



NEW METHODS FOR THE CALCULATION  
OF HOVER AIRLOADS

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# NEW METHODS FOR THE CALCULATION OF HOVER AIRLOADS

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## ABSTRACT

A new lifting-surface method that has currently been developed and mechanized for the prediction of hover airloads is described. The method includes both a prescribed wake and a relaxed wake representation, and the several unique features of wake modeling are discussed. Calculated results demonstrate that the fundamental explanation for discrepancies in hovering rotor airloads predicted by lifting-line and lifting-surface methods is the difference in the wake shedding model. Hovering performance correlations with experimental data for converged relaxed wake calculations are analyzed. Finally, a close-approach surface singularity model that is currently being developed to accurately model the detailed blade pressure distribution and wake trajectory when the blade is in close proximity to a vortex or vortices is discussed. Example calculations for wings demonstrate the capabilities of this new method.

## 1. Introduction

The latest generation of rotor blades that are being developed and tested for improved hover performance include planforms with appreciable amounts of taper or sweep as well as nonlinear twist schedules. The conventional analytic tools reviewed in References 1 and 2, including the lifting-line models of the blade aerodynamic loading, cannot, of course, accurately represent the newer designs that involve increasing spanwise flow effects or that require the detailed calculation of changes in chordwise loading. Consequently, at the very least, a simple lifting-surface theory that can represent changes in blade planform and also account for the spanwise flow in the tip region is required for the analysis of these designs. A more desirable

method for the detailed analysis of the aerodynamic loading near these newer blade tips would be a surface singularity method which includes a representation of the actual blade thickness and the generation of the tip vortex across the blade tip. The capability of calculating pressures around the tip edge and of accurately calculating detailed pressures near a free vortex would, therefore, be required.

In fact, the simple lifting-surface model of the rotor airloads has already been developed and programmed, and preliminary results for conventional rotors have been calculated and compared with experimental data. The details of the analysis and calculations are reported in References 3 and 4. Furthermore, the basic methodology for the close-approach, surface singularity model is also presently being developed for three-dimensional, high-lift wings (Ref. 5), and this technology will be utilized in the next year to build a rotary wing version. In both of these methods the lift and induced power effects are predicted, although the profile power must still be determined by reference to sectional data tables for the appropriate form drag. Ultimately, this last remaining empiricism will be eliminated when the calculation of the viscous flow effects can be coupled with the surface singularity model. The analysis for this part of the solution is already available and has been used to predict boundary layer and separated flow effects on complex configurations in References 6 and 7.

In this report, the theoretical development of the lifting-surface method, including the various refinements in the wake model and to wake relaxation techniques, is only briefly summarized. The theoretical description of the surface singularity model is also succinctly stated. The reader is referred to References 3 or 4 and 5 for the detailed development of these analyses. Here, the results of the data calculations are discussed. In particular, the differences in performance calculated using lifting-line and planar lifting-surface analyses are explored, and the hover performance correlations for converged, relaxed wake calculations with experimental data are analyzed. Finally, calculations by the surface singularity method for wings are presented to illustrate the unique capabilities of the method that will prove vitally useful in the development of a rotary version.

## 2. Vortex-Lattice Model

### 2.1 Blade Representation

A "linearized" lifting-surface representation of the rotor airloads is accomplished by a vortex lattice placed on the rotor planform area in the disk plane as illustrated in Figure 1. In the computer program, HOVER, the influences of individual panels in the blade lattice are computed by quadrilateral vortex rings; therefore, the basic unknowns in the flow tangency equations are the panel ring vortex strengths, or, equivalently, panel doublet strengths.

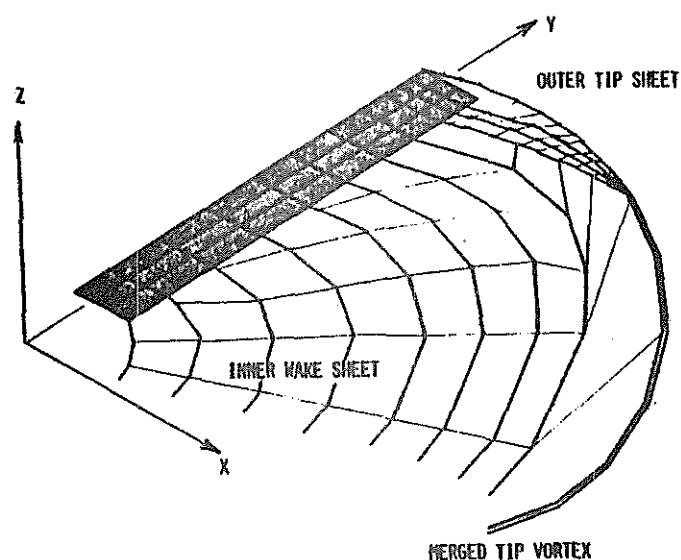


Fig. 1. Rotor Blade Vortex-Lattice Model.

### 2.2 Wake Representation

Of course, the discrete vortex filaments from the trailing edge of each blade represent the hovering rotor wake, which quickly separates into two parts--an inboard sheet of weaker vorticity and an outer tip sheet that rapidly rolls up to form a very strong tip vortex (Refs. 8 and 9). In HOVER, to reduce computational effort, the azimuthal step angle,  $\Delta\psi$ , of the inner sheet ( $\Delta\psi_s$ ) and the tip region ( $\Delta\psi_t$ ) can be specified independently as shown in Figure 1. Further, the wake azimuthal step increments can be changed to new values after the first blade passage. Also depicted in Figure 1 is a simple model of the tip vortex shedding across the chord that

is included in program HOVER in order to improve the prediction of the aerodynamic loading near the rotor tip. A straight vortex filament springs from the leading-edge bound vortex at an angle, say  $\beta_t$ , with respect to the vortex lattice.  $\beta_t$  is presently set to the low aspect ratio value of half the angle of attack at the tip.

