SYSTEMATIC DETERMINATION OF AIRCRAFT INERTIA PROPERTIES BY SELECTIVE ASSEMBLY OF OPTIONAL KITS AND PAYLOAD PERMUTATIONS

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ABSTRACT

This paper describes a methodology for the determination of the aircraft inertia properties based on the selective assembly of optional kits and payload permutations. The methodology allows to calculate the W-CG envelope along a selected reference axis by maximizing the CG excursion at each step of the configuration buildup. Since the number of possible configurations for a given set of options and payload arrangements could easily become very high, the method is conceived to work on boundary configurations only in order to optimize the computation time. In addition, different levels of approximation for the W-CG envelope are available, further reducing the computational effort.

NOTATION

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Symbols

CG_k Aircraft center of gravity position along k-axis, m

ixx Single item x-axis Moment of Inertia, kg·m^2

Ixx Aircraft total x-axis Moment of Inertia, kg·m^2

iyy Single item y-axis Moment of Inertia, kg·m^2

Iyy Aircraft total y-axis Moment of Inertia, kg·m^2

izz Single item z-axis Moment of Inertia, kg·m^2

Izz Aircraft total z-axis Moment of Inertia, kg·m^2

ixy Single item x/y-axes Product of Inertia, kg·m^2

Ixy Aircraft total x/y-axes Product of Inertia, kg·m^2

ixz Single item x/z-axes Product of Inertia, kg·m^2

Ixz Aircraft total x/z-axes Product of Inertia, kg·m^2

iyz Single item y/z-axes Product of Inertia, kg·m^2

Iyz Aircraft total y/z-axes Product of Inertia, kg·m^2

m Single item mass, kg

M Aircraft configuration total mass, kg

xcg Single item center of gravity position along x-axis, m

Xcg Aircraft center of gravity position along x-axis, m

ycg Single item center of gravity position along y-axis, m

Ycg Aircraft center of gravity position along y-axis, m

zcg Single item center of gravity position along z-axis, m

Zcg Aircraft center of gravity position along z-axis, m

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Acronyms

BEW Baseline Empty Weight

EEW Equipped Empty Weight

MFW Max Fuel Weight

MoI Moment of Inertia

MTOW Maximum Take-Off Weight

OEI Operational Equipment Items

OEW Operational Empty Weight

PoI Product of Inertia

W-CG Weight - Center of Gravity

ZFW Zero Fuel Weight

INTRODUCTION

Weight and balance data are fundamental inputs in the design of flying machines. Key parameters such as aircraft weight, center of gravity position, and moments of inertia are crucial for a proper sizing of power plants, flight controls, and critical structural elements, (Ref. 1). Additionally, for the end user, being aware of the weight and center of gravity position (W-CG) envelope limits is essential to plan and ensure safe, stable, and efficient flight operations, (Ref. 2).

The W-CG envelope of an aircraft shall be established by considering all possible loading combinations foreseen for a given program, including cabin layouts, optional kits, passengers, bags and fuel. This task can become cumbersome in helicopter applications, where the number of customizations in terms of layouts and equipment required to be competitive in the market might be considerable. Therefore, covering all possible loading conditions with a brute-force approach might require an unmanageably high number of cases.

This work deals with an innovative methodology based on selective assembly of the optional kits following the principle of maximizing the CG excursion. However, it covers all possible payload permutations, without any restrictions. The methodology allows for a significant reduction in computation time by limiting the calculation to the boundary configurations only and providing different levels of approximation of the resulting W-CG envelope.

AW09 AIRCRAFT OVERVIEW

The methodology has been developed and introduced within the kopter AW09 program. This platform is the only all-new design single engine helicopter in the last four decades, engineered using the latest technologies and advanced materials, and setting a new reference in the light single engine helicopters category. Figure 1 illustrates the main characteristics of the AW09, including crashworthy composite structure, fuel tank and interior solutions, IFR-compatible Garmin G3000H integrated flight deck system, low vibration five-blade main rotor and low noise signature shrouded tail rotor. The AW09 helicopter is designed to stand out from the competition, offering technologies, cabin volume and performance typically belonging to twin-engine helicopters, at the acquisition price and operating cost of a single-engine turbine one. Equipped with the Safran Helicopter Engine Arriel 2K turbine, the latest and most powerful evolution of the Arriel family for single engine aircrafts, the AW09 offers outstanding payload and sling-load capabilities. Thanks to its modularity, the AW09 cabin can be configured in multiple arrangements (see examples in Figure 2, 3, 4 and 5), accommodating one pilot and up to eight individual crash-worthy passenger seats, while ensuring unobstructed rear access through the clamshell doors for personal items and medical operations.

Figure 1. kopter AW09.

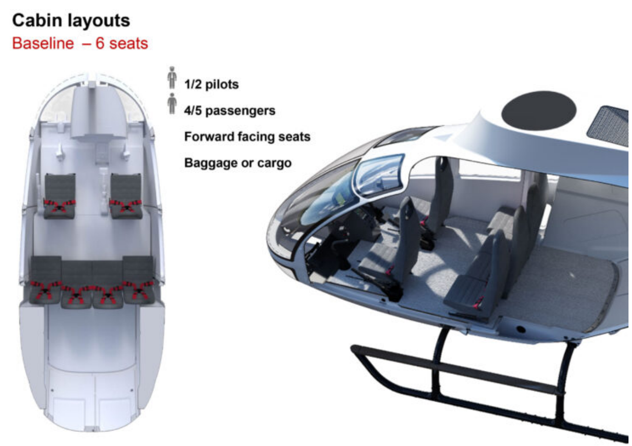

Figure 2. AW09 Baseline cabin layout.

