IDENTIFICATION OF RIGID BODY STATE SPACE MODELS FOR THE BO-105 HELICOPTER USING MIMO ALGORITHMS IN THE FREQUENCY DOMAIN

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Abstract

Parametric transfer function models are calculated for the BO-105 helicopter using iterative multiple input/multiple output algorithms in the frequency domain. These models are then combined to derive a state space model for the 6 DoF rigid body dynamics of the rotorcraft. The MIMO identification algorithms involved have been implemented in a program package for MATLAB using graphical user interfaces. The software design, structure and features are discussed and some of the identified models are presented in Bode plots. The time domain simulation using the state space model is presented and the identified modes are compared with those estimated by the working group 18 of AGARD.

1 Introduction

When applied in aerospace engineering, the objective of system identification is to estimate stability and control derivatives. These coefficients are the elements of the state matrix and input matrix in the state space equation that describes the dynamical behavior of the aircraft. The parameters are estimated using flight data that is recorded during special flight tests. Mainly due to the presence of the rotor, helicopters have to be treated differently than fixed wing aircraft. This leads to special approaches for the identification of rotary wing aircraft. Working Group 18 of NATO's Advisory Group on Aerospace Research and Development (AGARD) has investigated many of these procedures (Ref. 1). The procedure that is followed at the TU Delft is based on frequency domain methods, since these methods have several advantages for the identification of helicopters (Ref. 3, 4, 10).

1.1 Frequency Domain Methods

At the TUD and NLR, system identification for fixed wing aircraft has been practiced for many years. So far, time domain techniques have been the main approach (Ref. 2). However, for helicopters, the coupling between symmetric and asymmetric movements (mainly due to the presence of the rotor) and the extra degrees of freedom of the rotor (flapping and lead-lag hinges) result in models, with a large number of parameters. Simultaneous estimation of all these parameters poses a big problem. Some of these problems can be reduced, using frequency-domain techniques. Some of the key benefits of frequency-domain analyses are:

- MIMO systems can be decomposed into MISO and conditioned into separate SISO systems. This way, smaller models with less parameters can be identified. Afterwards, the SISO models can be combined into a MIMO model again.
- The 'quality' of the identified non-parametric transfer-functions can be assessed, via the coherence-function. The coherence functions can also be used for early evaluation of flight data, different input signals performance evaluation, ordering the inputs etc.
- The frequency range of fit can be restricted and/or frequency weighting can be applied.
- The model structure (transfer functions) can be based on visual inspection of the non-parametric transfer functions.
- The non-parametric transfer functions are unbiased in the presence of measurement and process noise.
- Time delays can be estimated directly.

Higher order models with widely spaced dynamic modes (e.g. fuselage and rotor modes) can be identified more easily. In this way, aeroelastic modes can be separated and analysed.

2 Identification Process

For identification a good approach with user friendly software is necessary. Therefore, a software-package for system identification for helicopters in frequency domain was developed on a personal computer, using Matlab 4.2 for Windows 95 (Ref. 6, 8). These algorithms are based on the new theoretical and software developments at TUD (Ref. 3 to 8). Extensive graphical user interfaces with on-line help facilities were built to facilitate the interaction between the analyst and the computer. As frequency domain analyses is graphics intensive, user friendly graphic routines are implemented for good insight. The package is built in a modular fashion.

Since the applied approach to system identification is entirely based on off-line methods, the identification procedure can be divided in a part concerning flight tests and data preprocessing, and a part concerning the actual system identification. In harmony with the main object of this study - the development of frequency domain algorithms - the latter part is emphasized here.

2.1 Flight Tests and Data Preprocessing

Stability and control derivatives are dependent on the aircraft's flight conditions, so the several flight tests for the identification of one model must be performed under the same conditions as much as possible. Preferably, the tests should be flown in calm air.

To insure roughly the same conditions, multiple test runs can be conducted during one flight. However, care must be taken to limit the changes in the aircraft's weight, so the duration of a flight may need to be restricted.

Data recording must include registration of at least the time, the four input signals - control positions - Euler angles, angular rates, angular accelerations, linear speeds and linear accelerations. In addition, atmospheric conditions can be recorded to allow for transformation of air data to standard atmosphere conditions and rotor speed and engine torque can be recorded to be able to identify the rotorcrafts heave axis response. Sample rates should be 100 Hz or higher.

Input signals should be entered in one input at a time. However, compensating inputs in the other three channels to prevent dangerous aircraft attitudes or too large deviations from the equilibrium position are acceptable. Test runs should start and end in trimmed conditions. If possible, tests runs should be repeated several times for each input. It is important that input signals excite all the dynamical modes of the rotorcraft. Frequency sweeps seem very well suited for this purpose. However, optimal input design may lead to a different approach. The use of 3211 signals or doublets is also possible, but not preferred. Frequency sweeps can be performed manually; no automatic signal generator is necessary.

The most important part of data preprocessing is conversion of the data to MATLAB data files and flight path reconstruction. If atmospheric information is available, recorded or indicated airspeeds can be converted to equivalent airspeeds. Of course, position errors in the accelerations must be removed. Initial filtering of the data is allowed and mostly necessary to remove high frequency noise. Removal of trends or average values from equilibrium conditions or biases is allowed as well, but not necessary, since this is taken care of during identification.

2.2 Interactive Identification

Starting with the flight data, the identification of a state space model takes place in a stepwise manner. The procedure is therefore divided into modules. Since frequency domain methods are very graphics intensive, every identification module is linked to a corresponding plotting module. These graphical modules allow the analyst to view the results of every step in the process and to decide whether these are satisfying. The procedure is depicted in figure 1. Basically, the same concept has been applied succesfully by Tischler (Ref. 9).

2.2.1 MISO selection

In order to identify a multiple input/single output state space model, a complete set of transfer functions must be estimated. A complete set covers transfer functions for each input/output pair.
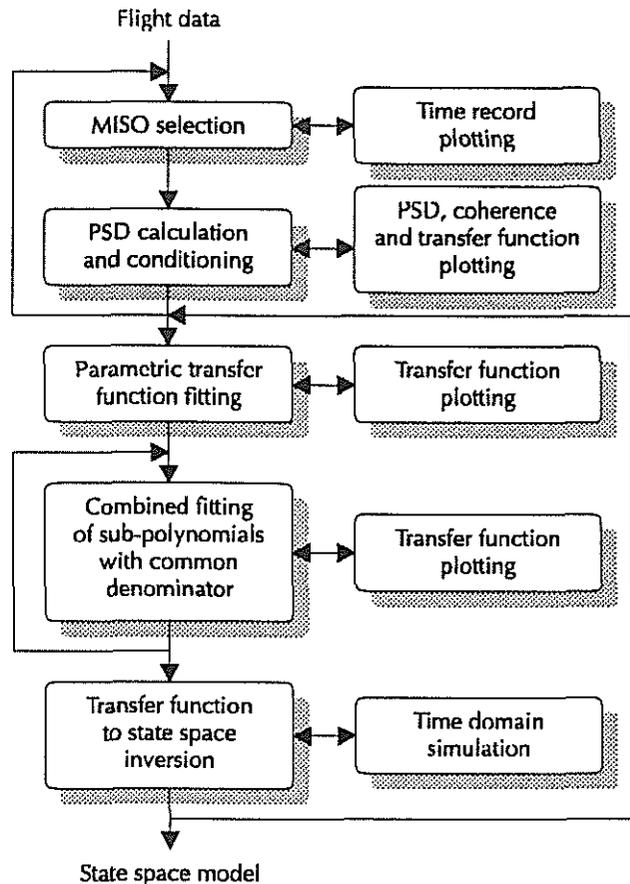

