A NEW INDIVIDUAL BLADE CONTROL APPROACH TO ATTENUATE ROTOR VIBRATION

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Abstract

This investigation deals with the development of an active control system designed to attenuate rotor vibration using a new concept. The system is aimed at the new bearingless rotors which use a flexbeam to command the blade collective and cyclic input flight controls. According to this concept, a higher harmonic signal is introduced through a special actuator device attached to the flexbeam in the blade main load path. The impedance of the flexbeam is actively changed in the rotating frame to attenuate the forced vibration of the blade at the critical harmonic frequency. A two-degree-of-freedom aeroelastic model of a single blade is developed incorporating unsteady aerodynamic effects. The mathematical model is solved using the integrating-matrix and harmonic balance techniques. The problem is cast into standard optimum control theory, and a systematic approach to determine the higher harmonic signal that drives the actuation device is developed.

Nomenclature

a	distance between the midchord and the pitch axis measured in semichords (positive aft)
a_o	blade section lift slope
B	IBC input matrix
B_0, B_1	boundary condition operators
b	blade semichord
C	row vector of output feedback states
$C(k), C_k$	Theodorsen's lift deficiency function; matrix collecting $C(k)$ at discretization points
c_ϑ, c_φ	flexbeam flexibility coefficients
D	matrix of compliance coefficients
D_{ij}	structure compliance coefficients ($i,j=1,3$)
EI	cross-section flatwise bending stiffness
E	vector of pilot input commands
F	dynamical matrix of the aeroelastic system
F_w	vector of applied forces per unit of length
G, G_i	matrix of applied loads ($-2\leq i\leq 2$)
H	cross-section shear force resultant, feedback transmission
I	unit matrix
I_b	blade mass moment of inertia, mR^3
J, J_i	integrating-matrix operators ($i=1,2,3$)
k	reduced frequency, control gain coefficients
L	lift per unit of length
M	cross-section flatwise bending moment, mass matrix
M_a	aerodynamic pitch moment per unit of length (about the pitch axis)
M_ϑ	vector of applied moments per unit of length (about the cross-section elastic center)
m	running mass
N	number of discretization points along the blade
n	harmonic frequency, ω/Ω
n_b	number of blades
P	solution of the algebraic Riccati equation
p	eigenvector of the dynamical matrix F
Q	matrix of airloads
q	vector of airloads
R	rotor radius, control vector weighting matrix
r	local radius
r_ϑ	cross-section radius of gyration about the elastic axis
S	state vector weighting matrix
s	Laplace variable
T	cross-section tension
U	airflow local velocity
V	rotor free stream velocity
w	cross-section flatwise bending displacement
x	state vector
Z	geometric stiffness matrix
z	vector collecting the of harmonics of x

Greek

α	rotor angle of attack
γ	Lock number, $\rho a_0(2b)R^4/I_b$
θ	cross-section torsion angle
θ_c	collective and cyclic commands
κ	control signal wave form
λ	eigenvalue of the dynamical matrix F
μ	rotor advance ratio, $V\cos\alpha/\Omega R$
ν	rotational parameter, $m\Omega^2R^4/EI_{ref}$
ρ	air density
τ	cross-section torque
Φ	resolvent of the open-loop system (forward transmission)
φ	cross-section flatwise bending slope
ψ	azimuth angle
Ω	rotor rotational frequency
ω	harmonic frequency

Superscripts

C, NC	circulatory, non-circulatory airload
D	disturbance or motion-independent airload
L, NL	linear, non-linear term
M	motion-dependent airload
Q	quasisteady airload
T	matrix transpose
—	non-dimensional quantity (see Appendix)
•	derivative with respect to azimuth angle, $\partial/\partial\psi$
$(\ )'$	derivative with respect to local radius, $\partial/\partial r$
$\grave{(\ )}$	diagonal or block-diagonal matrix
*	convolution, complex conjugate

Introduction

One of the most efficient ways of affecting the dynamic response characteristics of mechanical systems such as beams and plates is by modifying their boundary conditions. In helicopters, the main source of vibration is due to the unsteady aerodynamic loads produced by blade vortex interaction (BVI) and dynamic stall effects in the forward flight condition. These non-linear aerodynamic effects generate the periodic forced vibration of the blades that is characterized by a broadband spectrum. In general, the vibration is small in hover and increases in amplitude with the helicopter forward speed. The vibration produced along the blade is transmitted into the airframe through the rotor hub. However, it can be demonstrated that only the harmonics of the load spectrum that are multiples of the number of blades are actually transmitted into the fuselage. In modern bearingless rotors, a flexbeam is used to command the changes in the blade angle of attack that provide the cyclic and collective control to the helicopter. Since the flexbeam is in the main load path, it also provides the required attachment stiffness between the blade and the main shaft. Hence, by actively changing the dynamic stiffness of the flexbeam it is possible to control the vibration that is transmitted into the airframe. According to the individual blade control (IBC) technique, each blade is controlled individually in the rotating frame to achieve a global reduction in the vibration measured at the fuselage. Current investigations are underway at NRC to build an actuator device with the so-called "smart" materials that is able to control the stiffness of the flexbeam in an active way, providing a means of attenuating the transmission of vibration through the rotor hub at the harmonics of the number of blades. This paper deals with the mathematical modeling of an individual blade control system suitable for such a device. A two-degree-of-freedom (flatwise bending and torsion) model of a flexible rotating blade with variable root stiffness is constructed. The total airload is divided into two separated sets. The first set comprises the usual unsteady motion-dependent airloads, and the second set the disturbance airloads due to effects such as BVI and dynamic stall. The disturbance airloads are considered as an unknown spectrum that should be rejected by the IBC system. This paper will show that is possible to cast this problem into a classical feedback control system. The constant feedback gains are the required coefficients of the Fourier series expansion of the signal to be delivered by the actuator device.