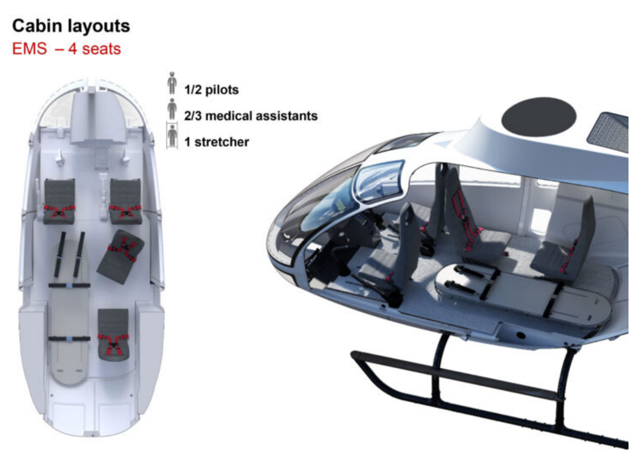

Figure 3. AW09 EMS cabin layout.

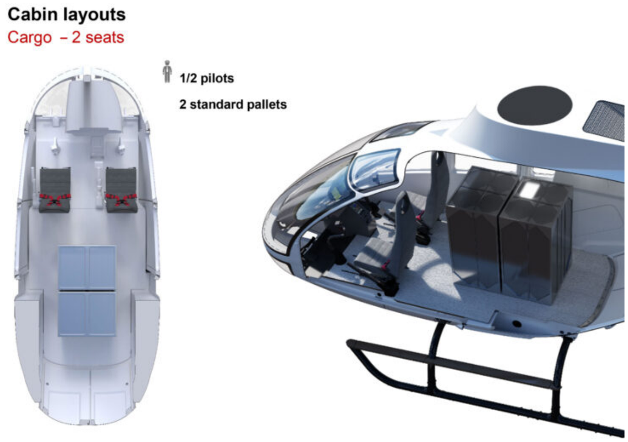

Figure 4. AW09 Cargo cabin layout.

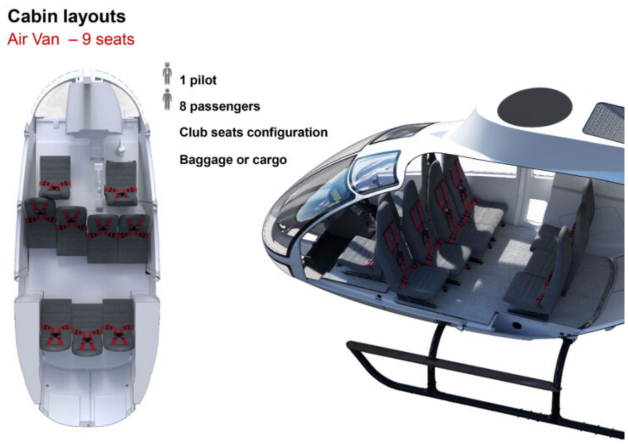

Figure 5. AW09 Air Van cabin layout.

METHODOLOGY DESCRIPTION

The rotorcraft configuration is subject to change depending on the field of application, the number of passengers and crew members, the seat layout, and the optional kits installed to perform specific missions. Generally, a multi-role helicopter can operate in various scenarios, including sightseeing, medical rescue, VIP transport, utility, law enforcement, and oil & gas applications. For each of these applications, here referred to as "roles", different cabin layouts can be adopted, each resulting in a specific configuration in terms of weight (W), center of gravity location (CG), and Moments and Products of Inertia (MoI/PoI).

Estimating the aircraft W-CG envelope has historically been one of the main challenges in any new project. Having a solid and systematic methodology to refine the envelope estimation at different stages of product development - especially in the early phases - can constitute a key element in ensuring the availability of up-to-date and more reliable design data.

The methodology described in this paper is based on two main pillars:

1) Selective options mounting: The different optional kits are mounted on the aircraft in a specific sequence, determined according to the principle of maximum CG excursion.

2) Computation time reduction: At a certain step in the calculation process — specifically, before the installation of the Operational Equipment Items (Step 3 in Figure 6) — a selection of the configurations to be considered for proceeding with the W-CG envelope calculation is made. In particular, only the boundary configurations calculated after Step 2 are retained, under the assumption that any other starting point would result in a configuration falling inside the final W-CG envelope.

Configuration build-up

The build-up process of any AW09 configuration starts always from the Baseline Empty Weight (BEW) and considers, in sequence, optional kits, cabin layout, pilot and Operational Equipment Items (OEI, i.e. essential equipments like medical and emergency kits), payload and, finally, fuel. At each step, Weight, Center of Gravity position and Moments and Products of Inertia are calculated by adding - in terms of inertial characteristics - the new element on top of the configuration built so far. Figure 6 illustrates the concept.

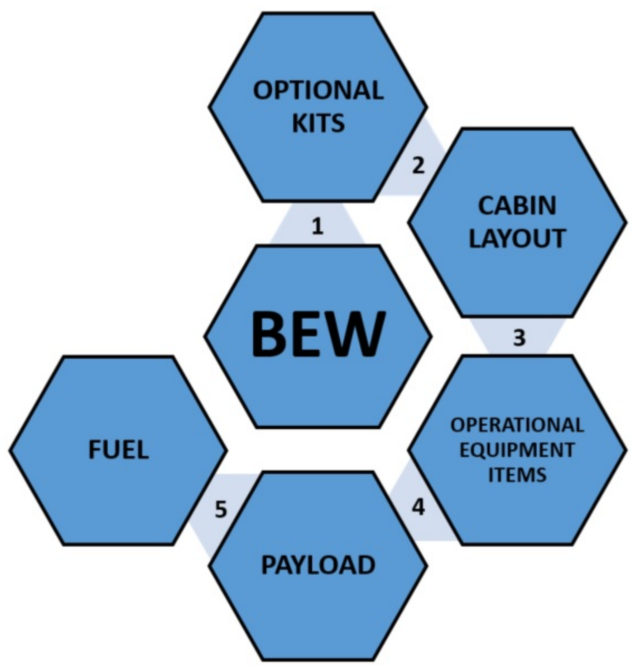

Figure 6. Aircraft configuration Build-Up

The reference frame used throughout the whole process is illustrated in Figure 7.

