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1. INTRODUCTION

This work deals with recent developments of a boundary element methodology for the unified analysis of aerodynamics and aeroacoustics of potential, transonic flows. This technique is adopted because, with respect to CFD approaches, it allows an easier and more computationally efficient analysis of flows around complex configurations like, such as a free-wake analysisi of a helicopter rotor in presence of the fuselage.

Boundary integral equations have been the first successfull approach for the analysis of transonic aerodynamics. In fact, the pioneering works of Oswatitsch [1] and Spreiter and Alskne [2], for steady, two-dimensional, transonic flows, precede the finite-difference work of Murman and Cole [3], which is considered a milestone in the development of numerical techniques for the solution of transonic flows.

Further boundary integral equation approaches are investigated by Nixon [4, 5] who presents a small perturbation scheme for the solution of unsteady two-dimensional and steady three-dimensional flows, and Piers and Sloof [6], who apply a shock capturing integral formulation to the TSP model. Tseng and Morino [7] present the first transonic results based on the formulation of Morino [8]. A different formulation for the full-potential equation has been presented by Sinclair [9, 10] and applied to two-dimensional and three-dimensional configurations, respectively. In contrast to Tseng and Morino [7], where the field sources are due to nonlinear terms, in Sinclair [9, 10] the field sources include all the compressibility terms (linear and nonlinear). This implies that the differential operator for the boundary integral formulation of Tseng and Morino [7] is that of the wave equation, whereas for Sinclair [9, 10] is the Laplacian.

Applying a TSP formulation similar to that used by Tseng and Morino [7], Iemma, Mastroddi, Morino, and Pecora [11], present the first validation for three-dimensional unsteady flows, whereas in Morino and Iemma [12] the extension to the full-potential analysis is considered for fixed wings. Preliminary results for the simulation of transonic flows around helicopter rotors in hover are presented in Morino, Gennaretti, Iemma, and Mastroddi [13], who include a derivation of the TSP model for rotors in hover.

Here, we derive a potential integral representation where the linear contributions are expressed by body and wake surface integrals, whereas a field term takes into account the non linear effects.

In particular, we investigate two different ways of treating the non linear term in the full-potential formulation, for application both in the aerodynamic and aeroacoustic analyses.

2. FULL-POTENTIAL TRANSONIC FLOWS

The governing equations for compressible, irrotational, isentropic flows of ideal gases are the expression for the velocity $\mathbf{v} = \nabla\phi$, the continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

Bernoulli's theorem,

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} v^2 + h = h_\infty \tag{2}$$

(where h is the hentalpy), and the isentropic law for ideal gases p/ρ^γ = constant. Combining the above equations, considering that $h = \gamma p / (\gamma - 1)\,\rho$, and moving all the nonlinear terms to the right hand side of the equation, we obtain the following form for the nonlinear equation of the velocity potential (in air frame of reference)

$$\nabla^2 \phi - \frac{1}{a_\infty^2} \frac{\partial^2 \phi}{\partial t^2} = \sigma \tag{3}$$

where $a_\infty^2 = \gamma p_\infty / \rho_\infty$ is the speed of sound in the undisturbed flow, and σ denotes all the nonlinear terms.

The full-potential form for σ is (see [14]):

$$\sigma = \nabla \cdot \left[\left(1 - \frac{\rho}{\rho_\infty} \right) \nabla \phi \right] + \frac{\partial}{\partial t} \left(1 - \frac{\rho}{\rho_\infty} - \frac{1}{a_\infty^2} \frac{\partial \phi}{\partial t} \right) = \nabla \cdot \tilde{\mathbf{b}} + \frac{\partial \hat{b}}{\partial t} \tag{4}$$

where $\rho/\rho_\infty = [1 - (\dot{\phi} + v^2/2)/h_\infty]^{1/(\gamma-1)}$.

