USE OF OPTIMISATION IN HELICOPTER VIBRATION CONTROL
BY STRUCTURAL MODIFICATION

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Abstract

The application of a mathematical optimisation process to heli-copter vibration control by structural modification is reported. Attention is focussed on the reduction of vibration in the crew area using stiffness parameters as design variables. Use is made of forced vibration response circles to identify the parameters most effective in controlling the response in the crew area, thereby reducing the number of available design variables to a tractable size. The problem of reducing vibration is then cast as a non-linear programming problem and a sequential unconstrained minimization technique incorporating an algorithm based on the methods of Davidon, Fletcher and Powell is used to determine the precise values of the parameters. The method is applied to a simple two-dimensional beam-element helicopter fuselage model, and the results discussed. Although the model is too simple for useful deductions of practical significance to be made in the strictly engineering sense, the exercise does demonstrate what can and cannot be done in controlling vibration by using an optimisation routine.

1.   Introduction

One of the many possibilities for vibration control of a heli-copter is to design the fuselage structure itself so that the vibration response in the more important areas such as crew and passenger spaces is minimised. Ideally this would involve optimising the separate ele-ments of the fuselage structure to achieve minimum response in the desired area. It is usual to perform structural analysis operations on an appropriate mathematical model, such as a finite element model of the structure comprised of many elements, but to treat each element as a variable for the purpose of optimisation would be beyond the scope of the average optimisation computer programme and could involve heavy computational costs. To avoid this, a subsidiary exercise is per-formed in which the best few elements that can be treated as variables for reducing vibration are chosen. When once the sensitive elements are identified, a formal optimisation can be used to fix the precise values of the parameters characterising these elements.

In the last decade there has been a considerable advance in the area of structural optimisation under dynamic constraints, as the recent comprehensive reviews by Rao [1] and Venkayya [2] will testify. A portion of the research work that has been done is concerned with optimisation of a structure under forced vibration, and this is rele-vant in the present case. Sciarra [3] provides the apparently sole example of a specific application to a helicopter fuselage, although he used an optimality criterion rather than a formal mathematical optimisa-tion procedure. Parameter selection before optimisation is not covered to any great extent in the literature; quite often, when the number of parameters needs to be limited the choice is simply made initially on the basis of dynamical experience.

2. Parameter selection

The identification of the best parameters for selection in the optimisation process is based on the way the response at a point behaves when a parameter is varied. If a structure is excited by a sinusoidal force, while either the mass at a point or the stiffness between two points (as represented by a spring) is continuously varied, then the response in the complex plane at some other point is seen to trace out a circular locus. The diameters of the response circles thus produced for each parameter can be taken as a simple yardstick for deciding which parameters are most effective in controlling the response at the point under consideration; for example, a parameter producing a large response circle diameter clearly has a relatively greater effect than one producing a small response circle diameter. From a listing of circle diameters in decreasing size order for different parameters, the top few parameters are those selected for the subsequent optimisation process.

The circular response property was originally used by Vincent [4], and adopted and utilised in the way just described by Done and Hughes [5,6] and Done et al. [7] on two simple practical cases.

3. Response optimisation

The response to harmonic excitation of a general structure modelled by finite elements and having n degrees of freedom is given by

$$\underline{R} = \underline{G}\,\underline{F} \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad (1)$$

$\underline{G}$ is the (nxn) complex receptance matrix between all the points concerned and is given by

$$\underline{G} = \left[\underline{K} - \omega^2 \underline{M} + i\omega\underline{C}\right]^{-1} \quad .. \qquad .. \qquad .. \qquad .. \qquad (2)$$

where $\underline{K}$, $\underline{M}$ and $\underline{C}$ are stiffness, mass and damping matrices respectively and ω is the circular frequency of the exciting force. $\underline{F}$ and $\underline{R}$ are the column vectors (order (nx1)) denoting force and response respectively.

If the structure is modified, say, by inserting linear springs of stiffness k_i (i=1,m) between the nodes that have compatible degrees of freedom, the stiffness matrix $\underline{K}$ becomes a function of these variable stiffness parameters k_i. Out of all possible k_i the best few are selected for optimisation by using the parameter selection programme. The particular responses $\underline{R}_p$ to be minimised are given by

$$\underline{R}_p = \underline{S}\,\underline{G}\,\underline{F} \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \qquad (3)$$

where $\underline{S}$ is a sorting matrix that picks out the associated points. Now in terms of mathematical programming the problem is to minimise an objective function which is a function of the elements of $\underline{R}_p$, using the best parameters as design variables. In evaluating $\underline{G}$ the damping matrix $\underline{C}$ is assumed to be zero, since for situations as in the present case where the forcing frequency is well removed from resonant frequencies its contribution is negligible compared with those from mass and stiffness. A major advantage is that complex matrices are thereby avoided. A further point is that the helicopter rotor speed is substantially constant, and therefore the excitation frequency is assumed constant.

