FOURTH EUROPEAN ROTORCRAFT AND POWERED LIFT AIRCRAFT FORUM

Paper No. 16

Rotor Blade Lag Plane Frequency Optimisation using Visco-Elastic Damping

A. H. VINCENT

Westland Helicopters Limited
Yeovil England

Sept. 13-15th 1978

STRESA ITALY

Rotor Blade Lag Plane Frequency Optimisation Using Visco-Elastic Damping

A. H. Vincent
Westland Helicopters Limited

1. INTRODUCTION

A study has been made of the effect of using visco-elastic damping materials on the choice of the rotor blade lag frequency for rotors with lag frequencies below rotor rotational frequency. Two apparently conflicting requirements determine the choice of lag frequency. One requirement, that of stability in the ground and air resonance modes, shows that the higher the non-dimensional lag frequency, the lower the damping required to stabilise ground resonance. The second conflicting requirement is that of in-flight lag plane loading acting on the hub and blades which demands a non-dimensional lag frequency which is as low as possible in order to minimise the amplification of lag loading due to resonance with the once per revolution rotor forcing frequency.

A detailed study of the first requirement has shown that it is only the non-dimensional damping parameter which reduces with increasing lag frequency, but that the actual damping required and particularly the volume of the visco-elastic damping material required to stabilise ground resonance may have a minimum in the range of lag frequencies under investigation or may indeed be decreasing with increasing lag frequency. The reason for this apparent contradiction is partly due to the increasing absolute frequency with increasing non-dimensional frequency, but is more significantly due to the dependence of the damper design on the in-flight lag amplitudes of the blade.

It has been shown that the shape of the non-dimensional damping versus frequency curve is well defined since both simplified Coleman theory and also a more exact model including blade flapping freedoms lead to similarly shaped curves. The shape of the curve also depends on the assumption that a frequency coincidence between fixed co-ordinate lag frequency and body frequency will occur for some permissible combination of helicopter all-up-weight, external stores configuration, rotor thrust and undercarriage state. This assumption may seem to be at variance with normal design process whereby fuselage and undercarriage configurations are constrained by ground resonance considerations, and where future developments of some designs may be limited by ground resonance limitations. In the opinion of the author, the advantages conveyed by providing sufficient damping to meet all eventualities far outweigh the modest penalties in rotor design.

Having determined the optimum choice of lag frequency, the study was continued in order to investigate the practicality of achieving the optimum solution. This part of the study gave valuable insight into the use of low damping - high fatigue strength materials which in general give very much smaller dampers but are sometimes difficult to apply because of their high stiffness to damping ratio.

The paper concludes with a set of design rules enabling the optimum damper design to be achieved. These rules define the dynamic characteristics of the rotor in the absence of damping so that the addition of a given damper at a given point achieves the required lag frequency.

2. DAMPER DESIGN STUDY

2.1 Damping Requirements

FIG. 1 shows the trend of decreasing non-dimensional lag damping required to stabilise ground and air resonance on Lynx. The dashed line shows the damping required for neutral stability and indicates clearly that for non-dimensional frequencies below $0.78\,\Omega$ the ground resonance instability dominates the damping requirement. The full lines indicate damping requirements for a one per cent margin in stability and again for lag frequencies below $\cdot 8\Omega$ ground resonance dominates the requirement. The curves are based on Lynx calculations using a four degree of freedom fuselage model and including blade flapping freedoms. The ground resonance curves are based on the assumption that the fuselage frequency coincides with the fixed co-ordinate blade lag frequency at normal operating rotor speed. A margin of one per cent critical has been used in the following analysis in order to ensure firstly that stability is still achieved if the frequency intersection occurs at maximum overspeed, and secondly to ensure that at normal rotor speed any oscillations initiated on the ground are damped out without undue annoyance to the pilot. It has also been assumed that the structural damping inherent in the blade will provide an added margin. Measurements indicate values in the range 0.5 - 1.0 per cent. Thus a total margin of approximately 1.5 per cent is assured. This represents a decay rate of 7 cycles to half amplitude, which by experience is acceptable.