The overall wake structure is illustrated in Figure 2 and consists of near-, intermediate-, and far-wake regions. The dimensionless axial coordinates at the start of the intermediate- and far-wake regions are ZFAR1 and ZFAR2, respectively. The near-wake region generally includes four vortex passes below the generating blade, and the near-wake filaments are geometrically represented by compound pitch, contracted helices in the conventional fashion (Ref. 9). The intermediate-wake region serves as a "buffer" zone between the near-wake and far-wake models. No wake contraction is allowed in this region so that wake filaments are fixed-pitch, constant radius vortex helices. The pitch and radius of each filament is determined by the final pitch and radius of the filament in the near wake. Finally, the far-wake model represents a semi-infinite continuation of the intermediate wake. The far-wake velocity contribution due to each helical vortex filament trailing behind the rotor is approximated by the velocity due to a cylindrical sheath of uniform vorticity. In this way, the far-wake model insures the continuity of vorticity in the wake.

The near-wake region is initially prescribed. Four prescribed wake options are available in HOVER for computing the constants in the helix equations and in generating the resulting wake coordinates. The four options include the Kocurek/Tangler wake (Ref. 10), the Landgrebe wake (Ref. 9) and two options for user input of the wake constants. The wake coordinates derived from these equations are then shifted according to the blade coning angle to preserve the relative positions of the wake to the blade vortex lattice.

If requested, the final prescribed wake geometry serves as an initial guess in an iterative scheme to obtain a true force-free wake. Usually, only a merged tip vortex relaxation is required, although the option of a full wake relaxation is

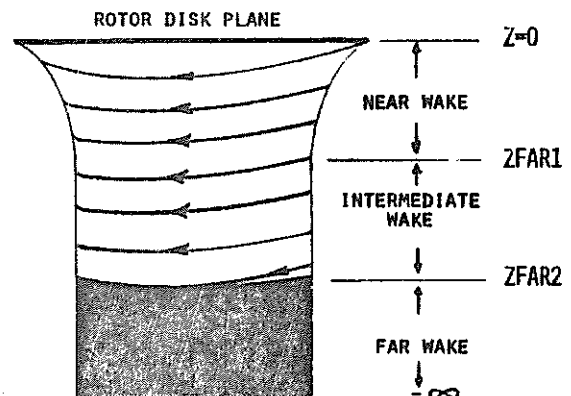


Fig. 2. Global Wake Model.

available in HOVER. The improvements necessary for meaningful free-wake results that were developed for the HOVER code are summarized briefly in the following.

(1) Vortex Core Model--

Usually, a Rankine (constant-vorticity) core model is used in three-dimensional rotary wing calculations; however, a new simple model that was suggested by Scully (Ref. 11) compares more favorably with experimental data and is used for all calculations in HOVER. The Scully core is a spread vorticity model that produces exactly one-half the maximum swirl velocity of a Rankine core of equal core radius.

(2) Numerical Integration Along Curved Segments--

Another improvement is the calculation of wake-induced velocities by numerical integration of the Biot-Savart Law along curved vortex filaments. Previous investigations of wake deformation have utilized straight vortex segments for the wake velocities with azimuthal step angles in the wake ranging from  $15^\circ$  to  $30^\circ$ ; however, it was shown in detail in Reference 3 that this approximation is too severe for the hover free-wake calculation. Here, the Romberg iteration method (Ref. 12) is used as a basic technique for integrating along curved filaments represented by a biquadratic (essentially a "safe" cubic) interpolation scheme (Ref. 4).

(3) Self-Induced Velocities--

Self-induced velocities (velocities induced by the filament on itself) are also calculated in a unique manner in HOVER. The basic procedure is detailed by Widnall in Reference 13, and the self-induced expression for a circular arc filament was derived in Reference 4 and is used in the present analysis.

(4) Integration for New Shape--

Finally, a new method was developed for the integration of the wake distortion velocities to obtain new wake geometries. Basically, a piecewise continuous biquadratic curve is fitted through the wake tangents (normalized wake velocities) and then integrated to obtain the new wake coordinates.

## 2.3 Loads Calculation

Once a converged wake geometry is computed by the prescribed wake or relaxed wake options, inviscid forces and moments on the blade bound vortex segments are then evaluated in the usual way by applying the Kutta-Joukowski Law. Of course, the chordwise and radial pressure jump distributions are also calculated, and the influence of compressibility is included in the manner described by Sopher in Reference 14.

Finally, with the sectional coefficient of lift distribution known from the "linearized" lifting-surface calculation, the profile drag and, hence, profile and total torque must be determined by falling back on empirical data. By using airfoil data from wind tunnel testing, the tables are entered at the lift coefficient calculated by the lifting-surface solution and the appropriate Mach number and the corresponding drag coefficient is read off. This reliance on empiricism can only be removed when a full thickness model such as that described in Section 3 is used in conjunction with a rigorous viscous flow analysis.

## 2.4 Data Correlations

In the process of verifying the lifting-surface method, several problem areas regarding data correlations with other programs have surfaced. A single example performance calculation for the OH58A rotor is presented here to illustrate the problem of data correlations with a simpler lifting-line method and to indicate the computational flexibility of HOVER. The power of the relaxed wake option is demonstrated in data calculations for the CH53A rotor.

