The 18th European Rotorcraft Forum

B - 18

Paper No 99

FACTOR ANALYSIS OF COAXIAL ROTORS AERODYNAMICS IN HOVER

VADIM N. KVOKOV

Kamov Helicopter Scientific & Technology Company

Avignon, France

September 15-18, 1992

FACTOR ANALYSIS OF COAXIAL ROTORS AERODYNAMICS IN HOVER

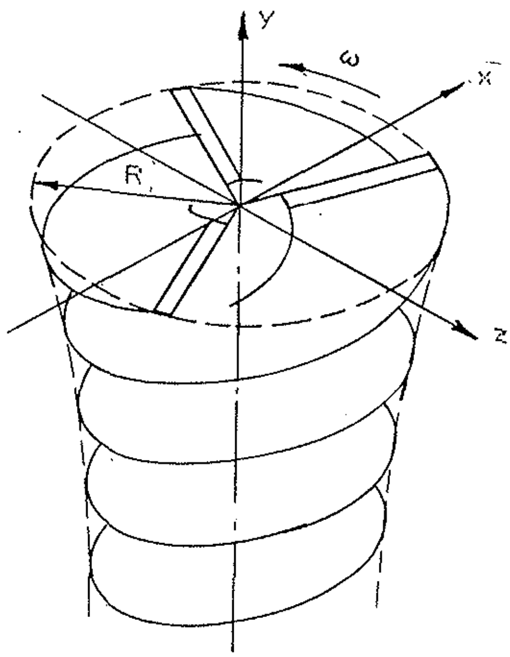
Vadim N.Kvokov

Kamov Helicopter Scientific & Technology Company

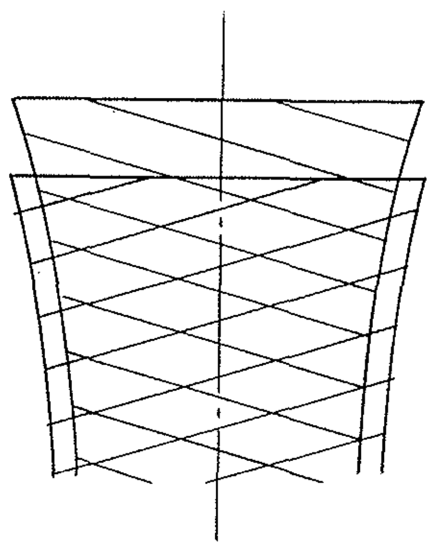
An aerodynamic design method for coaxial rotors operating at axial flight regimes was developed in Kamov Helicopter Scientific & Technology Company on the basis of a disc vortex theory.

The method consists in the following.

A rotating rotor is presented as a disk with uniformly distributed radial votices with radius-variable circulation.The airflow passes through the disc and the blade vortices are aligned with streamlines. The streamlines form a spiral surface for a general case of a variable pitch. The generating line of this surface (airstream shape) is cambered to the rotor axes. Vortices spatial position is considered to be fixed (Fig.1). If necessary coordinates of vortices can be refined (non-linear theory) using known relationships.



Single rotor



Coaxial rotor

Fig. 1.

Thus the vortex structures of upper and lower rotors are patterned.The induced velocities in the rotor plane and along optional section of the airflow are taken as the sum of the induced velocity proper ν_{ys} and additional ν_{ya} of a neighbouring rotor. It is assumed that free blade vortices move with a speed which is mean along the disc and variable along the airflow axis.

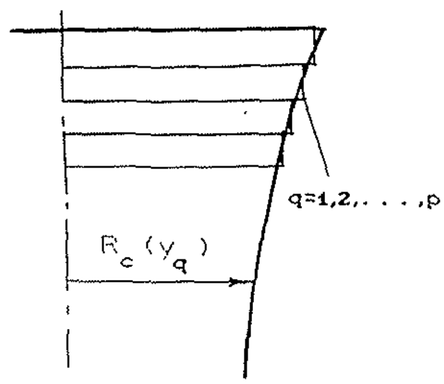
Blade section airflow is assumed to be two-dimensional (hypothesis of flat sections). In determining induced velocities the airflow is regarded as ideal ($Re = \infty$), non-compressibile ($M=0$), and the function $Cy(\alpha)$-as linear.

