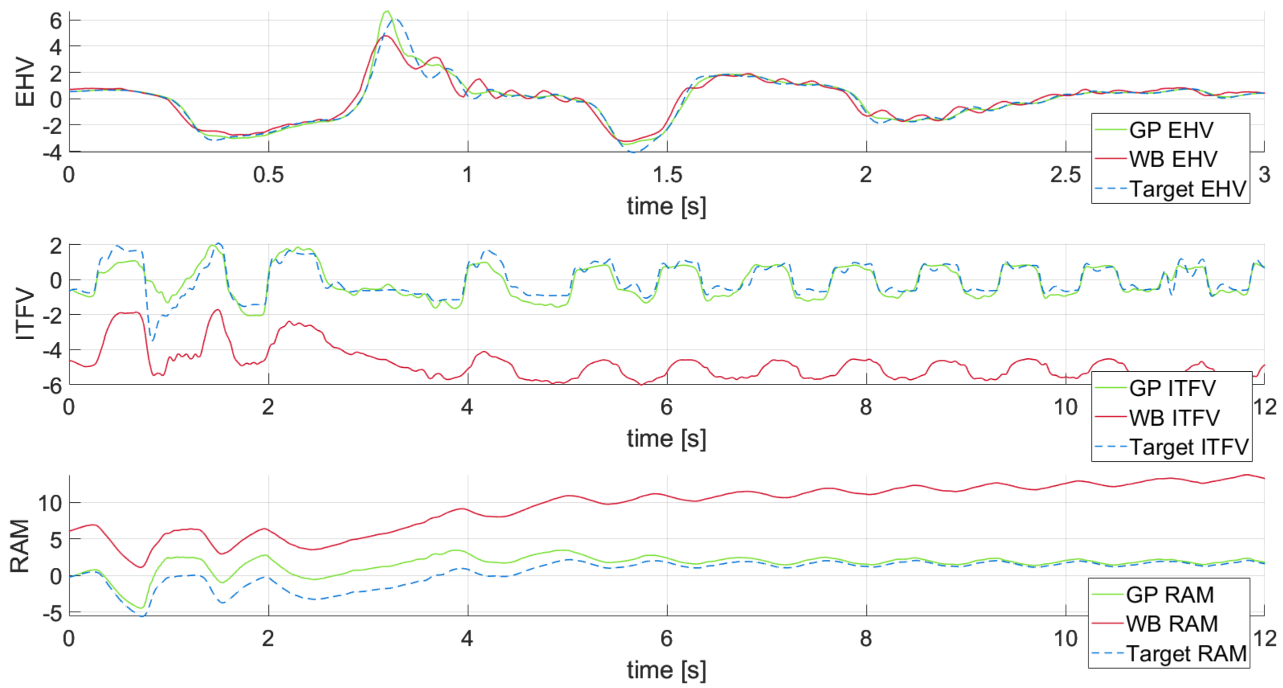

Figure 4: Comparative predictive results on real world test data, comparing original white-box model (red), target measurements performed on the real actuator during bench testing (dotted blue) and autoregressive Gaussian Processes predictions (green). Results for EHV stage are shown on a small portion of the validation sequence, to outline the actual difference between represented signals. Data are normalized over training target values.

	WB RMSE	GP RMSE	IMPROVEMENT
EHV	$2.5 \cdot 10^{-2}$	$1.7 \cdot 10^{-5}$	-99.9%
ITFV	$6.2 \cdot 10^{-1}$	$2.4 \cdot 10^{-1}$	-62.2%
RAM	$5.8 \cdot 10^{-2}$	$6.0 \cdot 10^{-3}$	-89.7%

Table 1: Comparative results of original white-box model and autoregressive grey-box model, for experiment performed on synthetic data. Data are normalized over training target values.

Validation of the regression performance is obtained on a 1300 validation dataset. The results obtained performing autoregressive predictions on the validation dataset are shown in Figure 3 for all actuator stages.

5.2. Test Bench Data

As the proposed autoregressive grey-box approach achieved promising results on synthetic data, it was also tested on a sample of data obtained from the real actuator during bench testing procedures. Even though the mismatch between the actuator behavior and the white-box model appeared to be more complex with respect to the simple proportional mismatch introduced during the previous experiment, the autoregressive grey-box approach obtained once again satisfactory results, by managing to significantly reduce the gap between the white-box model and the real world measurements, as shown in Figure 4.

In this case, the Gaussian Processes were trained on 3000 samples, while the prior was approximated with less than 50 inducing points with VFE approach. This encouraging result shows how the Gaussian Process regression approach can be both effective and data-efficient to tackle this kind of tasks. Notice also that an excess of inducing points does not always grants better predictions, as VFE does not dispose of unnecessary points, possibly leading to overfitting [9]. Hence, a careful selection of few inducing points can both result in efficient and yet satisfactory predictions and act as a regularization technique.

Validation of the regression performance is performed on almost 1300 sequential data, leading to the results shown in Figure 4 and summed up in Table 2. Both qualitative and numerical results show how the autoregressive grey-box approach outperforms the white-box original model, significantly improving the prediction accuracy.

	WB RMSE	GP RMSE	IMPROVEMENT
EHV	$3.0 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$	-48.5%
ITFV	4.9	$4.6 \cdot 10^{-1}$	-90.7%
RAM	9.3	1.5	-83.7%

Table 2: Comparative results of original white-box model and auto-regressive grey-box model, for experiment performed on real world bench testing data. Data are normalized over training target values.

6. APPLICATION TO SYSTEM ENGINEERING AND CERTIFICATION ACTIVITIES

During development stages of an aircraft, especially during Verification and Testing phase, the availability of actuator models in an integrated bench is almost fundamental.

To be used in verification and testing activities, the emulation model outputs have to be as representative of the real actuator as possible. Following the methodology proposed above, integration of real world data with prior knowledge leads to the creation of an actuator model that is significantly closer to the real actuator. However, to be used in certification activities the model must be validated against actual technical specification of the actuator. Any deviation from the real component must be recorded, to track what type of activities can be performed in simulation and what should be performed only on the real aircraft.

The use of a model instead of the real aircraft can lead to several benefits. First of all, a reduction of costs. Secondary, the simulated environment can be used to test safety critical conditions, in a completely safe environment. The enhanced models can be also used in validation and integration activities, like to simulate the integration of a new component—such as an updated control surface or actuator controller—allowing engineers to evaluate system behavior and performance before physical implementation, given the availability of a small amount of data sampled on the new component. In certification contexts, the increased fidelity of the models can aid in demonstrating compliance with safety and performance standards, especially in cases where full physical testing is impractical, constrained by cost or time, or when operating conditions are difficult to reproduce in a lab environment.

7. CONCLUDING REMARKS

The grey-box model presented in this paper exploits both prior knowledge and Gaussian Process regression in an autoregressive configuration to predict the behavior of a hydraulic actuator applied to the control surface of an aircraft. Validation results obtained both on synthetic data and real-world measurements show that the autoregressive grey-box model manage to significantly improve the performance of the original white-box model. While the obtained learning-based model outperforms the original white box model in the considered validation dataset, further effort should be applied in validation of the model for certification and system engineering testing. Specifically, the grey-box model may be assessed in terms of dynamics and frequency domain behavior, in order to assure that related characteristics for control design and testing purposes still reflects the behavior of the real actuator.

The promising results captured in this research encourages future application of this framework, both in the direction of further developing data-driven regression techniques for emulation and testing tools and in the field of certification and systems engineering.

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