### 2.4.1 OH58A Calculations

Results calculated using program HOVER for the case of the OH58A two-bladed rotor have been compared with data calculated using the UTRC lifting-line program (Ref. 9). For the comparison, only one chordwise panel and fifteen panels across the blade radius ( $NR = 1$ ,  $NC = 15$ ) were used in the vortex lattice. Further, the radial distribution of the panels and the prescribed wake constants (i.e., wake geometry) were the same for both programs, and the tip vortex angle was set to zero in the HOVER calculations. The calculated radial distributions of dimensionless bound circulation are compared in Figure 3 for a collective,  $\theta_{75}$ , equal to  $5.75^\circ$ . The blade coning angle,  $\beta$ , in both cases was  $3^\circ$  and wake azimuthal settings,  $\Delta\psi$ , were set to  $30^\circ$ . In the UTRC calculation, 16 revolutions of detailed wake were used. By contrast, the HOVER calculation required only  $2\frac{1}{2}$  revolutions of detailed wake ( $ZFAR2 = 0.55$ ) demonstrating the effectiveness of the far-wake model. Obviously, although blade and wake geometric representations are the same, the lifting-line and lifting-surface methods (even with  $NR = 1$ ) give quite different results. These differences have been traced to the method of wake shedding from the blade surface. In HOVER, the wake filaments leave the blade from the trailing edge, while the wake filaments are shed from the bound vortex itself in the lifting-line method as depicted in Figure 4. This basic difference in wake curvature accounts for the dissimilarity of rotor loading in Figure 3.

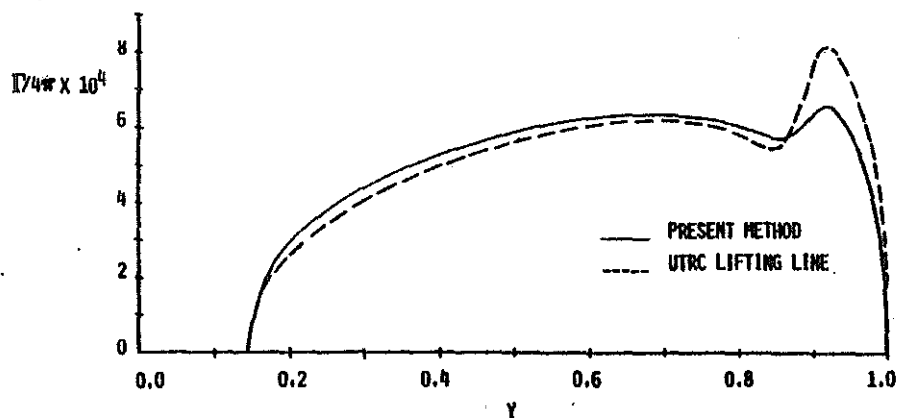


Fig. 3. Comparison of Predicted Bound Circulation Distribution for the OH58A Rotor ( $\theta_{75} = 5.75^\circ$ ,  $\Omega R = 655$  fps,  $\beta = 3^\circ$ , Fixed Wake Constants).

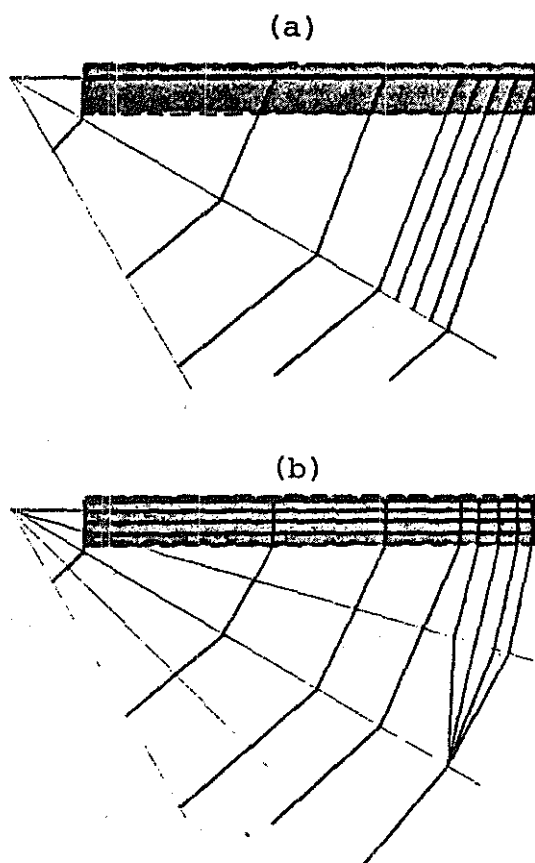


Fig. 4. Wake Shedding Model.

- (a) Lifting-Line Theory.
- (b) Lifting-Surface Theory.



The importance of the wake-shedding model is demonstrated in Figure 5. Here, the downwash calculated by the UTRC program, with  $\Delta\psi = 30^\circ$ , is indicated in the figure by the "dashed" line.