In order to complete the above differential formulation one needs boundary conditions. The surface of the body S_B is assumed to be impermeable, *i.e.*, $\mathbf{n} \cdot (\mathbf{v} - \mathbf{v}_B) = 0$, where $\mathbf{v}_B$ is the velocity of the point on the surface of the body S_B; at infinity the condition is $\mathbf{v} = 0$ (in the air frame of reference). In terms of the velocity potential, these boundary conditions are

$$\frac{\partial \phi}{\partial n} = \chi \qquad \text{for } \mathbf{x} \text{ on } S_B \tag{5}$$

where $\chi = \mathbf{v}_B \cdot \mathbf{n}$, and

$$\phi = 0 \qquad \text{at infinity} \tag{6}$$

The boundary conditions on the wake state the absence of penetration ($\mathbf{v} \cdot \mathbf{n} = \mathbf{v}_W \cdot \mathbf{n}$) and of pressure discontinuity ($\Delta p = 0$). These, in terms of the velocity potential, yield (see, *e.g.*, [14])

$$\Delta \left(\frac{\partial \phi}{\partial n} \right) = 0 \tag{7}$$

and

$$\frac{D_W(\Delta\phi)}{Dt} = 0 \tag{8}$$

where $D_W/Dt = \partial/\partial t + \mathbf{v}_W \cdot \nabla$, with $\mathbf{v}_W$ being the velocity of a point of the wake $\mathbf{x}_W$ (*i.e.*, the average of the velocity on the two sides of the wake). Equation 8 states that $\Delta\phi$ is constant in time following a wake point and equal to the value it had when $\mathbf{x}_W$ left the trailing edge. This value is obtained for $\Delta\phi$ on the body also at the trailing edge (trailing edge condition).

3. INTEGRAL FORMULATION FOR RIGID BODIES IN ARBITRARY MOTION

In this section we present a boundary integral formulation for isolated rigid bodies in arbitrary motion introduced by Gennaretti [15] and further developed in [16], in which the wake surface is assumed to be fixed with respect to the body. In order to accomplish this it is convenient to consider two different spaces: the air space (*i.e.*, the space rigidly connected with the undisturbed air) and the body space (*i.e.*, the space rigidly connected with the body). Let the transformation relating $\boldsymbol{\xi}$ to $\mathbf{x}$ be given by[1]

$$\boldsymbol{\xi} = \boldsymbol{\xi}(\mathbf{x},t) = \boldsymbol{\xi}_0(t) + \mathbf{R}(t)\mathbf{x} \tag{9}$$

where $\mathbf{R}(t)$ is rigid-body rotation tensor relating the two spaces, whereas $\boldsymbol{\xi}_0$ is the image of the point $\mathbf{x} = 0$. Note that Eq. 9 implies, for any $f = f[\boldsymbol{\xi}(\mathbf{x},t),t]$,

$$\frac{\partial f}{\partial t} = \left.\frac{\partial f}{\partial t}\right|_{\boldsymbol{\xi}=\text{const.}} + \mathbf{v}_\mathbf{x} \cdot \nabla f \tag{10}$$

where ∇f denotes the gradient of f in the body space, whereas $\mathbf{v}_\mathbf{x}$ denotes the body-space vector of the velocity of a body-space point $\mathbf{x}$ relative to the air space; $\mathbf{v}_\mathbf{x}$ is related to $\boldsymbol{v}_\mathbf{x}$ (air-space vector of the velocity of $\mathbf{x}$ relative to the air space) by

$$\boldsymbol{v}_\mathbf{x} = \frac{\partial}{\partial t}\boldsymbol{\xi}(\mathbf{x},t) = \mathbf{R}\mathbf{v}_\mathbf{x} \tag{11}$$

The equation for the velocity potential in the body space is given by

$$\nabla^2\phi - \frac{1}{a_\infty^2}\frac{\mathrm{d}_B^2\phi}{\mathrm{d}t^2} = \sigma \qquad \mathbf{x} \in \mathcal{V} \tag{12}$$

where

$$\frac{\mathrm{d}_B}{\mathrm{d}t} = \left.\frac{\partial}{\partial t}\right|_{\boldsymbol{\xi}=\text{const.}} = \frac{\partial}{\partial t} - \mathbf{v}_\mathbf{x} \cdot \nabla \tag{13}$$

denotes the body-space time derivative following a fixed point of the air space, whereas $\mathcal{V}$ denotes the volume where the flow is potential (*i.e.*, the whole space minus the solid volume and an infinitesimal volume that includes the wake surface).[2]

[1] In this section, the vectors in the body space are denoted with Latin boldface letters, whereas those in the air space are denoted with Greek boldface letters. In particular, $\boldsymbol{\xi}$ denotes the position vector in the air space, whereas $\mathbf{x}$ denotes the position vector in the body space.

