EIGHTH EUROPEAN ROTORCRAFT FORUM

Paper N° 4.2

STAHR : A PROGRAM FOR STABILITY AND TRIM ANALYSIS OF HELICOPTER ROTORS

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August 31 - September 3, 1982

Aix-en-Provence

ASSOCIATION AERONAUTIQUE ET ASTRONAUTIQUE DE FRANCE

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ABSTRACT

The design of modern helicopter rotors requires the use of fairly sophisticated design tools right from the beginning of the project development phase.

In fact the aeroelastic analysis can no more be confined only to detailed analyses, carried out in an advanced design and model refinement phase, but it has to be performed also in the preliminary design and performance evaluation as well.

There is then the need for an unified way of modeling the rotor, so that both the preliminary design, detailed analysis and flight test validation can be undertaken within the same computer program. This aim has led to a cooperation plan between Costruzioni Aeronautiche "G. AGUSTA" and Dipartimento di Ingegneria Aerospaziale del Politecnico di Milano, in order to develop a computer program for the analysis of different types of hinge arrangements or of hingeless or bearingless rotors.

The basic concepts on which this program has been built, were outlined in a paper presented at Seventh European Rotorcraft Forum, held in Garmisch.[1]

The original feature of the STAHR program is essentially related to the use of a direct space-time finite element idealization of the rotor blades, that allows the determination of a trimmed flight condition and the study of its stability within the same computer code. By means of this unified formulation, the set of equations describing the aeroelastic behaviour of the rotor blades, can be built up in a fully automatic way.

The present work stresses out the most relevant capabilities of the STAHR program, by illustrating the fundamental characteristics of the structural blade element and of the kinematic hinge elements, that can be used in modeling the rotor. The blade element is based on an original large displacements formulation, which takes into account all the stress-strain and dynamic couplings, typical of modern composite rotor blades.

The blade section properties, in terms of stiffness and mass matrices, can be obtained from HANBA (Hollow ANisotropic Beam Analysis) programs [2,3], by means of a unique two-dimensional finite element idealization of the blade section only.

The use of an unified data base, for the two programs, allows an easy CAD type approach to rotor optimization.

Moreover the choice made in STAHR, of the displacements and of the rotations of physical points of the blade as generalized coordinates, get rid of

all the troubles related to the assumption that flapwise/lagwise or flatwise/chordwise modal components remain constant with blade pitch variations.

Different types of kinematic and hinge elements, which need peculiar generalized coordinates, are easily matched with other structural elements, by use of appropriate constraint equations and of Lagrange's multipliers technique.

Finally, some applications of the STAHR program are presented, in order to demonstrate capabilities and flexibility of its use.

INTRODUCTION

During the last decade, the interest toward a general purpose program for comprehensive helicopter analysis has been increasing more and more. This trend is demonstrated by the proceedings of the various meetings on helicopter rotors, i.e. the Specialists Meeting on Helicopter Rotor Load Prediction Methods [4], the Conference on Rotorcraft Dynamics [5] and the AGARD Conference on Prediction of Aerodynamic Loads on Rotorcraft [6].

Only ten years ago the need for a general purpose program in helicopter design was not believed to be so stringent, as it is found in the survey of the panelists of Ref.[1]. Nowadays the development of the so called second generation programs [7] is clearly intended toward a computer software capable of dealing with the different configurations and problems typical in helicopters. This is due to both the new capabilities of the computers and to the progress in mathematical modeling.

The complexity of the rotary wing loads prediction problem is mainly related to the need of including the aeroelastic effects right from the preliminary design phase in order to properly take into account both the coupling between aerodynamic and structural blade properties and fuselage-rotor dynamics.

On the other hand the mathematical models, required at each design phase, must differ in degree of complexity and detail.

It is generally agreed that the main topics in helicopter analysis can be classified as steady state flight condition analysis, stability analysis and transient manoeuvre analysis. Each of these can be faced with three models of differing complexity, that is, isolated blade, isolated rotor and coupled rotor-fuselage.

All these various requirements, in terms of problem type and mathematical approximation, can be satisfied by many different specialized programs, whose degree of complexity follows the need of the design process.

The current tendency however is to develop a comprehensive analysis program for the rotorcraft design. This approach should offer the advantage of standardizing both the input-output data and the mathematical model set-up; thus, by making the management and support of the computing system more rational and efficient, the whole design is improved as well.

On this basis a cooperation plan between Costruzioni Aeronautiche "G. AGUSTA" and Dipartimento di Ingegneria Aerospaziale of the Politecnico di Milano was established in order to develop a comprehensive computer program for helicopter design.

Clearly this undertaking required plenty of time and money because of the overall complexity of the resulting computer program. In fact the design of the whole computer system could be heavy task for a single university department, and an expensive venture for a factory also. In this view a European cooperation would be desirable and could be profitable.

Nevertheless a strategic choice was made to obtain, consistent with the existing resources and in as a short time as possible, an operative module, that could satisfy the minimum needs of an isolated blade analysis, while performing at the same time the above mentioned duty of unifying and of directing the future software work.