It is worth noting that the need for a positive damping margin is a consequence of the linear nature of the damping. For the more conventional form of hydraulic damper with load limiting devices, the damping decreases with increasing amplitude and frequency. With this form of damping, adequate decay rates are achieved by virtue of the smallness of the normal disturbance and it is only for the very large initial disturbance that near zero decay rates are encountered.

FIG. 2 shows the damping versus lag frequency curve derived using a simple "Coleman" model. Assuming coincident body frequencies which are also coincident with fixed co-ordinate blade frequency, the damping required for neutral stability is given by:-

$$\frac{\sigma}{\omega_\zeta} = \frac{1}{4}\left(\frac{\Omega}{\omega_\zeta} - 1\right)^{3/2}\sqrt{\frac{n\,(M_b)^2}{M_e\,I_b}} \qquad (1)$$

where M_e = effective mass of fuselage

M_b = 1st moment of blade about lag hinge

I_b = 2nd moment of inertia blade about lag inge

n = number of blades

Ω = rotor speed

σ = real part of root

ω_ζ = lag frequency

Investigation showed that for a wide range of helicopter sizes and assuming a minimum ratio of effective to actual fuselage mass, then equation (1) reduced to:

$$\frac{\sigma}{\omega_\zeta} = \zeta = \cdot 077\left(\frac{\Omega}{\omega_\zeta} - 1\right)^{3/2} \qquad (2)$$

A comparison of equation (2) with the damping required for 1% margin is presented in FIG. 2. It is considered reasonable to compare the neutral stability boundary of the Coleman calculation with the 1% boundary of the more exact model in view of the pessimism of assuming coincident body frequencies with minimum effective mass in the Coleman calculation.

2.2 Damper Design Criteria

The two major criteria for damper design are firstly that the damper shall provide a specified minimum damping, and secondly that it shall have an adequate fatigue life. The first criterion has been covered in section 2.1 above. The second criterion can only be considered if the operating environment of the damper and the strength of the damping material is known.

It is assumed that for a given helicopter application, the in-flight amplitude of damper motion is governed by the fundamental lag frequency of the rotor. It is further assumed that the influence is the same for all conditions throughout the flight envelope, thus enabling the same factor to be applied to the equivalent blade lag angle derived by using a suitable S-N curve and Miners' Rule of Cumulative Damage. Consider an elastomeric damper which is attached to the blade in such a way that:

d = damper displacement/unit tip deflection

ζ = equivalent blade tip deflection for given life

$\frac{\sigma}{\omega_\zeta}$ = required lag damping

ε_s = allowable strain in damper for required life

t = thickness of damper

A = Area of damper

G = Shear modulus of rubber

η = Loss factor ($Q = \frac{1}{\eta}$)

K^1 = Real part of complex damper stiffness

K^{11} = Imaginary part of complex damper stiffness

m_e = effective mass of blade lag mode referred to unit tip deflection

ω_ζ = Blade lag frequency

Linear damping rate = $\dfrac{K^{11}}{\omega_\zeta} = \dfrac{K^1\eta}{\omega_\zeta} = \dfrac{GA}{t\omega_\zeta}\eta$ (3)

But required damping = $2\dfrac{\sigma}{\omega_\zeta}\cdot m_e\cdot\dfrac{\omega_\zeta}{d^2}$ (4)

Therefore $2\sigma\dfrac{m_e}{d^2} = \dfrac{GA\eta}{t\omega_\zeta}$ combining 3 and 4 (5)

But $t = \dfrac{\zeta d}{\varepsilon_s}$ from allowable strain (6)

and $A = \dfrac{2\sigma m_e\omega_\zeta t}{G d^2\eta}$ from (5) (7)

Therefore, Volume of damper = $At = 2\dfrac{\sigma m_e\omega_\zeta}{G\eta\varepsilon_s^2}\zeta^2$

However, the lag deflection ζ is dependent on lag frequency such that $\dfrac{\zeta}{\zeta_0} = \dfrac{1-\gamma_0^2}{1-\gamma^2}$ (8)

where $\gamma = \dfrac{\omega_\zeta}{\Omega}$ and subscript o refers to some arbitrary datum i.e. γ_0, ζ_0 constant.