Figure 1: Identification process

The total number of transfer functions to be identified is therefore equal to the number of inputs times the number of outputs.

Although transfer functions are single input/single output, using one input signal and one output signal leads to incorrect estimates when the multiple inputs of the real helicopter are mutually coherent during the test run. To remove this effect by conditioning, all secondary inputs that are coherent to the primary input must be selected. In addition, the single output signal has to be selected.

Time records of the flight data can be plotted, to help the analyst in deciding which input signals should be included for the conditioning process. However, this can really be determined only after the conditioning itself by looking at the coherence functions and power spectra. Therefore, the first two steps are combined in an iteration.

2.2.2 PSD calculation and conditioning

From the flight data, auto power spectral densities for each input and the output, and cross power spectral densities between all inputs and between each input and the output are calculated. From these, coherence functions are determined.

Coherence functions and power spectral densities are used to condition the multiple input system. The result is a set of partial coherence functions and conditioned power spectral densities.

These are used to calculate the unbiased single input/single output transfer function.

The conditioning process can be monitored by plotting the coherence functions and spectral densities. Until all linear coherent parts are removed, the multiple input/single output selection must be adapted and the calculation and conditioning must be repeated. Finally, the estimated transfer function can be viewed to check for excessive noise.

2.2.3 Parametric transfer function fitting

A parametric transfer function, described by a numerator and a denominator polynomial, a static gain and a time delay, is fitted to the estimated, non-parametric transfer function. The analyst can construct the parametric function in a stepwise manner, by adding first and second order subsystem to both the numerator and the denominator. Each subsystem corresponds to a specific dynamical mode of the rotorcraft. The result is shown instantaneously with updated Bode plots containing both the parametric and the non-parametric transfer function. After the construction of the transfer function using initial guesses, the parameters are optimized.

2.2.4 Combined fitting of sub-polynomials

When a complete set of parametric transfer function has been obtained, the denominators must be updated to contain the same subsystems. This is necessary for the conversion of the transfer functions into a state space equation. In addition, the time delays that are individually estimated for each input/output pair, must be equalized for each input.

Both operations are combined in a set of optimizations, resulting in a complete set of transfer functions with common dynamics. Subsystems that are not present in a specific transfer function, are added automatically and are used to improve the fit. On the other hand, excessive subsystems, for example a mode that is present in only one transfer function, can be removed to reduce the model order.

2.2.5 Transfer function inversion

A complete set of transfer functions with a common denominator is divided into single input/multiple output groups. Each group can be inverted to a single input/multiple output state space model. These models all have the same output vector, the same state matrix in phase canonical form and different output matrices. Since in phase or controller canonical form the states are excited by a fixed input matrix, the states are generally unknown quantities. By transforming the model such that the states are equal to the outputs, a set of single input/multiple output models is obtained with common states. However, the resulting estimates for the system matrix are generally inconsistent.

The inconsistency of the multiple input/multiple output model can usually be ignored, since the system matrix only serves as an initial guess for a numerical optimization. In this optimization, the output equation is known a-priori since the outputs are equal to the states and the optimal input matrix is calculated during optimization for the system matrix, to fit the original set of

transfer functions best. Hence, only the elements in the state matrix which are not known a-priori have to be varied.

Finally, the state space model can be validated by time domain simulation.

3   RSI Software

The spectral analysis and parameter estimation techniques that are referred to in the previous section, have been implemented in a computer program package called Rotorcraft System Identification (RSI). It uses MATLAB version 4 as a platform, applying the possibilities of this package for graphical user interfaces. Since MATLAB is available for different computer systems, the software can easily be ported to any system on which graphical user interfaces are supported.

The development of the package was performed on a personal computer, using MATLAB 4.2 for Windows. The 'touch and feel' of the interfaces was adapted to the Windows 95 standard, but portability of the program was not affected by this. Program testing took place on personal computers, using both Windows 3.1 and Windows 95, and on Hewlett-Packard workstations.

The structure of the program is discussed in section 3.1. Section 3.2 deals with the main menu for the package, introducing the graphical user interface. In sections 3.3 to 3.6, the several interactive environments are described which correspond to different stages in the identification process.

3.1   Program Structure

The program Rotorcraft System Identification consists of eighteen MATLAB functions, divided in thirteen program functions and five functions in a special toolbox. Since functions are precompiled by the MATLAB engine, using functions rather than scripts speeds up program execution extensively. Besides, functions offer the application of local variables, which leads to better structurized coding.

The toolbox functions cover the calculation of coherence functions, the application of a special moving average filter, the calculation of phase angles and gains and the application of zero-phase shift filters. The program functions provide the user interface, a context sensitive help function and a universal plotting module. A co-ordinating module and the algorithm modules are program functions as well.

The central module in the package is called *rsi*. It starts and closes down the package, controls the data flows and co-ordinates the interaction between the other modules. Information is passed between the modules by the function arguments. The user is shielded from the program by the graphical user interface gui. It passes all user input and presents all the program results. In addition, it is linked to the context sensitive help function help.