Thus the computer program STAHR (Stability and Trim Analysis of Helicopter Rotors) was developed and it is now fully operative for articulated and hingeless isolated blade analysis. From the system design point of view, the modularity and the data communication among modules is granted by an in core and out of core dynamic management package, making the possibility of future improvements to the program easier.

This paper briefly recalls the formulation of equations of motion, presented in Ref.[8], and shows the main features of the finite element used to simulate the blade behaviour.

Due to space limitations, only the fundamental ideas are recalled here, but however, they show the consistency of the formulation, especially in respect to the beam element used to model the elastic and dynamic characteristics of the blade.

Finally some numerical examples illustrate the program modeling flexibility. The standard problem of Ref.[9] was used in order to validate the results of this development phase.

HAMILTON'S PRINCIPLE

Even if the problem is restricted to an isolated blade analysis, the analytical burden, required to write the nonlinear differential equations of motion that appropriately take into account all the aeroelastic couplings and the geometrical and constitutive nonlinearities, could be cumbersome. To effectively face this problem in a fairly general way, Hamilton's generalized principle was used. This variational principle makes it possible to use the well established techniques of the finite element method, that allow us, due to the additivity property of the variational principle, to build up the set of the resulting equations in a completely automatic way, even in a complex system.

Moreover the use of a finite time numerical approximation of Hamilton's principle leads to a set of algebraic nonlinear equations that can be solved by means of a Newton-Raphson type technique. The tangent matrix required by this method can be automatically obtained by means of Hamilton's principle in its linearized form, as shown in Ref.[1].

The procedure outlined here is equivalent to the use of an implicit integration technique of the dynamic equations of motion; in fact, when only trim and response problems are implied, an explicit integration scheme would be more convenient, but, when the interest is focused on stability analysis, the use of implicit integration schemes becomes mandatory. This circumstance justifies the adoption from the beginning of such a complicated, yet more complete, scheme.

It must be emphasized that in the application of this method, the manual efforts are confined to writing the analytical expressions of the first and second variations δT and $\delta^2 T$ of the kinetic energy, of the potential energy δV and $\delta^2 V$ and of the virtual work $\delta \pounds_a$ and $\delta^2 \pounds_a$ of the aerodynamic forces, with respect to the virtual variation of the generalized coordinates. Clearly these quantities have to be written only at an arbitrary integration point of an element: the use of numerical integrations, both in space and time domain, and of assembling techniques, allows us to build up the equations of motion of the whole system.

Finally, it is remarkable to note that this formulation can be usefully applied to find out either the solution of periodic trim conditions or of transient response; moreover the same tangent matrix, required by the Newton-Raphson method, provides the needed information to evaluate the linearized stability.

BLADE ELEMENT

In order to adequately represent the aeroelastic behaviour of a helicopter rotor blade, an "ad hoc" beam finite element was developed, which takes into account the following items :

- different location of shear center, normal stress center and centroid of inertia within the same blade cross section;
- anisotropy and unhomogeneity typical of the composite blade design;
- geometrical nonlinearities, due to large displacements and rotations.

To this aim a 2, 3 or 4 nodes isoparametric beam element has been developed. Fig. 1 shows a 4 nodes beam element. In the following the geometrical and kinematic representation of such an element is presented.

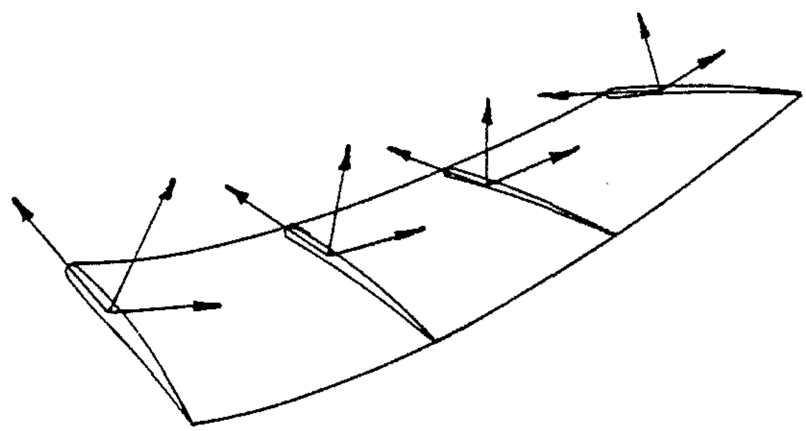


FIG.1 - FOUR NODES BEAM ELEMENT WITH CROSS SECTION REFERENCE FRAMES

In the rotating hub reference frame let $\{X\}_k$ and $[\alpha]_k$, (k=1,4) be the coordinates and the cosine directions of the nodal beam sections, $\{Y\}_k$ and $[\beta]_k$ have the following form :

$$
\begin{aligned}
\{Y\}_k &= \{X\}_k - \{X\}_1 \\
[\beta]_k &= [\alpha]_k [\alpha]_1^T
\end{aligned}
\qquad (1)
$$

Corresponding to each matrix $[\beta]_k$ there will be a finite rotation vector $\{\rho\}_k$, and we assume $\{\rho\}_1 = 0$. $\{Y\}$ and $\{\rho\}$ of an arbitrary point 0 belonging to the beam axis are expressed by an appropriate polynomial interpolation of the nodal values. Knowing in this way the values of $\{Y\}$and $\{\rho\}$, the corresponding coordinates $\{X\}$ and cosine directions $[\beta]$ will be immediately evaluated through Eq.(1).