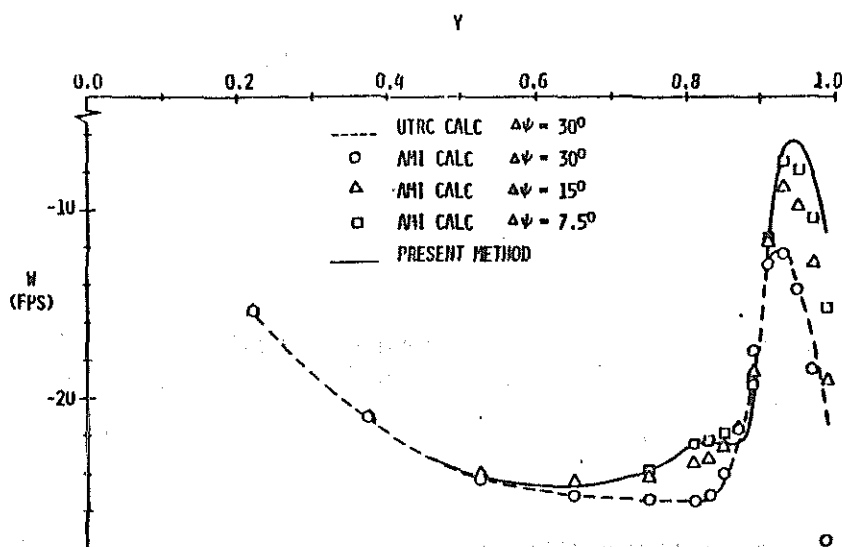


Fig. 5. Influence of Wake Segmentation on Rotor Inflow for Lifting-Line Approach.

For comparison, the HOVER wake model was modified to represent the lifting-line wake shedding (Figure 4(a)), the Mach number transformation was set to unity (i.e., incompressible), the far-wake model was eliminated, and 16 revolutions of detailed wake were represented. Finally, the UTRC bound circulation solution was read into HOVER, and the downwash along the bound vortex line calculated. The "circled" data are calculated values with this modified program for  $\Delta\psi = 30^\circ$ . The small difference in the "dashed" and "circled" data between 80 and 90%R have been shown to be within the round-off error of the basic circulation solution read from the UTRC data (Figure 3), while the differences in the "dashed" and "circled" data outboard of 95%R result from the tip inset that is automatic in HOVER. Consequently, this data verifies the aerodynamic matrix routines in HOVER. Also, by changing the wake azimuthal settings for the lifting-line model, a dramatic effect on the downwash distribution was found. These additional results for  $\Delta\psi = 15^\circ$  and  $\Delta\psi = 7.5^\circ$  are compared in Figure 5 with the data from the HOVER shedding model (UTRC circulation values still imposed).

Obviously, distribution data are very sensitive to wake azimuthal setting for the lifting-line model, and operating these programs in their present configurations with the small wake azimuthal settings required for accuracy would be expensive. More importantly, however, these results clearly show that the details of the wake shedding near the blade need to be carefully modeled for accurate results.

Based on these observations, the HOVER distributions are felt to provide more representative rotor performance for the given wake fairing constants. Further, these comparisons also illustrate the intimate relationship between the prescribed wake fairings and the method of representing the loading on the blades. In the past, for a given method of loads representation (i.e., lifting line), the wake fairings were "adjusted" within the scatter of experimental data until calculated and experimental integrated loads were in agreement. If the method of loads representation is changed, then it is not unreasonable to find that the prescribed wake fairings will have to be readjusted (again, within the scatter of the experimental data) to give accurate results.

#### 2.4.2 CH53A Calculations

A more challenging example for hover performance prediction is the CH53A six-bladed rotor. Whirl stand data for integrated loads and tip vortex wake geometry data for a thrust of 45,000 lbs. are available for this rotor in Reference 15. The detailed relaxed wake calculations for this case are illustrated in Figures 6 through 8. A cosine distribution of ninety panels per blade ( $NR = 3$ ,  $NC = 30$ ) was used in this exploratory calculation to insure sufficient detail near the tip, and a total of  $480^\circ$  of detailed wake ( $ZFAR2 = -0.76$ ) was included. Blade collective was  $11^\circ$  and blade coning angle was set to  $3.75^\circ$ . This coning angle was determined from the rotor tip deflection shown in Reference 16. The prescribed wake constants were estimated from the available wake geometry data, and five rotor wake relaxation iterations were calculated to demonstrate a converged solution. The prescribed wake rotor performance results were very optimistic (figure of merit  $> 0.78$ ); therefore, successful performance prediction relied completely on the relaxed wake calculation.

The behavior of key rotor performance parameters with relaxed wake iteration is illustrated in Figure 6. A highly damped convergent oscillation for each parameter is shown. The axial distance below the blade of the tip vortex at first blade passage,  $Z_{\psi_D}$ , is changed by more than 16%. Physically, this represents a shift of less than  $1\frac{1}{2}$  inches (or,  $0.3\%R$ ) further

from the rotor blade. The tip vortex helical pitch rate in the intermediate wake,  $K_{2t}$ , changed by 6.5%, bringing the wake after first blade passage closer to the blade. As a result of these wake changes (all within the scatter of the wake data), the dimensionless maximum blade circulation remained relatively constant, decreasing by only 1.7%, but the thrust coefficient per solidity,  $C_T/\sigma$ , decreased significantly by 5.8%. Further, the converged solution is essentially obtained after only three iterations, and calculations for other cases confirm this as a practical iteration limit.

The final predicted blade thrust, induced and profile torque distributions for this case are shown in Figures 7 and 8.