[2] In the following, $\mathcal{V}$ is assumed time independent in the body space; this implies that the formulation is applicable only to bodies without wake (non-lifting bodies) or the flows with a wake that does not move with respect to the body frame (*e.g.*, helicopter rotors in hover and propellers in axial flows). A general formulation has been presented by Gennaretti [17].

The fundamental solution of the operator on the left side of Eq. 12 satisfies the equation

$$\nabla^2 G - \frac{1}{a_\infty^2}\frac{\mathrm{d}_B^2 G}{\mathrm{d}t^2} = \delta(\mathbf{x}-\mathbf{x}_*)\,\delta(t-t_*) \qquad \mathbf{x}\in\mathcal{V} \tag{14}$$

with boundary condition $G(\infty,t)=0$ and initial conditions $G(\mathbf{x},\infty)=\dot{G}(\mathbf{x},\infty)=0$. For the cases of subsonic and transonic flows considered here, G has the expression

$$G(\mathbf{x},\mathbf{x}_*,t,t_*) = \frac{-1}{4\pi\hat{\varrho}}\delta(t-t_*+\theta) \tag{15}$$

where θ (the time required for a signal to propagate from $\mathbf{x}$ to $\mathbf{x}_*$) satisfies the equation $a_\infty\theta = \|\boldsymbol{\xi}(\mathbf{x},t_*-\theta)-\boldsymbol{\xi}(\mathbf{x}_*,t_*)\|$. In addition, $\hat{\varrho} = [\varrho\,|1+\boldsymbol{\varrho}\cdot\boldsymbol{v}_\mathbf{x}/a_\infty\varrho|]^\theta$, where $\boldsymbol{\varrho}=\hat{\boldsymbol{\xi}}(\mathbf{x},t)-\hat{\boldsymbol{\xi}}(\mathbf{x}_*,t_*)$ and $\varrho=\|\boldsymbol{\varrho}\|$, whereas $[...]^\theta$ denotes evaluation at time $t=t_*-\theta$.

Multiplying Eqs. 12 and 14 by G and ϕ respectively, subtracting, using Eq. 13, integrating with respect to time, and taking into account the initial conditions on ϕ and G, one obtains

$$\begin{aligned}\phi(\mathbf{x}_*,t_*) = &-\int_0^\infty \iiint_\mathcal{V} \nabla\cdot(G\nabla\phi-\phi\nabla G)\,d\mathcal{V}\;dt \\ &-\frac{1}{a_\infty^2}\int_0^\infty\iiint_\mathcal{V}\mathbf{v}_\mathbf{x}\cdot\nabla\left(G\frac{\mathrm{d}_B\phi}{\mathrm{d}t}-\phi\frac{\mathrm{d}_B G}{\mathrm{d}t}\right)d\mathcal{V}\;dt+\int_0^\infty\iiint_\mathcal{V} G\,\sigma\;d\mathcal{V}\;dt\end{aligned} \tag{16}$$

Next, note that for any f and $\mathbf{w}$, $\nabla\cdot(f\mathbf{w})=f\nabla\cdot\mathbf{w}+\nabla f\cdot\mathbf{w}$, and that $\nabla\cdot\mathbf{v}_\mathbf{x}=0$ (since the body space moves in rigid-body motion). Then, applying Gauss' theorem, Eq. 16 as well as the conditions at infinity for ϕ and G, yields

$$\begin{aligned}\phi(\mathbf{x}_*,t_*) = &\int_0^\infty\oiint_\mathcal{S}\left(G\,\frac{\partial\phi}{\partial n}-\phi\,\frac{\partial G}{\partial n}\right)d\mathcal{S}\;dt \\ &+\frac{1}{a_\infty^2}\int_0^\infty\oiint_\mathcal{S}\left(G\frac{\mathrm{d}_B\phi}{\mathrm{d}t}-\phi\frac{\mathrm{d}_B G}{\mathrm{d}t}\right)\mathbf{v}_\mathbf{x}\cdot\mathbf{n}\;d\mathcal{S}\;dt+\int_0^\infty\iiint_\mathcal{V} G\,\sigma\;d\mathcal{V}\;dt\end{aligned} \tag{17}$$

where $\mathcal{S}$ is the boundary of $\mathcal{V}$ and $\mathbf{n}$ is its outwardly directed unit normal.