It is clear that by means of this geometrical description we are able to easily model even curved and twisted beams. The same representation was adopted for the displacement and for the rotation fields, so each node has three translational and three rotational degrees of freedom. Moreover this choice of degrees of freedom get rid of all the troubles related to the assumption that flapwise/lagwise or flatwise/chordwise modal components remain constant with blade pitch variations. Since no constraint was "a priori" made between displacements and rotations, the behaviour of such an element was extensively tested and some significant results were reported in Ref.[1]. It was noted that the 2 nodes element was excessively stiff and thus it can only be used in those schemes where the influence of the elastic behaviour can be neglected, as for instance in performances calculations of the preliminary design phase.

Time discretization of the element is performed in the same manner, and Fig. 2 shows a beam element with three azimuthal nodes and four radial nodes.

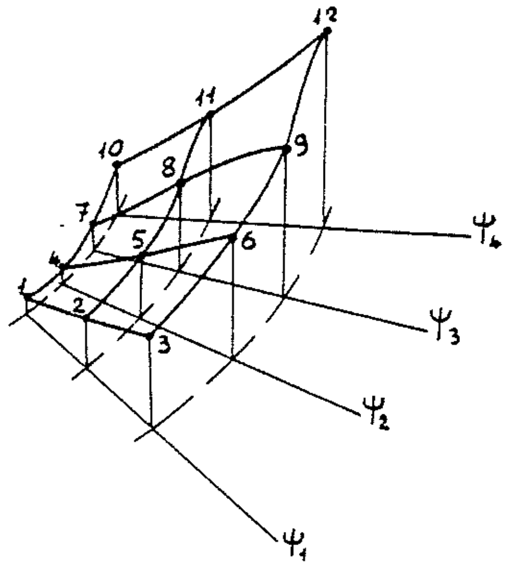


FIG.2 - SPACE-TIME BEAM ELEMENT (3X4 NODES)

Owing to the weak formulation afforded by Hamilton's principle, there is no need to guarantee the continuity of the time derivatives of the nodal unknowns, thus the efficiency of the formulation is greatly enhanced.

If δO and $\delta\Theta$ are the absolute virtual displacement and rotation of a beam section, we can resolve them into an entrainment hub displacement δH and a rotation $\delta\Phi$, plus a motion (δu and $\delta\phi$) relative to it, as follows :

$$\begin{aligned} \delta O &= \delta u + \delta H + \delta\Phi \wedge (O-H) \\ \delta\Theta &= \delta\phi + \delta\Phi \end{aligned} \tag{2}$$

or in matrix form :

$$\{\delta c\} = [S]\,\{\delta c_a\}$$

where :

$$\{\delta c\}^T = [\{\delta O\}^T, \{\delta\Theta\}^T]$$

$$\{\delta c_a\}^T = [\{\delta u\}^T, \{\delta\phi\}^T, \{\delta H\}^T, \{\delta\Phi\}^T]$$

$$[S] = \begin{bmatrix} I & \emptyset & I & Z^T \\ \emptyset & I & \emptyset & I \end{bmatrix}$$

with :

$$[Z] = [(O-H)\wedge]$$

If v and ω are the absolute linear and angular velocities of the blade cross section, and if w and Ω the linear and angular velocities of the hub frame, we have :

$$v = w + \Omega \wedge (O-H) + v_r$$

$$\omega = \Omega + \omega_r$$

where v_r and ω_r are the relative velocities of the blade cross section. Differentiating Eq.(2) with respect to time we have :

$$\frac{d}{dt}\,\delta O = \frac{d}{dt}\,\delta H + \frac{d}{dt}\,\delta\Phi \wedge (O-H) + \frac{d}{dt}\,\delta u + (w-v) \wedge \delta\Phi$$

then :

$$\frac{d}{dt}\,\delta O + v \wedge \delta\Theta = \frac{d}{dt}\,\delta u + v \wedge \delta\phi + \frac{d}{dt}\,\delta H + w \wedge \delta\Phi + \frac{d}{dt}\,\delta\Phi \wedge (O-H)$$

$$\frac{d}{dt}\,\delta\Theta = \frac{d}{dt}\,\delta\phi + \frac{d}{dt}\,\delta\Phi$$

or in matrix form :

$$\{\delta b\} = [S]\,\{\delta b_a\} \tag{3}$$

where :

$$\{\delta b\}^T = [\{\frac{d}{dt}\,\delta O + v \wedge \delta\Theta\}^T,\ \{\frac{d}{dt}\,\delta\Theta\}]^T$$

$$\{\delta b_a\}^T = [\{\frac{d}{dt}\,\delta u + v \wedge \delta\phi\}^T,\ \{\frac{d}{dt}\,\delta\phi\}^T,\ \{\frac{d}{dt}\,\delta H + w \wedge \delta\Phi\}^T,\{\frac{d}{dt}\,\delta\Phi\}^T]$$