The optimisation is done by using a readily available variable matrix algorithm supplied by R.A.E., Farnborough. The algorithm requires gradient information and is based on the methods of Davidon, Fletcher and Powell [8]; it is one of the most powerful gradient-based algorithms and has a quadratic convergence property. The gradients required can be obtained from differentiating eqn. (3):

$$\frac{\partial R_p}{\partial k_i} = - \underline{S} \, \underline{G} \frac{\partial \underline{K}}{\partial k_i} \, \underline{G} \, \underline{F} \qquad .. \qquad .. \qquad .. \qquad .. \qquad (4)$$

It can be seen from this that the receptance matrix $\underline{G}$ needs to be evaluated for each recomputation of the gradient vector, thus necessitating the inversion of a (nxn) matrix (eqn. (2)) each time. The number of degrees of freedom n may be large and there is the possibility of using a lot of computer time. However, an alternative expression to eqn. (2) can be used to calculate the receptance matrix which is derived in Ref. 5 and reproduced in the Appendix:

$$\underline{G} = \underline{G}_0 + \underline{A} \, \underline{V} \left[\underline{I} + \underline{D} \, \underline{V} \right]^{-1} \underline{B} \qquad .. \qquad .. \qquad .. \qquad (5)$$

where $\underline{G}_0$ is the receptance matrix of the unmodified structure i.e. with $k_i = 0$.

The elements of the matrices $\underline{A}$, $\underline{B}$ and $\underline{D}$ are formulated from the elements of the basic receptance matrix $\underline{G}_0$. $\underline{V}$ is a diagonal matrix of the variable parameters k_i and $\underline{I}$ is an identity matrix. Here the matrices $\underline{D}$ and $\underline{V}$ are of the order (mxm), where m is the number of variable parameters. Any sensible and practical modification would involve relatively few variable parameters compared with the number of degrees of freedom, so m is much less than n and hence the size of the matrix to be inverted is only (mxm). In the present work the gradients are computed directly from the expression resulting from differentiating eqn. (5).

4. Experience on a simple gear box/engine/fuselage model

As a test case for the application of the optimisation method to dynamic problems, a simple system representing a helicopter in which the gear box, engine and fuselage are replaced by rigid bodies as shown in Fig. 1 is considered. The gear box and engine mountings are treated as variable stiffness parameters. The model has eight degrees of freedom and six variable spring stiffnesses. The gear box is subjected to an oscillatory forcing moment and the aim is to find what variable stiffness values k_i are necessary to make the vibration at the pilot's seat, P, zero or as low as possible. The stiffnesses of the springs are given by $(k_b)_i + k_i$ (i=1,6), where (k_b) are the basic stiffness values. The stiffnesses k_1 and k_2 effectively act as one stiffness.

Realistic bounds on the stiffness values k_i are imposed and a basic objective function to be minimised is taken as $\phi = R_p^2$, where R_p is the vertical response at P.

Now with a simple objective function of this type it must be realised that a unique optimum does not exist. For example with only two variables k_3 and k_4 (the gradients with respect to k_1, k_2, k_5 and k_6 were very small and thus these variables are not important in the optimisation) zero vibration response is obtained when $k_4 = f(k_3)$ in the case of no damping; thus k_3 and k_4 are related for zero response.
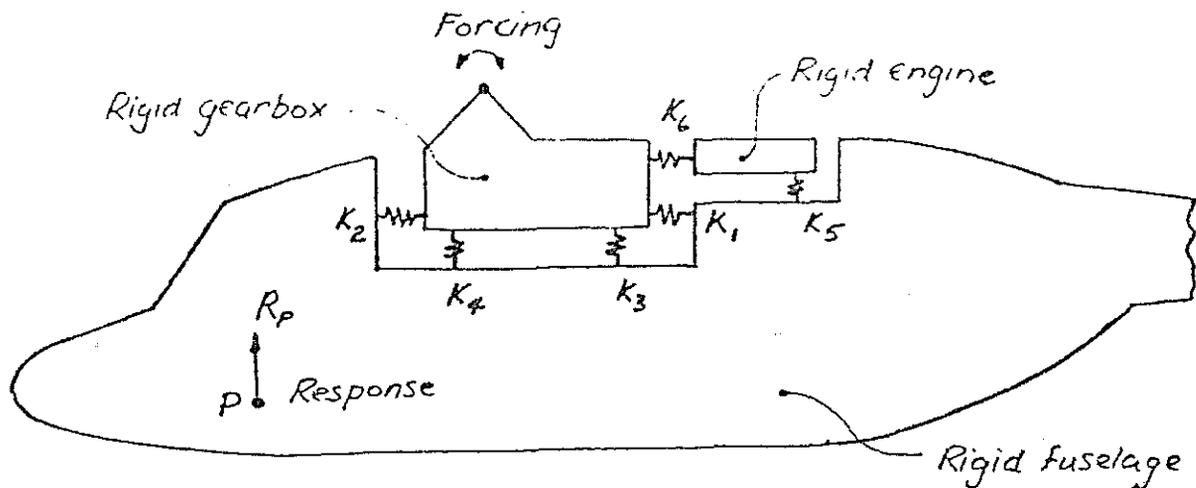
Stiffness parameter $k_i = (k_i - k_{ie})/(k_{iu} - k_{ie})$ where k_{iu}, k_{ie} are upper, lower bounds on i-th stiffness (i.e. $0 \leqslant k_i \leqslant 1$).

Fig. 1.   Gear Box/Engine/Fuselage Model.

Even in the presence of damping it can be shown by using response circles that there exist two optimum solutions for this case.   For small damping the shape of the objective function ϕ is such that the response is second order small along $k_4 = f(k_3)$ which makes it hard to find the minima numerically and the situation can be worse for the case of many variables.   Thus there is a need to choose the best solution out of all possible solutions.   The simplest means in the present case is to assume that an increase in stiffness normally implies an increase in weight and to seek the solution which provides the least weight penalty, i.e. the least total added stiffness.