Combining (8) and (9) gives:

$$\text{Volume} = 2 \frac{m_e \Omega^2}{G \varepsilon_s^2 \eta} (1-\gamma_o^2)^2 \frac{\zeta_o^2 \gamma^2}{(1-\gamma^2)^2} \left(\frac{\sigma}{\omega_s}\right) \qquad (9)$$

It can now be seen that the material properties affecting the volume of the required damper are Loss factor, Shear modulus and allowable strain. The factor $\frac{1}{G \varepsilon_s^2 \eta}$ is very sensitive to allowable strain, and it is for this reason that fatigue strength of the rubber plays a more important role than loss factor in determining the volume of damping material required.

FIG. 3 shows the effect of combining the volume of the damper defined by equation (9) as a function of lag damping and frequency with the lag damping required in FIG. 1. Also shown on FIG. 3 is the volume versus frequency curve based on the damping required from the simplified Coleman model.

The first conclusion to be reached from the examination of FIG. 3 is that whatever the assumption made about required damping, there is no advantage in damper design in increasing the lag frequency beyond 0.75Ω. Furthermore, for the practical Lynx case with 1% margin, an optimum frequency range exists, i.e. $0.50\Omega - 0.65\Omega$. In the case of damping derived from the Coleman model, the volume of the damper increases with increasing lag frequency throughout the frequency range of interest. The overall conclusion must therefore be that, both from consideration of overall lag plane loading and also from consideration of damper design, the lag frequency of the rotor should be less than 0.65Ω.

3. ROTOR DESIGN STUDY

In order to investigate the practicality of achieving the optimum solution indicated in the first part of the study, the simple blade model shown in FIG. 4 was used. This model is the simplest model which is capable of representing the range of blade lag plane characteristics from fully articulated through restrained articulated to semi-rigid.

The blade lag frequency ω_ζ is given by:-

$$\omega_\zeta^2 = \frac{m_b a b \Omega^2 + K}{I} \qquad \text{where} \qquad K = K_1 + \left\{\frac{1}{K_2} + \frac{1}{K_D}\right\}^{-1}$$

$$\gamma^2 = \Lambda_1 + \frac{K}{I\Omega^2} \qquad \text{where} \quad \gamma = \frac{\omega_\zeta}{\Omega} \qquad \text{and} \quad \Lambda_1 = \frac{m_b a b}{I} \tag{10}$$

Let $\gamma_0 = \left[\gamma\right]_{K_D \to 0}$ and $\gamma_\infty = \left[\gamma\right]_{K_D \to \infty}$

$$\gamma_0^2 = \Lambda_1 + K_1/I\Omega^2 \tag{11}$$

$$\text{and} \quad \gamma_\infty^2 = \Lambda_1 + (K_1 + K_2)/I\Omega^2 \tag{12}$$

The non-dimensional blade damping is given by:-

$$\frac{\sigma}{\omega_\zeta} = \frac{2K_D}{\omega_\zeta(2I_D\omega_\zeta)} \tag{13}$$

where I_D = blade inertia referred to damper displacement

$$\text{Therefore,} \quad I_D = I\, K_D^2\left(\frac{1}{K_D} + \frac{1}{K_2}\right)^2 \tag{14}$$

Combining (13), (14) and (10)

$$\gamma^2 = \Lambda_1 + \left[K_1 + \left(\frac{1}{K_2} + \frac{1}{K_D}\right)^{-1}\right]\frac{K_D}{2}\left(\frac{1}{K_D} + \frac{1}{K_2}\right)^2 2\gamma^2\frac{\sigma}{\omega_\zeta}$$

which reduces by suitable substitution to the form

$$\frac{\sigma}{\omega_\zeta} = \frac{2}{2\gamma^2}\,\frac{(\gamma^2 - \gamma_0^2)(\gamma_\infty^2 - \gamma^2)}{(\gamma_\infty^2 - \gamma_0^2)} \tag{15}$$

In addition it can be shown that

$$K_D = \frac{I\Omega^2(\gamma_\infty^2 - \gamma_0^2)(\gamma^2 - \gamma_0^2)}{(\gamma_\infty^2 - \gamma^2)} \tag{16}$$

It has been found convenient to plot curves of damping achieved in the form of $\frac{\sigma}{\eta \omega_s}$ versus frequency γ. At the same time the damping required may be presented in the same form for various values of loss factor η.