The areas under these curves give the integrated performance values per blade; that is, typically,

$$C_T = b \int_{Y_0}^1 C_{T_i} dy.$$

Here,  $b$  is the blade number;  $Y_0$  is the dimensionless root cutout. The increases in thrust loading and torque at the tip are due to the influence of the vortex shedding, and the "shaded" area in the profile torque distribution represents the variability in the available airfoil data sets. The predicted thrust and torque coefficients for

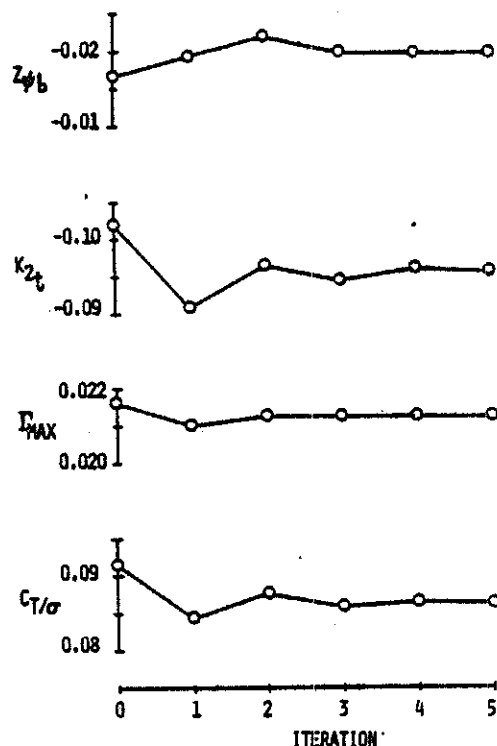


Fig. 6. Behavior of Key Rotor Performance Parameters with Relaxed Wake Iteration for the CH58A Rotor ( $\theta_{75} = 11^\circ$ ,  $\Omega R = 696$  fps,  $\beta = 3.75^\circ$ ).

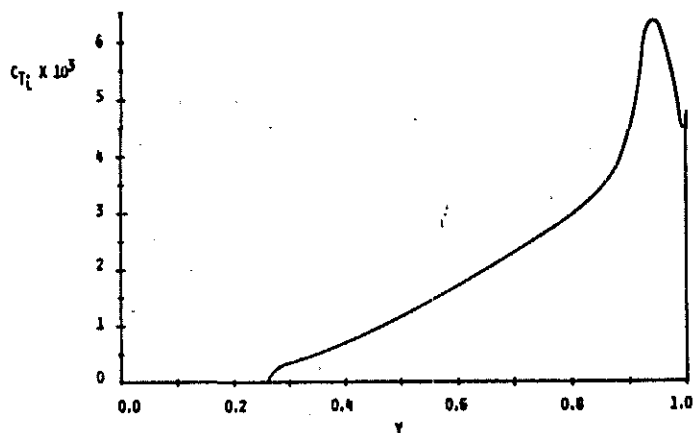


Fig. 7. Predicted Thrust Distribution for the CH53A Rotor.

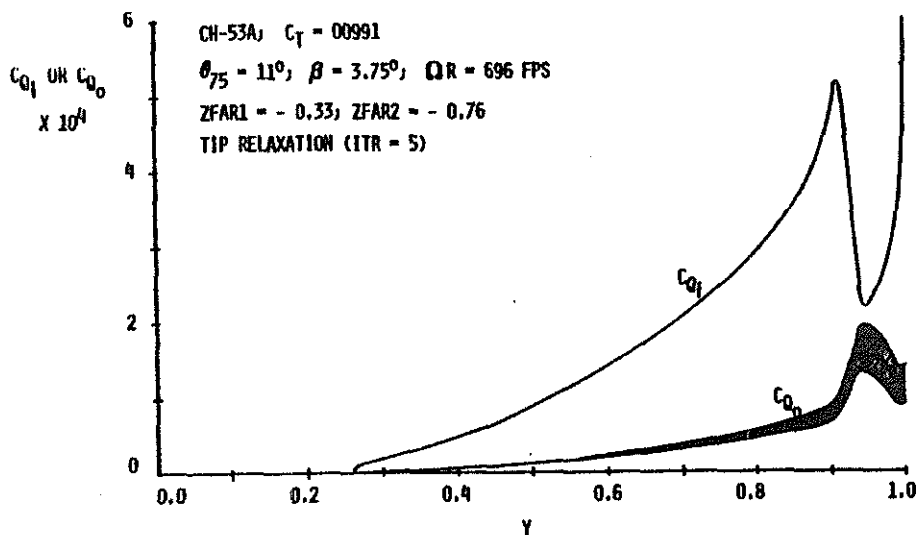


Fig. 8. Predicted Induced and Profile Torque Distribution for the CH53A Rotor.

this case are:

$$C_T = 0.00991, \text{ and}$$

$$C_Q = 0.000985 \pm 1.8\%.$$

The torque breakdown (based on the mean value) is approximately 81% induced and 19% profile. This breakdown and the loading distributions are, of course, very different than those obtained with a lifting-line method (Refs. 15, 16 and 17).

In Figure 9, the final integrated performance comparisons with experimental data (Ref. 15) for a range of thrust levels are illustrated. The experimental data symbols are sized according to the reported  $\pm 2\%$  accuracy and the shaded area for the theoretical calculations correspond once more to

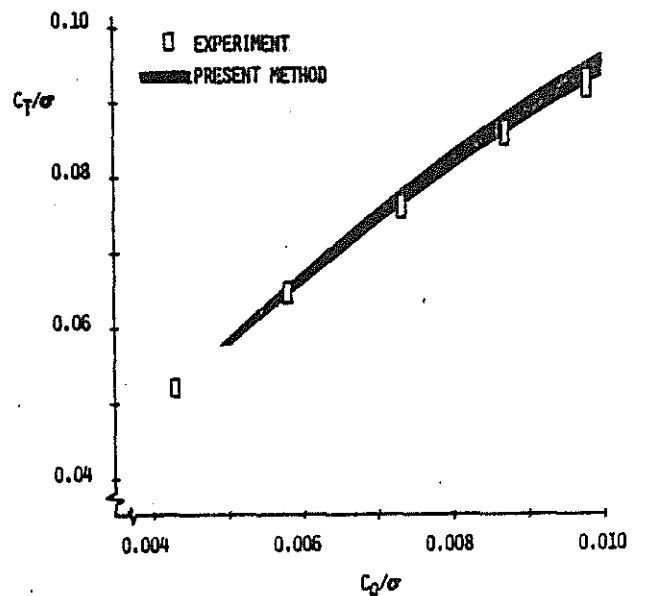


Fig. 9. Comparison of Integrated Performance Predictions with Experimental Results for the CH53A Rotor.