Moreover after having defined :

$$\{\delta a\}^T = [\{\delta b\}^T,\ \{\delta c\}^T]$$

$$\{\delta a_a\}^T = [\{\delta b_a\}^T,\ \{\delta c_a\}^T]$$

we have :

$$\{\delta a\} = [R]\,\{\delta a_a\} \tag{4}$$

where the matrix [R] is defined as follows :

$$[R] = \begin{bmatrix} S & \emptyset \\ \emptyset & S \end{bmatrix}$$

Eq. (2), (3) and (4) are very simple and they allow us to take into account, by means of simple matrix multiplications, the most general hub motions. This feature is mainly due to the fact that the accelerations have no place in Hamilton's principle, so that only the displacement and velocity fields have to be described.

INERTIA MODELING

Let us suppose the beam cross section performs a rigid motion (Fig. 3), then the kinetic energy dT of a blade element of width ds is given by :

$$dT = \tfrac{1}{2}\,(v \bullet Q \;+\; \omega \bullet \Gamma)\; ds$$

where Q and Γ are the momentum and the momentum moment of the beam section with respect to pint O and they can be given in matrix form by :

$$\begin{Bmatrix} Q \\ \Gamma \end{Bmatrix} = \begin{bmatrix} [m] & -[S] \\ [S] & [J] \end{bmatrix} \begin{Bmatrix} v \\ \omega \end{Bmatrix}$$

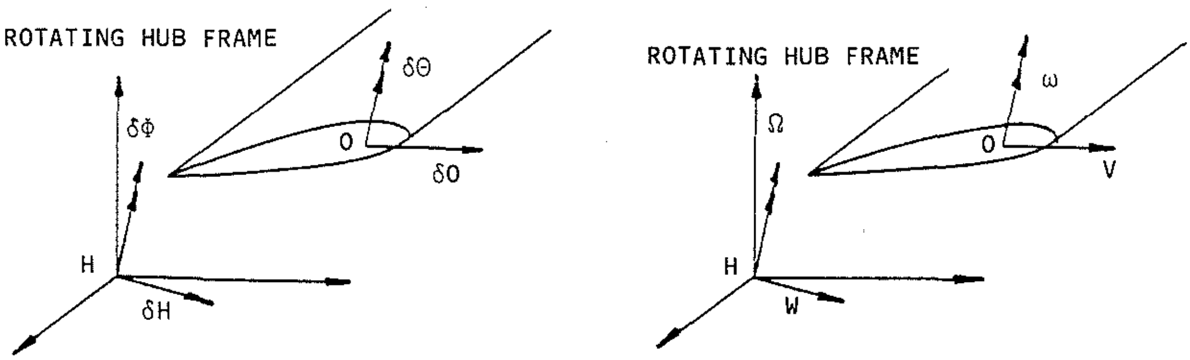
where :

$$[m] = [I]\int_A \rho \; dA$$

$$[S] = \int_A [(P-O)\wedge]\rho \; dA$$

$$[J] = -\int_A [(P-O)\wedge]\;[(P-O)\wedge]\rho \; dA$$

ρ is the mass density and A is the area of the cross section.



(A) - VIRTUAL DISPLACEMENTS

(B) - VELOCITY VECTORS

FIG.3 - KINEMATIC PARAMETERS

The first variation of the kinetic energy is :

$$\delta\,\frac{dT}{ds} = [(\frac{d}{dt}\,\delta O + v \wedge \delta\Theta)\bullet Q + \frac{d}{dt}\,\delta\Theta \bullet \Gamma]$$

or in matrix notation :

$$\delta \frac{dT}{ds} = \{\delta b\}^T \begin{matrix}\{Q\}\\ \{\Gamma\}\end{matrix}$$

The first variations δQ and $\delta\Gamma$ of the momentum and of the momentum moment are :

$$\delta Q = m \left(\frac{d}{dt} \delta O + v \wedge \delta\Theta \right) - S \frac{d}{dt} \delta\Theta - Q \wedge \delta\Theta$$

$$\delta\Gamma = S \left(\frac{d}{dt} \delta O + v \wedge \delta\Theta \right) + J \frac{d}{dt} \delta\Theta - \Gamma \wedge \delta\Theta$$

then the second variation $\delta^2 \frac{dT}{ds}$ of the kinetic energy can be put in the following form :

$$\delta^2 \frac{dT}{ds} = \{\delta a\}^T [j_\tau] \{\delta a\} \qquad (5)$$

where the matrix $[j_\tau]$ is the tangent inertia matrix and is composed of the matrices $[m]$, $[S]$, $[J]^\tau$ and of the componenets of the vectors Q and Γ, as well.
By applying the transformation equation (4), Eq.(5) becomes :

$$\delta^2 \frac{dT}{ds} = \{\delta a_a\}^T [J_\tau] \{\delta a_a\}$$

where :

$$[J_\tau] = [R]^T [j_\tau] [R]$$

By giving the shape functions in space and time domain, the use of this formulation allows us to develop the mass matrix for the space-time finite element of the blade, in quite a simple and consistent way, without any limitation to the type of motion that can be performed by the beam section.
It may be finally noted that this approach greatly reduces the manual effort required to derive in numerical form the final equations of motion and that the same formulation can be applied to any rigid body.