The identification procedure is performed by a series of algorithm functions, storing the intermediate results on disk. Starting
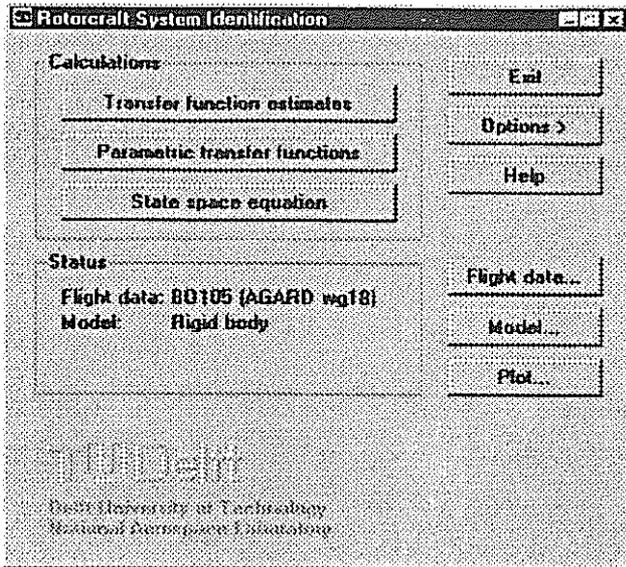
Figure 2: Main menu

with the flight data, the first module *nonpartf* calculates non-parametric transfer functions. They are fitted with parametric transfer functions in the module *fittf*. The next step is the combined fitting of a complete set of transfer functions and the conversion to a state space model in *fitglob*. Finally, the state space model is optimized in *fitss*.

The results are displayed by the module *gui*, but graphs are plotted by a universal plotting module. The data are loaded from disk by the module *plot* itself or are passed directly from the algorithm modules by the function parameters.

The modular structure of the package makes it very easy to adapt pieces of the code or to replace an entire module by one with a new algorithm. Extension of the possibilities, like adding a module for sub-space methods, only requires a change in the graphical user interface.

3.2 Main Menu, Plotting and Getting Help
Figure 2 shows the main menu for the RSI program. Like all menus in the package, it has a column of push buttons to the right and frames with information and options to the left. The upper push button is labeled 'exit' and can be used to terminate the program. For the other menus, the upper right push button is labeled 'cancel' and can be used to return to the previous menu. When another menu is entered from the main menu, the main menu is hidden till the other menu is left.

The push buttons 'plot' and 'help' in the main menu give access to the plotting menu and the help function. Both are context sensitive: from the main menu, a standard window is opened, while from any other point in the program the corresponding graphs can be plotted or related information is printed. For the help window, different topics can be recalled via the menu bar on top of the window.

The top left frame in the main menu shows an identifier of the current set of flight data that is being used for identification. The identifier string can be chosen by the user. The data can be selected and scale factors can be set in the source file management dialog box, as described in the next paragraph. The frame 'frequency domain methods' contains three push buttons, each for one step in the identification process.

The push buttons in the plot menu offer the possibility to return to the previous menu, to actually draw the selected graphs and to open the help window. The push button 'file' is only available when the plot menu is opened from the main menu. In that case, any RSI result file can be opened for plotting. From any other menu, the appropriate graphs are automatically passed to the plotting module.

The possible graph types for plotting are time records, coherence functions, power spectral densities and transfer functions. Time records show the original flight data. Coherence functions and power spectra can be inspected to verify the conditioning process. Coherence functions include the following:
- ordinary coherence functions between each input pair
- ordinary coherence functions between each input and the output
- partial coherence functions
- multiple coherence functions

The following power spectral densities can be plotted:
- auto power spectral densities of each input
- conditioned auto power spectral densities of each conditioned input
- auto power spectral density of the ouput
- coherent output spectrum contributions of each input
- auto power spectral density of the noise (uncoherent output spectrum)

The non-parametric transfer function estimate that is obtained from the conditioning process, can be plotted to verify the remaining frequency resolution after reducing the variance of the periodograms.

3.3 Managing Flight Data
The source file management dialog box is used to select MATLAB data files that contain flight data, and to group them into sets which are used for identification. All information on a set of flight data files is stored in a special file and loaded automatically when RSI is started again.

Changes to the flight data information file are only stored when the main menu is recalled using the push button 'ok'. Mistakes can be corrected by returning to the main menu using 'cancel'; the original information file is then left unchanged.

3.3.1 Setting an identifier
To identify different sets of flight data, the user can specify a different character string for each set. This string, which can be changed in the edit box below the number of data files in the set, is also printed in the main menu. The string can be longer than the size of the edit box, although the excessive characters are not printed in the main menu. This way, additional information

on the data - like flight test dates or flight conditions - can be stored.

3.3.2 Setting channels and scale factors

In the list of edit boxes, channel numbers for the flight data can be linked to the time and to the names of the different inputs and outputs. This way, the analyst is never faced with channel numbers during the identification procedure. Instead, signals can be selected by their names. In all list boxes throughout the package, the signal names always appear in the same sequence as in the source file management window.

Not every output needs to be present in a data set. A specific output can be removed from the set by blanking the channel number. For one data set, the assignment of channel numbers must be the same for every file. The program gives a warning when a channel is linked to more than one signal.

When information on biases in certain signals is available from flight path reconstruction, scale factors can be applied. Each signal is multiplied by the corresponding scaling factor before processing. A blank edit box indicates a scale factor of unity. Offset errors need not be corrected, since trends in the data are removed before transformation to the frequency domain. An offset error is part of the trend, so these errors have no influence.

3.3.3 Selecting and converting units

Three standardized unit systems can be selected from a list box:
- metric units (SI)
- US customary units based on feet
- US customary units based on inches

For US customary units, velocities should always be stored in feet per second. Linear accelerations and moments however can be based on either feet or inches. The state space model that is obtained, uses the same units for the inputs and outputs as the flight data signals, except when conversion to SI is switched on. Inputs are expected to be recorded as a percentage of the maximum control travel.

By enabling a checkbox, flight data in US customary units can be converted to SI before processing. Angles and angular rates or accelerations are always converted to radians. Times should always be recorded in seconds; the samplerate must be expressed in Hertz.

3.3.4 Opening existing data sets and creating new sets

When the program is used to work on several sets of flight data at the same time, a different set can be opened by selecting the corresponding information file. All information, like channels, scale factors, the samplerate and directories, is adapted automatically.

New sets can be created by selecting all the flight data files which are part of the set and specifying the location for the result files. The flight data is analyzed and when a list of (abbreviated) channel names is stored with the data, the program tries to match these with the names of the inputs and outputs. The channel numbers are then inserted in the edit boxes automatically
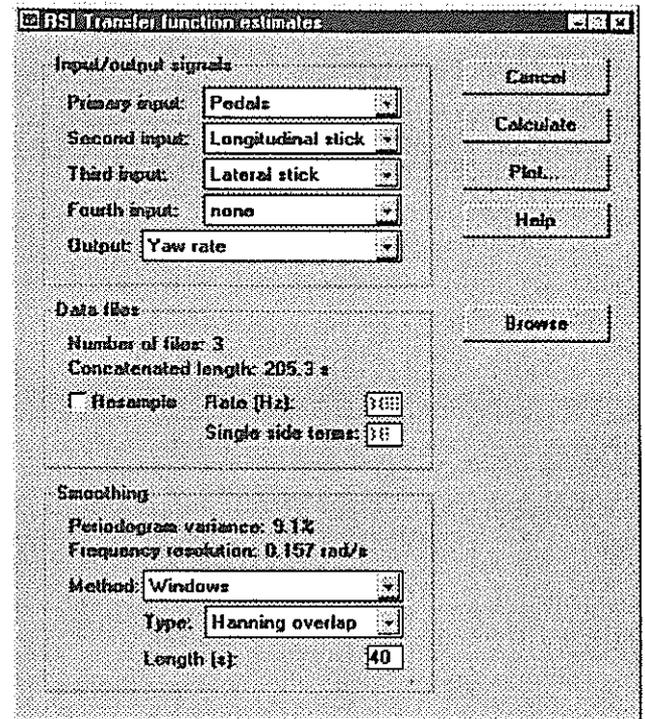

Figure 3: Transfer function calculation dialog box

and only unidentified channels need to be assigned. The samplerate is calculated from the time signal.