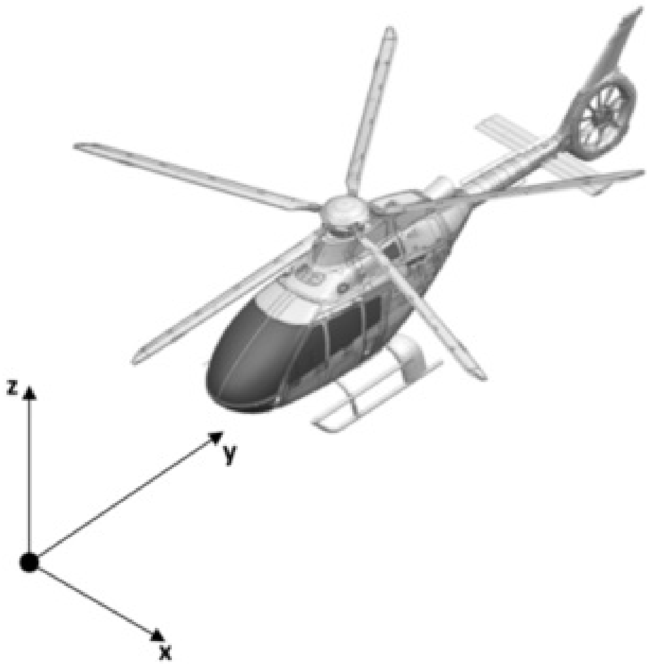

Figure 7. Reference frame for inertial characteristics determination.

When a new element is added, e.g. at step *(i)*, to the configuration obtained up to step *(i-1)*, the inertial characteristics are updated as per equation 1 to 10, with obvious notation.

$$M_{(i)} = M_{(i-1)} + m \tag{1}$$

$$Xcg_{(i)} = \frac{M_{(i-1)}Xcg_{(i-1)} + mx_{cg}}{M_{(i-1)} + m} \tag{2}$$

$$Ycg_{(i)} = \frac{M_{(i-1)}Ycg_{(i-1)} + my_{cg}}{M_{(i-1)} + m} \tag{3}$$

$$Zcg_{(i)} = \frac{M_{(i-1)}Zcg_{(i-1)} + mz_{cg}}{M_{(i-1)} + m} \tag{4}$$

$$\begin{aligned} Ixx_{(i)} = Ixx_{(i-1)} + ixx + \\ + M_{(i-1)}\left((Ycg_{(i-1)} - Ycg_{(i)})^2 + (Zcg_{(i-1)} - Zcg_{(i)})^2\right) \\ + m\left((ycg - Ycg_{(i)})^2 + (zcg - Zcg_{(i)})^2\right) \end{aligned} \tag{5}$$

$$\begin{aligned} Iyy_{(i)} = Iyy_{(i-1)} + iyy \\ + M_{(i-1)}\left((Xcg_{(i-1)} - Xcg_{(i)})^2 + (Zcg_{(i-1)} - Zcg_{(i)})^2\right) \\ + m\left((xcg - Xcg_{(i)})^2 + (zcg - Zcg_{(i)})^2\right) \end{aligned} \tag{6}$$

$$\begin{aligned} Izz_{(i)} = Izz_{(i-1)} + izz \\ + M_{(i-1)}\left((Xcg_{(i-1)} - Xcg_{(i)})^2 + (Ycg_{(i-1)} - Ycg_{(i)})^2\right) \\ + m\left((xcg - Xcg_{(i)})^2 + (ycg - Ycg_{(i)})^2\right) \end{aligned} \tag{7}$$

$$\begin{aligned} Ixy_{(i)} = Ixy_{(i-1)} + ixy \\ + M_{(i-1)}\left((Xcg_{(i-1)} - Xcg_{(i)})(Ycg_{(i-1)} - Ycg_{(i)})\right) \\ + m\left((xcg - Xcg_{(i)})(ycg - Ycg_{(i)})\right) \end{aligned} \tag{8}$$

$$\begin{aligned} Ixz_{(i)} = Ixz_{(i-1)} + ixz \\ + M_{(i-1)}\left((Xcg_{(i-1)} - Xcg_{(i)})(Zcg_{(i-1)} - Zcg_{(i)})\right) \\ + m\left((xcg - Xcg_{(i)})(zcg - Zcg_{(i)})\right) \end{aligned} \tag{9}$$

$$\begin{aligned} Iyz_{(i)} = Iyz_{(i-1)} + iyz \\ + M_{(i-1)}\left((Ycg_{(i-1)} - Ycg_{(i)})(Zcg_{(i-1)} - Zcg_{(i)})\right) \\ + m\left((ycg - Ycg_{(i)})(zcg - Zcg_{(i)})\right) \end{aligned} \tag{10}$$

Options management and mounting sequence

The first items added on top of the BEW in the configuration build-up are the optional kits, whose selection and installation sequence represent a core aspect of the proposed methodology. In particular, the following two steps are considered crucial to ensure efficient input data:

1) Define options requirements and incompatibilities: Among the available options, some items require the presence of others for their installation. These dependent items are referred to as *requirements* (e.g., fixed and removable parts of specific equipment: the fixed parts are considered requirements for the removable ones). In contrast, the installation of certain items automatically excludes others; in such cases, we refer to *incompatibilities* (e.g., different components that share the same mounting interface, or alternative items like mirrors or cameras). A precise definition of requirements and incompatibilities is essential to support options selection during the configuration build-up.

2) Merge options: Once all the requirements have been identified, the involved options are merged together - applying the same formulae of equations 1–10 - to form a single item. If multiple items share the same requirements, the process will result in several combined elements, as illustrated in Figure 8. These combined elements will, in turn, become mutually incompatible.