FIG. 5 shows this comparison for a typical articulated rotor configuration where $\gamma_o = 0.25$. Two values of loss factor are presented for the damping required curves. Solutions using the lower loss factor are preferred due to the higher fatigue strength of this material. As indicated in the previous section, damper volume is inversely proportional to loss factor and allowable strain squared. This effect gives rise to a reduction in damper volume by a factor of approximately four for current materials. The penalty of achieving the minimum volume solution is, as can be seen from FIG. 5, an increase in the resultant lag frequency of the blade. In fact for some values of γ_o and γ_∞ no solutions exist using high fatigue strength rubber.

The curves on FIG. 5 give emphasis to what is perhaps intuitively obvious, that is, that to achieve an optimum overall solution, a low value of γ_o combined with a high value of γ_∞ is required. FIG. 3 indicates that from a damper design point of view, a lag frequency below $0.65\,\Omega$ is required. FIG. 5 shows that for $\gamma_o = 0.25$, a value of $\gamma_\infty > 0.75$ is required to achieve a solution using high fatigue strength rubber.

The significance of γ_o, the non-dimensional lag frequency with the damper disconnected, is shown in FIG. 6 where a range of values of γ_∞ is plotted for a value of $\gamma_o = 0.65$. For this value of γ_o, which is typical of a semi-rigid or hingeless rotor, clearly no solution exists which gives a resulting lag frequency less than $0.65\,\Omega$.

FIG. 6 shows that the minimum resultant frequency that can be achieved is approximately $0.70\,\Omega$, and this only with low fatigue strength rubber combined with a γ_∞ of 0.9. Using the same value of γ_∞ but combined with high fatigue strength rubber, a resultant lag frequency of 0.74 Ω is required, with consequent effects on blade and hub lag plane loading. Since the value of γ_∞ is primarily determined by blade chordwise stiffness, the effect of using visco-elastic damping for semi-rigid or hingeless rotors is to put more stringent requirements on the design of the blade. FIG. 7 shows more clearly the effect of varying γ_o while keeping γ_∞ constant. For instance, should a final frequency of 0.65 Ω be required, together with γ_∞ = 0.9, then the required value for γ_o is 0.59 for high loss factor material and 0.53 for low damping rubber.

FIG. 8 presents the intersections of required and available damping from FIGS. 6 and 7 in the form of carpet plots of γ versus γ_o and γ_∞. Two values of loss factor are shown together with the limit line beyond which no solutions exist. By cross-plotting pairs of values of γ_o, γ_∞ which give constant values of γ, the relationship between γ_o and γ_∞ for any value of γ can be evaluated. An example of this is given in FIG. 9 where a value for γ of 0.65 has been chosen. It is interesting to note that the curves terminate at the lower values of γ_∞ where the damping requirement curve is tangential to the damping available curve.

FIG. 9 clearly shows the penalties of using low loss factor material since for typically achievable values of γ_∞ say 0.8 - 0.9, the required values of γ_o are 0.45 - 0.52 for a loss factor of 0.25, while for a loss factor of 0.4, a value of γ_o of 0.58 would be acceptable. This difference is clearly critical for a semi-rigid or hingeless design of rotor.

4. CONCLUSION

The first conclusion to be drawn from the damper design study is that from purely damper design considerations, and given the assumption of a frequency coincidence within the rotor operating speed range, the rotor lag frequency should be less than $0.65\,\Omega$. Since from a rotor design point of view the lower the lag frequency the lower the overall blade and hub lag loading, the overall conclusion must be that the optimum lag frequency lies below the value of $0.65\,\Omega$.

