

Finite State Inflow Models for a Coaxial Rotor in Hover

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ABSTRACT

Formation of a robust, accurate, and computationally efficient induced inflow model is needed to support real time coaxial rotorcraft simulations. Two generalized coaxial rotor models are developed to handle hovering flight condition. One inflow model is obtained by extending the popular Peters-He single rotor dynamic inflow model. In this extension of the model, two pressure fields are superimposed to create the desired dual actuator disk representation of a coaxial rotor configuration. The second model is based on the recently developed inflow model, where Galerkin weighted residuals are used to develop a finite state inflow theory, which is applied to a coaxial rotor configuration. Because of the importance of the upper rotor's wake interaction with the lower rotor's inflow, the rigid wake assumption of the models cannot be justified for coaxial rotor modeling, and a wake contraction method is developed through incompressible flow equations and is used in model correlations. Thrust coefficient, torque coefficient, and rotor inflow results are compared with data from the US Army Aeroflightdynamics Directorate (AFDD) experiment based on a small scale model rotor with blade geometry similar to that of XV-15. The two models are shown to correlate well with experimental hover data.

1. NOMENCLATURE

C_T	thrust coefficient
d	distance along a streamline
D	rotor diameter
L	Inflow influence coefficient matrix
$\vec{w}$	velocity vector
w_z	axial induced velocity
w_r	radial induced velocity
r_{wake}	wake radius
R	rotor radius
α_1, α_2	vectors of inflow coefficients of rotors 1 and 2
$\hat{\alpha}_1, \hat{\alpha}_2$	Coefficients of derived velocity potentials of rotors 1 and 2
Φ_1, Φ_2	derived velocity potential functions of rotors 1 and 2
σ	rotor solidity
ζ	exponential decay constant in the empirical viscous correction
∇	gradient operator

2. INTRODUCTION

In the 1970's, the next big advancement in helicopters was coaxial rotor designs. With the new advancing blade concept being developed, coaxial rotor helicopters were expected to compete with conventional single main rotor/tail rotor helicopters. Unfortunately, the progress towards a fully developed coaxial helicopter system was slow, partly due to a lack of comprehensive understanding of the complex flow behaviors of a coaxial rotor system.

Now, 40 years later, Sikorsky Aircraft Corporation has built and tested their X-2 Technology Demonstrator and they are now pursuing a government contract with their S-97 Raider high speed scout and attack helicopter. Good modeling will be needed as a cornerstone for further development of coaxial rotorcraft.

The present work looks into the capabilities of current finite state inflow modeling theories and their adaptability to perform modeling of coaxial rotor configurations. Finite state dynamic inflow modeling has for a long time

held a special niche in rotor inflow modeling. While CFD and wake vortex theory can provide more detailed results, and have been shown to be able to capture coaxial rotor aerodynamics,^{1, 2} dynamic inflow models have the ability to outperform these bulkier codes when a lean and more efficient code is required for real time simulations. The scope of this paper is the development of finite state inflow models for a coaxial rotor in hover and is aimed towards the future development of a comprehensive model for its use in real time simulations as well as for the rapid turnaround of maneuverability results from coaxial rotorcraft simulations.

Experimental studies conducted by McAlister and Tung³ using scaled highly twisted bladed rotors based on the XV-15 tilt rotor in both single and coaxial configurations provided the data for model correlations in the current study.

3. MODELING

The work documented herein expands two forms of the finite state dynamic inflow modeling, the Peters and He inflow model⁴ and the more recent Galerkin method based inflow model⁵, for use in inflow modeling of coaxial rotors. Each of the models is based on actuator disk theory and provides identical on-disk inflow results for an isolated rotor in hover. They differ in terms of their handling of off-disk inflow and flexibility.

3.1 Peters-He Inflow Model

The Peters-He inflow model⁴ is used currently in several simulation programs, for example, FLIGHTLAB, RCAS, etc. Based on the actuator disk concepts and the momentum theory equations, it uses the odd functions of the Legendre polynomials and Fourier harmonics to model the pressure perturbations along the radial and azimuthal directions of a rotor disk. It then uses orthogonality relations to form closed form matrices based on free stream skew angle and flight speed which relate pressure perturbations at the disk to inflow velocities at the disk. Off disk velocities can be calculated once pressure potential coefficients are known through integration of the derivative of the pressure

potential along rigid streamlines from infinity to the point at which inflow is desired⁶.