Next, using Eq. 15, integrating with respect to time, and setting $\hat{G}=-1/4\pi\hat{\varrho}$, one finally obtains the following boundary integral representation, in the body frame of reference, for the velocity potential ϕ, for a body in arbitrary rigid motion:

$$\begin{aligned}\phi(\mathbf{x}_*,t_*) = &\oiint_\mathcal{S}\left[\hat{G}\frac{\partial\phi}{\partial\tilde{n}}-\frac{\partial\hat{G}}{\partial\tilde{n}}\phi+\frac{\partial\phi}{\partial t}\hat{G}\;\left(\frac{\partial\theta}{\partial\tilde{n}}+\frac{2}{a_\infty^2}\mathbf{v}_\mathbf{x}\cdot\mathbf{n}\right)\right]^\theta d\mathcal{S} \\ &-\frac{1}{a_\infty^2}\oiint_\mathcal{S}\left[\hat{G}\;\phi\frac{\partial}{\partial t}\left(\mathbf{v}_\mathbf{x}\cdot\mathbf{n}\;\mathbf{v}_\mathbf{x}\cdot\nabla\theta-\mathbf{v}_\mathbf{x}\cdot\mathbf{n}\right)\right]^\theta d\mathcal{S}+\iiint_\mathcal{V}\left[\hat{G}\sigma\right]^\theta d\mathcal{V}\end{aligned} \tag{18}$$

where

$$\frac{\partial}{\partial\tilde{n}} = \frac{\partial}{\partial n}-\frac{1}{a_\infty^2}\mathbf{v}_\mathbf{x}\cdot\mathbf{n}\;\mathbf{v}_\mathbf{x}\cdot\nabla \tag{19}$$

If $\mathbf{x}_*$ is in $\mathcal{V}$, Eq. 18 is an integral representation for $\phi(\mathbf{x}_*,t_*)$ in terms of ϕ, $\partial\phi/\partial\tilde{n}$ and $\partial\phi/\partial t$ on $\mathcal{S}$. On the other hand, if $\mathbf{x}_*$ is on $\mathcal{S}$, Eq. 18 represents a compatibility condition between ϕ, $\partial\phi/\partial\tilde{n}$, and $\partial\phi/\partial t$ for any function ϕ satisfying Eq. 12.

In the case of non-lifting bodies, we identify $\mathcal{S}$ with the surface of the body, $\mathcal{S}_B$. Then, $\partial\phi/\partial n$ is known from the boundary condition, and the compatibility condition yields an integral-differential-delay equation which may be used to obtain the values of ϕ on $\mathcal{S}$ from those of $\partial\phi/\partial n$. For the case of lifting bodies, an explicit treatment of the wake is required. Consider an isolated rotor in hover or an isolated propeller in axial flow and identify $\mathcal{S}$ in Eq. 18 with a surface $\mathcal{S}_{BW}$ surrounding body and wake. Letting the portion of $\mathcal{S}_{BW}$ that surrounds the wake approach $\mathcal{S}_W$ we obtain a contribution on the wake surface that is related only to $\Delta\phi$ (as $\Delta(\partial\phi/\partial n) = 0$ on $\mathcal{S}_W$, see Eq. 7); $\Delta\phi$ is obtained from Eq. 8.

It should be emphasized that, once the potential is known on $\mathcal{S}$, Eq. 18 with $\mathbf{x}_*$ in the field yields the value of ϕ anywhere in the field. Therefore, we evaluate the aeroacoustic field, *i.e.*, the pressure in the field around the body, by using the potential solution in the Bernoulli theorem.

4. FIELD TERM CONTRIBUTION

In this section we present two different approaches for the evaluation of the non linear field term which is present in Eq. 18.

Noting that the term σ in the volume integral may be written in the form $\sigma = \nabla\cdot\mathbf{b} + \partial\hat{b}/\partial t$ where $\mathbf{b} = \tilde{\mathbf{b}} - \hat{b}\,\mathbf{v}_{\mathbf{x}}$ and the time derivative is calculated in the body space, in order to avoid the calculation of the divergence, the formulation introduced in [7] consists of an integration by parts that yields

$$\iiint_{\mathcal{V}} [\sigma]^\theta\,\hat{G}\,d\mathcal{V} = -\oiint_{\mathcal{S}_B} [\mathbf{n}\cdot\mathbf{b}]^\theta\,\hat{G}\,d\mathcal{S} + \iiint_{\mathcal{V}} \left[\frac{\partial\hat{b}}{\partial t} + \frac{\partial\mathbf{b}}{\partial t}\cdot\nabla\theta\right]^\theta \hat{G}\,d\mathcal{V} - \iiint_{\mathcal{V}} [\mathbf{b}]^\theta\cdot\nabla\hat{G}\,d\mathcal{V} \qquad (20)$$

having considered that

$$\nabla\cdot[\mathbf{b}]^\theta = [\nabla\cdot\mathbf{b}]^\theta - \left[\frac{\partial\mathbf{b}}{\partial t}\right]^\theta\cdot\nabla\theta \qquad (21)$$