The second conclusion, drawn from section 3 above, is that in order to utilise the full potential of visco-elastic damping materials, the rotor design must take into account at the outset that such materials will be used. In other words, the application of elastomeric dampers to an existing design will certainly not be an optimum solution and may indeed be impossible; for instance, hingeless rotors with relatively low inplane blade stiffness.

FIGS. 5-9 should enable the rotor designer to select the optimum type of rotor, which largely determines the value of γ_0 and also to select the optimum position of the damper and its attachments which together with the blade lag plane stiffness determine the value of γ_∞.

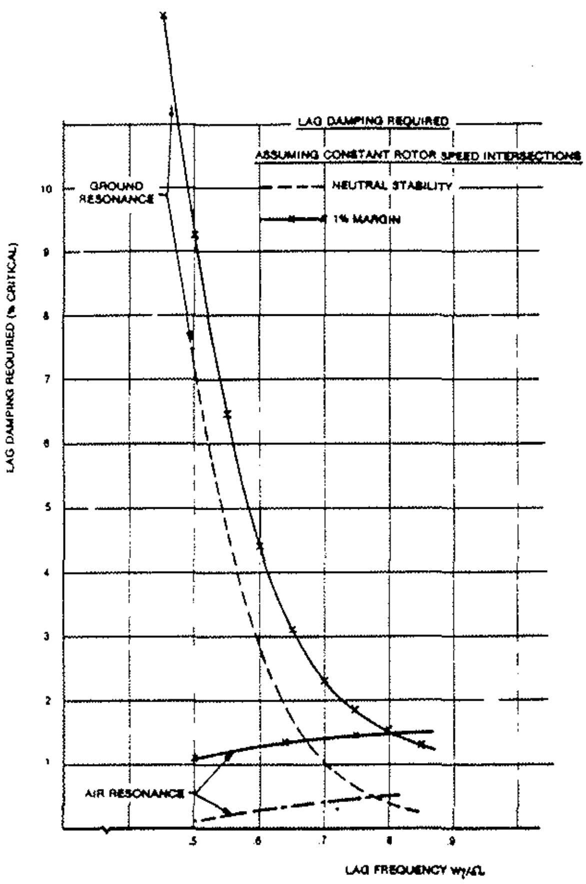


FIGURE 1

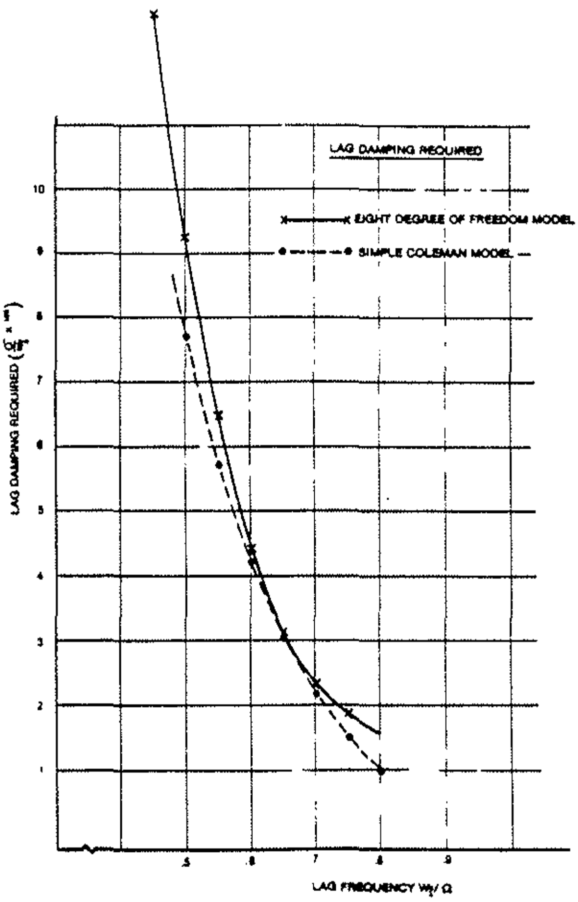


FIGURE 2

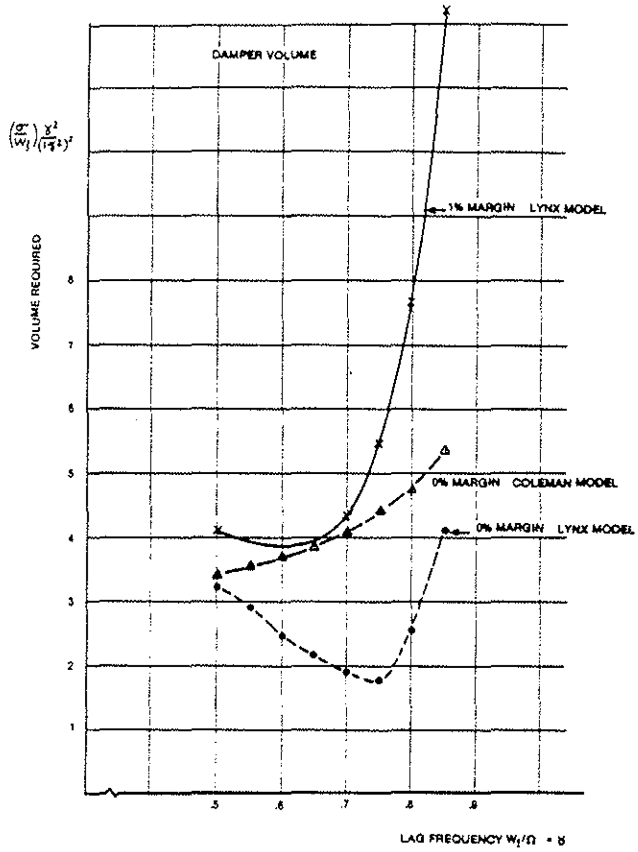
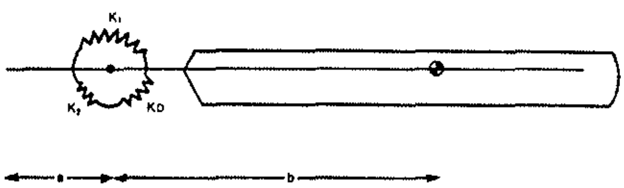