When a blade element lift/drag are determined, viscosity and compressbility of the airflow are taken into ac c ount, as well as non-linearity $Cy(\alpha)$ taken from the profile wind-tunnel test results $Cy, Cx = f(\alpha, M, Re)$. Within the scope of the adopted model, the formulae to determine mean induced velocity generated by the rotor vortex structure in an arbitrary point in space (x_*, y_*, z_*) take the following forms:

$$\bar{\nu}_y = \frac{K}{8\pi^2 \bar{V}_{mean}} \int_{\bar{\rho}_0}^{1} \bar{\Gamma}_\rho(\bar{\rho})\, K(\bar{\rho}, \bar{r})\, d\bar{\rho} \quad ;$$

$$K(\bar{\rho}, \bar{r}) = \int_0^{2\pi} \sum_{q=1}^{p+1} \frac{\bar{\rho}_q^2 - \bar{\rho}_q \bar{r}\cos\theta}{B_q L_q^2} (\gamma_q I_q - I_{q-1})\, d\theta \quad ;$$

$$I_q = \frac{\bar{y}_* - \bar{y}_q}{\sqrt{(\bar{y}_* - \bar{y}_q)^2 + L_q^2}} \quad ;$$



$$L_q^2 = \bar{\rho}_q^2 + \bar{r}^2 - \bar{\rho}_q \bar{r}\cos\theta \quad ;$$

$$\gamma_q = \begin{cases} 1 , & q < p+1 ; \\ \dfrac{1}{I_{p+1}} , & q = p+1 ; \end{cases}$$

$$\bar{\Gamma}_\rho = \frac{d\bar{\Gamma}}{d\bar{\rho}} \quad ;$$

Fig. 2.

$$\bar{\Gamma}(\bar{r}) = f_1(\bar{r})\left(\varphi - \frac{\bar{V}_\infty}{\bar{r}}\right) + \frac{f_2(\bar{r})}{\bar{V}_{mean}} \int_{\bar{\rho}_0}^{1} \bar{\Gamma}_\rho(\rho)\, K(\bar{\rho}, \bar{r},)\, d\bar{\rho} \quad ;$$

$$\bar{V}_{mean} = \bar{V}_{\infty} - \upsilon_{ymean} \quad (y=0) = \bar{V}_{\infty} - \frac{1}{2}\sqrt{C_T} \quad ;$$

$$f_1(\bar{r}) = \frac{a\bar{b}\bar{W}}{2} \quad ; \quad f_2(\bar{r}) = f_1(\bar{r}) * \frac{\kappa}{8\bar{\pi}^2\bar{r}} \quad ;$$

$$\bar{\rho}_q = \bar{\rho}(\bar{y}=0) * \bar{R}_c(\bar{y}_q) = \mathrm{const}(\bar{y}_q, \bar{y}_{q-1}) \quad ;$$

$$\bar{V}_q = \bar{V}(\bar{y}=0) * \bar{G}\ (\bar{y}_q) = \mathrm{const}(\bar{y}_q, \bar{y}_{q-1}) \quad ;$$

$$\bar{R}_c(\bar{y}) \le 1 \quad ;$$

Here, $\bar{y}$ – is the distance of the wake cross section from the rotor rotational plane related to the rotor radius.

The function $\bar{R}_c(\bar{y})$ takes into account the change in a jet radius along the height of a vortex column and the function $G(\bar{y})$ – the change in the vortices velocity. These functions define the rotor vortex structure and can be refined during the calculations.

With account for inductive velocity a blade section angle of attack α is found (Fig.3), which is necessary to define lift and drag coefficients. Then, by way of numerical integration we get the total coefficients of thrust C_T and torque m_K.

Coaxial rotors characteristics within the linear model are determined by way of solving a system of linear integro-differential equations.

