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Design of a feedback active control regulator for reduction of vibrations or noise in a helicopter cabin

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ABSTRACT :
The present paper is concerned with vibrations reduction of a flat honeycomb panel representative of a helicopter mechanical deck. The panel is equipped with 8 PVDF patches as sensors and with 8 PZT patches used as actuators. Vibrations are induced into the panel by an electro-mechanical shaker. The frequency range of interest (0-1000Hz) makes the problem difficult to treat by classical methods. Here, the panel is first identified in the time domain by performing successive single-input, single-output experiments. These experiments are analyzed by the discrete time Correlation-Chebyshev method. A very large number of parameters is necessary for an accurate modeling of the panel. Still, numerical simulations are readily performed. A simplified form of the standard LQG regulator is used and a solution is obtained by numerical optimization. The realization of the controller is detailed and a comparison between theory and experiment is given.

INTRODUCTION.

The reduction of vibrations or noise inside a helicopter is a challenging problem. Passive attenuation by appropriate materials which introduce some damping in the desired range of frequencies is currently used. Nevertheless, this adds weight and active control has been suggested. At ONERA, this problem has been under study for some years (see for example [3] [4]). In the present paper, another approach to broad band active control by means of piezoelectric ceramic actuators is presented. The structure considered is a flat panel of dimension 0.90m×0.90m and thickness 2 cm made from an honeycomb « Nida Nomex » central part covered on each side by a fiber glass skin. This panel is representative of a helicopter mechanical deck. In this article, the method of system identification of the panel is detailed and different controllers are designed. One of these controllers has been experimented.

PRESENTATION OF THE EXPERIMENTAL MODEL.

The honeycomb panel is flat. It is, as tightly as possible, cantilevered on its perimeter. The remaining free surface is a square of dimension 0.84m×0.84m. The panel is equipped with 8 piezo-electric actuators made of a thick PZT ceramic film (1mm thickness) located on one of the sides of the panel. On the other side and at the same geometrical positions are 8 sensors made of a thin (40 μm) PVDF film. The sensors are used to record the deformations of the panel. The position of the actuators and sensors piezo-elements is given in Figure 1 (taken from [4]).

On the panel, a classical electro-mechanical shaker is also attached. The point where the shaker is attached is also indicated in Figure 1. The whole structure made from the shaker and panel may be considered as a kind of giant loud-speaker. As the panel deforms, it acts as the membrane of the loud-speaker and signals are induced into the 8 PVDF transducers. The goal of the study is to use the signals given by the 8 PVDF transducers to compute inputs for the 8 PZT actuators. With the right algorithm, it is expected that the noise radiated by the panel may be reduced.

One possible way to reach this goal is to attenuate structural vibrations. A broad enough range of frequencies is necessary (here from 0 to 1000 Hertz). The present paper is limited to the study of attenuation of the vibrations. The first step is the modeling of the flat panel in the frequency range of interest (0-1000 Hz). The second step is the design of a digital controller, the output of which will, in turn, drive the 8 PZT actuators.

IDENTIFICATION AND MODELING OF THE SYSTEM.

The modeling of the whole system, instrumented panel with associated electronic plus electro-mechanical shaker, is made by identification of open loop experimental results. The experiments have been carried out as follow :

1) A single actuator is selected, it may be the electro-mechanical shaker or one of the 8 PZT actuators.

2) A random generator with a limited frequency band output (Figure 2) is used to drive the selected actuator.

3) The input of the actuator and the responses of the 8 thin PVDF films are digitized and recorded.

There are 9 experiments of this kind (one for the electro-mechanical shaker and 8 for the PZT actuators). Each experiment has 1 input and 8 outputs (response of the 8 PVDF films). The whole data is considered to be the result of 72 SISO (single input - single output) independent experiments. Each experiment gives a transfer function by the Correlation-Chebyshev method of Mäkilä [1-2]. As the number of natural modes in the frequency band of interest is large (over 15), the modeling of one single SISO experiment necessitates a large number of parameters. So the complete modeling of the structure obtained in this way is not suitable for the design of a classical LQG (Linear Quadratic Gaussian) controller. Nevertheless the modeling allows simple and fast simulations of the system.