Then, dividing the surface of the body into M surface elements $\mathcal{S}_m$, the fluid volume into Q volume elements $\mathcal{V}_q$ and applying a zeroth-order discretization, the discretized version of the non linear term is given by

$$\begin{aligned}\iiint_{\mathcal{V}} [\sigma]^\theta\,\hat{G}\,d\mathcal{V}\Big|_{\mathbf{x}_*=\mathbf{x}_k} &\approx \sum_m^M B_{km}\,[\hat{\chi}_m]^{\theta_{km}} + \sum_q^Q \mathbf{H}_{kq}\cdot[\mathbf{b}_q]^{\theta_{kq}} \\ &+ \sum_q^Q \bar{\mathbf{H}}_{kq}\cdot[\dot{\mathbf{b}}_q]^{\theta_{kq}} + \sum_q^Q \hat{H}_{kq}[\dot{\hat{b}}_q]^{\theta_{kq}}\end{aligned} \qquad (22)$$

where $\hat{\chi}_m = -\mathbf{n}(\mathbf{x}_m)\cdot\mathbf{b}(\mathbf{x}_m)$, $[\ldots]^{\theta_{km}}$ denotes evaluation at the retarded time $t - \theta_{km}$, and the coefficients are defined in the following way

$$B_{km} = \iint_{\mathcal{S}_m} \hat{G}_k\,d\mathcal{S}; \quad \mathbf{H}_{kq} = -\iiint_{\mathcal{V}_q} \nabla\hat{G}_k\,d\mathcal{V}; \quad \bar{\mathbf{H}}_{kq} = \iiint_{\mathcal{V}_q} \hat{G}_k\,\nabla\theta_k\,d\mathcal{V}; \quad \hat{H}_{kq} = \iiint_{\mathcal{V}_q} \hat{G}_k\,d\mathcal{V} \qquad (23)$$

where $\hat{G}_k = \hat{G}\Big|_{\mathbf{x}_*=\mathbf{x}_k}$.

In the alternative approach, the field term is discretized in its original form. Therefore, following the discretization criteria described above, and remembering the expression of the space coefficients yield

$$\iiint_{\mathcal{V}} [\sigma]^{\theta} \hat{G} d\mathcal{V}\bigg|_{\mathbf{x}_*=\mathbf{x}_k} \approx \sum_{q}^{Q} \hat{H}_{kq} \left(\overline{[\nabla \cdot \mathbf{b}]^{\theta_k}}\right)_q + \sum_{q}^{Q} \hat{H}_{kq} [\dot{\hat{b}}_q]^{\theta_{kq}} \tag{24}$$

where $\left(\overline{[\nabla \cdot \mathbf{b}]^{\theta_k}}\right)_q$ denotes the mean value inside the volume element $\mathcal{V}_q$ of the term $[\nabla \cdot \mathbf{b}]^{\theta}$, with the delay θ evaluated with respect to the point $\mathbf{x}_* = \mathbf{x}_k$. Next, in order to avoid the evaluation of the divergence operator, we consider Eq. 21 and perform the following convenient transformation

$$\left(\overline{[\nabla \cdot \mathbf{b}]^{\theta_k}}\right)_q = \frac{1}{\mathcal{V}_q} \iiint_{\mathcal{V}_q} [\nabla \cdot \mathbf{b}]^{\theta_k} d\mathcal{V} = \frac{1}{\mathcal{V}_q} \oiint_{\partial \mathcal{V}_q} [\mathbf{b}]^{\theta_k} \cdot \mathbf{n}\, dS + \left(\overline{\left[\dot{\mathbf{b}}\right]^{\theta_k} \cdot \nabla \theta_k}\right)_q \tag{25}$$

where $\mathbf{n}$ is the outwardly directed unit normal to the surface $\partial \mathcal{V}_q$, whereas $\left(\overline{\left[\dot{\mathbf{b}}\right]^{\theta_k} \cdot \nabla \theta_k}\right)_q$ denotes the mean value inside the volume element $\mathcal{V}_q$ of the term $\left[\dot{\mathbf{b}}\right]^{\theta} \cdot \nabla \theta$, with the delay θ evaluated with respect to the point $\mathbf{x}_* = \mathbf{x}_k$.