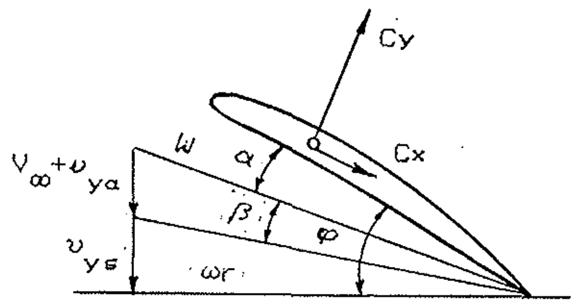


Fig. 3.

$$\bar{\upsilon}_{yn} = \bar{\upsilon}_{ysn} + \bar{\upsilon}_{yan} \quad ;$$

$$\bar{V}_n(y) = \bar{V}_{\infty} - \bar{\upsilon}_{ysn}(\bar{y}) - \upsilon_{yan}(y) = \bar{V}_n^{mean}\quad (\bar{y}=0) * G_n(\bar{y}) \quad ;$$

$$\bar{\upsilon}_{ysn}(\bar{y}) = \bar{\upsilon}_{ysn}(\bar{y}=0) * G_n(\bar{y}) = \frac{1}{2}\sqrt{C_T} * G_n(\bar{y}) \quad ;$$

$$\bar{\upsilon}_{yan}(\bar{y}) = \left[\frac{1}{2\pi(1-\bar{\rho}_o)} \int_{\bar{\rho}_o}^{1} \int_{0}^{2\pi} \bar{\upsilon}_{yan}(\bar{y}=0, \bar{\rho}, \theta)\, d\theta d\bar{\rho} \right] * G_n(\bar{y}) \quad ;$$

$$\bar{\Gamma}_n(\bar{r}) = f_{1n}(\bar{r})\,(\varphi_n - \frac{\bar{V}_{\infty}}{\bar{r}}) + \sum_m \frac{f_{2m}(\bar{r})}{\bar{V}_{mean\ m}} \int_{\bar{\rho}_o}^{1} \bar{\Gamma}_{\rho m}(\bar{\rho}) K_{nm}(\bar{y}_{on}, \bar{\rho}, \bar{r})\, d\bar{\rho} \quad ;$$

$n = 1,2\ ; \quad m = 1,2\ ;$

$$f_{1n}(\bar{r}) = \left(\frac{a\bar{b}\bar{W}}{2}\right)_n\ ; \quad f_{2m}(\bar{r}) = f_{1m}(\bar{r})\,\frac{\kappa}{8\pi^2\bar{r}}\ ;$$

$$K_{nm}(\bar{y}_{on},\bar{\rho},\bar{r}) = \int_0^{2\pi}\sum_{q=1}^{p+1}\frac{\bar{\rho}^2_{qm}-\bar{\rho}_{qm}\bar{r}\cos\theta}{G_{qm}L^2_{qm}}\left(\gamma_{qm}I^m_{qm}-I^m_{q-1}\right)d\theta\ ;$$

$$I^m_q = \frac{\bar{y}_m-\bar{y}_q}{\sqrt{(\bar{y}_m-\bar{y}_q)^2+L^2_{qm}}}\ ; \quad \bar{\rho}_{qm} = \bar{\rho}(\bar{y}=0)*\bar{R}^m_c(\bar{y}_q)\ ;$$

$$\gamma_q = \begin{cases} 1, & q<p+1\ ; \\ \dfrac{1}{I^m_{p+1}}, & q=p+1\ ; \end{cases}$$

$$L^2_{qm} = \bar{\rho}^2_{qm}+\bar{r}^2-\bar{\rho}_{qm}\bar{r}\cos\theta\ ;$$

Here $\bar{y}_{on}$- is the distance of the n-th rotor rotational plane from the neighbouring rotor: $n=1 \longrightarrow$ upper rotor $\longrightarrow \bar{y}_{on}<0$

$n=2 \longrightarrow$ lower rotor $\longrightarrow \bar{y}_{on}>0$

The solution of this system of equations amounts to solving a set of linear algebraic equations.