For the convenience of the reader, an outline of the Correlation-Chebyshev method is given below. More details can be found in [1].

We consider a stable, causal, linear time-invariant discrete system where the output y is related to the input u by the equation :

(1)

$$y(t+1) = \sum_{k \geq 0} g_k u(t-k) \quad + \quad v(t+1)$$

In this equation v is an additive error. The parameters t and k take positive integer values. The coefficients g_k are those of a classical filter with a Finite-duration Impulse Response (FIR filter). The sequences $\{y(t+1)\}$ and $\{u(t)\}$ are known for $0 \leq t \leq N-1$. When $t<0$ we suppose $u(t)=0$. If this is not the case, the error introduced by the hypothesis is added to v. The auto-correlation function θ is defined as :

$$\theta(k,l) \quad = \quad \frac{1}{N} \sum_{t=0}^{N-1} u(t-k)\, u(t-l)$$

The cross-correlation function ϕ between the output and the input is given by :

$$\phi(l) \quad = \quad \frac{1}{N} \sum_{t=0}^{N-1} y(t+1)\, u(t-l)$$

The sequences $\{y(t+1)\}$ and $\{u(t)\}$ are known and the $\{g_k\}$ are to be determined. In fact, only a finite number n of coefficient can be considered. The problem is to determine the $\{\hat{g}_k\}$ for $0 \leq k \leq n-1$, which explain the sequence $\{y(t+1)\}$ from the sequence $\{u(t)\}$. In the Correlation-Chebyshev method, the $\{\hat{g}_k\}$ minimize the criterion :

$$J(\hat{g}) = \left[\frac{1}{n} \sum_{l=0}^{n-1} \left| \phi(l) - \sum_{k=0}^{n-1} \hat{g}_k \theta(k,l) \right|^p \right]^{1/p}$$

where $1 \leq p < \infty$. For any p satisfying the condition $1 \leq p < \infty$, the solution is given by :

$$(2)\quad \begin{pmatrix} \hat{g}_0 \\ \hat{g}_1 \\ \vdots \\ \hat{g}_{n-1} \end{pmatrix} = \begin{pmatrix} & & \\ & \Theta & \\ & & \end{pmatrix}^{-1} \begin{pmatrix} \phi_0 \\ \phi_1 \\ \vdots \\ \varphi_{n-1} \end{pmatrix}$$

where the element Θ_{ij} on the row i and column j of matrix (Θ) is given by :

$$\Theta_{ij} = \theta(i-1\,, j-1)$$

If the input sequence $\{u(t)\}$ is chosen such that the matrix (Θ) in (2) is non singular, the $\{\hat{g}_k\}$ are determined by (2). Then the sequence $\{\hat{y}(t+1)\}$ given by :

$$\hat{y}(t+1) = \sum_{k=0}^{n-1} \hat{g}_k u(t-k)$$

is an approximation of $\{y(t+1)\}$.

If two others input-output sequences $\{y'(t+1)\}$ and $\{u'(t)\}$ for $0 \leq t \leq N'-1$ are available and are independent of the previous $\{y(t+1)\}$ and $\{u(t)\}$, the $\{\hat{g}_k\}$ may be used to compute an approximate value of the output $\{y'\}$ by :

$$\hat{y}'(t+1) = \sum_{k=0}^{n-1} \hat{g}_k u'(t-k)$$

It is then possible to define a Root Mean Square (RMS) error by :

$$E(n) = \sqrt{\frac{1}{N'} \sum_{i=0}^{N'-1} (y'(i) - \hat{y}'(i))^2}$$

The error $E(n)$ depends on the number n of parameters $\hat{g}_k$ used. The number n can be chosen to minimize $E(n)$.

One can also define a relative error by :

$$R(n) = \frac{E(n)}{\sqrt{\frac{1}{N'} \sum_{i=0}^{N'-1} (y'(i))^2}}$$

Remark : As p approaches infinity, the criterion $J(\hat{g})$ tends to the Chebyshev criterion :

$$J_C(\hat{g}) = \sup_{l\,=\,0,1,\ldots,n-1} \left| \phi(l) - \sum_{k=0}^{n-1} \hat{g}_k \theta(k,l) \right|$$

This remark explains the name, Correlation-Chebyshev, given to Mäkilä's method.