the variability of the airfoil data. The agreement of the relaxed wake calculations is excellent over the entire performance chart. Finally, it is pointed out that a calculation carried out with sixty panels per blade ( $NR = 3$ ,  $NC = 20$ ) and three relaxation iterations required only 30 seconds of CDC 7600 CPU time. This brings it into the realm of practicality for use as a design tool and is in marked contrast to the early wake relaxation models which required up to one hour for each relaxation step.

### 3. Surface Singularity Model

#### 3.1 The Method

In the last section, it was demonstrated that the aerodynamic loading of a rotor in hover is very sensitive to small shifts in axial position of the tip vortex at first blade passage. Additionally, the vortex passage distance from the blade may be on the order of the blade thickness. In these cases, conventional vortex-lattice or even surface singularity methods cannot, in general, accurately represent the local solution. The closest approach between a vortex and a lifting surface for maintaining accuracy was shown in Reference 18 to be approximately the same as the panel spacing. Consequently, a surface-singularity method that will adapt automatically for the close-vortex problem is required in the hover case. The basic techniques for this new method have recently been developed for three-dimensional high-lift wings (Ref. 5), and will be useful in constructing a method for rotor performance estimation.

The new method is based on a surface doublet distribution on panels. Various forms of the model are being evaluated. Influence calculations in the far-field use the basic panels, while for near-field calculations, each local panel is divided into a set of subpanels (Ref. 5). The position and singularity strength of each subpanel are obtained using biquadratic interpolation through the surface geometry and panel singularity strengths, respectively. Thus, as the surface is approached, the singularity representation becomes closer to the smooth (biquadratic) variation because of the increasing number of smaller steps.

The basic panel and subpanel representation of a wing tip is illustrated in Figure 10. The closer geometric representation offered by the subpanel scheme is obvious in this case. (Note: the control point locations for each panel are taken from the central subpanel on that panel. Also, the panel influence on itself always uses its basic subpanel set.)

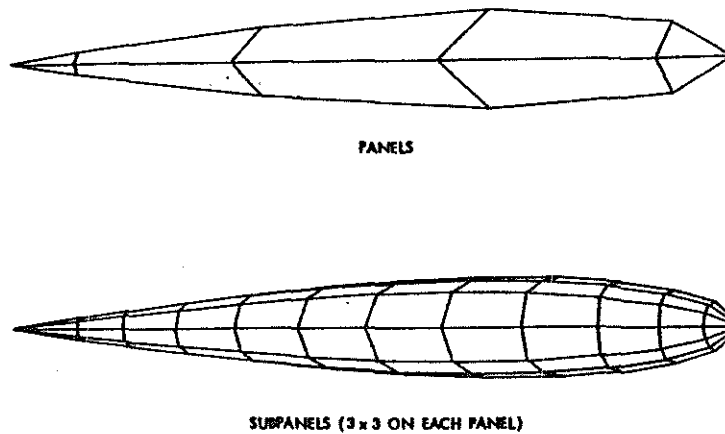


Fig. 10. Panels and Subpanels on a Tip Edge.

The main features of this technique that are essential to the vortex/surface interaction problem are summarized below.

- (1) Subpanels offer a closer representation of curved surfaces and smooth singularity distributions than is possible with practical panel densities.
- (2) Subpanels give the effect of higher panel density without increasing the number of unknowns.
- (3) Subpanels give a "higher-order effect", yet maintain simple influence coefficient expressions.
- (4) A panel's subpanel set is used only when a velocity calculation is performed within a small near-field radius from the panel's center (e.g., within three panel sizes away). This minimizes computing effort.
- (5) Smooth velocity and pressure calculations are obtained with reasonable panel density, even in the case of the vortex/surface interference problem.
- (6) Detailed pressures can be calculated at any point including the possibility of calculations around actual blade or wing tips.

These features are illustrated in the final example calculations shown here.

### 3.2 A Vortex/Surface Interaction Calculation

As a searching test of the subpanel technique, a vortex/surface interaction calculation was chosen in which a prescribed vortex was positioned close to a Joukowski airfoil. The vortex location was  $x = .15c$ ,  $z = .125c$ , and its strength was  $.2\pi$ . The vortex flow was combined with an  $\alpha = 10^\circ$  onset flow. Thirty panels were used in a cosine spacing, and the near-field radius factor was set to 3.

The ability of the subpaneling scheme to provide smooth velocity calculations anywhere is very apparent in Figure 11(a), which shows calculated streamlines. The streamline calculation method employs a second-order variable step integration of calculated velocities. Three starting points were selected as shown in Figure 11(a). The forward point gives a streamline that on the upstream part passes very close to the leading edge, and in the downstream part climbs over the vortex before dropping to the airfoil surface which it follows very closely back to the trailing edge. Details of this streamline (and the second streamline) in the leading-edge region are given in the inset in Figure 11(a). The first streamline passes very close to the surface, well within the spacing of the control points. The line is very smooth, even though the velocity calculations have been performed at a number of "arbitrary" positions. The second streamline is clearly very close to the stagnation streamline and essentially follows the surface with one or two minor oscillations. As the calculation proceeds from the starting point, this second streamline hits the airfoil very steeply, and yet quickly takes up the surface direction, a very searching test for both the streamline calculation procedure and the velocity calculation routine. On the downstream side, this second streamline follows the surface back to the trailing edge.