STRUCTURAL MODELING

Let O_o be an arbitrary point of the reference beam axis in the starting configuration and O the corresponding point in the deformed one, $\{X\}_o$ and $\{X\}$ their coordinates in a rotating hub frame, $[\alpha]_o$ and $[\alpha]$ the cosine directions of the corresponding cross section, then the constitutive structural equations assume the following form :

$$\begin{matrix}\{N\} - \{N\}_o \\ \{M\} - \{M\}_o\end{matrix} = \begin{bmatrix} E \end{bmatrix} \begin{matrix}\{d\} \\ \{r\}\end{matrix} \qquad (6)$$

where $\{N\}_o$,$\{M\}_o$,$\{N\}$ and $\{M\}$ are the beam elastic forces and moments related to the starting and deformed configuration, $[E]$ is the cross section stiffness matrix, while $\{d\}$ and $\{r\}$ are the translational and rotational global strains of the section. Eq.(6) is intended in the cross section reference frame. The global strains of the section are defined as follows :

$$\{d\} = [\alpha]^T \{X\}' - [\alpha]_\circ^T \{X\}_\circ' \qquad (7)$$

$$\{r\} = [\alpha]^T \{c\} - [\alpha]_\circ^T \{c\}_\circ$$

where the prime denotes differentiation with respect to the curvilinear abscissa s of the beam axis, and $\{c\}_\circ$ and $\{c\}$ are two vectors related to the curvature of the beam in the starting and in the deformed configuration. Their definitions follow from the well known formulas :

$$j_\circ' = c_\circ \wedge j_\circ$$

$$j' = c \wedge j$$

where $j_\circ$ and j are two vectors jointly connected with the beam cross section.

The previous formulas are very general and extend the validity of the engineering beam theory to twisted and curved beams, in the range of small strains-large displacements and rotations. No limitations are made on the form of the section stiffness matrix [E], so that any kind of elastic coupling can be taken into account. The numerical evaluation of this matrix can be performed by the HANBA2 code [3], which employs a 2D finite element discretization of the blade section, as exemplified in Fig.4 .

The virtual variation of elastic energy can be expressed in the following form :

$$\delta dV = (\delta\{d\}^T\{N\} + \delta\{r\}^T\{M\}) \, ds$$

where $\delta\{d\}$ and $\delta\{r\}$, through relations (7), are expressed by :

$$\delta\{d\} = [\alpha]^T \{\delta X'\} + [X'\wedge] \{\delta\phi\}$$

$$\delta\{r\} = [\alpha]^T \{\delta\phi'\}$$

where $\{\delta X\}$ and $\{\delta\phi\}$ denote the virtual displacements and rotations of the beam section, and again the prime means differentiation with respect to the curvilinear abscissa s.

The second variation of elastic energy assumes the following form :

$$\delta^2 dV = \{\delta e\}^T [K_T] \{\delta e\} \, ds$$

where :

$$\{\delta e\}^T = [\{\delta X'\}^T, \{\delta\phi\ \}^T, \{\delta\phi\}^T]$$

and the matrix $[K_T]$ is the tangent elastic matrix of the section, which can be easily obtained by using the previous equations and fully takes into account all the geometrical nonlinearity effects.

AERODYNAMIC AND INFLOW MODELING

It is well known that rotary wing aerodynamics is very difficult to be modeled in a precise manner. The major difficulties arise from the inflow model and it is clear that the full coupling between the wake analysis and the aeroelastic blade analysis constitutes an expensive and time consuming task, even for the high speed computers available today. Since the set of equations involved is very large and fully coupled, some assumptions and approximations have inevitably to be taken. The common trend is to split the problem into two parts, i.e. rotor induced inflow module analysis and rotor aeroelastic module analysis, whose equations may be independently derived. Such an approach also offers the advantage of developing each part independently and, since it is not yet clear which model can be conveniently applied, it frees us from any constraint on the structure of the aeroelastic analysis module.

During the STAHR development the major concern, as mentioned above, was to produce, in the shortest possible time, an operative module for the simple case of an isolated blade problem. So the major attentions were focused on the structural and dynamics aspects, while a simple two dimensional aerodynamics blade element theory was employed, with the usual corrections for sweep, compressibility, unsteadiness and dynamic stall effects [10-12], but without any kind of corrections for tip losses.