The first approach represents the extension to the full-potential description of the formulation already used in [7], [11], [12] and [13] under the assumption of the TSP approximation. The second one, to the authors knowledge, is here adopted for the first time. The comparison between the space coefficients used in Eq. 22 and those used in Eq. 24 evidences that, at least from the computational point of view, the approach here introduced appears to be more convenient to apply, since does not involve vectorial coefficients. Therefore, it is particularly suitable for the analysis of three-dimensonal flows around wings and rotors.

5. NUMERICAL RESULTS

First, we consider the subsonic propeller aeroacoustic problem examined experimentally by Magliozzi [18] and computationally by Farassat and Succi [19]. This case consists of a three-bladed propeller, having radius $R = 1.295m$, average chord $c = .1538m$, and non-linear twist varying from 50° at the root to 23° at the tip (and by us approximated with a linear twist varying from 50° at the root to 18° at the tip). The tip Mach number is $M_{TIP} = .71$, corresponding to a rotational speed of $1,760rpm$. Figure 1 presents the aeroacoustic pressure in the plane of the rotor, at $7.28m$ from the axis of the rotor, as a function of time: the curve identified as "Method 1" is obtained with the approach discussed at the end of Section 3 (*i.e.*, by evaluating ϕ in the field using the integral representation, Eq. 18, and then applying Bernoulli's theorem). The curve identified as "Method 2" was obtained using the integral representation of Morino and Gennaretti [20] (very similar to that of Section 3) of the differential equations of Ffowcs-Williams and Hawkings [21]. The pressure is determined by the present methodology neglecting the contribution of the non linear terms. Both results are compared to the experimental data of Magliozzi [18] and the numerical results of Farassat and Succi [19], (who use a very simple theory for the aerodynamic calculations and the integral representation of Ffowcs-Williams and Hawkings [21]). This result shows the capability of the aeroacoustic analysis here discussed to capture the acoustic signal generated by a propeller, with an accuracy comparable with that of the methodologies commonly used.

In the following we shall consider transonic results, where the contribution of the nonlinearities has to be included. The above formulation, valid for three-dimensional unsteady flows, has been applied to the analysis of rotary wings, under the assumption of transonic small perturbation (TSP), and to the analysis of two-dimensional steady flows in the full potential form. Figure 2 deals with a very preliminar result for the analysis of a steady flow around a non-lifting rotor (collective pitch $\theta = 0$), under the assumption of transonic small perturbation (TSP). The tip Mach number is $M_{tip} = 0.83$. The pressure distribution at the tip section is compared with results obtained by FDM [22].

The steady state is reached by marching in time, and the two-dimensional flow is approximated with a three-dimensional one with very high aspect ratio. The numerical applications presented here deal with supercritical flows around a NACA 0012 airfoil with zero angle of attack. Figures 3 and 4 show the solution obtained for a free stream mach number $M_\infty = 0.8$. The formulation for the non linear terms is that of eq. 20. The pressure distribution is compared with the results of Ref.[9]. The mesh used for the result presented is $n_x = 40$, $n_z = 10$, where n_x and n_y represent the number of elements in the x and z directions, respectively. The pressure distribution appears to be in excellent agreement with the reference solution. The shock predicted by the present formulation (dotted line) is correctly located and present an slightly higher intensity. Note that the pressure discontinuity is confined within only one element. This is accomplished by an accurate evaluation of the velocity in the vicinity of the shock; in fact, the chordwise distribution of the velocity potential (Fig. 3) presents an actual discontinuity of its slope, smoothed-out only by the discretization.

Figures 5 and 6 present the solution obtained for the same test case, using the formulation of Eq. 24. The reference solution for the pressure distribution is the same of Fig. 4. The dimensions of the grid for the BIE solution are $n_x = 36$, $n_z = 9$. The comparison reveals an anomalous behaviour of the solution near the discontinuity, whereas in the remaining portion of the domain the solution appears to be correctly predicted. This effect is currently under investigation.

6. CONCLUDING REMARKS

A boundary element method for the unified aerodynamic and aeroacoustic analysis of rotors has been presented. The formulation is valid for isolated rigid bodies in arbitrary motion in transonic, potential flows.