Thus the problem can be recast as a constrained problem, viz.

Minimise Σk_i, subject to the constraint $\phi = \underline{R}_p^T \underline{Q} \, \underline{R}_p = 0$

where $\underline{Q}$ is a matrix which allows a quadratic function of response magnitudes at various points to be formed and Σk_i is the sum of added stiffnesses.   This optimisation algorithm is used sequentially in the present problem on a modified objective function defined by

$$\psi = \Sigma_i k_i + \beta \phi^2 \qquad .. \qquad .. \qquad ... \qquad .. \qquad (6)$$

where β is a penalty parameter.   Starting with a small value of β and an initial design vector of variables k_i, ψ is minimised to arrive at a new design and taking this as the initial design and with an increased value of β the process is repeated sequentially until convergence is achieved.   If a zero response is obtainable, the least added weight solution is obtained.   If however there is no zero response solution then the sequentially increasing values of β ensure the minimum possible response within the stiffness (or weight) bounds allowed.

The nature of the result obtained for the two variable case is shown in Fig. 2.   This represents a case where zero response can be obtained within the range of variation of k_3 and k_4 considered.   The optimisation process has been started arbitrarily at $k_3 = k_4 = 0.1$ and it is clear from study of the Figure that the minimum value of ψ occurs roughly at the intersection of $k_3 = 0$ and $R_p = f(k_3,k_4) = 0$.   For an initial low value of β the solution moves to point 2; thereafter, β is increased sequentially (steepening the slope of the penalty function surface) and the solution moves to the final one, satisfactory convergence being achieved after 5 steps.
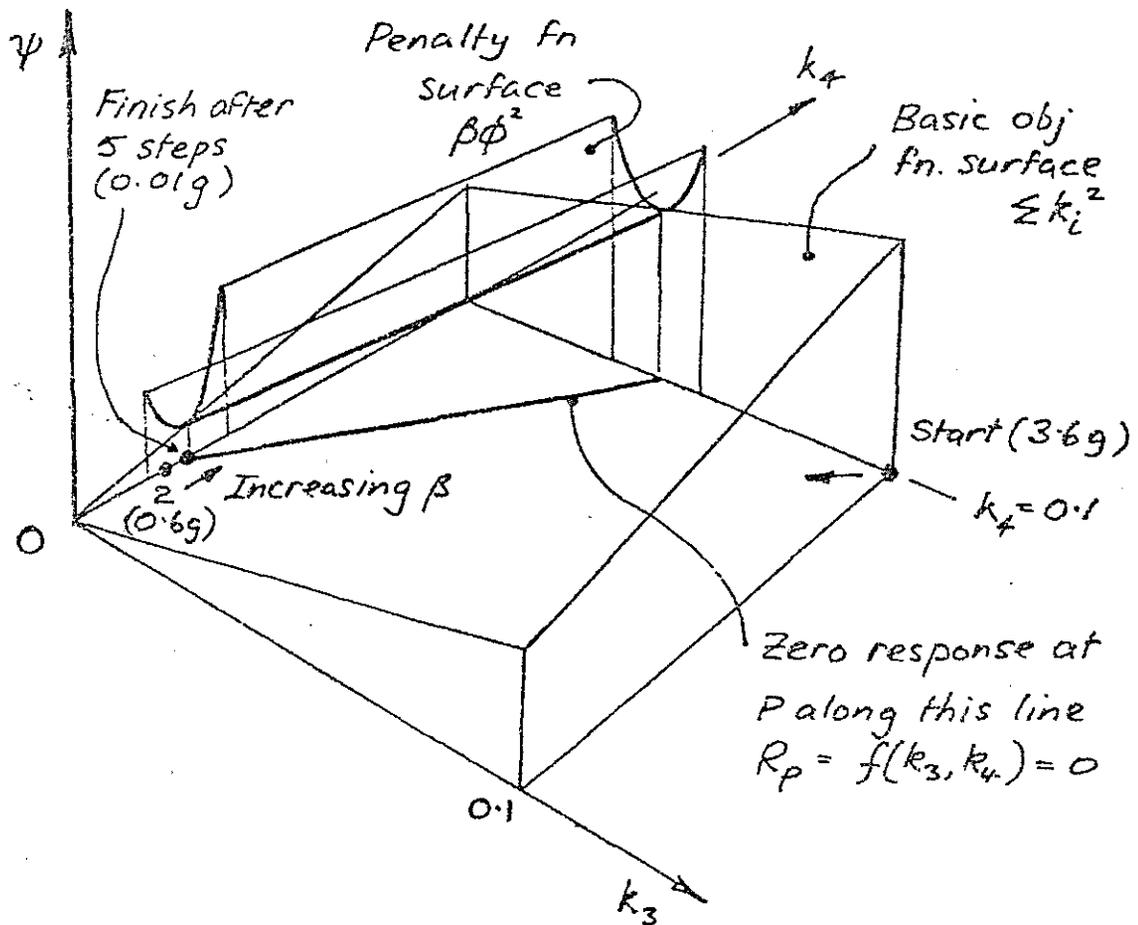
Fig. 2. Visualisation of Optimisation.

The model considered here is very simple and to assess how this method works for a more complex model, a two-dimensional beam element model ("stick" model) of the Westland Lynx helicopter is studied. The details of the model and the results of application are presented in the next section.