An extension of the Peters-He inflow model is utilized for inflow modeling of coaxial rotor configurations in hover in this study. The extended model is developed by superposition of two actuator disk pressure potentials. The inflow at each rotor disk produced by each of the two pressure fields is calculated using an inflow influence coefficient matrix (L) which relates the coefficients of inflow at the two rotors to the coefficients of pressure fields from the two rotors. This relationship can be written as

$$(1) \quad \begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix} = \frac{1}{V_m} [L] \begin{Bmatrix} \tau_1/2 \\ \tau_2/2 \end{Bmatrix}$$

where α_1 and α_2 are vector coefficients of the distributed inflow at rotors 1 and 2, respectively, τ_1 and τ_2 are vector coefficients of pressure fields from rotors 1 and 2, respectively, and V_m is the mass flow parameter. The inflow influence coefficient matrix L has four quadrants.

$$(2) \quad [L] = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$$

The diagonal quadrants L_{11} and L_{22} relate the self induced inflow to the aerodynamic loading on each rotor, and hence, they are the same as a single rotor Peters-He inflow influence coefficient matrix. The off-diagonal blocks L_{12} and L_{21} model the inflow coupling between the two rotors, and they are obtained by integrating the derivative of the individual pressure potentials from infinity to the other rotor disk and taking its integral with the inflow shaping functions across that disk. While closed form expressions for the diagonal blocks are readily available from the Peters-He inflow model, unfortunately, no known closed form solutions have been found for the off-diagonal blocks, and hence, they are computed numerically for each of the co-axial rotor configurations considered in this study.

3.2 Galerkin Method Based Inflow Model

The Galerkin method based inflow model⁵ is a more recent advancement in the finite state inflow modeling. It uses the actuator disk theory and state space formulation, similar to Peters-He

inflow model, but allows for the use of both even and odd functions of Legendre polynomials in order to model the combined effects of pressure discontinuity as well as mass source effects at a rotor disk. It uses Galerkin weighted residuals to discretize the momentum equation. This formulation allows for the solution of velocities in all three dimensions anywhere in the flow field by taking a velocity potential perturbation in the desired direction. Closed form expressions for computing flow velocities on the disk as well as off the disk are, perhaps, one of the most persuasive draws of this new formulation in its application to a co-axial rotor configuration. This is in contrast to the previously described extension of the Peters-He inflow model which requires numerical computations of the combined inflow coefficient matrix for a coaxial rotor configuration.

The solution for a coaxial rotor using the Galerkin method based inflow model is obtained by superposition of the velocity potentials from the two rotors in computing flow velocity ($\vec{w}$) at any point (x, y, z) in the flow field as given by

$$(3) \quad \vec{w}(x, y, z) = \sum_m \sum_{n=m+1, m+3, \dots}^{\infty} \hat{\alpha}_{n1}^m \nabla \hat{\Psi}_{n1}^m + \hat{\alpha}_{n2}^m \nabla \hat{\Psi}_{n2}^m$$

where $\hat{\Psi}_1$ and $\hat{\Psi}_2$ are non-dimensional derived velocity potentials⁵ for rotors 1 and 2, respectively, $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are associated coefficients and ∇ is the gradient operator.

3.3 Viscous Effects

Viscous effects are taken into account through an empirical exponential decay model⁷ of the form $e^{-\zeta d}$ where ζ is the exponential decay constant and d is the distance along the streamline from the rotor disk to the point of interest.

3.4 Wake Contraction

Both the Peters-He inflow model and the Galerkin method based inflow model have an inherent assumption of rigid wake and straight streamlines. For off disk flow in the wake of the rotor, this assumption falls apart and a significant departure from realistic results is obtained when modeling coaxial rotor inflows.

In order to compensate for this assumption, the wake is contracted using the incompressible mass flow continuity equation. The extended Peters-He inflow model utilized a contraction ratio which was updated using the continuity equation as part of the solution process. In order to increase the flexibility of the Galerkin method based inflow module for use in future simulations, a wake table was developed instead. The wake table used in this study was derived using the velocity potentials of the Galerkin method based inflow model.