FIGURE 3



K_D = Damper Stiffness =

K_2 = Damper Support Stiffness

K_1 = Structural Stiffness in parallel with damper

a = Hinge offset

b = Blade centre of gravity offset from hinge

M_b = Blade mass

I = Blade moment of inertia about hinge

FIGURE 4

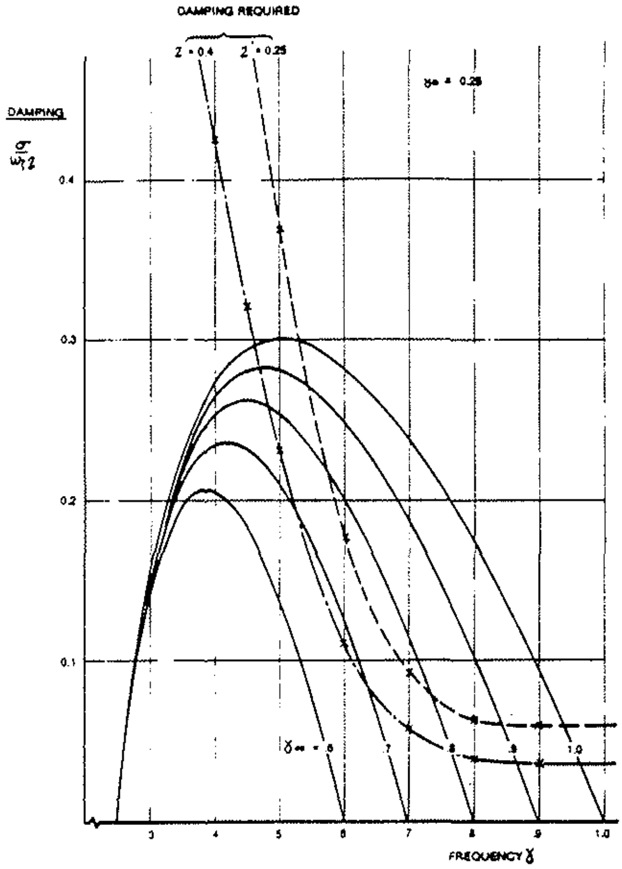


FIGURE 5