5. Application of Lynx "stick" model

The aforementioned optimisation technique using the modified objective function is now applied to the problem of minimising the vibrational response in the region of the pilot's seat of the Westland Lynx helicopter. The model used is shown in Fig. 3. This "stick" model has 25 tapered beam elements and 60 degrees of freedom with two translational and one rotational at each node. The excitation on the structure is an oscillatory couple of frequency 21.7 Hz applied to the rotor head (node 8) as shown in the Figure. Only stiffness changes corresponding to the adjacent nodes are considered, and the vertical response at the pilot's seat (node 18) denoted by R_{53} is minimised. The suffix "53" refers to the degree of freedom number - its relation to structure node number is found by reference to Fig. 3.

The first twenty parameters obtained from a response circle diameter listing are given in Table 1 and the parameters are indicated by the respective degree of freedom numbers. A modal damping factor of 2% critical is assumed for getting the parameter listing. The parameters are also indicated in order of importance in Fig. 4 and are those used in the optimisation.
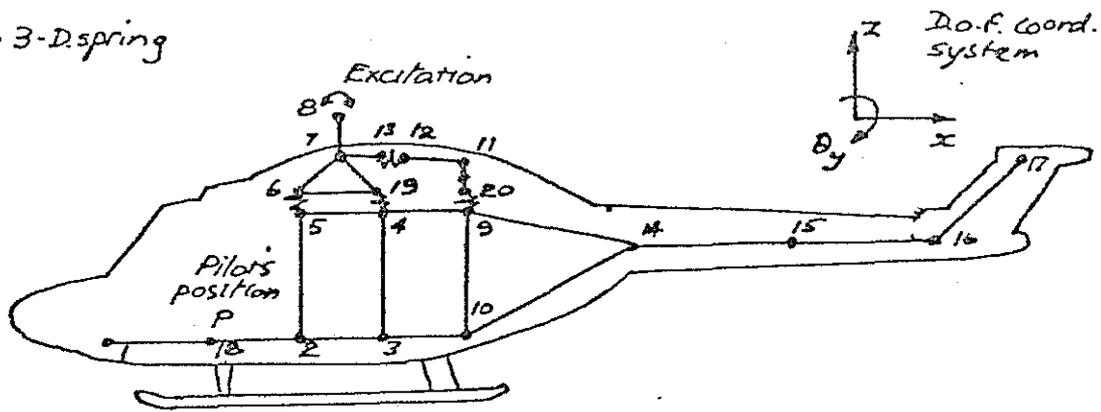
Degree of freedom co-ordinates for node i are (3i-2,3i-1,3i).
Thus, e.g. co-ordinates for node 18 are (52,53,54) and for
node 2 (4,5,6).

Fig. 3.   Lynx "Stick" Model - Node & d.o.f. Numbering.

TABLE 1

Parameter listing for the response R_{53} based on circle diameters

| Parameter identifying no.	Parameter co-ordinates		Direction	Corresponding structural nodes	Unmodified stiffness value k_i (N/m)
1	25	28	X	3-10	0.398620×10^7
2	7	10	X	3- 4	0.398620×10^7
3	4	13	Z	2- 5	0.398620×10^7
4	14	17	Z	5- 6	0.525380×10^8
5	5	14	Z	2- 5	0.250400×10^8
6	28	40	X	10-14	0.703606×10^7
7	12	57	θ_y	4-19	0.175127×10^4
8	36	39	θ_y	12-13	0.175127×10^4
9	6	15	θ_y	2- 5	0.165430×10^9
10	9	30	θ_y	3-10	0.277979×10^{11}
11	9	12	θ_y	3- 4	0.165430×10^9
12	20	23	Z	7- 8	0.965550×10^{12}
13	19	22	X	7- 8	0.406040×10^{11}
14	16	19	X	6- 7	0.384125×10^{12}
15	19	55	X	7-19	0.384125×10^{12}
16	17	20	Z	6- 7	0.434304×10^{12}
17	17	56	Z	6-19	0.778341×10^{10}
18	20	56	Z	7-19	0.434304×10^{12}
19	11	56	Z	4-19	0.210152×10^{10}
20	2	53	Z	1-18	0.216385×10^8
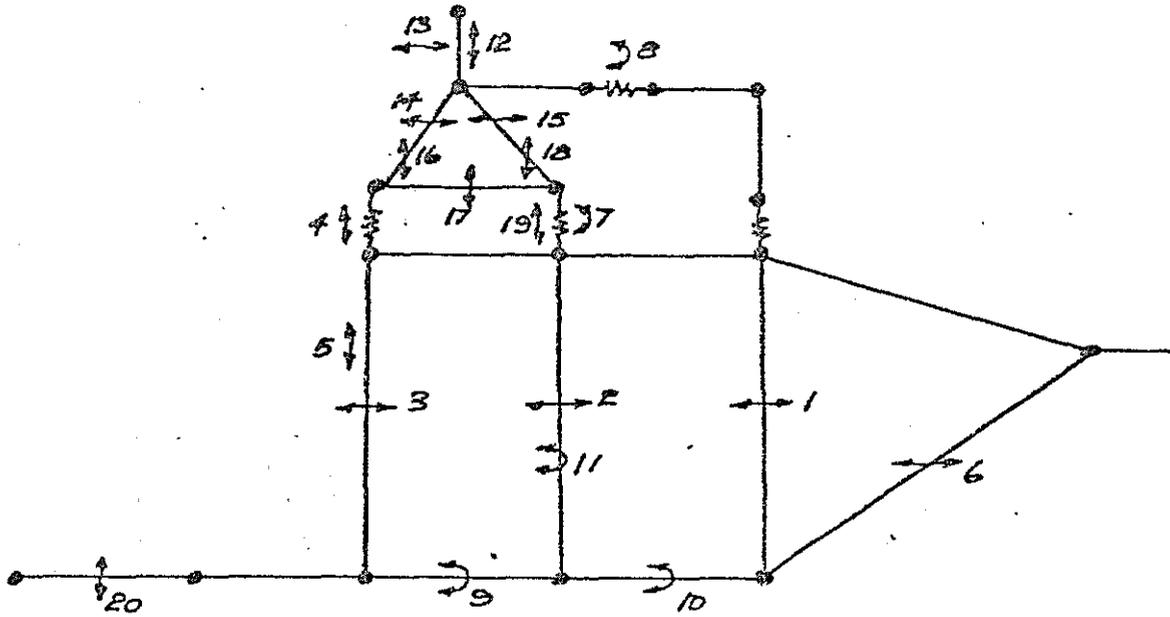
Fig. 4.   Stiffness Parameters (Identification Numbers).