4. EXPERIMENTAL DATA

4.1 AFDD Experimental Data

In 2005, McAlister and Tung³ developed and performed experimental testing on an AFDD coaxial rotor at NASA Ames Research Center. Each rotor used was a generic scaled down version of a tilt rotor, loosely based on a 1/7th scaled XV-15 rotor. Each scaled rotor had three blades, radius of 2.025 feet, and solidity of .077. Each blade had a root cutout of 17% to accommodate the hub setup as well as a non-linear twist of roughly 37 degrees, a constant chord over the outer 69% of 1.95 inches, and blade section profiles based on the NACA 64 series which included the 64-X08, 64-X12, 64-X19, 64-X26, 64-X27, and 64-X32 airfoils.

Each rotor was mounted on an adjustable arm, allowing for independent vertical positioning so that variable separation distance and distance from the ground plane could be obtained. Each rotor setup was equipped with a blade pitch servo for adjusting the rotor collective. A torque sensor mounted on each electric motor and a thrust load cell mounted on each rotor support allowed for collection of individual rotor loading and power requirements. A PIV system was used for flow measurements.

4.2 Airfoil Tables

As rotor size is decreased, the Reynolds number also decreases and its effects on airfoil data become more significant. With the scaling of the rotors in the AFDD experimental data, the Reynolds number is in the range of 36,000-180,000, just 3% of the full scale XV-15 rotor

Reynolds number. This low range of Reynolds number required a reformulation of the airfoil tables. The new airfoil tables were developed by Lim, et al.⁸ using the MSES code. The MSES code utilizes compressible Euler equations on an inviscid flow field along with suction and pressure solutions of the viscous boundary layer to produce airfoil tables corrected for Reynolds number effects. In all, six airfoil tables with a range of Reynolds numbers from 12,000 to 570,000 were developed for each of the airfoil sections and provided a lookup table for the range of flight conditions considered in this study.

5. MODEL CORRELATION

5.1 Single Rotor

The two rotors for the AFDD coaxial setup were designed to be mirror images of each other and had identical aerodynamic characteristics. They were tested on their respective test stands in individual rotor configurations. Comparisons showed that the upper rotor experienced a slight performance advantage over the lower rotor, though it was within expected variations due to manufacturing discrepancies and rotor mount configuration differences with the upper rotor being mounted from above and the lower rotor being mounted from below.

All the tests used for correlation in this study were performed at a rotor nominal speed of 800 rpm. The isolated rotor results of $\frac{C_T}{\sigma}$ for the upper and lower rotors had a difference of roughly five percent for the nominal rotor speed of 800 rpm, equivalent to 1.2 degrees of collective difference. The predicted $\frac{C_T}{\sigma}$ results using the Peters-He inflow model and the Galerkin method based inflow model for the individual rotors were found to be nearly same, falling in between the experimental data.

5.2 Coaxial Rotor

The extended Peters-He and the Galerkin method based inflow models for the coaxial rotor configuration considered in this study are correlated with the experimental data for different values of rotor spacing. During the experimental test, the lower rotor was fixed at one rotor diameter ($1.0D$) distance from the

ground plane. The position of the upper rotor was then swept through a range of separation distances between $0.1D$ to $0.8D$. It is felt that the ground effect due to the lower rotor location at $1.0D$ from the ground plane would be small, and hence, ground effect modeling is not considered in this study.

For model predictions, the upper rotor collective is adjusted so that the computed torque coefficient at $0.1D$ separation distance is same as the experimental data. For all separation distances, the upper rotor collective is held fixed at the value selected for the $0.1D$ separation distance case, and the lower rotor collective is adjusted for torque balancing between the upper and lower rotors.

6. RESULTS

With the viscous effect taken into account, wake contraction results were developed for the inflow wake region. The modeled wake contraction show good correlation with the experimentally measured wake contraction results over the region available for comparison as seen in Fig. 1. The axial location, z , shown in Fig. 1 is the distance from the lower rotor with z positive below and negative above. The upper rotor predictions show more contraction than the isolated rotor case and the lower rotor predictions show less contraction than the isolated rotor case, which is in agreement with what was observed in the experimental results.

The Galerkin model predictions of axial and radial flow velocities at various axial locations from the lower rotor for a separation distance of $0.1D$ are compared with PIV data from the experiment in Fig. 2. The model predictions in Fig. 2 use an empirical exponential viscous decay model constant ζ value of 0.25. In Fig. 2, positive z values represent axial locations below the lower rotor and negative z values represent axial locations above the lower rotor. From the results shown in Fig. 2, the Galerkin method based inflow model predictions of flow velocities, in general, are found to correlate well, both qualitatively and quantitatively, with experimental data.