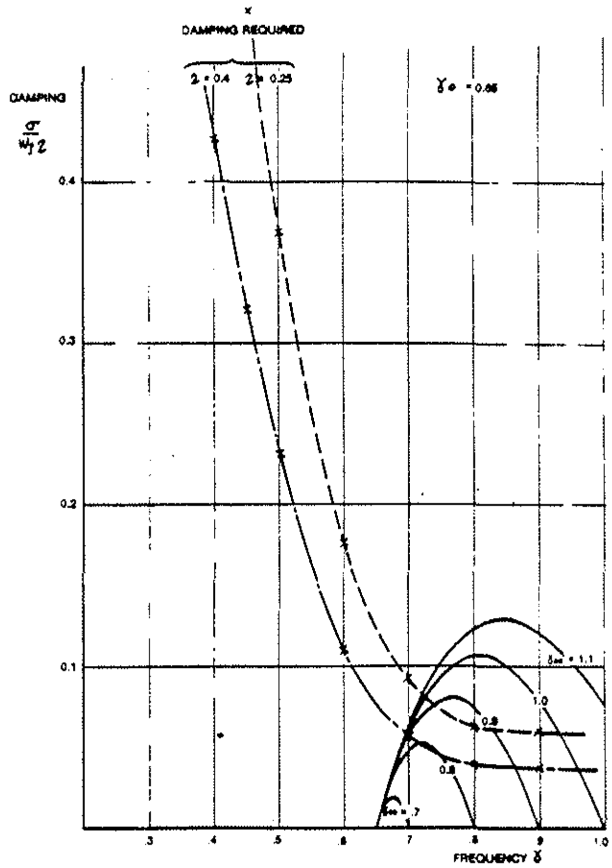


FIGURE 6

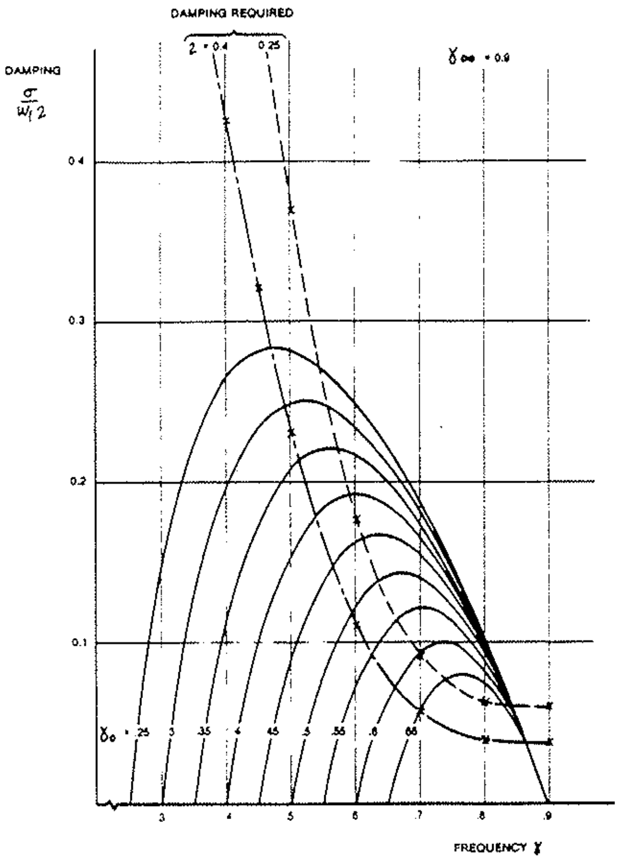


FIGURE 7

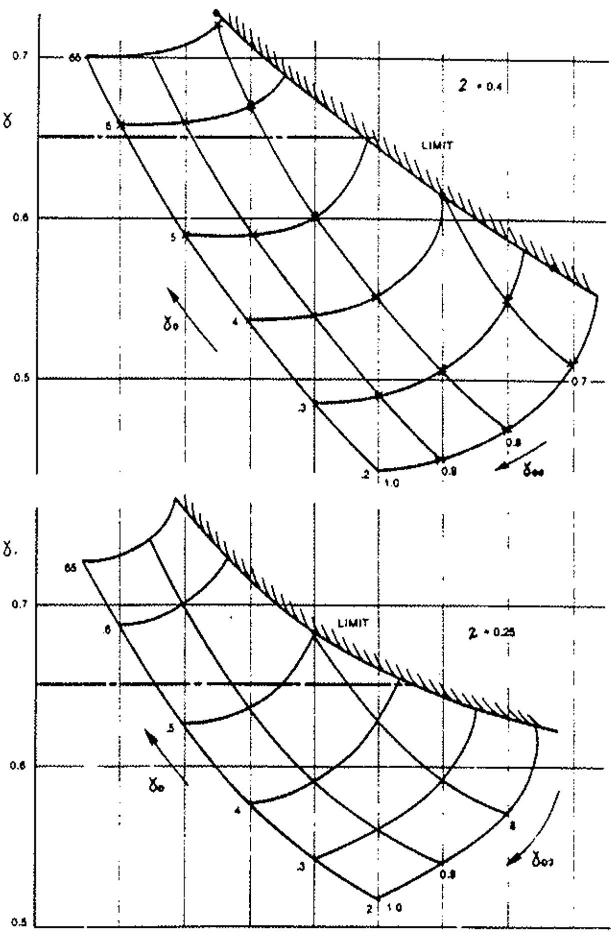


FIGURE 8

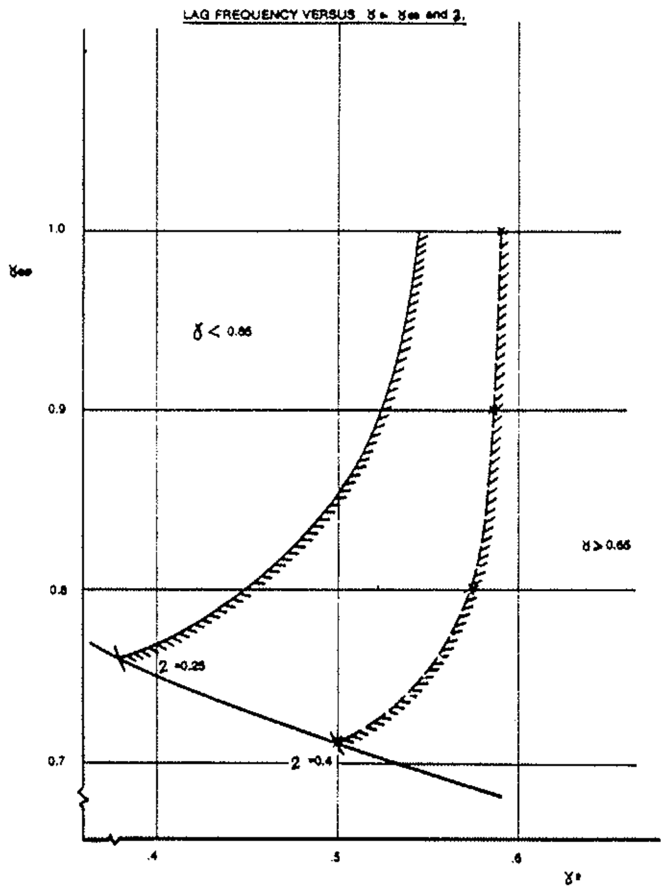

LAG FREQUENCY VERSUS γ*, γee and ζ.
γee
1.0
0.9
0.8
0.7
γ < 0.65
γ > 0.65
ζ =0.25
ζ =0.4
.4
.5
.6
γ*

FIGURE 9