Mathematical Model

In this section, the mathematical model of a single bearingless blade modeled as a flexible rotating beam with two degrees of freedom (flatwise bending and torsion) is described. In the first part, the linearized structural dynamics of a blade for which the stiffness of the flexbeam may be actively changed is developed. In the second part, an unsteady aerodynamic model for the forward flight condition based on Theodorsen's theory (Ref. 1) is introduced. According to this model, the airloads are divided into two sets. The first set comprises the motion-dependent, and the second set the disturbance airloads. Finally, the assembly of the aeroelastic model is performed in the last part of the section.

Blade Structural Dynamics

In previous works (e.g. Ref. 2), a two-degree-of-freedom mathematical model of a rotary wing including the elastic flatwise bending and torsion was developed. The linearized equations were cast in a state vector form where the dependent variables are local quantities associated with the cross-section flatwise bending moment, shear force resultant, flatwise bending slope, flatwise bending displacement, torque resultant and torsion angle, respectively in Eqs. 1a-f. The independent variables are the spanwise coordinate and time. The equations are written in a non-dimensional form (bars were omitted for sake of simplicity), using the normalization shown in the Appendix.

$$\begin{aligned}
M' &= H + \nu T\varphi \\
H' &= \nu m\ddot{w} - F_w \\
\varphi' &= D_{11}M + D_{13}\tau \\
w' &= -\varphi \\
\tau' &= \nu m r_\theta^{\,2}\left(\theta + \ddot{\theta}\right) - M_\theta \\
\theta' &= D_{31}M + D_{33}\tau
\end{aligned} \qquad (1a\text{-}f)$$

The boundary conditions associated with the above set of six first-order differential equations are listed in Eqs. 2a-f, where they are normalized for the interval $0 \le r/R \le 1$.

$$
\begin{aligned}
&M(0) = 1/c_\varphi\, \varphi(0) \\
&M(1) = 0 \\
&H(1) = 0 \\
&w(0) = 0 \\
&\tau(1) = 0 \\
&\tau(0) = 1/c_\theta \left(\theta(0) - \theta_c\right)
\end{aligned}
\tag{2a-f}
$$

The lumped flexibility constants of the flexbeam in flatwise bending and torsion appear explicitly in Eqs. 2a and 2f, respectively. By varying these constants (c_φ and c_θ) from zero to infinity, boundary conditions spanning from the perfect cantilever to the hinged blade can be simulated. More interesting, IBC can be introduced via the latter two parameters. According to the proposed method, the mechanical impedance of the flexbeam is actively controlled to tailor the blade aeroelastic response. In order to accomplish this task, the flexibility constants are adapted by a function dependent on the azimuth angle:

$$
\begin{Bmatrix} c_\varphi \\ c_\theta \end{Bmatrix} = \begin{Bmatrix} c_{\varphi 0} \\ c_{\theta 0} \end{Bmatrix} + k(\psi) \begin{Bmatrix} \Delta c_\varphi \\ \Delta c_\theta \end{Bmatrix}
\tag{3}
$$

The objective of the present study is to develop a method to design the open-loop control law represented by Eqs. 3a-b. In Eq. 2f the collective and cyclic pitch control are introduced:

$$
\theta_c = \theta_0 + \theta_{1c} \cos\psi + \theta_{1s} \sin\psi
\tag{4}
$$

The integrating-matrix method (Ref. 3) offers a systematic means of integration for the governing equations in space. According to this method, the equations are discretized at N equally-spaced co-location points taken along the integration path. An $N{\times}N$ matrix operator J based on polynomial interpolation is introduced. If $f(x)$ is a non-dimensional function defined in the interval $0 \le x \le 1$, J has the property:

$$
\{f\} = J\{\partial f/\partial x\} + f(0)\{1\}
\tag{5}
$$

where f is $N{\times}1$ vector of discrete quantities. The boundary conditions are introduced via two additional $N{\times}N$ matrix operators such that:

$$
\begin{aligned}
&B_0\{f\} = f(0)\{1\};\, B_0 J \equiv 0 \\
&B_1\{f\} = f(1)\{1\}
\end{aligned}
\tag{6a-c}
$$