The optimisation algorithm requires the lower and upper limits on the variables. Two types of bound are chosen, firstly, all the stiffness parameters are each allowed a uniform variation of 0–1.75 x 10^9 N/m (0–10^7 lbf/in) and, secondly, a range of variation of 20% of the basic stiffness values is also taken. For each set of bounds different response cases are considered although the listing of best parameters is not strictly applicable to response cases other than R_{53}.

The basic objective functions ϕ_i corresponding to the different response cases (see Fig. 5) considered are

$$\phi_1 = R_{53}^2 \qquad , \qquad \phi_2 = R_{53}^2 + R_{52}^2$$

$$\phi_3 = R_{53}^2 + R_5^2 \qquad , \qquad \phi_4 = R_{53}^2 + R_{52}^2 + R_5^2 + R_4^2$$

where the suffices on the R's refer to degree of freedom numbers. Minimising a multiple response involving more than one node as in the case of ϕ_3 and ϕ_4 can be thought of as reducing the vibratory response over a region of the structure. These response cases are illustrated in Fig. 5.

The computer results for the different cases of response and stiffness parameter constraints follow in Tables 2 to 9 inclusive. For each case the response function ϕ_i used is stated, and also type of upper bound $\Delta k_{i_{max}}$ on the additional stiffness variations Δk_i. This is either simply 1.75 x 10^9 N/m throughout or $0.2k_i$, where k_i is the existing stiffness parameter value of the unmodified structure. A typical value is obtained from the stiffness matrix; for example, a stiffness connecting points p and q on the structure is given by the negative of the pq-th element in the matrix. The lower bounds on Δk_i are all zero.
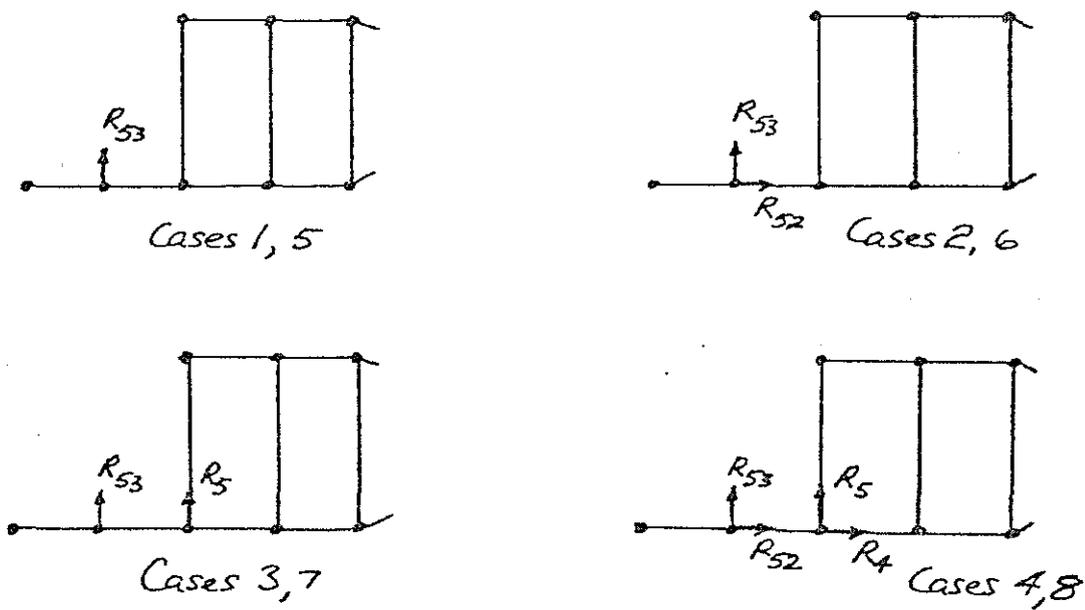
Fig. 5.   Response Cases Considered.

The magnitude of the response or composite response $(\phi_i)_o^{\frac{1}{2}}$ is given as an acceleration for a nominal excitation pitching moment at the rotor head of 5650 N m.