The thrust coefficient predictions for the coaxial rotor configuration as a function of separation distance are compared with

experimental data in Figs. 3 and 4 for the extended Peters-He inflow model and the Galerkin method based inflow model, respectively. The torque predictions are also shown as sub-plots in Figs. 3 and 4. It is seen from Figs. 3 and 4 that both the extended Peters-He inflow model and the Galerkin method based inflow model correlate well with the experimental thrust measurements over the range of separation distances considered in this study. While the extended Peters-He inflow model appears to provide a slightly better correlation, it involves much more computational complexity because of the elaborate numerical integrations involved.

7. CONCLUSIONS

Both the extended Peters-He inflow model and the Galerkin method based inflow model for coaxial rotor configurations are found to correlate well with model test data for the hover case considered in this study when viscous and wake contraction considerations are taken into account.

From the model predictions in hover for the coaxial rotor configuration considered in this study, it is found that the total rotor thrust and power are less sensitive to the separation distance, similar to the experimental data. The thrust sharing ratio is found to be affected by the separation distance, especially when the separation distance is small. The flow velocity predictions from the Galerkin method based inflow model are found to be both qualitatively and quantitatively consistent with the experimental data, indicating that it could provide other valuable interference effects and modeling capabilities without too much difficulty.

The Galerkin method based inflow model lends itself better to running many variable cases quickly and to its adaptation for forward flight cases due to having entirely closed form solutions. The extended Peters-He inflow model requires a numerical integration as part of the pre-processing for each of the coaxial rotor configuration cases. For the hover case considered in this study, it would be fair to assess that the advantages of the Galerkin method based inflow model in flexibility and

speed allow it to outperform the extended Peters-He inflow model. While these advantages should translate well into the forward flight condition where constantly changing skew angles make the closed form solutions of the Galerkin method based inflow model more attractive, previous research results for an isolated rotor⁵ indicate that the Galerkin method based inflow model in its current form may not predict forward flight results as accurately as the Peters-He inflow model, especially in unsteady high skew angle flight. Therefore, it may be prudent to pursue extensions of both models for forward flight studies of coaxial rotor configurations.

8. ACKNOWLEDGMENTS

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experimental data along with the rotor specification data used in this study.