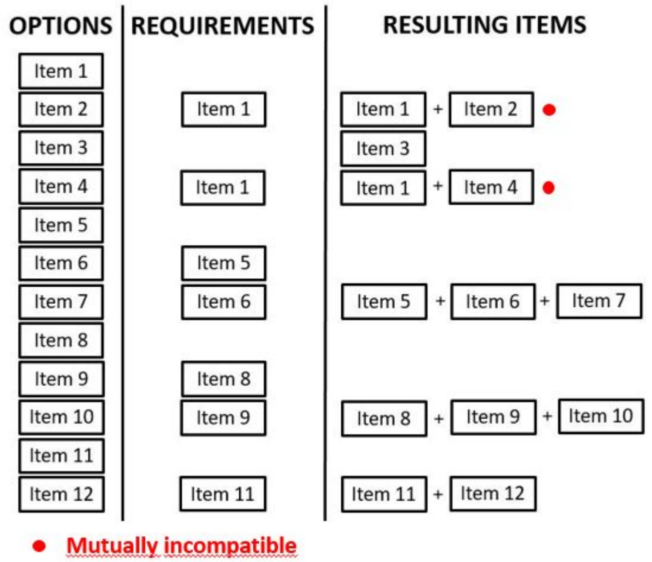

Figure 8. Example of options merging results

Once the options have been successfully merged and any potential incompatibilities have been clearly identified, the process of mounting the options onto the BEW configuration begins. In this phase, the set of optional kits is not treated as a single unit and mounted all at once; instead, the options are mounted sequentially, each time selecting the one that leads to the maximum CG excursion.

Depending on the considered axis and direction - typically, most applications involve the longitudinal one - this results in the solution of one of the two problems

$$\max_{i=1:N}\left(\frac{dCG_k}{dm_i}\right) \tag{11}$$

$$\min_{i=1:N}\left(\frac{dCG_k}{dm_i}\right) \tag{12}$$

where m_i is the mass of the $i-th$ option, CG_k is the CG excursion along axis k and N is the number of remaining options, which decreases at each step according to the logic illustrated in Figure 9. The figure also shows how incompatibilities are handled: once an item is added to the configuration, all incompatible options are subsequently discarded.

Figure 10 provides a graphical representation of the option selection by minimization of the derivative dCG_y/dm, which allows to obtain the forward branch of the envelope.

Once the options list have been cleared, meaning that all the possible options leading to the maximum CG excursion have been installed, a branch of the envelope along the selected axis has been defined.

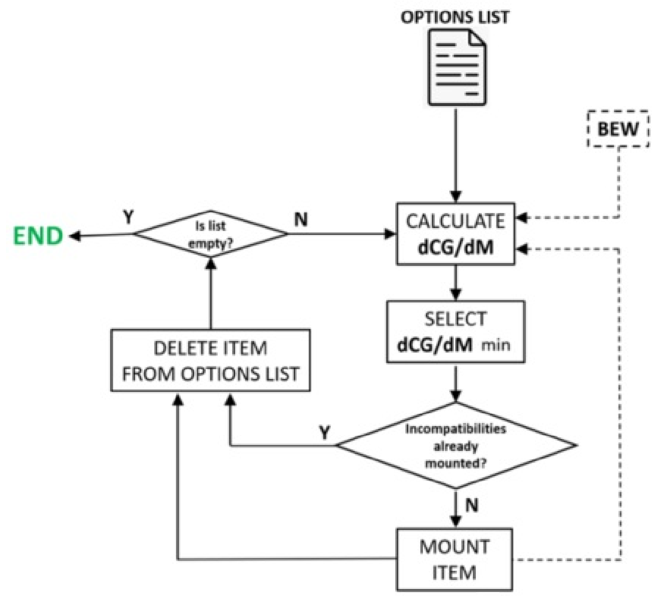

Figure 9. Options mounting process

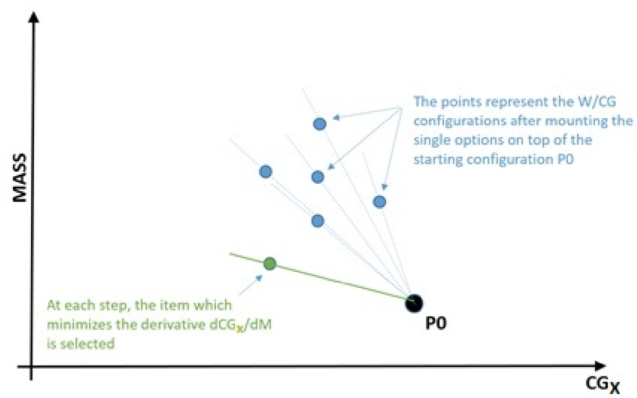

Figure 10. Example of options selection

The mounting sequence dictated by the logic of Figure 9 results in the maximum CG excursion along the selected axis and direction (e.g., minimizing dCG_y/dM_i leads to the maximum forward CG excursion). To obtain the maximum excursion in the opposite direction using the same set of options, it is sufficient to mount them in reverse order.

Mounting the optional kits onto the aircraft BEW leads to the Equipped Empty Weight (EEW) configuration.

Cabin Layouts W-CG envelope and approximations

Different cabin layouts - i.e. seats arrangements and special equipments and/or kits - can be available for operating across the multiple roles foreseen for the platform.

In the configuration build-up process, a cabin layout is considered as a single item, with its own inertial properties.

A single cabin layout can be used in combination with different optional kits to operate across the various platform roles, resulting in a multiplicity of configurations that can easily reach a very high number: for this reason, before Step 3 of the configuration build-up process illustrated in Figure 6, a selection of the boundary configurations for each cabin layout is performed, in order to reduce the computation time of the remaining steps (i.e., the installation of OEI, plus payload and fuel).

Figure 11 illustrates the concept of boundary configurations: among all possible configurations, only those lying on the enveloping curve are considered in the subsequent steps of the configuration build-up. This significantly reduces computation time, by up to a factor of 10^3 depending on the number of considered layouts, roles and optional kits.