The third streamline forms a closed loop around the vortex and does several turns (total streamline length specified is 2.5 chords) before accumulating errors eventually allow it to escape downstream along the airfoil surface.

The surface pressure distribution corresponding to this calculation is shown in Figure 11(b). Intermediate velocity calculations are indicated by triangles to distinguish them from the basic control point values. These additional calculations, made possible by the subpaneling technique, clearly define the details of the three suction peaks and three stagnation points. The control point values in some of these areas would have been inadequate--particularly in defining the suction peak located beneath the vortex.

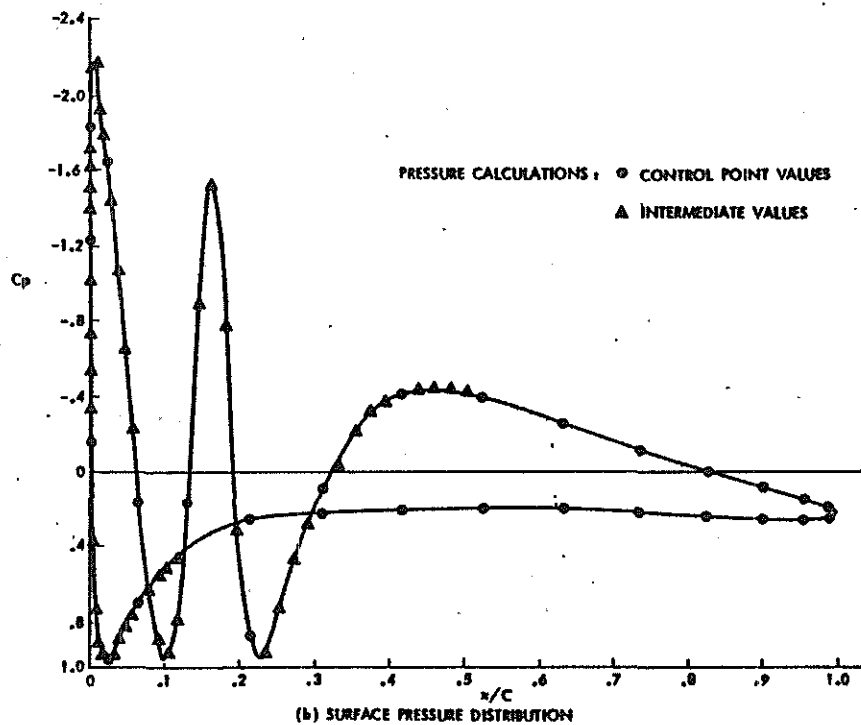
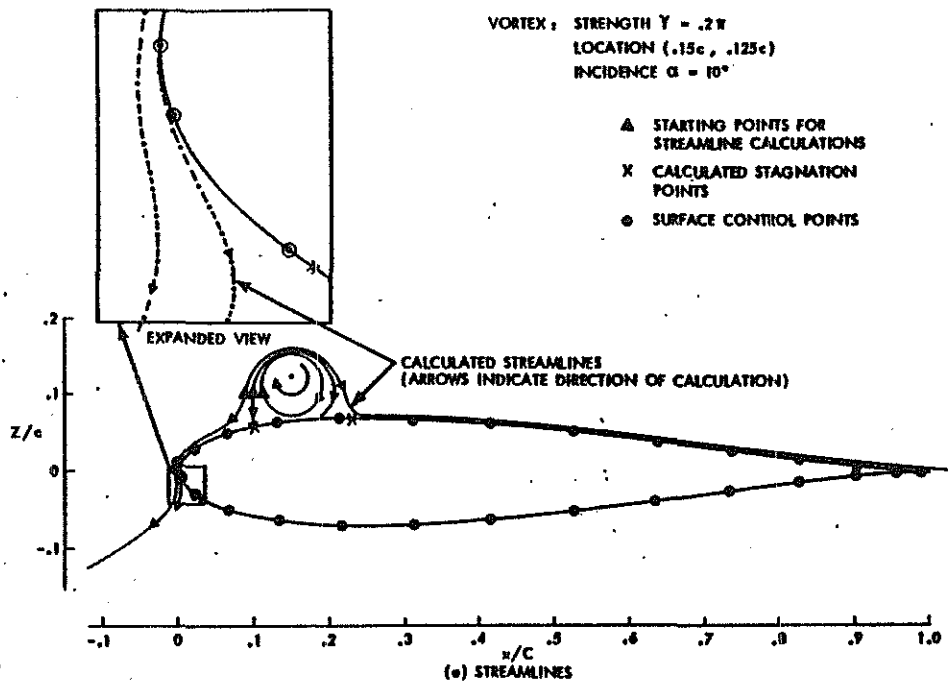


Fig. 11(a), (b). Calculations for a Joukowski Airfoil in the Presence of a Vortex.