In the present version STAHR assumes a prescribed induced flow distribution, whose amplification factor, i.e. the mean induced velocity, is determined by Glauert's momentum balance equation.

The local wind velocity denoted by v^* and the local wind velocity in the blade cross section by v^*_p, the aerodynamic forces and moments are then :

Lift

$$dL = \tfrac{1}{2}\rho c\ ds\ |v^*_p|\ C_L\ k_3 \wedge v^*$$

Friction Drag

$$dD_f = -\tfrac{1}{2}\rho c\ ds\ |v^*|\ C_{D_f}\ v^*$$

Pressure Drag

$$dD_p = -\tfrac{1}{2}\rho c\ ds\ |v^*_p|\ C_{D_p}\ v^*_p$$

Pitching Moment

$$dM = \tfrac{1}{2}\rho c\ ds\ |v^*_p|\ C_{M_{c/4}}\ k_3$$

where k_3 is the unit vector normal to the blade cross section in its actual configuration. These forces can conveniently be rearranged in the force vector F and moment M about reference point O of the beam axis; then the virtual aerodynamic work can be written :

$$\delta d\pounds_a = (dO \bullet F + d\Theta \bullet M)\ ds$$

while the first variation of the vectors F and M assumes the form:

$$\delta F + F \wedge \delta\Theta = [a_1]\ \left(\frac{d}{dt}\ \delta O + v^* \wedge \delta\Theta\right) + [a_2]\ \frac{d}{dt}\ \delta\Theta$$

$$\delta M + M \wedge \delta\Theta = [a_3]\ \left(\frac{d}{dt}\ \delta O + v \wedge \delta\Theta\right) + [a_4]\ \frac{d}{dt}\ \delta\Theta$$

where the matrices $[a_1]$, $[a_2]$, $[a_3]$ and $[a_4]$ are functions of the aerodynamic coefficients C_L, C_D, C_M and of the velocities v^* and v_p^* as well.

Then the second variation of the aerodynamic virtual work can be written in the following matrix form :

$$\delta^2 d\pounds_a = \{\delta c\}^T [a_\tau] \{\delta a^*\} \quad (8)$$

where $\{\delta c\}$ and $\{\delta a^*\}$ have the same definition as before and the matrix $[a_\tau]$ is the tangent aerodynamic matrix and is composed by the matrices $[a_1]$, $[a_2]$, $[a_3]$ and $[a_4]$. By applying the transformation equation (4), Eq.(8) becomes :

$$\delta^2 d\pounds_a = \{\delta c_a\}^T [A_\tau] \{\delta a_a^*\}$$

where obviously :

$$[A_\tau] = [S]^T [a_\tau] [R]$$

From the prescribed inflow distribution we are able to express the vector $\{\delta a_a^*\}$ by means of the vector $\{\delta a_a\}$ itself and of the induced velocity variation $\delta\nu$. Glauert's momentum balance in the linearized form provides the necessary equation for $\delta\nu$ in the incremental form.

HINGES AND CONTROL MODELING

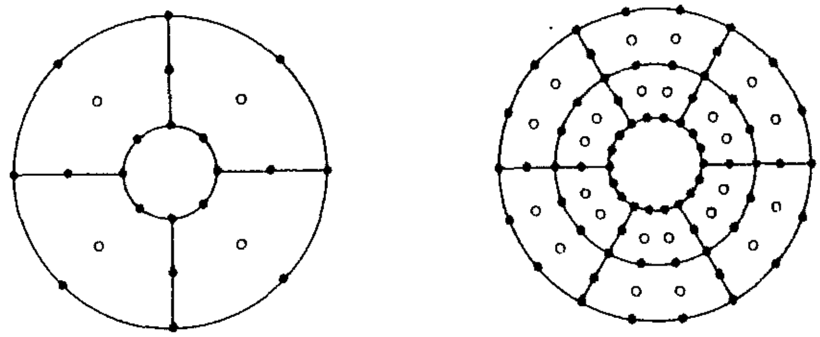
An articulated hub can be modeled by a sequence of rotational and/or translational linkages, each having a single degree of freedom, so that both flap, drag and pitch hinges, in whatever their position, and elastomeric bearing displacement, can be easily taken into account.

The choice of the degree of freedom related to such complex linkages, in general, is quite evident and the correct matching between these degrees of freedom and those of the blade is performed by means of Lagrange's multipliers technique.

Moreover drag dampers and flap stiffnesses, when present, can be included by appropriate isoparametric scalar elements.

Control inputs are provided by the user, which specifies position and orientation of the swashplate, together with the complete geometry of the pushrod and of the link lever. The swashplate, the pushrod and the link lever are assumed as rigid elements and their kinematics are taken into account by appropriate algebraic equations. This way of applying the control input, instead of setting the blade pitch angle, allows us to analyze all of the pitch-flap and/or pitch-lag couplings, originating from the control geometry.

FIG.4 - TYPICAL 2D FINITE ELEMENT MESH FOR COMPUTING BLADE CROSS SECTION PROPERTIES BY HANBA2 PROGRAM.