3.4 Calculating Transfer Function Estimates

The dialog box that is used to calculate transfer function estimates from flight data is shown in figure 3. Source files can be selected and viewed using the push buttons. The transfer function to be calculated is then selected, together with a smoothing method for variance reduction. If wanted, the flight data can be resampled before processing. After calculating the transfer function, the results can be plotted. Several transfer functions - with the same input and different outputs - can be calculated without selecting a new set of source files in between.

3.4.1 Selecting an output and inputs

The output for the single input/single output transfer function can be selected from a list box. A group of four inputs can also be chosen from list boxes, but not all four need to be included, since for the secondary inputs, 'none' is a possibility as well. The first input is the primary input for the conditioning process and this is the input for which the transfer function is obtained. The second, third and fourth input are used to remove coherent parts from the output spectrum.

The mutual ordering of the secondary inputs is of no importance. For the original conditioning process as described by Bendat (Ref. 10), the ordering of the inputs plays an important role in the accuracy of the several transfer function estimates. The inputs should be ordered such, that if a correlation between two inputs is known to be caused by one of them, this input should be put prior to the other input. In that case, all transfer functions are calculated from one time series and applying the right ordering
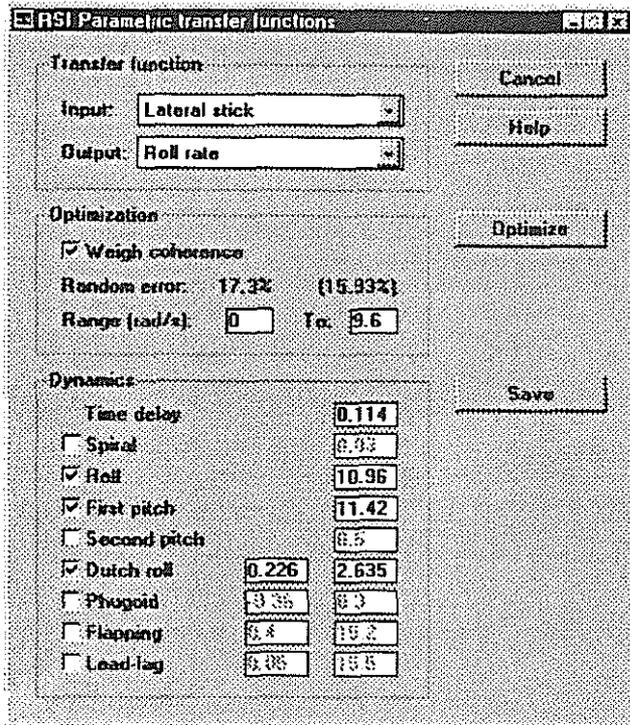
Figure 4: Tranfer function fitting dialog box

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causes influences in the inputs to be conditioned correctly and leads to physically correct transfer functions. For the RSI program however, the conditioning is repeated with a different ordering for each input, since only the transfer function from the primary input is obtained. The transfer functions for the secondary inputs would be erroneous due to the pilot feedback. When the inputs are reordered for each transfer function and when the corresponding flight data is used - with the test signal in the primary input - the larger part of the coherent output spectrum is always due to the first input. Since the primary input, which is generated by the test pilot, is expected to be uninfluenced by the secondary inputs, this leads to the best possible transfer function estimate, independent from the mutual ordering of the secondary inputs.

3.4.2 Resampling flight data

When flight data is sampled at rates much higher than is required for the frequency resolution — although this is unlikely since the resolution can be 'converted' in a reduction of the estimate variance — the calculation time for the power spectral density estimates can be reduced by resampling the data before processing. The new samplerate can be chosen arbitrarily; it does not need to be an integer part of the original samplerate. The number of single side terms that is used by the MATLAB resample function can be selected seperately.

3.4.3 Selecting a smoothing method

For the smoothing of power spectral density estimates, both time domain and frequency domain methods are available. Time domain smoothing uses the Welch method, for which the following windows can be used:

● Bartlett

● Blackman
● Hamming
● Hanning

The window type can be selected from a list box; the window length in seconds is specified in an edit box. The program divides the original time records in portions close to the specified window length, with a number of time points per section resulting in a relatively fast calculation of the Fourier transforms.

Smoothing methods in the frequency domain can also be selected from a list box. Two smoothers have been implemented:

● moving average (mixed radix)
● Butterworth filter

For the moving average filter, the number of points to be included in the averaging for each frequency point must be specified. For the Butterworth filter, the cut-off time as a number of time points must be entered.

3.5 Fitting Individual Transfer Functions

Figure 4 shows the menu for fitting parametric transfer functions. A single input/single output transfer function for which the non-parametric estimate has been calculated, can be selected from the list boxes in the upper left frame. The Bode plot is brought up automatically and the model structure can be constructed and the parameters can be optimized.

If the results of a parametric fit are saved before returning to the main menu, the model structure, the values for the parameters and the frequency range are automatically reloaded the next time the same transfer function is opened. By providing a warning, the program protects the analyst from unintentionally overwriting previous results by saving parameters for the same function again.

3.5.1 Viewing transfer functions

Both the non-parametric and the parametric transfer function are displayed in a Bode plot. On the screen, the calculated estimate and the fit are drawn in different colors. From the menu bar, the graphs can be saved, printed or copied. Saved graphs can be reproduced within MATLAB without starting RSI. Thus, different graphs can be compared easily.

Options include the possibility to draw a grid behind the graphs, to show the 95%-confidence range for the estimates and to extend the Bode plot by a plot of the coherence function. In addition, the frequency scale can be set to linear or logarithmic and the magnitude scale can be set to linear, logarithmic or deciBel.