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10. FIGURES

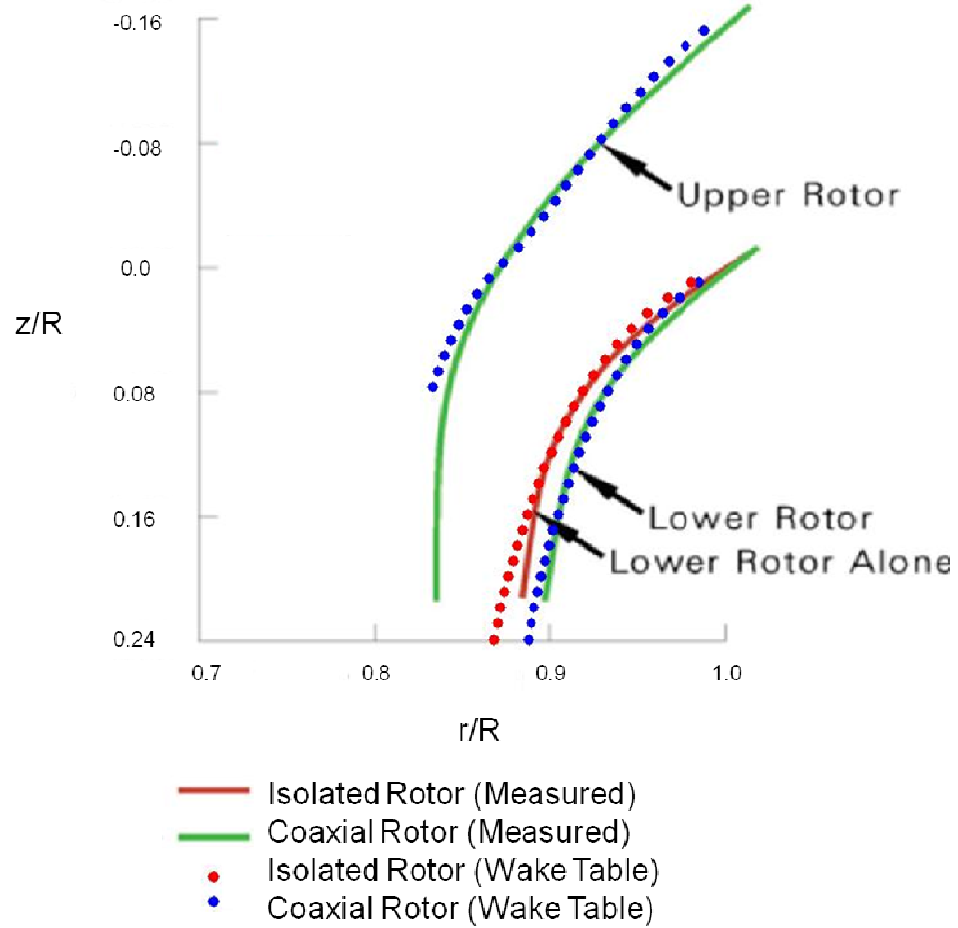


Figure 1. Comparison of modeled wake contraction with data for a coaxial rotor configuration of $0.1D$ separation distance. (The lower rotor is located at $z=0$ with z positive below the lower rotor and negative above it.)