The mathematical programme developed for IBM PC/XT computers allow to define the characteristics of a coaxial rotor and also of a single rotor by separating coaxial rotors at a fairly large distance. It ensures the specified difference in torque moments of the upper and lower rotors $M_{Kn}-M_{Kl}$=const.

As a result we get the total thrust $C_T = C_{Tl} + C_{Tn}$ and torque moment coefficients $m_K = m_{Kl} + m_{Kn}$, where

$$C_{Tl(n)} = \frac{T_{l(n)}}{\frac{\rho}{2}(\omega R)^2 F} \quad \text{and} \quad m_{Kl(n)} = \frac{M_{Kl(n)}}{\frac{\rho}{2}(\omega R)^2 RF}$$

are coefficients of the lower and upper rotors respectively.

To analize the results of the computations and to compare them with the experimental data the rotor figure of merit is used:

$$\eta_o = \frac{C_T^{3/2}}{2m_K}$$

The rotor characteristics C_T, m_K and η_o derived from the computations are the first approximations because there exists a number of factors which are not accounted for in the mathematical model.

To take into account this condition, the coefficients $\varkappa$, Δm_{KP}, $\varkappa_{\eta}$ are introduced into C_T, m_K and η_o:

$$C_T = C_T * \varkappa\ ; \quad m_K = m_K + \Delta m_{KP}; \quad \eta_o = \eta_o * \varkappa_{\eta}\ ;$$

Then

$$\eta_o = \frac{(C_T \varkappa)^{3/2}}{2m_k} * \varkappa_{\eta}.$$

Here Δm_{KP} -is an additional value of a profile component of torque moment coefficient, arising when the aerodynamic profile operates on a rotating blade. To determine Δm_{KP} special rig tests were conducted using rotor models. Flat blades of specific profiles were installed at an angle such that rotor thrust was zero and simultaneously the torque moment m_K^e was measured.

Then $m_{KPO} = \frac{1}{4}\sigma\, C_{xpo}$ was subtracted from it, where C_{xpo} is profile drag coefficient received for the given profile in wind tunnel. The difference $\Delta m_{KP} = m_K^1 - m_{KPO}$ was added to the calculated value: $m_K = m_K + \Delta m_{KP}$. The experiment has revealed that Δm_{KP} is practically constant within the operating range of tip Mach numbers.

The $\varkappa$ value is usually related to the so-called thrust tip losses. We can assume after Prandtl:

$$\varkappa = B^2 = 1 - \frac{\sqrt{2C_T}}{\kappa}$$

Then the influence of the rest neglected factors (due to load non-uniformity along the blade length and three-dimensional nature of the flow over the blade) is concentrated in $\varkappa_{\eta}$.

By way of comparing computational results and experimental data we get $\varkappa_{\eta}$ as a discrepancy between the computation and experiment.

For a single rotor the $\varkappa_{\eta}$ values are taken under the following conditions. Vortex drift velocities dependant on the function

$$G = 1 - \frac{\bar{y}}{\sqrt{1+\bar{y}^2}}$$

and the wake shape (wake radius) - on the continuity equation $\bar{R}_c = \sqrt{1/G}$. (Fig. 4).