The parameters listed in Table 1 are used for a sequence of runs in which the first two, four, six, ten etc. parameters are used in the optimisation, the optimum values being shown.   The response is given as a ratio of that for the unmodified structure, and the sum of the additional stiffnesses indicated.   Also given are the C.P.U. times. The parameter identifying number appearing in the first column of Tables 2-9 is that which appears in the first column of Table 1 and also in Fig. 4.

TABLE 2

Case 1   Response function $\phi_1 = R_{53}^2$

Stiffness bounds $0 \leq \Delta k_i \leq 1.75 \times 10^9$ N/m

Response magnitude $(\phi_1)_o^{\frac{1}{2}} \rightarrow 0.132g$

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.						Resp. ratio $\left\{\dfrac{\phi_1}{(\phi_1)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	2	4	6	10	15	20		
1	0	0	0	0	0	0		
2	0	0	0	0	0	0	1.0	0.025
3		0	0	0	0	0		
4		0.1936	0.0059	0.0059	0.0059	0	0	3.5
5			0.0161	0.0161	0.0161	0		
6			0	0	0	0	0	3.2
7				0	0	0		
8				0	0	0		
9				0	0	0		
10				0	0	0	0	4.0
11					0	0		
12					0	0		
13					0	0		
14					0	0		
15					0	0	0	15
16						0		
17						0		
18						0		
19						0		
20						0.0037	0	45
$\Sigma(\Delta k_i \times 10^{-7})$	0	0.1936	0.022	0.022	0.022	0.0037		

TABLE 3

Case 2   Response function $\phi_2 = R_{53}^2 + R_{52}^2$

Stiffness bounds $0 \leq \Delta k_i \leq 1.75 \times 10^9$ N/m

Response magnitude $(\phi_2)_o^{\frac{1}{2}} \rightarrow 0.403g$

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.					Resp. ratio $\left\{\dfrac{\phi_2}{(\phi_2)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	2	4	6	10	15		
1	0	1.0	0	0	0		
2	0	1.0	0	0	0	1.0	0.03
3		1.0	0	0	0		
4		1.0	1.0	1.0	0	0.322	0.12
5			0	1.0	0		
6			0	0	0	0.870	1.0
7				1.0	0		
8				0	0		
9				1.0	1.0		
10				1.0	0	0.939	1.7
11					0		
12					1.0		
13					1.0		
14					1.0		
15					1.0	0.011	12
$\Sigma(\Delta k_i \times 10^{-7})$	0	4.0	1.0	5.0	5.0		

TABLE 4

Case 3   Response function $\phi_3 = R_{53}^2 + R_5^2$

Stiffness bounds $0 \leqslant \Delta k_i \leqslant 1.75 \times 10^9$ N/m

Response magnitude $(\phi_3)_o^{\frac{1}{2}} \Rightarrow 0.190g$

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.					Resp. ratio $\left\{\dfrac{\phi_3}{(\phi_3)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	2	4	6	10	15		
1	0	1.0	0	0	0		
2	0	1.0	0	0	0	1.0	0.025
3		1.0	0	0	0		
4		0.0173	1.0	0.3570	0.3757	0.818	0.5
5			1.0	0.2374	0.2545		
6			0.0037	0.0035	0.0036	0	1.7
7				0	0		
8				0.7790	0.7467		
9				0	0		
10				0	0	0	5
11					0		
12					0		
13					0		
14					0		
15					0	0	57
$\Sigma(\Delta k_i \times 10^{-7})$	0	3.0173	2.0037	1.3769	1.3805		

TABLE 5

Case 4   Response function $\phi_4 = R_{53}^2 + R_{52}^2 + R_5^2 + R_4^2$

Stiffness bounds $0 \leqslant \Delta k_i \leqslant 1.75 \times 10^9$ N/m

Response magnitude $(\phi_4)_o^{\frac{1}{2}} \Rightarrow 0.571g$

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.				Resp. ratio $\left\{\dfrac{\phi_4}{(\phi_4)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	4	6	10	15		
1	0	0	0	0		
2	0	0	0	0		
3	0	0	0	0		
4	1.0	1.0	1.0	0	0.868	0.7
5		0.0004	0.0008	0		
6		0	0	0	0.867	1.1
7			0	0		
8			0	0		
9			0	1.0		
10			1.0	0	0.863	4.5
11				0		
12				1.0		
13				1.0		
14				1.0		
15				1.0	0.011	14
$\Sigma(\Delta k_i \times 10^{-7})$	1.0	1.0004	1.0008	5.0		

TABLE 6

Case 5   Response function $\phi_1 = R^2_{53}$

Stiffness bounds $0 \leqslant \Delta k_i \leqslant 0.2k_i$

Response magnitude $(\phi_1)^{\frac{1}{2}}_o \Rightarrow 0.132g$

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.					Resp. ratio $\left\{\dfrac{\phi_1}{(\phi_1)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	2	4	6	10	15		
1	0	0	0	0	0		
2	0	0	0	0	0	1.0	0.027
3		0	0	0	0		
4		1	1	1	1	0.781	0.24
5			1	1	1		
6			0	0	0	0.538	0.34
7				1	1		
8				1	0		
9				1	1		
10				0	0	0.537	1.24
11					1		
12					1		
13					1		
14					1		
15					1	0.002	33
$\Sigma(\Delta k_i \times 10^{-7})$	0	0.0060	0.0089	0.0278	202.7		