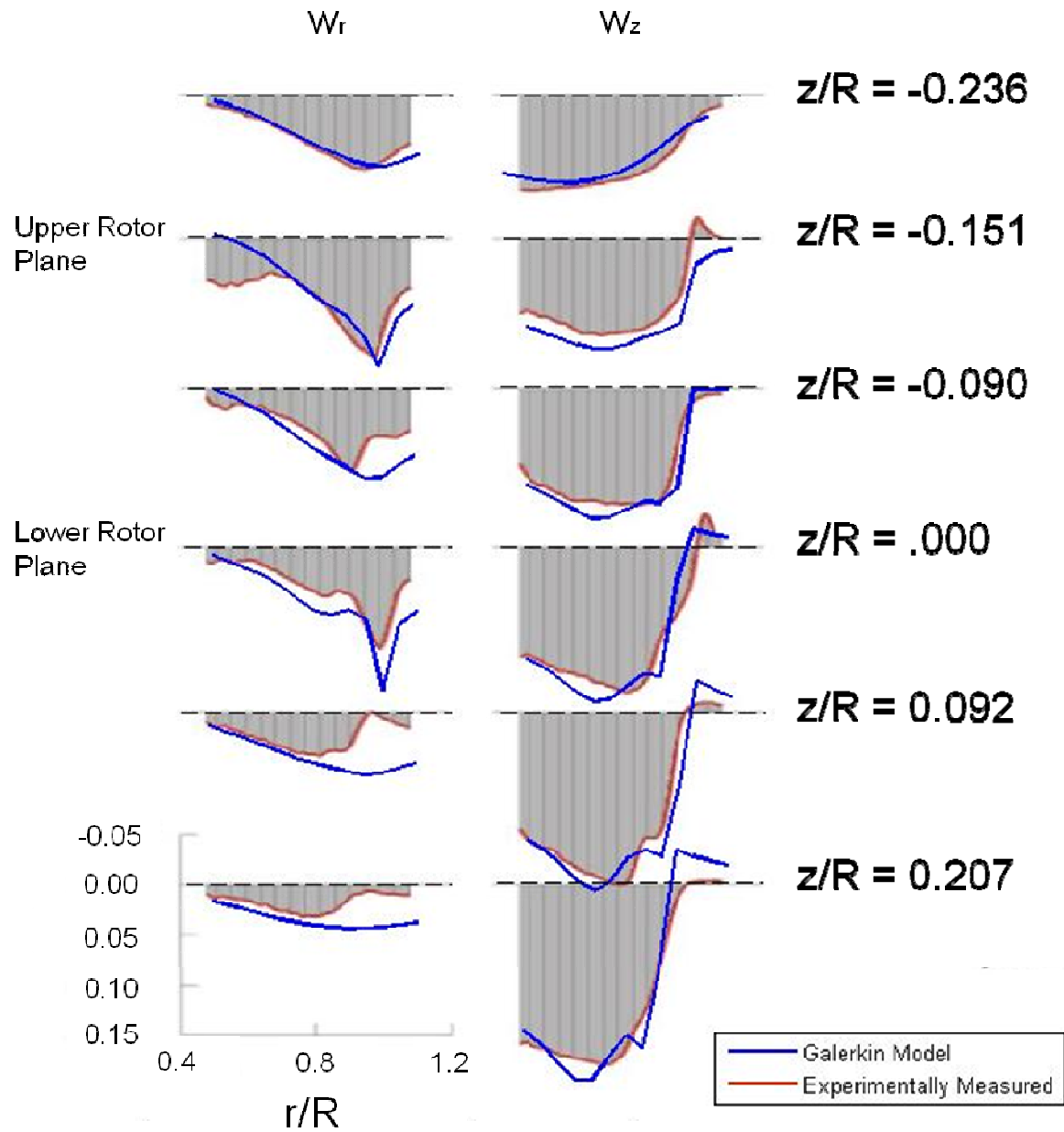


Figure 2. Comparison of predicted axial and radial induced velocities at various axial locations with PIV data for a coaxial rotor configuration of $0.1D$ separation distance. (The lower rotor is located at $z = 0$ with positive z below the lower rotor and negative z above it.)

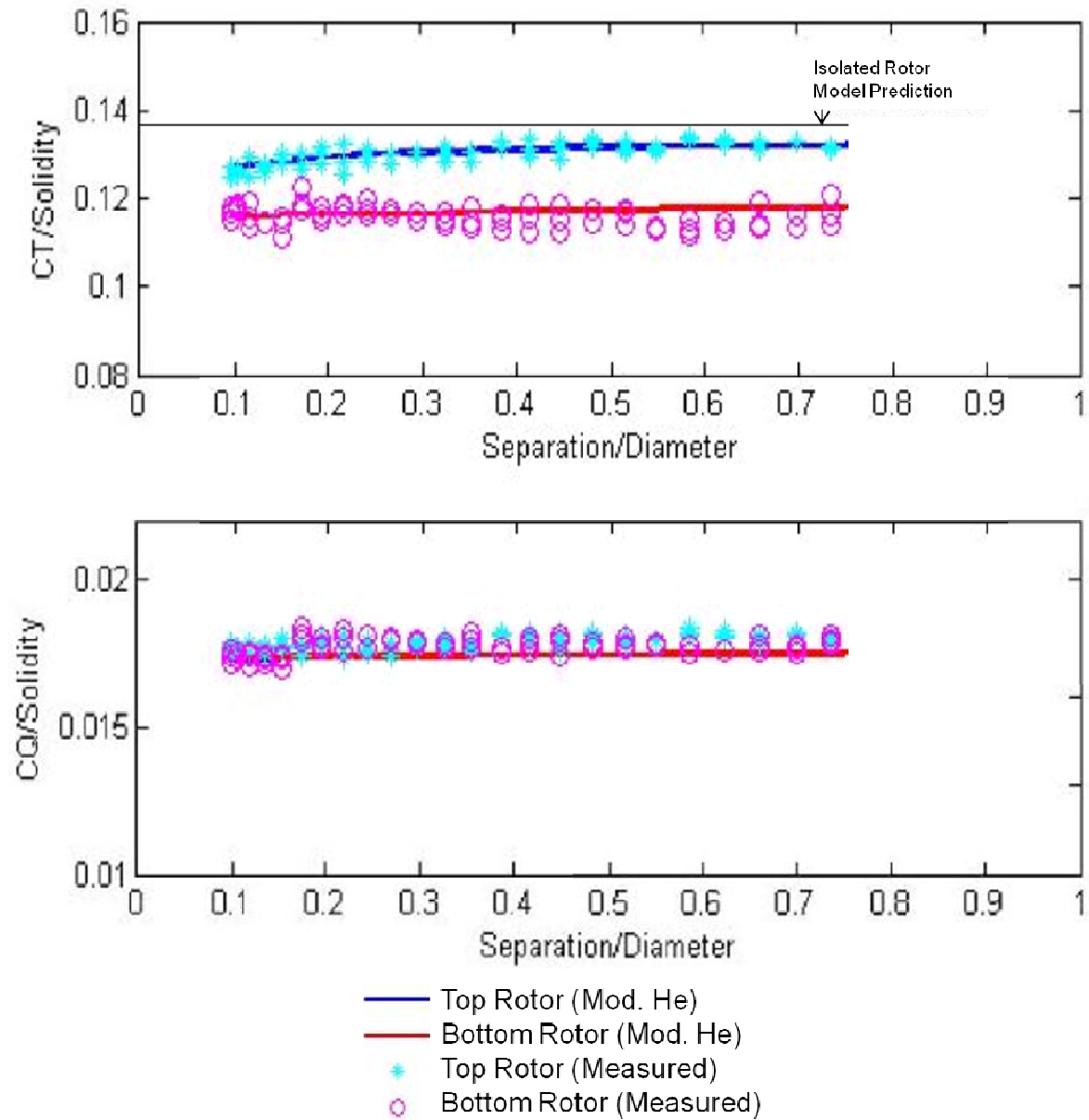


Figure 3. Comparison of thrust and torque predictions using the extended Peters-He inflow model with test data.