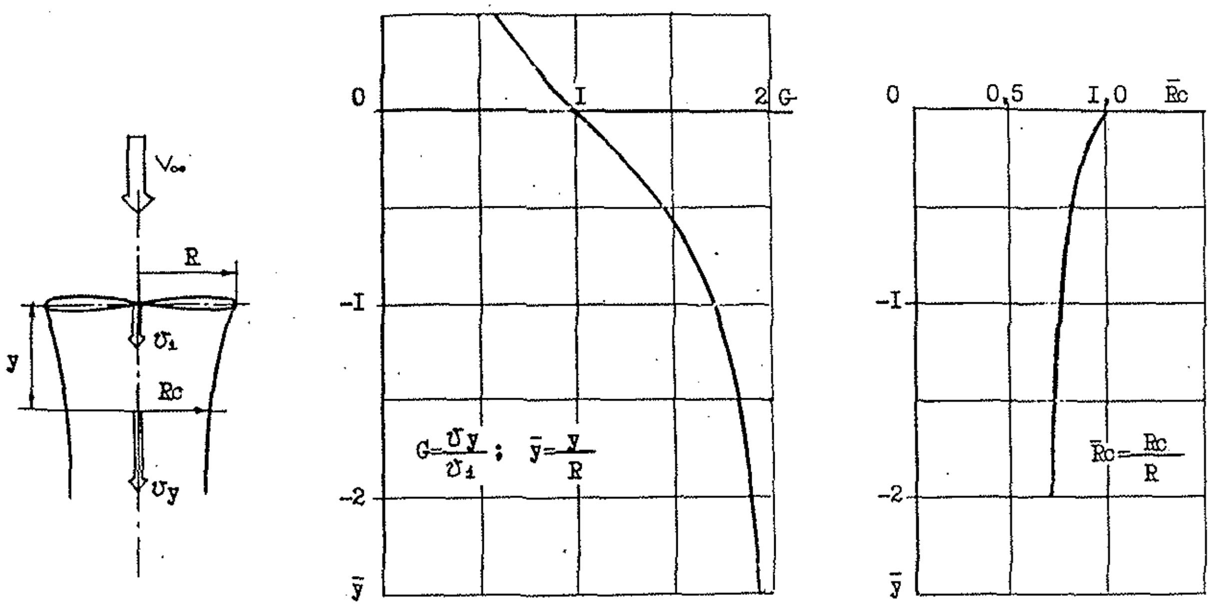


Fig. 4.

A large amount of experimental data has been processed using this method. Comparing the computed values of

$$\eta_o = \frac{(C_T \varkappa)^{3/2}}{2m_K}$$

with experimental η_o^e values we get for the specified rotor configuration and mode of operation:

$$\varkappa_\eta = \frac{\eta_o^e}{\eta_o} = \eta_o^e \frac{2m_K}{(C_T \varkappa)^{3/2}} .$$

By way of illustration Fig.5,6 show the plotted dependencies of $\varkappa_\eta = f(C_T/\sigma)$ for different values: $\Delta\varphi$ -blade geometric twist; ωR- -blade tip speed.