(A) - COARSE MESH (B) - REFINED MESH

FIG.5 - SPACE-TIME MESHES USED IN NUMERICAL EXAMPLE

NUMERICAL EXAMPLE

In the following some results are shown which are related to the A3 case (without unsteady stall) of the so called standard problem of Ref.[9], with the trim parameters listed below :

Thrust	=	15530. lb
Propulsive Force	=	1795. lb
Side Force	=	143. lb
$\theta_{.75}$	=	10.07deg
B_{1S}	=	12. deg
$-A_{1S}$	=	2. deg

The referred test case has been solved with the two meshes shown in Fig.5, which involve 3 azimuthal x 3 radial and 4 azimuthal x 3 radial nodes/element for a total of 4 and 12 elements respectively.

Static airfoil properties are based on Ref.[13] and it can be seen that they are a rather crude approximation of those used in Ref.[9], especially with respect to the maximum lift and moment behaviour (Fig.6).

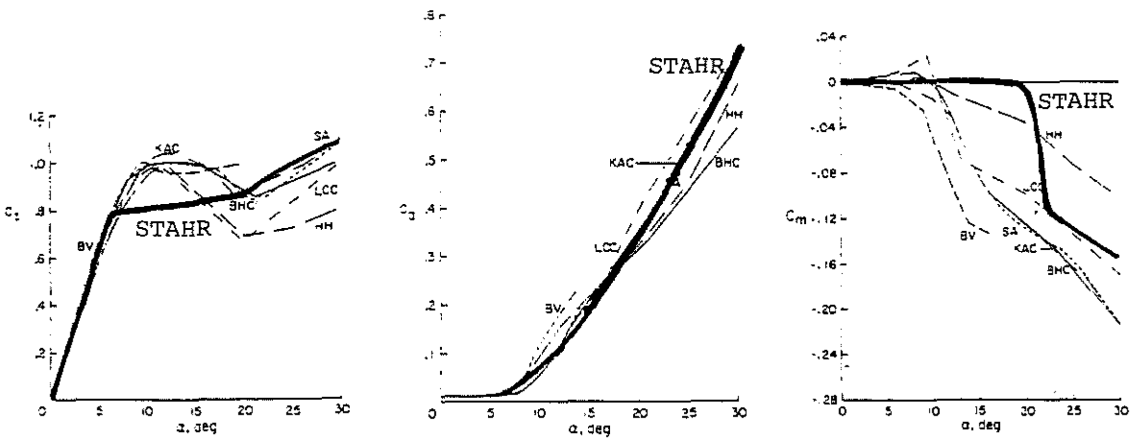


FIG.6 - NACA 0012 AIRFOIL SECTION LIFT, DRAG AND PITCHING MOMENT COEFFICIENTS AT M = .5 (REF.[9])

In view of the large scatter of results shown in Ref.[9], these crude approximations are deemed to be acceptable, as the test application was not aimed to any particular result, but it had to show that the developed procedure is suitable for practical applications.

In all of the following figures the continous line is related to the fine mesh of Fig.5b, while the ball thicks point to those of the coarse discretization of Fig. 5a.

It can in general be seen that the results of the simpler mesh are fairly well converged, when related to kinematical quantities, while they are far from an acceptable convergence, if related to stresses.

If one compares the diagrams shown here, with those of the many programs compared in Ref.[9], he can note that these are nothing else that a new addition to a set of rather scattered results, and in some sense, the procedure developed here is neither better nor worst than any of the there reported programs.

Nevertheless some comment can be made on the operation of the method of this paper as related to its use in design and analysis practice.

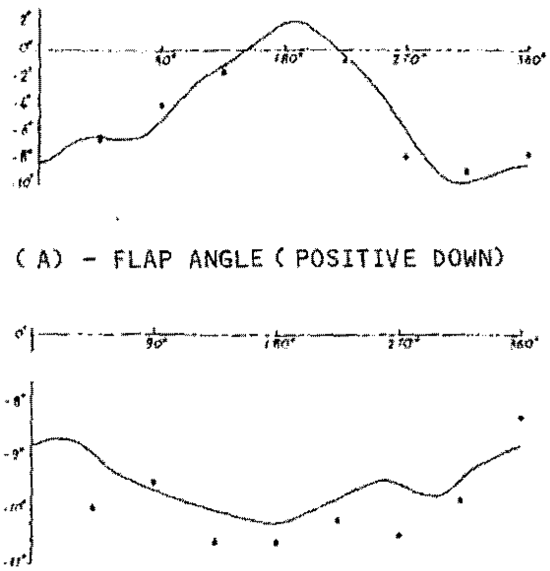
First of all it has been noted that the space-time beam element, despite to its sophisticated formulation, requires rather fine radial discretization in order to give converged acceptable stresses. This fact implies an heavy burden in term of computer resources, so making the use of the program tiresome for most of the design phases in which one need to manage fast computer responses with acceptable results. Moreover the unified approach to trim and stability outlined in Ref.[1] poses some problems in relation to latter one, as the inclusion of all of the nodal degrees of freedom in stabilty analyses shows up in numerical instabilities, which are related to the method, when is used as an explicit integration formula. Thus some judgement is required in accepting stability trends.