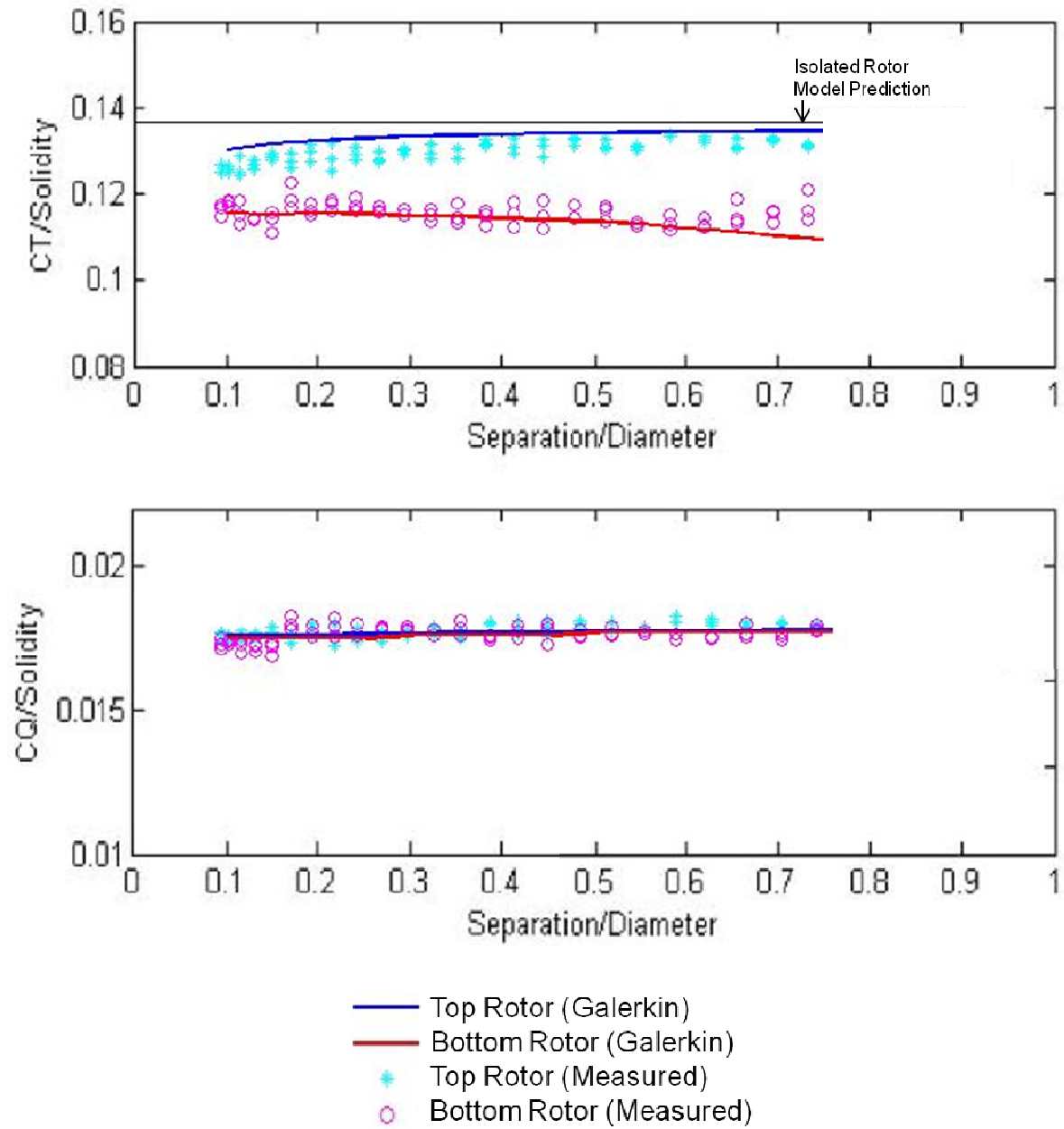


Figure 4. Comparison of thrust and torque predictions using the Galerkin method based inflow model with test data.