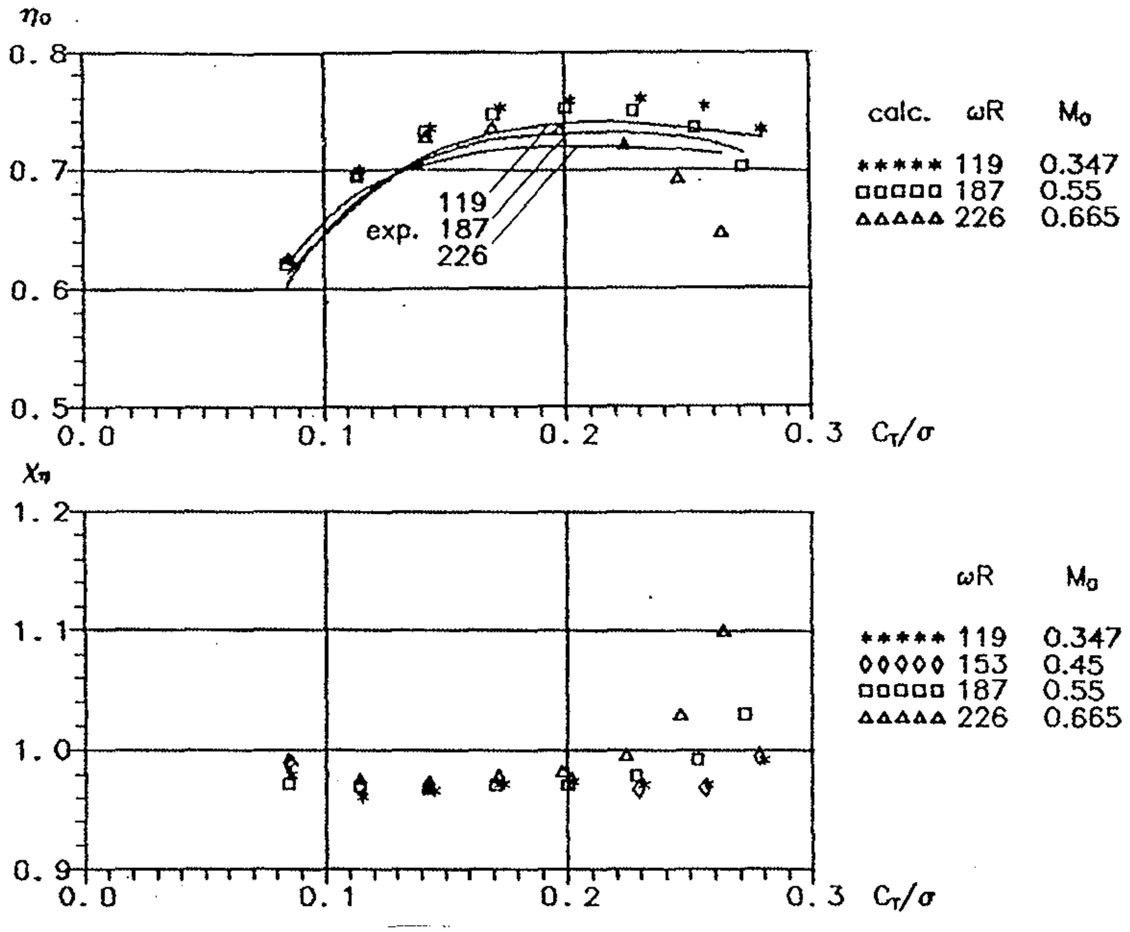


Fig. 5.

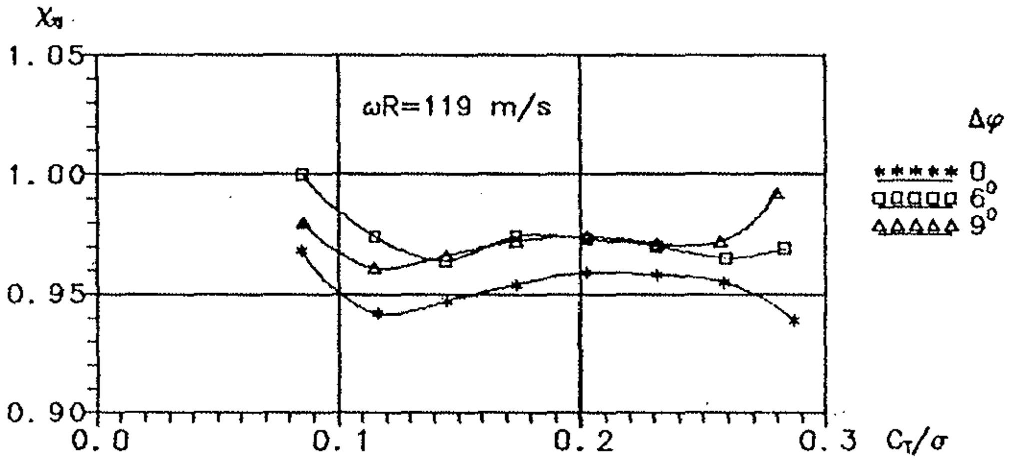


Fig. 6.

It is assumed for a coaxial rotor that tip losses values $\varkappa$ and $\varkappa_\eta$ will be the same for coaxial rotors as for a single rotor, and the wake shape is taken as an identification factor. For this purpose a multiplier **A** is introduced into the function **G** of the upper rotor:

$$G = 1 - \frac{\bar{y}^*}{\sqrt{1+\bar{y}^{*2}}} ; \qquad \bar{y}^* = A * \bar{y} ,$$

whereas the boundary wake line of the lower rotor is taken equidistant to the wake boundary of the upper rotor.

For the investigation the experimental data obtained from wind tunnel testing performed in TsAGI for a coaxial rotor model were used. Fig.7 presents two experimental curves $\eta_o = f(C_T/\sigma)$ for coaxial rotor and for the equivalent single rotor (as to the rotor solidity $\sigma_S = \sigma_C$). In this case the single rotor mathematical model was identified for $\varkappa_\eta$ with the experimental dependency of a single rotor. Then with the known function $\varkappa_\eta = f(C_T/\sigma)$, such values of $A = f(C_T/\sigma)$ were selected so as the design values of η_o of coaxial rotors became close to the experimental ones (Fig.7).