TABLE 7

Case 6   Response function $\phi_2 = R^2_{53} + R^2_{52}$

Stiffness bounds $0 \leqslant \Delta k_i \leqslant 0.2k_i$

Response magnitude $(\phi_2)^{\frac{1}{2}}_o \Rightarrow 0.403$ g

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.					Resp. ratio $\left\{\dfrac{\phi_2}{(\phi_2)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	2	4	6	10	15		
1	0	1	0	0	0		
2	0	1	0	0	0	1.0	0.033
3		1	0	0	0		
4		0	1	1	1	0.991	0.067
5			1	1	1		
6			0	0	0	0.944	0.15
7				1	0		
8				0	1		
9				1	1		
10				1	0	0.938	0.55
11					1		
12					1		
13					1		
14					1		
15					1	0.001	27
$\Sigma(\Delta k_i \times 10^{-7})$	0	0.0137	0.0089	3.202	202.7		

TABLE 8

<u>Case 7</u>   Response function $\phi_3 = R_{53}^2 + R_5^2$

Stiffness bounds $0 \leqslant \Delta k_i \leqslant 0.2 k_i$

Response magnitude $(\phi_3)_o^{\frac{1}{2}} \Rightarrow 0.190g$

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.					Resp. ratio $\left\{\dfrac{\phi_3}{(\phi_3)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	2	4	6	10	15		
1	0	1	0	0	0		
2	0	1	0	0	0	1.0	0.033
3		1	0	0	0		
4		0	1	1	1	0.994	0.12
5			1	1	1		
6			0	0	0	0.573	0.33
7				1	1		
8				1	1		.
9				1	1		
10				0	0	0.572	0.27
11					1		
12					1	.	
13					1		
14					1		
15					1	0.001	28
$\Sigma(\Delta k_i \times 10^{-7})$	0	0.0014	0.0089	0.0278	202.7		

TABLE 9

<u>Case 8</u>   Response function $\phi_4 = R_{53}^2 + R_{52}^2 + R_5^2 + R_4^2$

Stiffness bounds $0 \leqslant \Delta k_i \leqslant 0.2 k_i$

Response magnitude $(\phi_4)_o^{\frac{1}{2}} \Rightarrow 0.571g$

Param. ident. no.	Opt. values of $\Delta k_i/\Delta k_{i_{max}}$ for diff. nos. of params.				Resp. ratio $\left\{\dfrac{\phi_4}{(\phi_4)_o}\right\}^{\frac{1}{2}}$	CPU time (min)
	4	6	10	15		
1	0	0	0	0		
2	0	0	0	0		
3	0	0	0	0		
4	0	1	1	1	1.0	0.067
5		1	1	1		
6		0	0	0	0.945	0.15
7			1	0		.
8			1	1		
9			1	1		
10			1	0	0.938	0.68
11				1		
12				1		
13				1		
14				1		
15				1	0.001	25
$\Sigma(\Delta k_i \times 10^{-7})$	0	0.0089	3.202	202.7		

6.   Discussion and conclusions

6.1   The optimisation process

The problem of minimising the vibration on the Lynx model has
been formulated so that a straightforward optimisation process can be
applied.   In doing this, many practical aspects have had to be
ignored, but discussion of this is presented in the following section.

The optimisation exercise has been extremely useful in that an
idea of the limitations, and hence its future usefulness can be formed.
The most important limitation is that the process itself, or rather the
particular process adopted, is very extravagant of computer time.
This has been found to vary approximately with the cube of the number
of variables considered, thereby imposing a practical top limit of
about 20 variables.   This represented a serious drawback because the
present exercise was essentially exploratory and the time left for
development was thus drastically curtailed.

Before discussing further drawbacks, it is instructive to
examine individually each case studied.   In Case 1 (Table 2), zero
response was obtainable and the optimisation proceeded without compli-
cations.   Parameters 4 and 5 are seen to provide all that is necessary,
until the final introduction of parameter 20 which gives a solution of
lesser total added stiffness (discussion of the added weight is
reserved for the following section).   Case 2 illustrates that a problem
of finding a global maximum has occurred.   This case was run several
times with different input conditions and for the 10 and 15 parameter
cases, a variety of solutions was obtained.   In the event, a low
response solution was found (15 parameters), but the results taken
together do not represent a sensible and ordered sequence.   Case 3, on
the other hand, shows a successful sequence, as does Case 4, although
the latter indicates a rather heavy penalty (in added stiffness) to pay
for the reduced response.   The remaining Cases 5 to 8 may be con-
sidered together.   In these the stiffness bounds were taken to be 20%
of the existing stiffnesses, and throughout, the variable stiffnesses
went to either their upper or lower limits.   It is interesting that in
these cases, regardless of the response function taken, the final
stiffness configuration obtained is roughly the same.

From this study of the separate cases it may be seen that a
second important drawback is the inability in Case 2 to find global
minima.   When it is suspected or known that a global minimum has not
been found, then the case under investigation must be re-run with dif-
ferent starting values of the parameters and of the penalty function
coefficient β.   Thus, the human operator enters into the picture and
judgement, backed up by experience, must be used to obtain a better
answer.   In cases where a zero or near-zero response was obtainable,
finding the global minimum did not seem to be a problem.   However, in
the opposite situation, there was a tendency for all parameters to
adopt either their upper or lower limits, and, particularly in Case 2,
a number of different minima were found.   To some extent, the par-
ticular minimum found is pre-determined at the first and lowest value
of β;   thereafter, for increasing values of β the response part of the
objective function dominates at the expense of the added weight part.