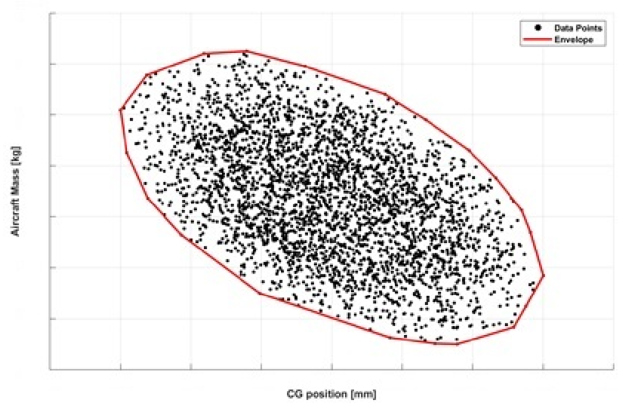

Figure 11. Example of W-CG points envelope for a single layout

To further limit the computational effort — particularly during project phases when reliable data may not be available and extensive computation would therefore not be justified — three levels of approximation for the envelope depicted in Figure 11 have been defined:

1) **LOW (Approx 1):** The low level approximation consists of the points having the maximum/minimum weight and the maximum/minimum CG position along the considered axis. Theoretically, the maximum number of points for this approximation is four, as shown in the example of Figure 12.

2) **MIDDLE (Approx 2):** The middle-level approximation builds upon the low-level one by adding one point to each side. The added points are the farthest possible from each side of the low-level approximation, resulting in a theoretical maximum of four additional points, for a total of eight. Figures 13 and 14 illustrate an example of point selection and the resulting envelope.

3) **HIGH (Approx 3):** The high-level approximation builds upon the middle-level one by adding points in the same manner, with the only difference being that a point is added only if it increases the envelope coverage by more than 0.5%. The theoretical maximum number of points for this approximation is sixteen. Figure 15 shows the result on the example envelope

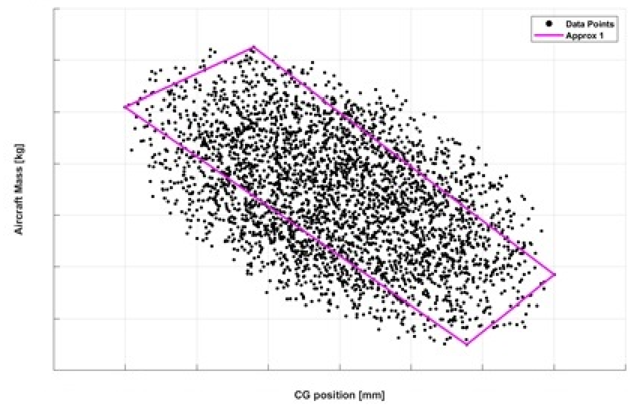

Figure 12. Low Level W-CG envelope approximation

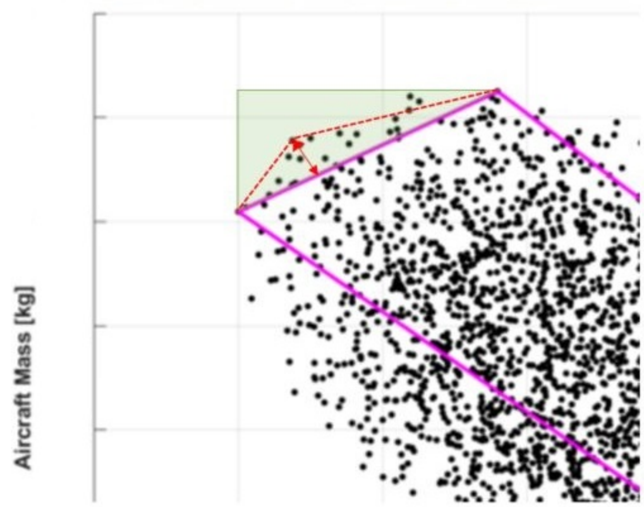

Figure 13. Example of point selection in envelope Approximation

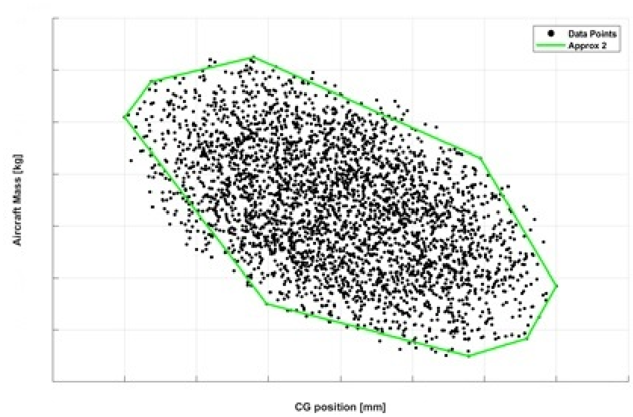

Figure 14. Middle Level W-CG envelope approximation

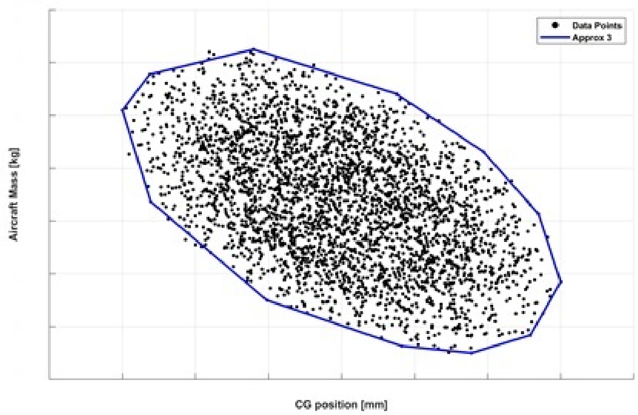

Figure 15. High Level W-CG envelope approximation

Figure 16 shows that the number of points characterizing the high level approximation falls in most cases in the range [11, 13] ($\pm\sigma$).

Figure 17, 18 and 19 show the probability density of the envelope coverage for the three different levels of approximations, while Table 1 reports the $\pm 3\sigma$ ranges calculated from a statistical database of 20,000 cases.