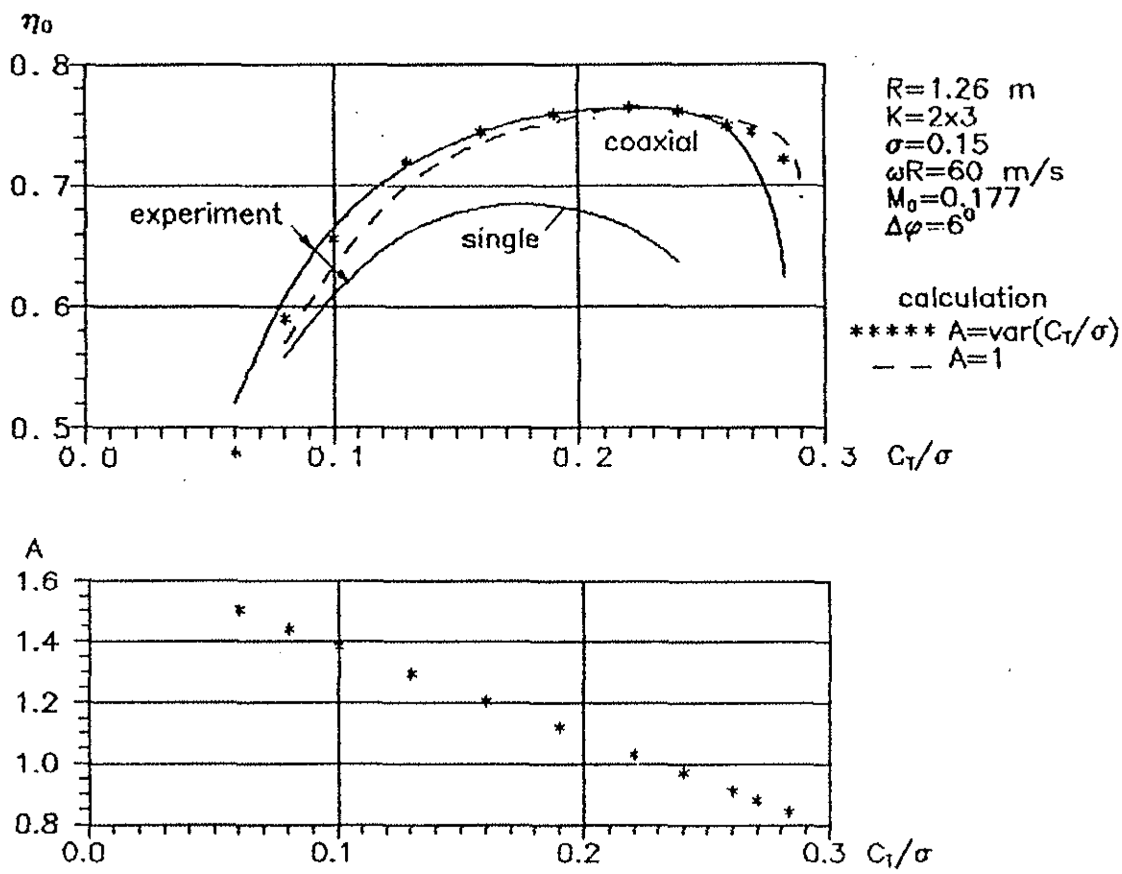


Fig. 7.

This function is not absolute since it corresponds to a specific experimental rotor configuration. What is more important is that the developed mathematical model of a coaxial rotor basically adequately reflects the fact (which is well known from flight testing) that the figure of merit of a coaxial rotor is drastically higher as compared with a single rotor.

Similarly the results of other model tests were handled.

As a result extensive statistical materials were received for $\varkappa_\eta$ and **A**. When used for calculating aerodynamic characteristics of the rotors these materials improve the authenticity of the results.