In particular, the full-potential conservative form of the non-linear field term has been studied. Its contribution has been developed following two different formulations: the first one stems from an integration by parts and is the extension to the full-potential case of the formulation introduced in [7] for the TSP analysis, whereas the second one has been introduced here and is particularly suitable for the analysis of three-dimensional flows.

The numerical results presented indicate that the new formulation for the evaluation of the field term needs to be improved. In particular, the behaviour of the solution in the shock region is not in good agreement with the one obtained by the formulation using integration by parts, and investigation upon shock jump conditions is necessary.

Finally, it has also been demonstrated the capability of the formulation presented to predict with a good accuracy the acoustic signal genarated by a rotor.

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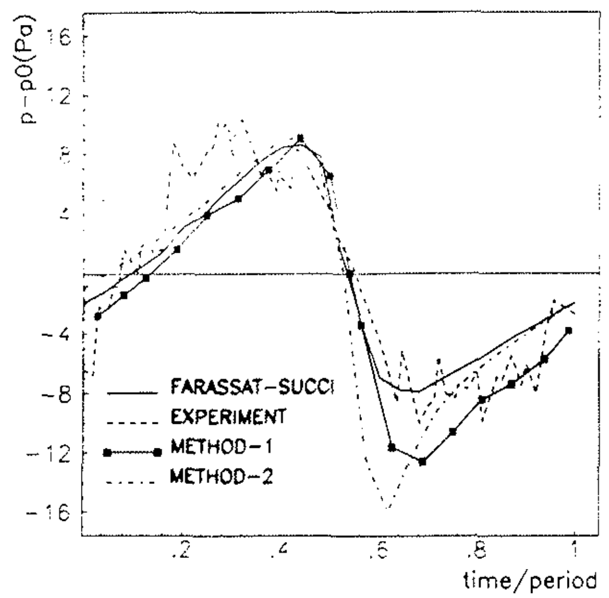


Figure 1. Aeroacoustic pressure in the plane of the rotor, at $7.28m$ from the axis of the rotor, as a function of time. The three-bladed propeller has radius $R = 1.295m$, average chord $c = .1538m$, and *non-linear* twist varying from 50° at the root to 23° at the tip. The tip Mach number is $M_{TIP} = .71$, corresponding to a rotational speed of $1,760rpm$.

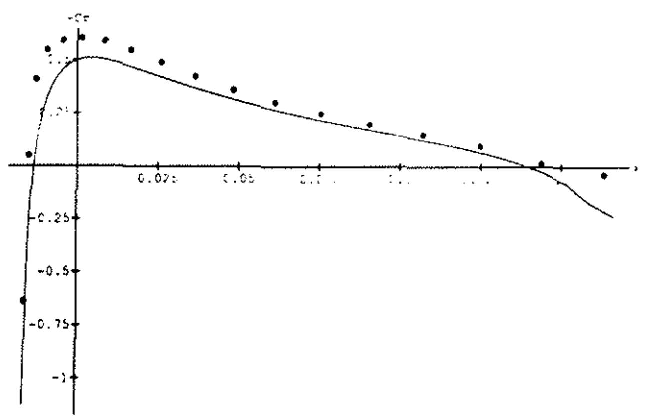


Figure 2. Non-lifting rotor in transonic range. Chordwise pressure distribution at the tip section for a monoblade rotor. $M_{TIP} = 0.830$, $\theta_c = 0°$. Present analysis *(dotted line)* is compared to the results of a FDM code [22], (solid line).

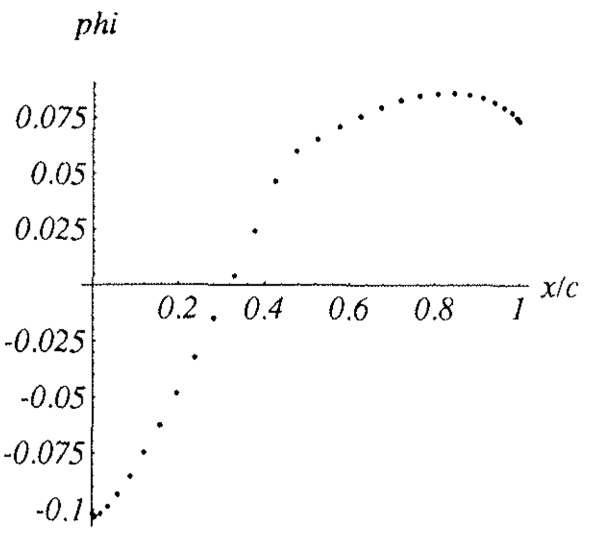


Figure 3. Velocity potential distribution in transonic regime. NACA 0012 airfoil, incidence $\alpha = 0°$, free stream mach number $M_\infty = 0.8$. Nonlinear term from Eq. 20.