Each one of Eqs. 1a-f is pre-multiplied by J, resulting in a set of algebraic equations where the unknowns are the constants of integration in Eq. 5. Next, the latter equations are successively pre-multiplied by B_0 and B_1, and the boundary conditions in Eqs. 2a-f are used to solve for the constants of integration, yielding:

$$
\begin{aligned}
&\{M(0)\} = -B_1 J\{H\} - \nu B_1 \mathcal{J}T\{\varphi\} \\
&\{H(0)\} = -\nu B_1 \mathcal{J} m\{\ddot{w}\} + B_1 J\{F_w\} \\
&\{\varphi(0)\} = c_\varphi\{M(0)\} \\
&\{w(0)\} = 0 \\
&\{\tau(0)\} = -\nu B_1 \mathcal{J} m r_0^2\{\theta + \ddot{\theta}\} + B_1 J\{M_\theta\} \\
&\{\theta(0)\} = c_\theta\{\tau(0)\} + \{1\}\theta_c
\end{aligned}
\tag{7a-f}
$$

where the diagonal matrices are constructed with the discrete values of the parameters defined at the N co-location points taken along the blade. In Eq.. 7a, the geometric stiffness term due to the blade local tension can be readily calculated:

$$
\grave{\nu}T = \mathrm{diag}\{\nu T\} = \mathrm{diag}\{\nu J_1 \grave{}m\{r\}\}
\tag{8}
$$

where another operator is introduced:

$$
J_1 = (B_1 - I)J
\tag{9}
$$

Substitution of the constants of integration into the algebraic equations yields:

$$
\begin{aligned}
&\{M\} = -J_1(\{H\} + \grave{\nu}T\{\varphi\}) \\
&\{H\} = -J_1(\grave{\nu}m\{\ddot{w}\} - \{F_w\}) \\
&(I + \nu c_\varphi B_1 \mathcal{J}T)\{\varphi\} = \mathcal{J}D_{11}\{M\} - \\
&\qquad c_\varphi B_1 J\{H\} + \mathcal{J}D_{13}\{\tau\} \\
&\{w\} = -J\{\varphi\} \\
&\{\tau\} = -J_1(\grave{\nu}m r_0^2\{\theta + \ddot{\theta}\} - \{M_\theta\}) \\
&(I + \nu c_\theta B_1 \mathcal{J} m r_0^2)\{\theta\} = \mathcal{J}D_{31}\{M\} - \\
&\qquad c_\theta B_1 J(\grave{\nu}m r_0^2\{\ddot{\theta}\} - \{M_\theta\}) + \\
&\qquad \mathcal{J}D_{33}\{\tau\} + \{1\}\theta_c
\end{aligned}
\tag{10a-f}
$$

From the six dependent variables presented in Eqs. 10a-f, only two (associated with the primary degrees of freedom) are sufficient to close the formulation of the problem. Here, the $N{\times}1$ vectors defining the flatwise bending slope and torsion angle distribution along the blade are chosen as the dependent variables. After straight-forward algebra, the remaining states are solved in terms of the chosen ones, yielding:

$$
\begin{aligned}
&\nu DM\ddot{x} + (I + \nu DZ)x + DJ_2 q + \\
&k(\psi)\Delta D(\nu M\ddot{x} + \nu Zx + J_2 q) = E\theta_c
\end{aligned}
\tag{11}
$$

where

$$
x^T = \left[\{\varphi\}^T \quad \{\theta\}^T\right]
\tag{12}
$$

$$
q^T = \left[\{F_w\}^T \quad \{M_\theta\}^T\right]
\tag{13}
$$

$$
E^T = \left[\{0\}^T \quad \{1\}^T\right]
\tag{14}
$$

$$D = \begin{bmatrix} JD_{11}J_1 + c_{\varphi 0}B_1 J & JD_{13}J_1 \\ JD_{31}J_1 & JD_{33}J_1 + c_{\theta 0}B_1 J \end{bmatrix} \tag{15}$$

$$M = \begin{bmatrix} J_1^{`}mJ & 0 \\ 0 & {`}mr_\theta^2 \end{bmatrix} \tag{16}$$

$$Z = \begin{bmatrix} {`}T & 0 \\ 0 & {`}mr_\theta^2 \end{bmatrix} \tag{17}$$

$$\Delta D = \begin{bmatrix} \Delta c_\varphi B_1 J & 0 \\ 0 & \Delta c_\theta B_1 J \end{bmatrix} \tag{18}$$

$$J_2 = \begin{bmatrix} J_1 & 0 \\ 0 & -I \end{bmatrix} \tag{19}$$

Blade Aerodynamics

The unsteady aerodynamic loads will be divided into two sets; the first set represents the airloads dependent on the airfoil motion (flatwise bending and torsion); the second set collects airloads that can be considered motion-independent due to effects such as BVI and dynamic stall. However, other airloads that are due to non-modeled effects could also be included in the second set, as the objective of the proposed IBC system is basically "disturbance rejection" under an unknown (broadband) spectrum of exogenous excitation. The presence of the non-modeled airloads in the control system can be accessed through robustness considerations, as it will be demonstrated in a later section.

Motion-dependent Airloads: The present model is based on the classical unsteady aerodynamic theory for two-dimensional thin airfoils in incompressible flow due to Theodorsen (Ref. 1). Corrections to account for reverse flow, compressibility and finite span effects can be accomplished by adapting the lift-curve slope to the azimuth angle. However, these corrections will not be made in the present work. An approximation is also introduced in the lift deficiency function due to the time-varying free stream velocity in forward flight. For relatively small variations in the free stream velocity (small advance ratio or radial stations not very close to the hub), the lift deficiency function is nearly the same as the original Theodorsen function $C(k)$ if the reduced frequency k is based on the local velocity. Hence, from the three effects that a time-varying free stream is recognized to produce in the motion-dependent unsteady aerodynamics of a two-dimensional airfoil, i.e. (1) the non-circulatory lift and moment due to the time rate of change of the local velocity, (2) the coupling by the wake of all harmonics of the circulation terms, and (3) the above referred influence on the lift deficiency function, only the last effect is not fully accounted for in the present model. In fact, its inclusion is also possible by augmenting the number of states, but one can also argue that the latter effect may be as well considered noise in the IBC synthesis.