3.5.2 Setting the frequency range

The frequency range over which the Bode plot is drawn can be selected in the menu of the graph. The range over which the cost function for the fit is evaluated need not be the same. The lower and upper boundary for this range can be set in the edit boxes in the top left frame in the dialog box. Changes to the range cause an automatic update of the cost function and a redraw of the Bode plot.
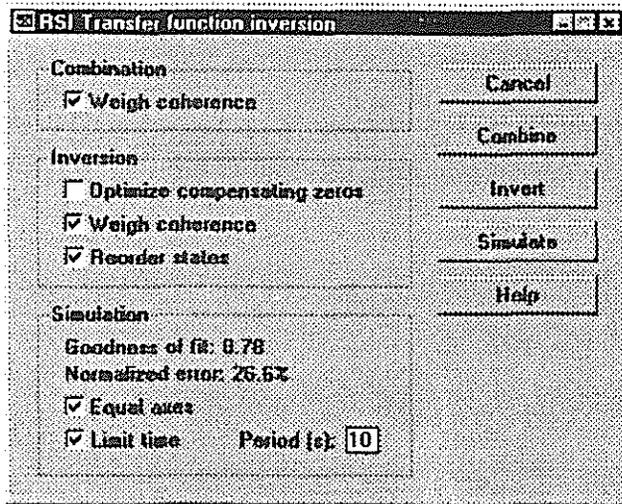
Figure 5: Tranfer function inversion dialog box

3.5.3 Selecting coherence weighting

The cost function for the fit is evaluated over the selected frequency range. Before it is printed in the middle left frame of the dialog box however, it is divided by the number of frequency points and the magnitude of the transfer function. In addition, the square root is taken and the result is multiplied by hundred. This way, the indicated cost is independent from the number of frequency points. Besides, it is approximately equal to the magnitude of the weighted average error as a percentage of the magnitude of the transfer function estimate. An estimate for this value are derived from the non-parametric data is shown between parentheses.

The weighting functions that is used is normalized to avoid scaling of the cost function. It automatically includes frequency weighting to account for the increasing number of frequency points per decade on a logaritmic scale with equally spaced frequenc points. In addition, coherence weighting can be selected to emphasize the fit where coherence is high.

3.5.4 Constructing a parametric model

The model for a transfer function consists of a time delay, a static gain and a numerator and a denominator polynomial. The denominator polynomial is made up of first and second order subsystems which correspond to the aircraft dynamics. The optimal numerator polynomial is calculated instantly each time a change is made. The time delay and the subsystems are listed in the lower frame of the dialog box. Items can be added to or removed from the model by enabling or disabling the checkbox in front. The time delay can not be disabled, but setting it to zero has the same effect. Every time an item is added to or removed from the model and every time a parameter is changed, the Bode plot and cost function are automatically updated, to demonstrate the effect of the change.

First order systems are characterized by one parameter, the time constant. Two parameters, the neutral frequency and the damping ratio, characterize second order systems. These parameters are printed in edit boxes next to the subsystem names. Each

checkbox contains an initial guess, giving insight in the location of the different systems in the frequency domain and helping the analyst to decide on the right model structure. After an item is added to the model, the parameters can be adapted to the actual system parameters. When an item is disabled, the customized parameters are retained.

The available subsystems include six rigid body modes, two second order and four first order systems, and two flexible modes, rotor lead-lag movements and rotor flapping. Which modes are included in the model, is determined by the state space model that is aimed for. For example, identification of a 6 degree-of-freedom rigid body model implies including the eight rigid body modes only.

3.6 Inverting to a State Space Model

The multiple input/multiple output model that can be constructed from the individual transfer functions, is calculated using the menu of figure 5. The global fitting procedure adds missing modes to individual transfer function denominators and removes excessive modes. When modes are added, a compensating system is added in the numerator as well. All parameters are then optimized simultaneously. Time delays are optimized to be the same for each output from one input.

On inverting tranfer functions to a state space model, the states are ordered in the same sequence as the list in the RSI program. Conversion to the standardized sequence for aircraft state space models can be applied by enabling the checkbox in the top frame. Velocities are then ordered before rates, only then followed by the Euler angles.

The state space model can be validated by simulation and comparison of the outputs with original flight data. The appropriate signals are selected by the program automatically; choosing a model and a flight data file is sufficient. The identified model is then driven by the measured control input signals. The resulting simulation is plotted in a set of time record graphs, one for each input and output, together with the measured output signals.

4 Results and Discussion

To verify the spectral analysis and parameter estimation algorithms, as well as the implementation in a MATLAB program, the software is applied to the identification of a rigid body model for the BO-105 helicopter. In section 4.1, the helicopter is introduced and the flight test database is briefly discussed. The state space model that is aimed for, is presented in section 4.2. Section 4.3 deals with the calculation of frequency response function estimates and the fitted transfer function models in the light of two examples. The identified modes and time delays are presented in section 4.4. The resulting state space model is validated and discussed in section 4.5. Finally, section 4.6 contains the main conclusions of this study.

4.1 Helicopter and Flight Tests

Since the BO-105 helicopter was one of the case studies of working group 18, an extensive set of identification results is

Figure 6: DLR operated BO-105

available. This makes the BO-105 a good subject for verification of the algorithms and implementation, because results can be compared to those of several other research institutes. The flight data that was used by the working group, was available for this study as well.

The BO-105 is a small, multi purpose helicopter built by MBB. It is often used by the police, ambulance services and search and rescue services. A military version, called PAH-1, is also available. The BO-105 is powered by two Allison 250-C20B turboshaft engines. The four bladed hingeless main rotor has a diameter of 9.84 m; the two bladed teetering tail rotor has a diameter of 1.90 m. The equivalent hinge offset of the composite main rotor blades is 14%. This large value makes the BO-105 well maneuverable in the pitch and roll axes. The basic empty weight is 1189 kg. The maximum gross weight is 2300 kg, resulting in a disc loading of 26.5 kgm^{-2}.

The flight data for the BO-105 were provided by the DLR Braunschweig. The DLR operated BO-105C S123, a civil version which is used for research activities only, is shown in figure 6. The data were recorded with a digital data acquisition system and were preprocessed by the DLR.

The flight test database contains a grand total of 28 data runs. Frequency sweeps, doublets and 3211 input signals are available for all four inputs. Doublets and 3211's are performed in both directions; frequency sweeps are repeated three times. Frequency sweeps have been designed to serve as input signals for frequency domain analysis. Therefore, the frequency sweeps are used for identification. A typical frequency sweep in the lateral cyclic stick is shown in figure 7. Data records with doublet inputs are used for validation.