CONCLUDING REMARKS

It can be stated that the formulation of trim and stability analysis of helicopter rotors, as outlined here and in Ref.[1], can afford a powerful tool for the easy development of a working computer program to be used in different design and analysis phases.

The actual implementation of the approach needs to be improved with respect to beam element optimization and avoidance of spurious stability results.

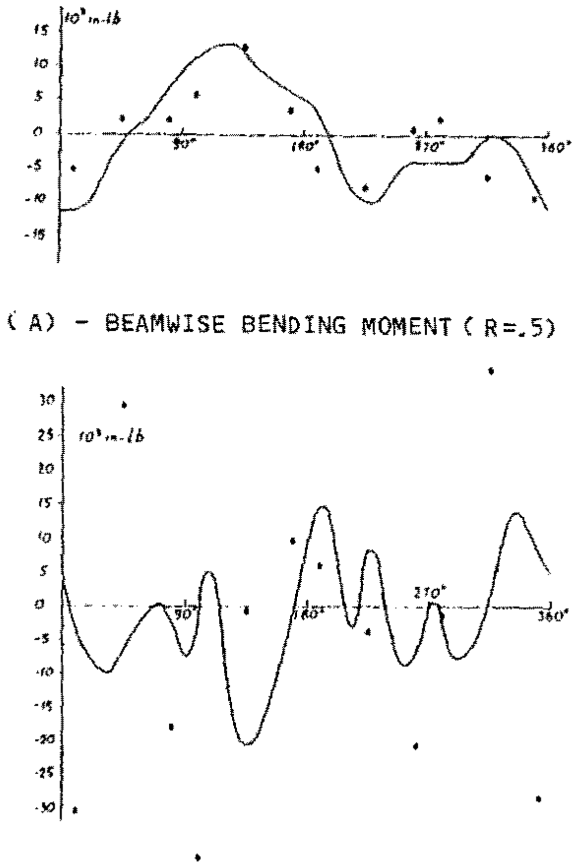
This items, together with the adoption of the improved system solution, outlined in Ref.[1], and extrapolation of coarse mesh results for affording starting solutions to refined meshes, should be implemented, in order to improve program performances and to reach the stated goal of a single and unified computer program for different design phases.



(A) - FLAP ANGLE (POSITIVE DOWN)

(B) - LEAD-LAG ANGLE (POSITIVE FORWARD)

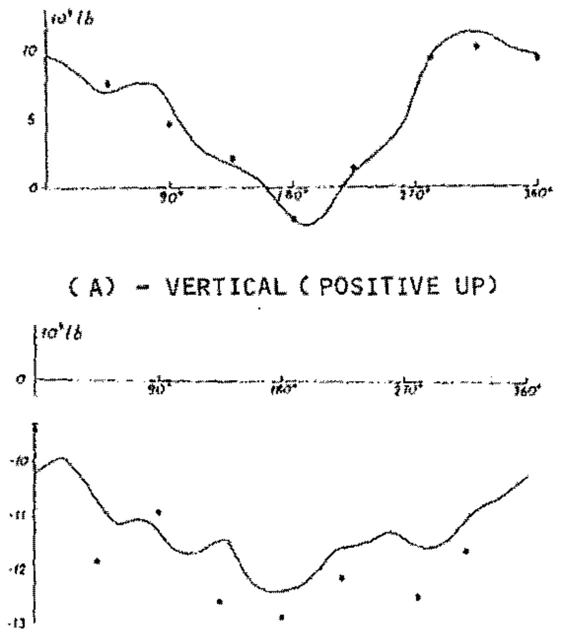
FIG.7 - FLAP AND LEAD-LAG ANGLES VS. AZIMUTH



(A) - BEAMWISE BENDING MOMENT (R=.5)

(B) - CHORDWISE BENDING MOMENT (R=.5)

FIG.8 - BEAMWISE AND CHORDWISE BENDING MOMENTS VS. AZIMUTH



(A) - VERTICAL (POSITIVE UP)

(B) - HORIZONTAL (POSITIVE FORWARD)

FIG.9 - HUB SHEARS VS. AZIMUTH

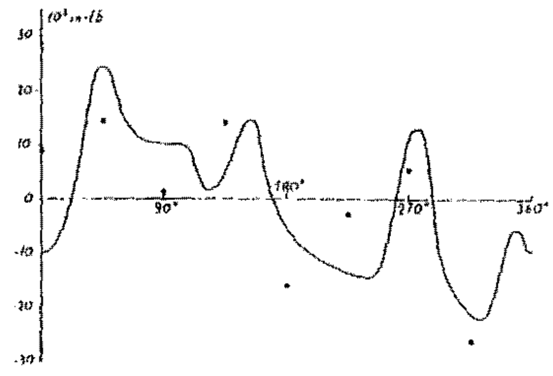


FIG.10 - MOMENT OF LEAD-LAG DAMPER

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