The non-dimensional lift and pitch moment (about a point distant ab from the mid-chord position) per unit of blade span can be expressed by (Ref. 4):

$$\overline{L}^M = \overline{L}^C + \overline{L}^{NC} = C(k)\overline{L}^Q + \overline{L}^{NC} \tag{20}$$

$$\overline{M}_a^M = (\frac{1}{2}+a)\overline{L}^C + \pi\overline{b}(a\ddot{\overline{w}} - (\frac{1}{2}-a)\overline{U}\dot{\theta} + \dot{\overline{U}}\theta - \overline{b}(\frac{1}{8}+a^2)\ddot{\theta}) \tag{21}$$

where the quasisteady circulatory lift is

$$\overline{L}^Q = a_0(\overline{U}^2\theta + \overline{U}\dot{\overline{w}} + \overline{b}(\frac{1}{2}-a)\overline{U}\dot{\theta}) \tag{22}$$

and the non-circulatory lift is

$$L^{NC} = \pi\overline{b}(\dot{\overline{U}}\theta + \overline{U}\dot{\theta} + \ddot{\overline{w}} - a\overline{b}\ddot{\theta}) \tag{23}$$

In forward flight,

$$\overline{U} = \overline{r} + \mu\sin\psi \tag{24}$$

giving

$$\overline{U}^2 = (\overline{r}^2 + \mu^2/2) + 2\mu\overline{r}\sin\psi - \mu^2/2\cos 2\psi \tag{25}$$

and

$$\dot{\overline{U}} = \mu\cos\psi \tag{26}$$

According to the previously stated justification, the reduced frequency will be based on the local velocity,

$$k = \omega b/U = n\overline{b}/\overline{U} \cong n\overline{b}/\overline{r} \tag{27}$$

The latter approximation is satisfactory for the blade radial stations such that $0 \leq \mu R/r \leq 0.7$ (Ref. 5). For the plausible advance ratio of 0.14, this restriction is such that the airloads produced at stations less than $r/R=0.2$ are inaccurate, and, thus, will be neglected.

Motion-independent (Disturbance) Airloads: At this point, the only assumption that is made about the disturbance airloads is that they may be expressed as a Fourier series of the harmonics of the blade revolution:

$$\begin{Bmatrix} \overline{L} \\ \overline{M}_a \end{Bmatrix}^D = \sum_{n=-\infty}^{\infty} \begin{Bmatrix} \overline{L} \\ \overline{M}_a \end{Bmatrix}_n^D e^{in\psi} \tag{28}$$

where

$$\begin{Bmatrix}\bar{L}\\ \bar{M}_a\end{Bmatrix}_n^D = \frac{1}{2\pi}\int_0^{2\pi}\begin{Bmatrix}\bar{L}\\ \bar{M}_a\end{Bmatrix}^D(\psi; Mach, \mu, k, \ldots)e^{-in\psi}d\psi \qquad (29)$$

$$(n = 0, \pm 1, \ldots, \pm\infty)$$

Aeroelastic Model Assembly

In Eqs. 20 and 21, the lift is defined positive upwards, and the pitch moment positive nose-up. Following the sign convention adopted in the previous Blade Structural Dynamics section, $F_w = -L$ and $M_\theta = M_a$. Using the normalization shown in the Appendix,

$$\begin{aligned} q &= q^M + q^D \\ &= \frac{\nu\gamma}{2} I_b \begin{bmatrix}-I & 0\\ 0 & \bar{b}I\end{bmatrix}\left(\frac{1}{a_0}\begin{Bmatrix}\bar{L}\\ \bar{M}_a\end{Bmatrix}^M + \begin{Bmatrix}\bar{L}\\ \bar{M}_a\end{Bmatrix}^D\right) \end{aligned} \qquad (30)$$

Recalling Eq. 10-d, and using Eqs. 20-26, after the discretization procedure Eq. 30 yields a set of $2N$ differential equations where the values of φ and θ at the discretization points are the dependent variables (Eq. 31). These equations can be appended to Eq. 11 to obtain the complete set of aeroelastic governing equations. The non-dimensional time (azimuth angle) is the remaining independent variable.