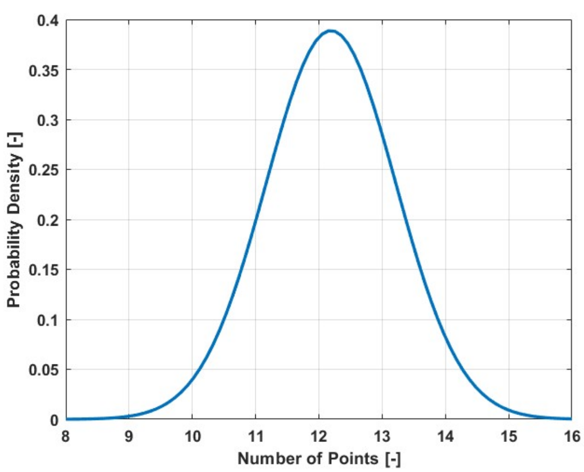

Figure 16. High Level W-CG envelope approximation number of points

Table 1. Envelope coverage at $\pm 3\sigma$

Approx Level	Min cover [%]	Max cover [%]
No Approx (Fig. 11)	-	100
Low Level (Fig. 12)	33	81
Middle Level (Fig. 14)	81	99
High Level (Fig. 15)	95	99.9

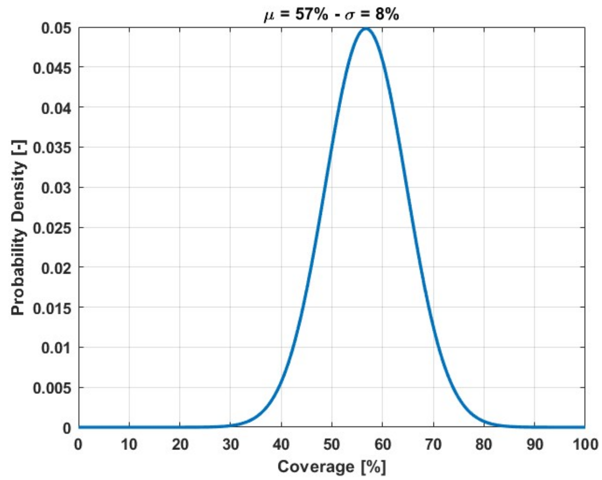

Figure 17. Low Level W-CG envelope coverage

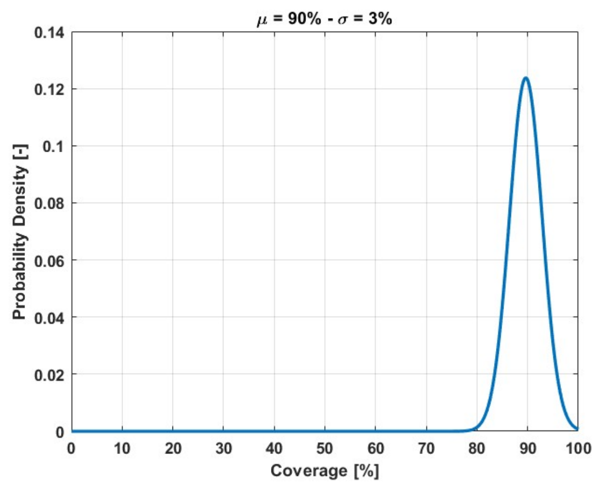

Figure 18. Middle Level W-CG envelope coverage

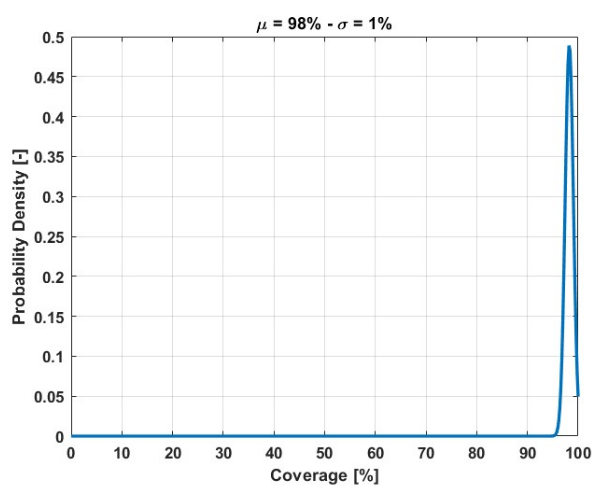

Figure 19. High Level W-CG envelope coverage

OEI and Payload mounting

Once the cabin layout envelopes have been calculated and the level of approximation has been selected, the next steps involve mounting the OEI — treated as a single item with its inertial properties — and accommodating the payload. These two steps bring first to the Operational Empty Weight (OEW), and then to the Zero Fuel Weight (ZFW).

Each cabin layout provides a number n of seats, and the arrangement of k passengers on the available seats results in a number of configurations determined by the well-known binomial formula shown in equation 13:

$$N_P = \frac{n!}{k!(n-k)!} \tag{13}$$

In this phase, an average weight of 75 kg per passenger is considered, plus optional 20 kg per passenger to account for their luggage. Each payload permutation results therefore in a single item with its own inertial properties, determined by the way passengers are accommodated within the cabin. All the resulting payload configurations, both with and without baggage, are then added on top of each point of the OEW envelope, to get to the ZFW configurations.

Fuel loading

The last step to get to the final aircraft configuration is the fuel loading, which brings from the ZFW to the Max Fuel Weight (MFW).

The fuel curve is a characteristic of the platform and depends on the shape and capacity of the installed tanks. An example for the AW09 is provided in Figure 20.