Finally, there must remain the question of whether or not the
optimisation process used here was the right one.   Time did not allow
the development of other methods.

6.2   The use of optimisation in practical problems

It is clear from Tables 2 to 9 that using the response circle diameters for initially selecting the best parameters for optimisation is not altogether satisfactory.   Ideally, the parameters that turned out to be important in the optimisation should have appeared at the top of the circle listing.   However, the circle diameter method as used here is very simple, and a more realistic criterion, still based on circle response properties, can easily be employed.   This is something that warrants further investigation.

The gear box-engine-fuselage model of Fig. 1 was simply used to test out and gain experience with the optimisation routine.   The "stick" model of the Lynx provided, however, scope for a more serious attempt at structural optimisation.   Even here, though, the model was not sufficiently representative for useful practical deductions to be made.   The major drawback was the absence of a relationship between element weight and element stiffness.   Future models, even if simplified by restricting the number of degrees of freedom, should have the element weights involved in a proper manner.

Unlike flutter optimisation for fixed wing aircraft, the basic objective function is somewhat arbitrary.   Human judgement is required to formulate a suitable response function for embodiment in the objective function.   Clearly, it is impossible to reduce the response in the fuselage everywhere to zero, so it has to be decided where the response should be small, or zero, and in which directions.   If a composite response is to be used, some sort of weighting has to be decided upon for the various components.

A further practical consideration is that the optimisation should be terminated when the payoff between added weight and response reduction becomes unacceptable.   Such a bounding relationship could be provided by information obtained from industry.

7.   Acknowledgements

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8.   References

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Appendix

Matrix expression for the modified receptance

A brief derivation of eqn. (5) of Section 3 is given here. Consider a structure modified by inserting two linear springs of rates k_1 and k_2 between points a and b, and c and d respectively, and excited by a harmonic force F_p at p.   The objective is to obtain the receptance G_{pq} between point p and a further point q when the structure is thus modified.

The forces exerted by the springs F_a, F_b and F_c, F_d at the points of attachment a, b and c, d can be written as

$$F_a = k_1(R_b - R_a) = - F_b$$
$$F_c = k_2(R_d - R_c) = - F_d$$

$$\tag{A1}$$

where R denotes the displacement response.

The response R_q can now be written as

$$R_q = G_{qp}F_p + G_{qa}F_a + G_{qb}F_b + G_{qc}F_c + G_{qd}F_d$$

where G_{ij} is the receptance between the points i and j.

Using eqns. (A1) this can be re-written as

$$R_q = G_{qp}F_p + (G_{qa} - G_{qb})k_1\delta_1 + (G_{qc} - G_{qd})k_2\delta_2 \qquad \tag{A2}$$

where $\delta_1 = R_b - R_a$ and $\delta_2 = R_d - R_c$ are the relative displacements of the springs.

Similar expressions for the displacements at a, b, c and d can also be formed:

$$R_a = G_{ap}F_p + (G_{aa} - G_{ab})k_1\delta_1 + (G_{ac} - G_{ad})k_2\delta_2$$

$$R_b = G_{bp}F_p + (G_{ba} - G_{bb})k_1\delta_1 + (G_{bc} - G_{bd})k_2\delta_2$$

$$R_c = G_{cp}F_p + (G_{ca} - G_{cb})k_1\delta_1 + (G_{cc} - G_{cd})k_2\delta_2$$

$$R_d = G_{dp}F_p + (G_{da} - G_{db})k_1\delta_1 + (G_{dc} - G_{dd})k_2\delta_2$$

and by combining these the relative displacements δ_1 and δ_2 can be expressed

$$\underline{\delta} = F_p\underline{B} - \underline{D}\,\underline{V}\,\underline{\delta} \qquad \cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad (A3)$$

where

$$\underline{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} R_b - R_a \\ R_d - R_c \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} G_{bp} - G_{ap} \\ G_{dp} - G_{cp} \end{bmatrix}$$

$$\underline{D} = \begin{bmatrix} (G_{bb} - G_{ba}) - (G_{ab} - G_{aa}) & (G_{bd} - G_{bc}) - (G_{ad} - G_{ac}) \\ (G_{bd} - G_{ad}) - (G_{bc} - G_{ac}) & (G_{dd} - G_{dc}) \quad (G_{cd} - G_{cc}) \end{bmatrix}$$

$$\underline{V} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$$

Rewriting eqn. (A2) as

$$R_q = G_{qP}F_p + \underline{A}\,\underline{V}\,\underline{\delta} \qquad \cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad (A4)$$

where $\underline{A} = (G_{qa} - G_{qb} \quad G_{qc} - G_{qd})$, $\underline{\delta}$ can now be eliminated using eqn. (A3) to provide the new receptance G_{new} between p and q, i.e.

$$G_{new} = R_q/F_p$$

$$= G_{qp} + \underline{A}\,\underline{V}\,(\underline{I} + \underline{D}\,\underline{V})^{-1}\underline{B} \qquad \cdots \qquad \cdots \qquad \cdots \qquad (A5)$$

The expression can be generalised to include any number of variables and $\underline{A}$, $\underline{B}$ and $\underline{D}$ can be easily formulated from the receptance matrix of the unmodified structure. The matrix to be inverted is only as large as the number of variable spring stiffnesses considered.