$$\begin{aligned} J_2 q = \frac{\nu\gamma}{2} I_b (&Q_2^L \ddot{x} + Q_1^L \dot{x} + Q_0^L x + \\ &\sin\psi Q_{1s}^{NL}\dot{x} + \sin\psi Q_{0s}^{NL} x + \\ &\cos 2\psi Q_{02c}^{NL} x + \cos\psi Q_{01c}^{NL} x + J_2 q^D) \end{aligned} \qquad (31)$$

where

$$Q_2^L = \pi/a_0 \; J_2 Q_2^{NC} J_3 \qquad (32)$$

$$Q_1^L = J_2({}^{\backprime}C_k Q_1^C + \pi/a_0 \; Q_1^{NC}{}^{\backprime})^{\backprime}r J_3 \qquad (33)$$

$$Q_0^L = J_2 {}^{\backprime}C_k Q_0^C({}^{\backprime}r^2 + 1/2{}^{\backprime}\mu^2)J_3 \qquad (34)$$

$$Q_{1s}^{NL} = J_2({}^{\backprime}C_k Q_1^C + \pi/a_0 \; Q_1^{NC}{}^{\backprime})^{\backprime}\mu J_3 \qquad (35)$$

$$Q_{0s}^{NL} = 2J_2 {}^{\backprime}C_k Q_0^{C\backprime}{}^{\backprime}\mu{}^{\backprime}r J_3 \qquad (36)$$

$$Q_{02c}^{NL} = -J_2 {}^{\backprime}C_k Q_0^C \, 1/2{}^{\backprime}\mu^2 J_3 \qquad (37)$$

$$Q_{01c}^{NL} = \pi/a_0 \; J_2 Q_0^{NC\backprime}\mu J_3 \qquad (38)$$

$$Q_2^{NC} = \begin{bmatrix} {}^{\backprime}\bar{b} & {}^{\backprime}a\bar{b}^2 \\ -{}^{\backprime}a\bar{b}^2 & -{}^{\backprime}(1/8 + a^2)\bar{b}^3 \end{bmatrix} \qquad (39)$$

$$Q_1^{NC} = \begin{bmatrix} 0 & -{}^{\backprime}\bar{b} \\ 0 & -{}^{\backprime}(1/2 - a)\bar{b}^2 \end{bmatrix} \qquad (40)$$

$$Q_0^{NC} = \begin{bmatrix} 0 & -{}^{\backprime}\bar{b} \\ 0 & {}^{\backprime}\bar{b}^2 \end{bmatrix} \qquad (41)$$

$$Q_1^C = \begin{bmatrix} I & -{}^{\backprime}(1/2 - a)\bar{b} \\ -{}^{\backprime}(1/2 + a)\bar{b} & {}^{\backprime}(1/4 - a^2)\bar{b}^2 \end{bmatrix} \qquad (42)$$

$$Q_0^C = \begin{bmatrix} 0 & -I \\ 0 & {}^{\backprime}(1/2 + a)\bar{b} \end{bmatrix} \qquad (43)$$

and

$${}^{\backprime}C_k = \begin{bmatrix} {}^{\backprime}C(n\bar{b}/\bar{r}) & 0 \\ 0 & {}^{\backprime}C(n\bar{b}/\bar{r}) \end{bmatrix} \qquad (44)$$

$${}^{\backprime}r = \begin{bmatrix} {}^{\backprime}\bar{r} & 0 \\ 0 & {}^{\backprime}\bar{r} \end{bmatrix} \qquad (45)$$

$${}^{\backprime}\mu = \begin{bmatrix} {}^{\backprime}\mu & 0 \\ 0 & {}^{\backprime}\mu \end{bmatrix} \qquad (46)$$

$$J_3 = \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix} \qquad (47)$$

In the former equations, it is worthwhile to note that when the advance ratio equals to zero (hover condition), the "non-linear" terms (denoted by the superscript *NL*) reduce to zero, resulting in a set of linear, constant coefficient differential equations. The Laplace transform in time can be taken, and the control problem may be cast in the frequency domain, yielding the transfer function:

$$x(s) = (I + \Phi H)^{-1}\Phi\begin{bmatrix} E & J_2 \end{bmatrix}\begin{Bmatrix} \theta_c(s) \\ q^D(s) \end{Bmatrix} \qquad (48)$$

where Φ is the forward transmission or resolvent of the system, involving the open-loop transfer function (can always be inverted):

$$\Phi(s) = (I + DG_0(s))^{-1} \qquad (49)$$

and H is the feedback transmission transfer function, involving the convolution between the signal generated by the actuation device and G_0:

$$H(s) = k(s) * \Delta DG_0(s) \qquad (50)$$

where

$$G_0(s) = \nu M s^2 + \nu Z + Q^M(s) \qquad (51)$$

and

$$Q^M(s)x(s) = \frac{V\gamma}{2} I_b \mathcal{L}\left(Q_2^L \ddot{x} + Q_1^L \dot{x} + Q_0^L x\right) \tag{52}$$

involves only Laplace transformation of the "linear" aerodynamic matrices in Eq. 31. Hence, the feedback nature of the proposed IBC system now becomes evident as depicted in Fig. 1. Two inputs to the full-state feedback system are considered: (1) the pilot command, and (2) the exogenous perturbation due to the disturbance airloads. The design of the control system resumes in finding a control law $k(s)$ which minimizes the state vector $x(s)$ under the perturbation $q^D(s)$. If k is constant, a classical linear quadratic regulator problem is recovered. This solution would yield no active control, only the aeroelastic optimization of the flexibility of the flexbeam to reject the disturbance airloads in hover. However, such a solution has very limited practical applications as it is well recognized that in helicopters the vibration is low in hover and becomes severe as the forward velocity increases. The conclusion is that $q^D(s)$ becomes significant for the higher advance ratios, where the "non-linear" aerodynamic effects cannot be neglected. The IBC system should be designed to operate under the latter conditions, and, thus, in the next section a solution not restricted to hover will be sought.