4.2 State space model structure

The model to be identified is an 8th order rigid body model. The eight quantities which characterize the state of the aircraft, are the longitudinal, lateral and vertical speeds, the roll, pitch and yaw rates and the roll and pitch angle. However, a state space model is a linear model that is expressed in quantities which are deviations with respect to the values in trimmed flight. For the equilibrium condition — i.e. trimmed flight — all states are

equal to zero. Therefore, not the actual speeds, angular rates and angles are used as states, but differences with respect to the values in trimmed flight:

$$
x = \begin{pmatrix}
\Delta u \\
\Delta v \\
\Delta w \\
\Delta p \\
\Delta q \\
\Delta r \\
\Delta \varphi \\
\Delta \theta
\end{pmatrix}
$$

Except for the forward speed, the values in trimmed flight are zero. The four control inputs are combined in the input vector:

$$
u = \begin{pmatrix}
\delta_{lon} \\
\delta_{lat} \\
\delta_{ped} \\
\delta_{col}
\end{pmatrix}
$$

Thus, the rigid body model is a model with four inputs and eight outputs. The state matrix contains 36 stability derivatives in a 6 by 6 submatrix. The last two rows only link the roll and pitch angle to the roll and pitch rate. The corresponding columns contain a gravity term which expresses the influence of the attitude on the speeds:

$$
A = \begin{pmatrix}
X_u & X_v & X_w & X_p & X_q{-}w_0 & X_r{+}v_0 & 0 & -g \\
Y_u & Y_v & Y_w & Y_p{+}w_0 & Y_q & Y_r{-}u_0 & g & 0 \\
Z_u & Z_v & Z_w & Z_p{-}v_0 & Z_q{+}u_0 & Z_r & 0 & -g\theta_0 \\
L_u & L_v & L_w & L_p & L_q & L_r & 0 & 0 \\
M_u & M_v & M_w & M_p & M_q & M_r & 0 & 0 \\
N_u & N_v & N_w & N_p & N_q & N_r & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
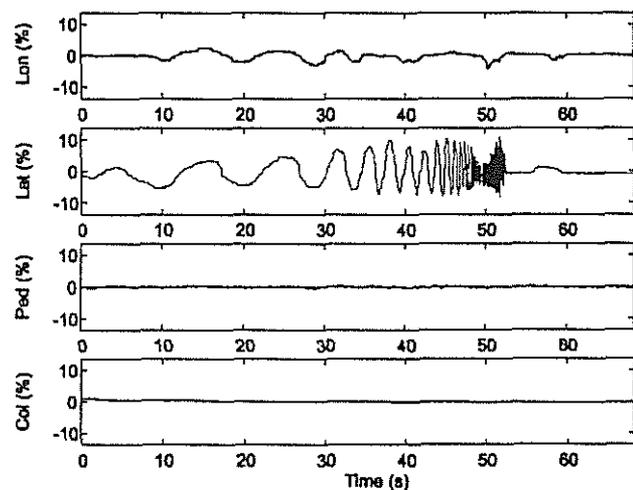0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
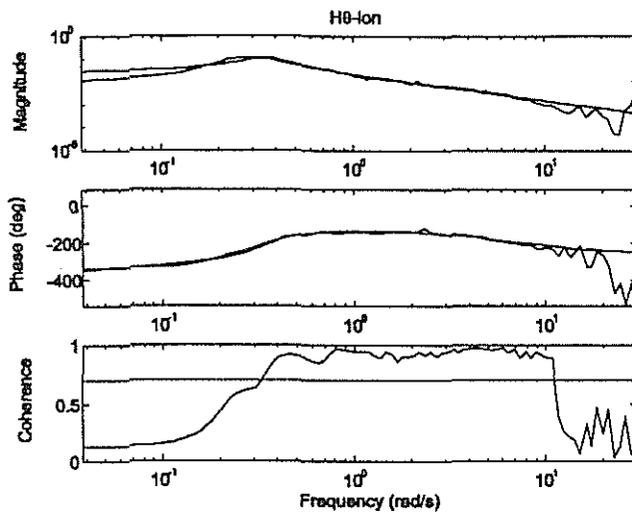$$

Figure 7: Typical lateral stick frequency sweep

103.8

Figure 8: Frequency response function fit ($H\theta$-δ_{lon})

Since the angles are completely controlled by the corresponding rates, the inputs only influence the first six states. Therefore, the input matrix contains 24 control derivatives:

$$B = \begin{pmatrix} X_{\delta_{lon}} & X_{\delta_{lat}} & X_{\delta_{ped}} & X_{\delta_{col}} \\ Y_{\delta_{lon}} & Y_{\delta_{lat}} & Y_{\delta_{ped}} & Y_{\delta_{col}} \\ Z_{\delta_{lon}} & Z_{\delta_{lat}} & Z_{\delta_{ped}} & Z_{\delta_{col}} \\ L_{\delta_{lon}} & L_{\delta_{lat}} & L_{\delta_{ped}} & L_{\delta_{col}} \\ M_{\delta_{lon}} & M_{\delta_{lat}} & M_{\delta_{ped}} & M_{\delta_{col}} \\ N_{\delta_{lon}} & N_{\delta_{lat}} & N_{\delta_{ped}} & N_{\delta_{col}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

4.3 Transfer function models

The parametric transfer function models were obtained from conditioned flight test data (Ref. 11). That is, the secondary inputs were removed by a conditioning process to keep the estimates unbiased and the variances to a minimum possible. In each evaluation of a transfer function model, the judgement of the accuracy was made on the obtained coherence relationship. Different smoothing parameters were set for on-axis inputs and off-axis inputs. Similarly, a relatively higher degree of smoothing was used as a result of the relatively poor accuracy of airspeed measurements. In all models, two or three inputs were taken into account and were found to be sufficient. This judgement was exercised on the basis of output spectral decomposition.

All 32 transfer functions for the four input/eight output model were calculated and fitted with parametric models. Two of those are shown in Bode plots here. The longitudinal on-axis response from longitudinal stick to pitch angle is shown in figure 8. A good coherence between input and output is achieved in the range from about 0.4 rad/s up to about 10 rad/s. Since rotor modes — which are characterized by crossover frequencies at about 15 rad/s — are not included in the rigid body model, this covers almost the complete range of interest. A parametric trans-

fer function model, shown in the same graph, was found using two longitudinal modes of the rotorcraft: the phugoid at 0.3 rad/s with a negative damping and a pitch mode at 5.6 rad/s.

Figure 9 shows the transfer function from lateral cyclic stick to yaw rate. Again, a good coherence over the complete frequency range of interest has been achieved. All four lateral modes of the rotorcraft were included in the model to fit the non-parametric transfer function. The dutch roll mode is the clearest at a frequency of 2.7 rad/s. In addition, the first order spiral at 0.08 rad/s and aperiodic roll at 8.5 rad/s were included. Note that the location of the spiral mode is below the frequency range for fitting. This mode can therefore hardly be distinguished from a pure integrator and the identified time constant is unreliable.