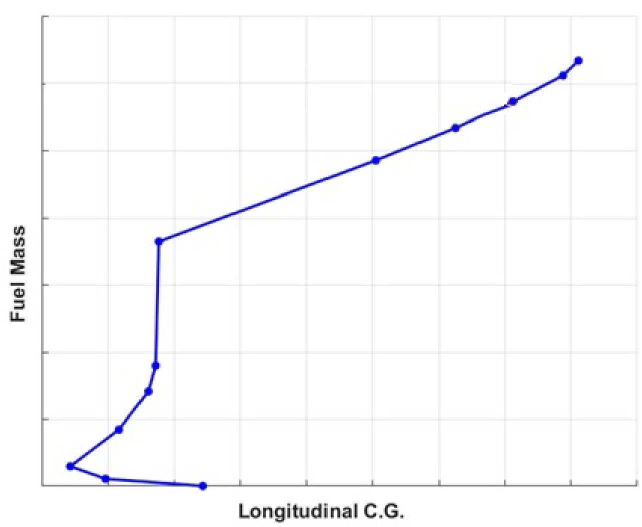

Figure 20. Example of fuel curve

The fuel curve is loaded on top of each ZFW configuration, resulting in the final cloud of aircraft configurations, with a total number of points possibly reaching the order of 10^6.

Figure 21, 22 and 23 provide a visual representation of the obtained results for the Air Van cabin layout of Figure 5, showing that, for this single cabin layout, the number of possible configurations reaches nearly 620 000 in the case of no envelope approximation.

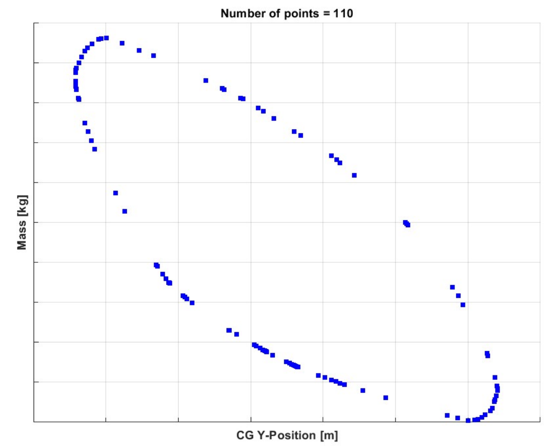

Figure 21. Example of Air-Van Layout Envelope - OEW

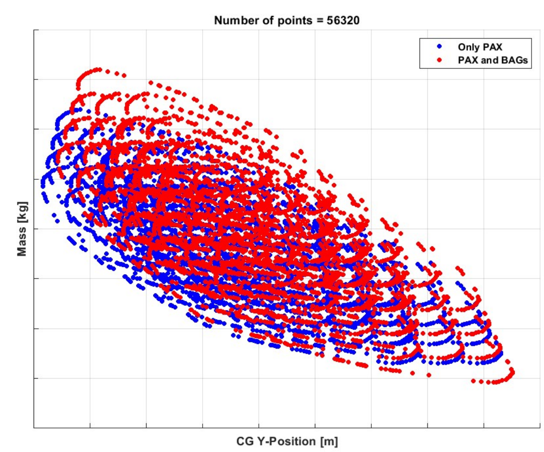

Figure 22. Example of Air-Van Layout Envelope - ZFW

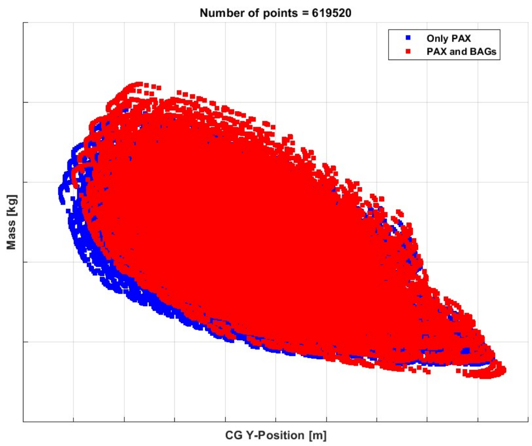

Figure 23. Example of Air-Van Layout Envelope - MFW

In general, the total number of configurations is expressed by equation 14:

$$NC = \sum_{i=1}^{n} N_{Li} \cdot (2 \cdot N_{Pi}) \cdot N_F \tag{14}$$

where n is the number of cabin layouts, N_{Li} is the number of points in the envelope of each layout, N_{Pi} is the number of payload permutations for each layout (multiplied by 2 to account for the baggage options), and N_F is the number of points of the fuel curve.

Final W-CG envelope determination

Once the MFW configurations have been calculated for all the cabin layouts, their superposition leads to the final cloud of points on the W-CG position plane, which represents all the possible aircraft configurations, covering all the combinations of roles, cabin layouts, payload and optional kits.

A curve enveloping these points, such as the one shown in Figure 24, represents the actual W-CG envelope along the selected axis (note that there is a W-CG envelope for each reference axis), upper limited by either the MFW or the Maximum Take-Off Weight (MTOW), whichever is lower. This curve is exact and does not involve any approximation, meaning that no configurations are excluded. However, it may consist of a large number of points (95 in the example of Figure 24), which is impractical for real-world applications. Typically, the W-CG corner points are used in the aircraft design phase to size controls and structural elements: in these contexts, it is preferable to work with a lower number of points in order to reduce the amount of analysis to be performed, ensuring at the same time a conservative approach.

During certain phases of a project, when fresh and reliable data may not be available, it could be advantageous to define the envelope using the middle or high approximation levels in the same way as is done for cabin layouts, as both provide extensive envelope coverage while using a known — and relatively low — number of points representing feasible aircraft configurations. In the examples of Figures 25 and 26, the resulting coverage are 89% and 98%, with a number of points equal to 8 and 12 respectively.