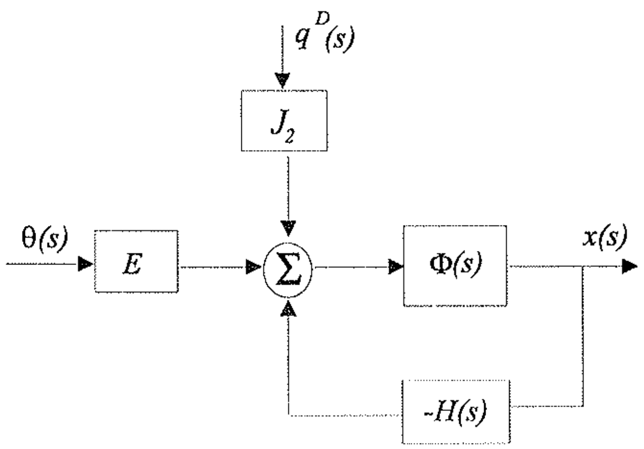

Fig. 1: Full-state feedback IBC system for the hover condition.

IBC for the Forward-flight Condition

The state vector can be expanded into complex Fourier series of the harmonics of the blade revolutions:

$$x = \sum_{n=-\infty}^{\infty} x_n e^{in\psi} \tag{53}$$

where the complex coefficients, given by

$$x_n = 1/2\pi \int_0^{2\pi} x(\psi) e^{-in\psi} d\psi \qquad (n = 0, \pm 1, \ldots, \pm\infty) \tag{54}$$

are related to the coefficients of the usual sine (sub 's') and cosine (sub 'c') Fourier expansions:

$$x_n = 1/2\,(x_{nc} - i x_{ns}) \qquad x_{-n} = x_n^* \tag{55a-b}$$

The Fourier series expansion (Eq. 53) is substituted into Eqs. 11 and 31, noting that

$$\dot{x} = i \sum_{n=-\infty}^{\infty} n x_n e^{in\psi} ; \ddot{x} = -\sum_{n=-\infty}^{\infty} n^2 x_n e^{in\psi} \tag{56a-b}$$

and that the non-linear terms yield Eqs. 57a-c.

$$\begin{aligned} x \cos m\psi &= \frac{1}{2} \sum_{n=-\infty}^{\infty} x_{n-m} e^{in\psi} + \frac{1}{2} \sum_{n=-\infty}^{\infty} x_{n+m} e^{in\psi} \\ x \sin\psi &= \frac{1}{2i} \sum_{n=-\infty}^{\infty} x_{n-1} e^{in\psi} - \frac{1}{2i} \sum_{n=-\infty}^{\infty} x_{n+1} e^{in\psi} \\ \dot{x} \sin\psi &= \frac{1}{2i} \sum_{n=-\infty}^{\infty} (n-1) x_{n-1} e^{in\psi} - \frac{1}{2i} \sum_{n=-\infty}^{\infty} (n+1) x_{n+1} e^{in\psi} \end{aligned} \tag{57a-c}$$

For the control signal:

$$k = \sum_{p=-\infty}^{\infty} k_p e^{ip\psi} \tag{58}$$

where a phase lag is incorporated in the complex constant:

$$k_p = \kappa_p e^{ip\Delta\psi} \tag{59}$$

The substitution of Eq. 58 into Eq. 11 yields products involving Fourier series e.g. Eq. 60. Other series products are treated similarly. These products replace, and in fact simplify, the convolution suggested in Eq. 50.

$$kx = \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} k_p x_{n-p} e^{in\psi} \tag{60}$$

The harmonic balance technique is used to eliminate the time dependence. A set of algebraic equations is obtained after some algebra:

$$x_n + D\sum_{j=-2}^{2} G_j(n)x_{n+j} +$$

$$\Delta D\sum_{p=-\infty}^{\infty} k_p \sum_{j=-2}^{2} G_j(n-p)x_{n+j-p} = \tag{61}$$

$$E\theta_c + DJ_2 q_n^D + \Delta DJ_2 \sum_{p=-\infty}^{\infty} k_p q_{n-p}^D$$

$$(n = 0, \pm 1, \ldots, \pm\infty)$$

where

$$G_0(n) = -vn^2 M + vZ +$$

$$\frac{v\gamma}{2} I_b(-n^2 Q_2^L + inQ_1^L + Q_0^L)$$

$$G_1(n) = \frac{v\gamma}{4} I_b(-(n+1)Q_{1s}^{NL} + iQ_{0s}^{NL} + Q_{01c}^{NL})$$

$$G_{-1}(n) = \frac{v\gamma}{4} I_b((n-1)Q_{1s}^{NL} - iQ_{0s}^{NL} + Q_{01c}^{NL}) \tag{62a-e}$$

$$G_2(n) = G_{-2}(n) = \frac{v\gamma}{4} Q_{02c}^{NL}$$

A clear way of achieving control authority at the critical frequency $n=n_b$ is to set $j=p$ in Eq. 61. In this case, the useful harmonics of the control signal are limited to the first 2 ($-2\leq p\leq 2$):

$$\kappa_p = 1/2\pi \int_0^{2\pi} \kappa(\psi) e^{-ip\psi} d\psi \tag{63}$$

$$(n = 0, \pm 1, \pm 2)$$

where $\kappa(\psi)$ is the signal driving the actuator device. Moreover, at the critical frequency the collective and cyclic control inputs are not affected by the IBC input (as suggested in Eq. 61). Hence,