### 3.3 A Three-Dimensional Wing Calculation

The doublet potential flow code was applied to a rectangular wing of aspect ratio 2 to check the detailed pressure calculation. The wing section was the 11.1% t/c, Boeing Section TR 17, and the angle of attack was  $7.73^\circ$ . Figure 12(a) shows the chordwise pressure distribution at .125 semispan calculated using panels distributed in a  $24 \times 4$  array on the main surface patch and a  $2 \times 12$  array on a tip patch with semicircular sections. For comparison, Figure 12(a) also shows solutions from the original VIP3D (Ref. 19) program ( $36 \times 4$  array) and also from the USSAERO program (Ref. 20). There is

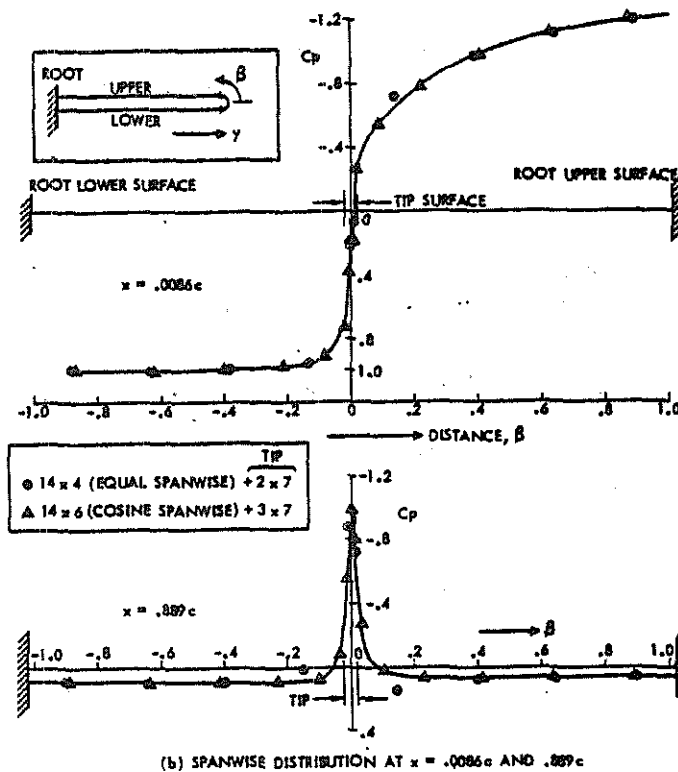
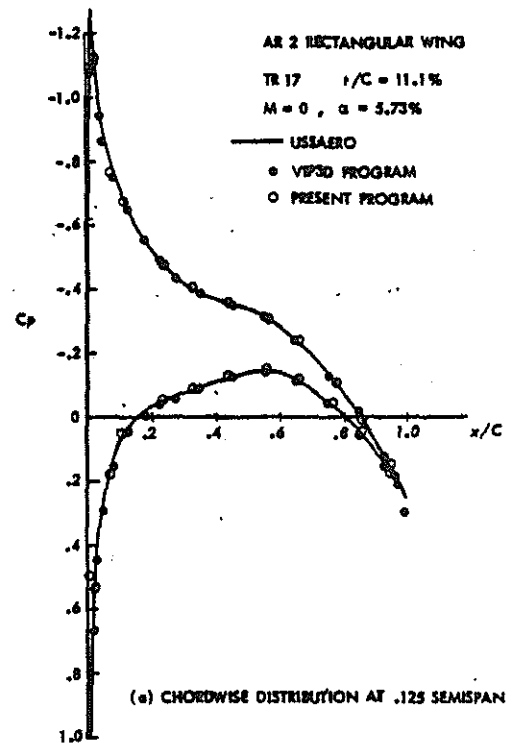


Fig. 12(a), (b). Calculated Pressures for an Aspect Ratio 2 Rectangular Wing.

very close agreement between all three programs. This is very encouraging because the doublet solution used a less dense panel system than the others.

The tip patch paneling in the doublet model allows pressures to be calculated round the tip edge. Figure 12(b) shows pressure distributions plotted in the spanwise direction from lower surface round the tip and back along the upper surface at x-wise stations .0086 and .889. Values are plotted from two panel distributions, one with 4 equal spanwise intervals and one with 6 spanwise intervals with cosine distribution giving increased density towards the tip. The latter improves the matching in panel size between the main surface and tip patch compared with the first case which has panel size ratios of the order 50 passing onto the tip patch; this probably accounts for the discrepancies between the two solutions near the tip in Figure 12(b). Large and sudden changes in interval size can cause numerical error when interpolating or differentiating the surface doublet distribution.

At the forward station, the spanwise flow from lower surface onto upper surface clearly has a monotonically decreasing pressure. Towards the trailing edge, however, the upper surface suction level has disappeared while a peak suction has developed on the tip surface, Figure 12(b). At this station, therefore, the spanwise flow is suddenly faced with a strong adverse pressure gradient as it climbs around the tip edge and will lead to the conditions for tip-edge separation.

#### 4. Conclusions and Recommendations

In this report, two new methods for the prediction of hover airloads have been discussed. Calculated results with the first, a lifting-surface model, have demonstrated that the discrepancies in hovering rotor airloads predicted by lifting-line and lifting-surface methods are due to differences in the wake shedding model. Additional performance correlations with experimental data for this method have shown the ability of new relaxed wake techniques to obtain accurate hover performance predictions. Further, the refinements in the wake model and to the wake relaxation techniques have decreased the computation time required for the wake calculation such that the second method, a surface-singularity model, is a practical prospect. Example calculations with the surface singularity method have demonstrated the unique capabilities of this method to accurately model the detailed solutions when a vortex is close to a wing surface. It is hoped that the experimental programs planned for the future will provide the basis on which these new analytic methods can be evaluated and developed into useful tools for the designer.

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