Transfer functions for off-axis responses show much smaller coherences than the on-axis responses. Therefore, all modes were identified from on-axis responses. Only then the transfer functions for the off-axis responses were fitted, while the dynamics were fixed. Although coherence was low, a good fit of off-axis responses was still possible because transfer function estimates are unbiased for all coherences. Only variances increase with degrading coherence, but since the dynamics were fixed, the number of unknowns is limited — only the numerator polynomials needed to be determined — and accurate determination of the numerator zeros remained possible.

4.4 Identified modes and time delays

The identified modes and time delays are listed in table 1 and 2, together with values obtained by the working group members. All identified modes are consistent with those from the working group, only showing comparatively small differences. Time delay estimates are generally larger than the values obtained by the working group members. However, since time delays are used to model the aerodynamics of the rotor, these values do not correspond to physically meaningful quantities. Surprisingly, a relatively small value was found for the collective lever input. Since heave axis response of the BO-105 is rather slow, a larger value is more likely to be correct.
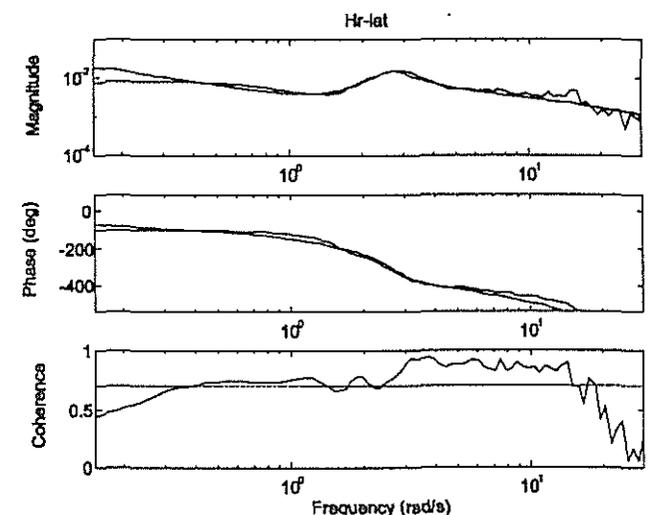

Figure 9: Frequency response function fit (Hr-δ_{lat})

	TUD	AFDD	CERT	DLR	Glasgow	NAE	NLR
Phugoid	[-0.40 0.36]	[-0.36 0.30]	[-0.17 0.32]	[-0.15 0.33]	[-0.01 0.35]	[-0.14 0.33]	[-0.07 0.33]
Dutch roll	[0.18 2.66]	[0.22 2.60]	[0.13 2.51]	[0.14 2.50]	[0.16 2.27]	[0.13 2.58]	[0.17 2.17]
Roll	(8.48)	(8.32)	[0.99 2.89]	(8.49)	(5.12)	(8.47)	(2.38)
First pitch	(5.60)	(6.04)	*	(4.36)	(1.98)	(4.38)	(1.37)
Second pitch	(0.67)	(0.49)	(0.66)	(0.60)	(0.64)	(0.63)	(0.71)
Spiral	(0.076)	(0.03)	(-0.05)	(0.02)	(-0.007)	(0.03)	(0.04)

$[\zeta\ \omega]$ indicates $s^2 + 2\zeta\omega s + \omega^2$ Frequencies in rad/s

$(1/T)$ indicates $s + 1/T$ Time constants in s

* Combined with roll mode to second order

Table 1: Identified time constants, damping ratios and undamped natural frequencies (Ref. 1; table 6.2.13)

	TUD	AFDD	CERT	DLR	Glasgow	NAE	NLR
Longitudinal	0.0751	0.113	0	0.100	0.044	0.100*	0.100*
Lateral	0.1502	0.062	0	0.060	0.074	0.060*	0.060*
Pedals	0.1425	0.044	0	0.040	0	0.040*	0.040*
Collective	0.0713	0.168	0	0.040	0.102	0.040*	0.040*

* From DLR value   Time delays in s

Table 2: Identified time delays (Ref. 1; table 6.2.12)

4.5 State space model simulation

After combination of the transfer function into a multiple input/multiple output state space model, the result is used for simulation. The results for a longitudinal and a lateral stick doublet input are shown in figure 10 and 11. Problem areas are clearly the longitudinal speed and the yaw rate. For the longitudinal speed, this is probably caused by the divergence from the point of equilibrium due to the phugoid and the non-linear effects which gain in influence from this effect. The yaw rate was identified less accurate than the pitch and roll rate, since for the latter two, redundant information in the form of the corresponding angles was included in the model. This leads to a more accurate optimization.

4.6 Conclusions

The iterative algorithms for transfer function estimation, fitting and state space estimation that were developed are applicable to helicopter flight data. The software works reliably and yields consistent results. Finally, the identified modes and state space simulations are consistent with the values obtained by AGARD working group 18. The results are under further improvement.

Acknowledgements

Abbreviations

AFDD   Aeroflightdynamics Directorate (US Army), USA
AGARD   Advisory Group for Aerospace Research and Development
CERT   Centre d'Etudes et de Recherches de Toulouse, France
DLR   German Aerospace Research Establishment, Germany
DoF   Degrees of Freedom
Glasgow   Glasgow university
MIMO   Multi-Input/Multi-Output
NAE   National Aeronautical Establishment, Canada
NLR   National Aerospace Laboratory, the Netherlands
PSD   Power Spectral Density
RSI   Rotorcraft System Identification
SISO   Single-Input/Single-Output
TUD   Delft University of Technology, the Netherlands
WG-18   Working Group 18, on Rotorcraft system identification