$$\left[I + DG_0(n) + \Delta D\sum_{j=-2}^{2} k_j G_j(n-j) \right] x_n +$$

$$D\sum_{j=-2}^{2} G_j(n)x_{n+j} = \tag{64}$$

$$DJ_2 q_n^D + \Delta DJ_2 \sum_{j=-2}^{2} k_j q_{n-j}^D$$

$$(n = 0, \pm 1, \ldots, \pm\infty)$$

The complete solution of the problem demands the infinite set of algebraic equations be solved. Of course, the set can be truncated at a finite number of harmonics, but because one is only interested in obtaining results for a determined harmonic ($n=n_b$), Eq. 64 needs to be solved for a reduced set of frequencies ($n_b-2\leq n\leq n_b+2$). It is worthwhile to point out that this procedure yields a set which is complete only for the critical frequency, and inaccurate results are obtained for the remaining harmonics. Hence, the *10N* algebraic equations involving the central, and the neighboring upper two and lower two harmonics of the state vector are cast into a matrix form:

$$Az = b \tag{65}$$

where

$$z^T = \begin{bmatrix} x_{n_b-2}^T & x_{n_b-1}^T & x_{n_b}^T & x_{n_b+1}^T & x_{n_b+2}^T \end{bmatrix} \tag{66}$$

$$b^T = \begin{bmatrix} b_{n_b-2}^T & b_{n_b-1}^T & b_{n_b}^T & b_{n_b+1}^T & b_{n_b+2}^T \end{bmatrix} \tag{67}$$

with

$$b_n = DJ_2 q_n^D + \Delta DJ_2 \sum_{j=-2}^{2} k_j q_{n-j}^D \tag{68}$$

In Eq. 65, A is a $10N\times 10N$ band matrix that can be always inverted:

$A=$

$$\begin{bmatrix} A_0(n_b-2) & A_1(n_b-2) & A_2(n_b-2) & 0 & 0 \\ A_{-1}(n_b-1) & A_0(n_b-1) & A_1(n_b-1) & A_2(n_b-1) & 0 \\ A_{-2}(n_b) & A_{-1}(n_b) & A_0(n_b) & A_1(n_b) & A_2(n_b) \\ 0 & A_{-2}(n_b+1) & A_{-1}(n_b+1) & A_0(n_b+1) & A_1(n_b+1) \\ 0 & 0 & A_{-2}(n_b+2) & A_{-1}(n_b+2) & A_0(n_b+2) \end{bmatrix} \tag{69}$$

where

$$A_0(n) = I + DG_0(n) + \Delta D\sum_{j=-2}^{2} k_j G_0(n-j)$$

$$A_j(n) = DG_j(n); j = \pm 2, \pm 1 \tag{70a-b}$$

The equations are still written in physical coordinates, i.e., no model reduction procedure was necessary to obtain Eq. 64. However, a sub-space representation of the latter equations such as a produced by the introduction of modal coordinates, drastically reduces the number of algebraic equations. Furthermore, a convenient "spatial weighting" of the state vector may in fact improve the IBC performance at determined blade spanwise locations.

The solution of the problem may proceed at this point by taking different control signals (defined by the assumed 5 complex constants), e.g. sinusoidal and square waves with corresponding phase lags, and comparing the IBC results with the "open-loop" situation. However, a few remarkable characteristics on the nature of Eqs. 64 and 65 are expected to provide a means of systematically obtain an optimum solution for the problem.

Observing Eq. 69 and 70, Eq. 65 may be rewritten as:

$$(A_o + \Delta A)z = b \tag{71}$$

where A_o is formed with the first two terms of Eq. 70-a, and Eq. 70-b; and ΔA is formed with the last term of Eq. 70-a. Therefore, the control signal may be factored as an independent term. Next, matrix A_o is factored out of Eq. 71, yielding:

$$A_o(I + A_o^{-1}\Delta A)z = b \tag{72}$$

Hence,

$$z = (I + \Phi\Delta A)^{-1}\Phi b \tag{73}$$

where Φ is the resolvent of the system:

$$\Phi = A_o^{-1} = (I + {}`DG)^{-1} \tag{74}$$

A feedback control system similar to Fig. 1, for which Eq. 73 is now the transfer function between the state-vector z and the disturbance b is recovered. There is a "forward path" given by Eq. 74, and a "feedback path" given by Eq. 75. However, as suggested in Fig. 2, a multiple-loop feedback system based on output states is obtained as opposed to the single loop, full-state feedback system shown in Fig. 1. For this,

$$\Delta A = \sum_{j=-2}^{2} H_j = \sum_{j=-2}^{2} B_j k C_j \tag{75}$$

where the unknown complex constants associated with the control signal are collected in k, and

$$B_j = {}`\Delta D \begin{bmatrix} G_{0_{-2}} \\ G_{0_{-1}} \\ G_{0_0} \\ G_{0_1} \\ G_{0_2} \end{bmatrix}; \tag{76}$$

$$G_{0_\ell} = \begin{cases} 0; \ell \neq j \\ \left[G_0(n_b+1-\ell) \ \cdots \ G_0(n_b+5-\ell)\right]; \ell = j \end{cases}$$

$$C_j = \begin{bmatrix} c_{-2} & \cdots & c_2 \end{bmatrix}; c_\ell = \begin{cases} 0; \ell \neq j \\ 1; \ell = j \end{cases} \tag{77}$$