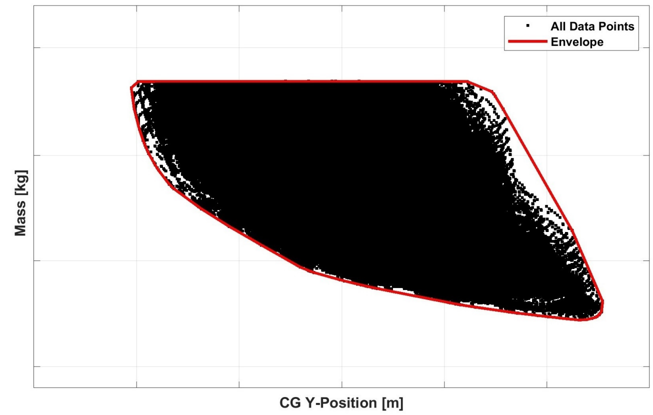

Figure 24. Example of W-CG envelope - No Approximation

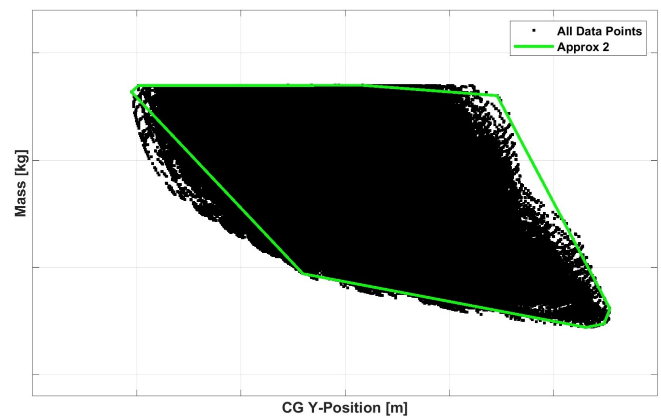

Figure 25. Example of W-CG envelope - Middle level Approximation

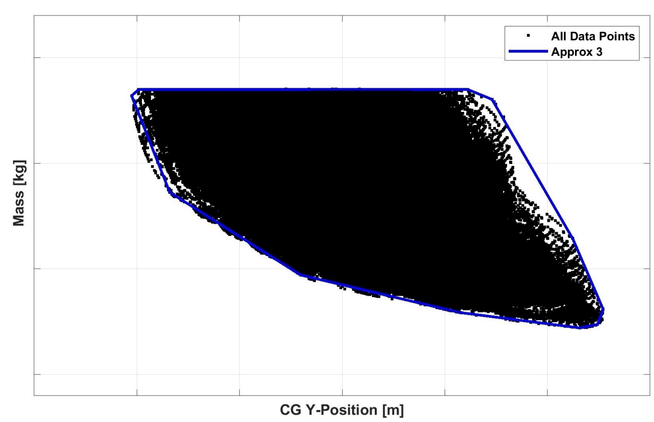

Figure 26. Example of W-CG envelope - High Level Approximation

One of the main advantages of the presented methodology is that any configuration — meaning any point in the cloud — can be uniquely identified thanks to the underlying data structure, which is updated at each step of the configuration build-up - see Figure 27 - and includes:

- **Inertial Properties:** Mass, CG position, MoI, PoI.
- **Role Index:** Each role foreseen for the platform has an associated index which is stored in the data structure.
- **Layout Index:** Each cabin layout has an associated index which is stored in the data structure.
- **Option Index:** Each step in the optional kits mounting is indexed, differently for the two envelope branches. This index is stored in the data structure.
- **Number of passengers**
- **Payload Permutation Index:** Each payload permutation has an associated index, which is stored in the data structure.
- **Bags Flag:** A flag indicating if the baggage are considered or not.
- **Fuel quantity**

This enables the designer to identify and retain any configuration that may initially fall outside the envelope approximation, but is considered meaningful based on its likelihood of use and importance in the intended applications. This will add another corner point to the envelope.

		Inertial Prop	Role Idx	Layout Idx	Option Idx	N pax	Perm Idx	Bags y/n	Fuel Qty
STEP #	1	•			•				
	2	•	•	•	•				
	3	•	•	•	•				
	4	•	•	•	•	•	•	•	
	5	•	•	•	•	•	•	•	•

Figure 27. W-CG envelope data Structure

An alternative approach to guarantee 100% coverage with a reasonably low number of points is to consider the middle-level approximation and apply suitable margins to some of its corners - ensuring that the resulting configurations are feasible for flight testing - so as to encompass all possible configurations. This can further reduce the number of corner points, at the expense of no longer representing actual operating aircraft configurations. Figure 28 clarifies the concept.

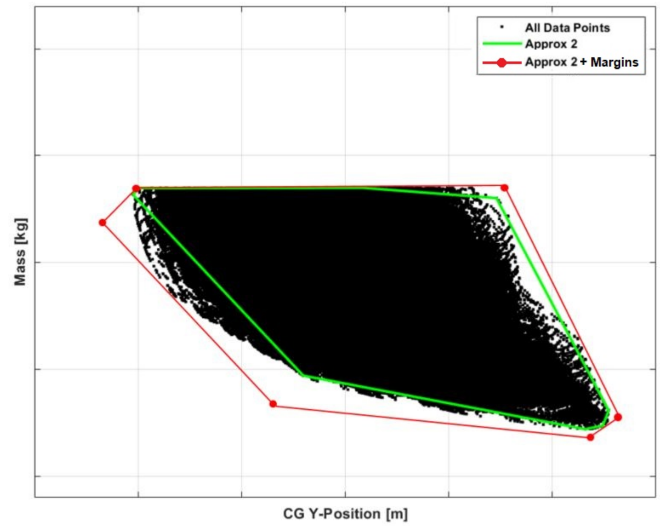

Figure 28. Example of W-CG envelope - Middle Level Approximation with margins

CONCLUDING REMARKS

The presented methodology allows to derive the aircraft W-CG envelope along a selected axis starting from an input dataset including the inertial characteristics of:

1) Empty aircraft
2) Optional kits
3) Cabin layouts
4) Operational Equipment Items
5) Payload
6) Fuel

The entire computation process is supported by a data structure that retains all relevant information about how each aircraft configuration is generated. This enables the designer to easily:

a) Identify every configuration within the W-CG envelope, including corner points
b) Identify any configuration(s) falling outside a predefined W-CG envelope

During the design of a new aircraft, the significant reduction in computation time — ensured by the presented methodology — allows for rapid refinement of the W–CG envelope boundaries as new and more mature data become available.

Future developments will focus on defining suitable ranges of inertial characteristics for the envelope corner points, which directly impact aircraft design and dynamic behavior.

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