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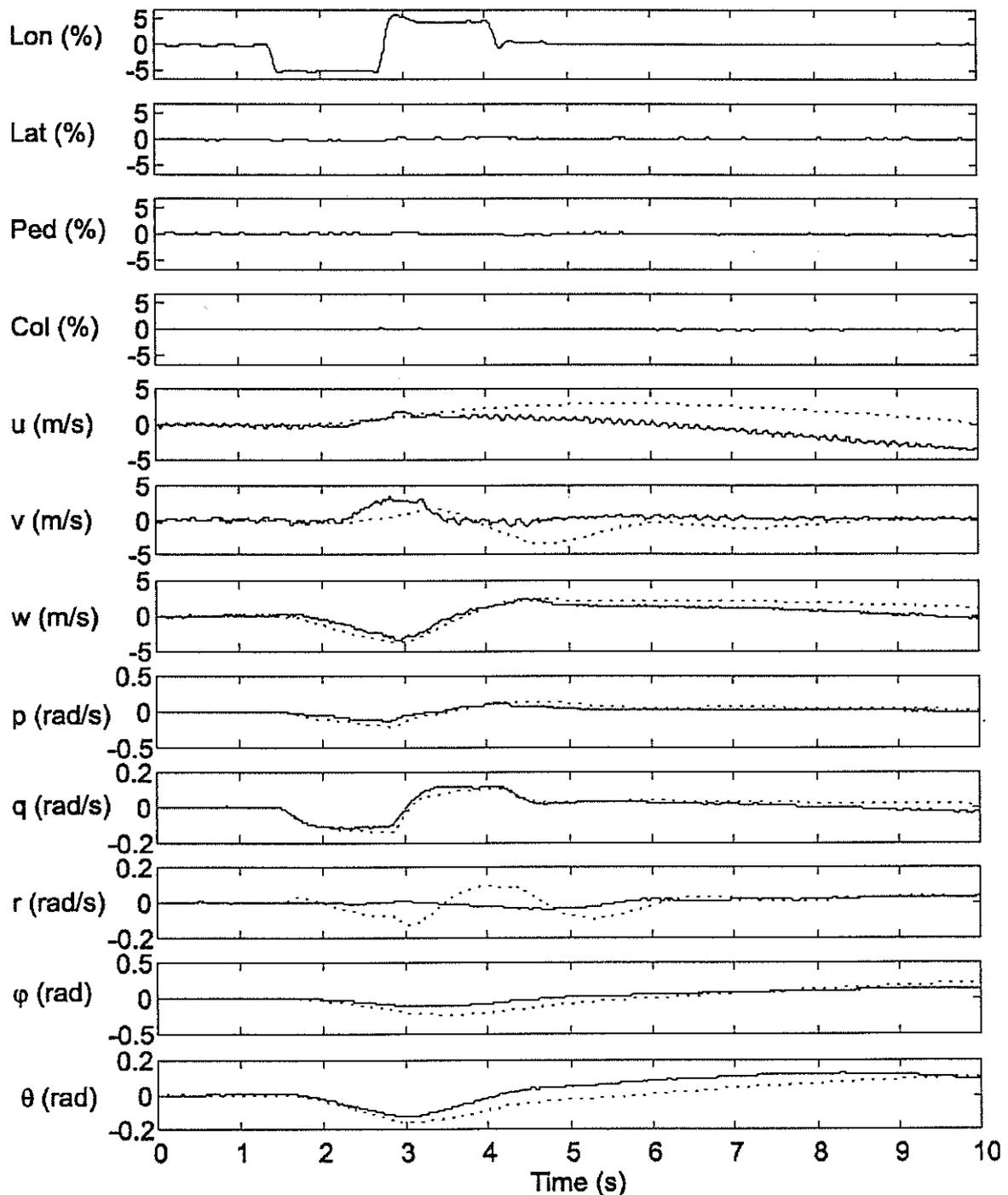   1995, paper no VII-11.
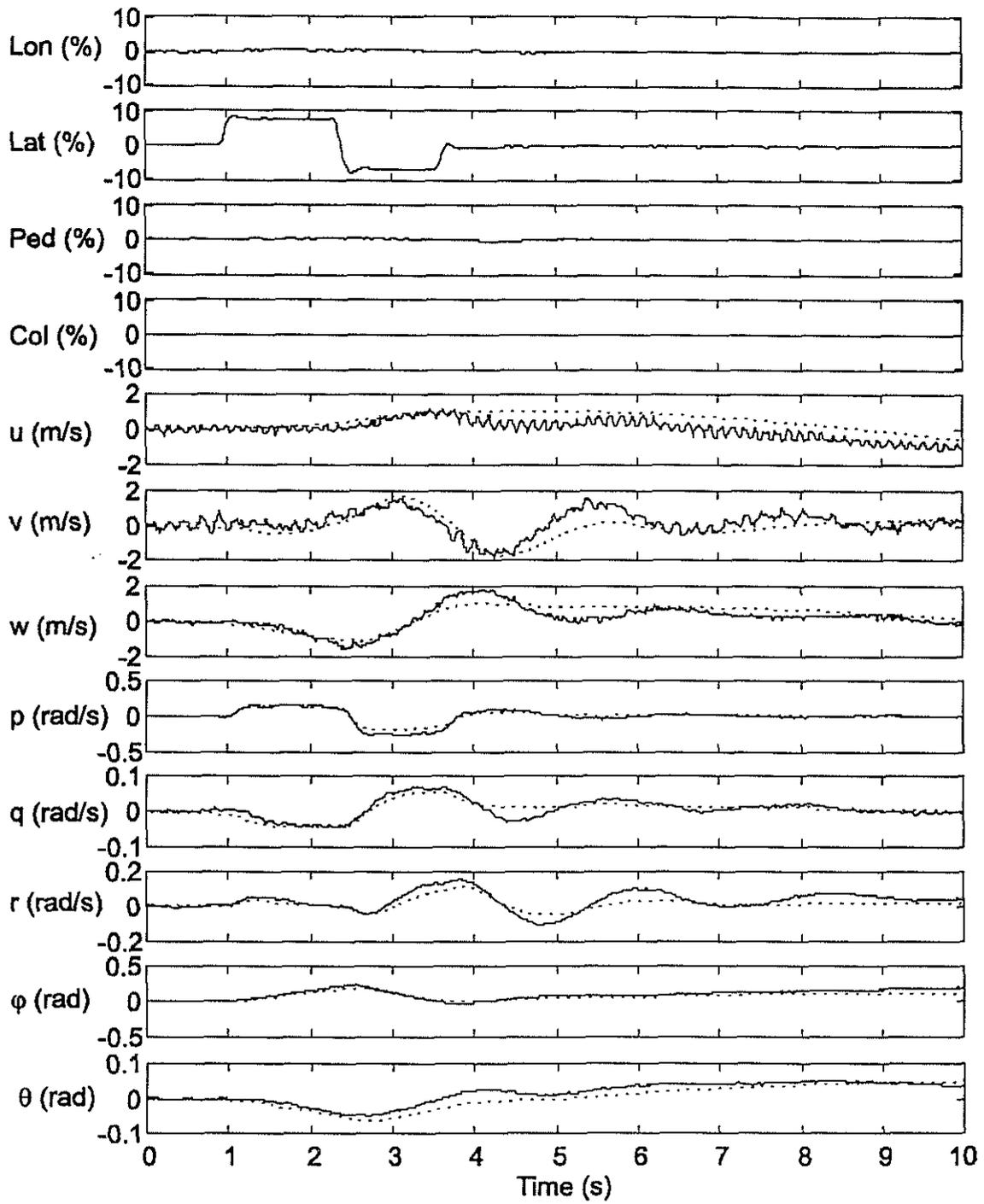
Figure 10: Time response simulation of longitudinal stick doublet

Figure 11: Time response simulation of lateral stick doublet