Therefore, related to the "open-loop" solution,

$$A_o z_o = (I + {}`DG)z_o = b \tag{78}$$

i.e.

$$z_o = \Phi b \tag{79}$$

an important eigenvalue problem is introduced:

$$\lambda p = -{}`DGp = Fp \tag{80}$$

where p is an eigenvector of F associated with the eigenvalue λ. Due to its nature, F can be identified to the "dynamical matrix" referred by Meirovitch (Ref. 6). Here, besides the inertial, loads due to the geometric stiffness (centrifugal) and the motion-dependent aerodynamics are included in F. The eigenvectors p have the interesting property of decoupling the non-linear aeroelastic system into a set of 5 independent systems.

The above formulation of the problem in terms of a feedback control system has to be taken with caution as no signal flow is present in this case. However, the analogy exists and tools available from the optimum control theory can be used to obtain valid solutions for the control signal. The complex constants collected in k may be regarded as control gains that are sought to minimize the component of the state vector z associated with critical frequency n_b under a broadband and persistent noise environment due to the disturbance airloads. Since there is no time dependence, the control interval is infinite (steady state regulator) and the solution of the problem is given by the algebraic Riccati equation,

$$0 = PF + F^T P - PBR^{-1}B^T P + C^T SC \tag{81}$$

yielding:

$$K = R^{-1}B^T P \tag{82}$$

It is important to note that the above solution involves a more extensive optimization procedure because there are multiple feedback loops present in the non-linear (forward flight) case as opposed to the linear (hover) situation. Hence, k is determined by an "optimum choice" of the weighting matrices R and S in Eq. 81.

Conclusions

In the present investigation a mathematical model of a rotary wing with time-dependent boundary conditions was developed. The aeroelastic model incorporates the elastic flatwise bending and torsion degrees of freedom of a single blade and full unsteady aerodynamics. It is demonstrated that is possible to design a control system to attenuate helicopter rotor vibration based on active adaptation of the dynamic stiffness of the flexbeam. This control system is useful in the development of an actuator device designed to be attached to the flexbeam and adjust its impedance according to a prescribed control law. The control law is synthesized to reject a broadband spectrum of disturbance airloads due to non-linear aerodynamic effects such as BVI and dynamic stall. The problem has been cast into the standard optimum control theory as the control signal is expanded in terms of complex Fourier series of the harmonics of the blade passage. A general method to solve the aeroelastic equations for the complex coefficients of the series has been derived.

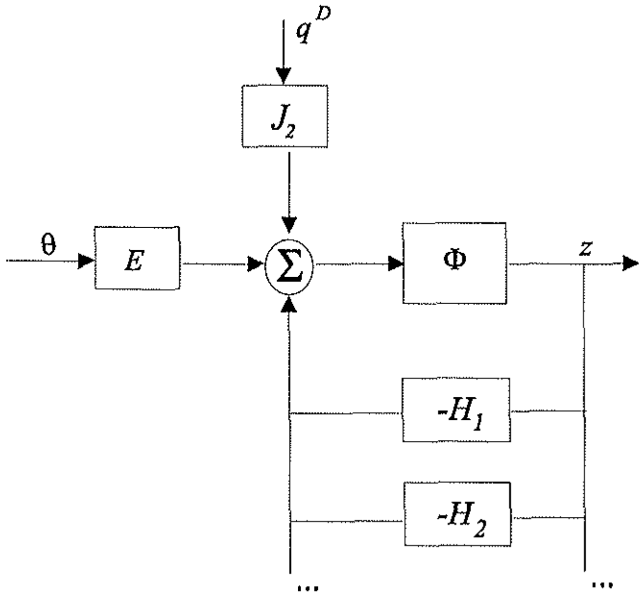

Fig. 2: Multiple-loop output "feedback" IBC system for the forward-flight condition.

Appendix: Definition of Non-dimensional Quantities

$$
\begin{gathered}
\bar{b} = b/R \\
\bar{c}_\theta, \bar{c}_\varphi = c_\theta, c_\varphi \times EI_{ref}/R^2 \\
\bar{D}_{ij} = D_{ij} R/EI_{ref}\,; i, j = 1,3 \\
\bar{F}_w = R^3 F_w/EI_{ref} \\
\bar{H} = R^2 H/EI_{ref} \\
\bar{I}_b = I_b/mR^3 \\
\bar{L} = L/\Omega^2 R^2 \rho b \\
\bar{M} = RM/EI_{ref} \\
\bar{M}_a = M_a/\Omega^2 R^2 \rho b^2 \\
\bar{M}_\theta = R^2 M_\theta/EI_{ref} \\
\bar{m} = m/m_{ref} \\
\bar{r} = r/R \\
\bar{r}_\theta = r_\theta/R \\
\bar{s} = s/\Omega \\
\bar{T} = T/\Omega^2 R^2 m_{ref} \\
\bar{U} = U/\Omega R \\
\bar{w} = w/R \\
\bar{\tau} = R\tau/EI_{ref}
\end{gathered}
$$

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