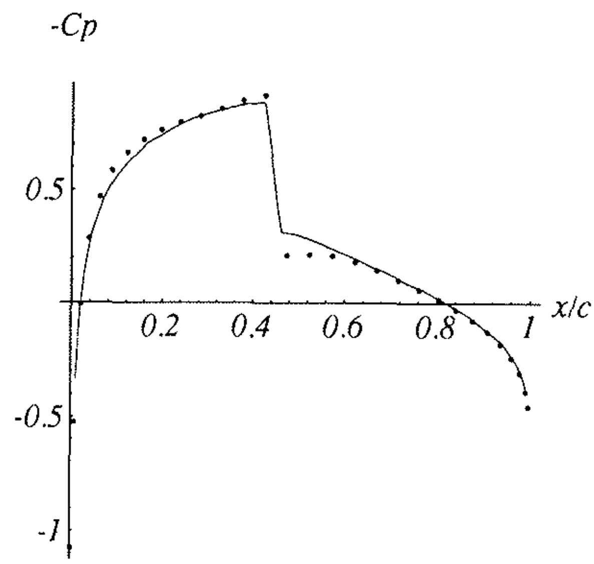
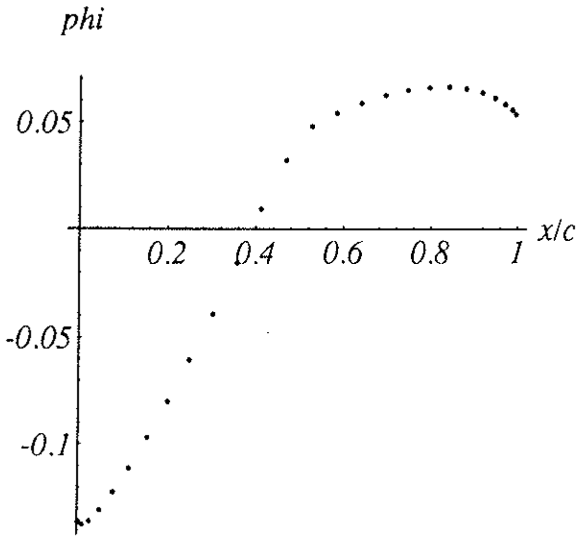


Figure 4. Pressure coefficient along the chord for the same case of Fig.1. The result of the formulation from Eq. 20 (dots) is compared to the results of Ref. [9].



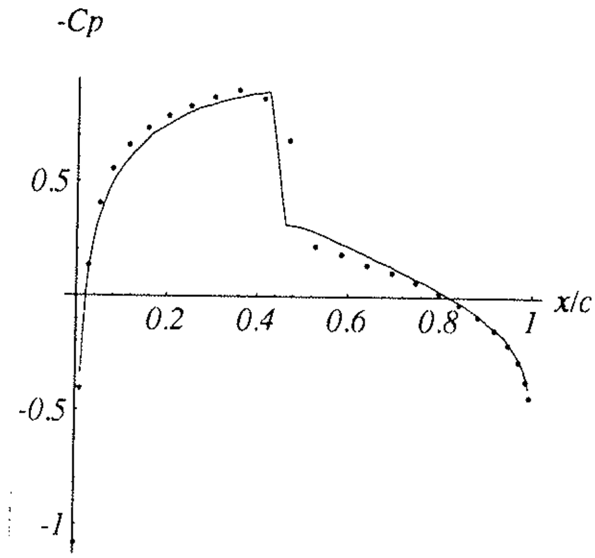


Figure 5. Velocity potential distribution in transonic regime. NACA 0012 airfoil, incidence $\alpha = 0^{o}$, free stream mach number $M_{\infty} = 0.8$. Nonlinear term from Eq. 24.

Figure 6. Pressure coefficient along the chord for the same case of Fig.3. The result of the formulation from Eq. 24 (dots) is compared to the results of Ref. [9].