Application and validation of the method: The Correlation-Chebyshev method has been applied to the identification of the instrumented panel. For each open loop experiment, the input and all the 8 outputs are digitized and recorded with a sampling frequency close to 4000Hz. There is 32768 consecutive time steps in each record. The power spectrum of the input is shown in Figure 2 . This curve has been obtained by the Moving-Blocks method. The number of points in a block and the value of the shift are optimized to reduce noise in the averaged spectrum. In Figure 2 , there is still much noise in the result, nevertheless the overall shape of the power spectrum is well defined.

In the identification of the instrumented panel by the Correlation-Chebyshev method, the number of unknown parameters $\hat{g}_k$ must be optimized. It has been found that this number is about 600. The optimal number may slightly vary with the transfer function considered. However, for the convenience of the programming, the number n has been fixed to 600. The error thus introduced is negligible.

The identified model must also be validated. One first validation is the computation of the relative error $R(n)$ which, generally, is less than 10% . However, the relative error is quite global and does not give any information on the frequency content of the reconstructed signal. As the Correlation-Chebyshev method identifies the system to FIR filters, the Transfer Function of these filters may be compared to direct measurements. As usual, the direct measurements are obtained by the quotient of the input/output cross spectrum and the input power spectrum (Welch's method) and averaged by a Moving-Blocks process. Generally, Correlation-Chebyshev and direct measurement are in very close agreement. Nevertheless, when the actuator and the sensor are collocated , that is to say when they are at the same location on opposite sides of the panel, discrepancies are larger. In this case, the relative error $R(n)$ is also larger. For example $R(n)$ is equal to 15.23% for actuator/sensor 5 and 16.57% for actuator/sensor 6. The most important discrepancies are for the highest frequencies in the case of collocation (see Figure 3). This is not explained though local non linear effects and low level high frequency input are suspected. As seen in Figure 2, when the frequency is larger than 1500Hz, the level of the input is reduced by the low-pass filter in the random generator. This increases the level of noise in the final result as shown in Figure 3 specially for the Moving-Blocks method. It is not known which method provides the best results. The Correlation-Chebyshev method seems nevertheless to be very effective when the recorded sequences are of short duration. This method is also very good at filtering. This may be seen in Figure 3 for the actuator/sensor 5, where the effect of the electrical current at 50Hz is entirely eliminated. At least for frequencies inferior to 1500Hz, the Correlation-Chebyshev method is superior to the direct Moving-Blocks identification as it is easier to use and it gives more compact results. For the Correlation-Chebyshev method, there is only one parameter, the number n of $\hat{g}_k$ parameters to be optimized instead of two, the size of the blocks and the number of points shifted for the Moving-Blocks method. Moreover, only 600 real numbers are necessary for the Correlation-Chebyshev method instead of 2048 complex numbers for the Moving-Blocks Welch's method. Yet, even with Correlation-Chebyshev, the complete modeling of the panel necessitate 9×8×600 real parameters and this is much too large a number for the design of an optimized controller by any classical method.

DESIGN OF THE REGULATOR. THEORETICAL ASPECT.

Following the standard LQG design (Kwakernaak [5]), let us consider the following discrete time system :

$$\begin{aligned} x_{i+1} &= A\,x_i + B\,u_i + w_i \\ y_i &= C\,x_i \end{aligned}$$

where vectors $x \in R(n \times 1)$ and $y \in R(m \times 1)$ are the state and output of the system, respectively. The vectors $u \in R(l \times 1)$ and $w \in R(n \times 1)$ are the command and noise, respectively. As the state is not directly known, an observer is designed to estimate the state. The equations of the observer are :

$$\begin{aligned} \hat{x}_{i+1} &= A\,\hat{x}_i + B\,u_i + D\,(y_i - \hat{y}_i) \\ \hat{y}_i &= C\,\hat{x}_i \end{aligned}$$

The vector $\hat{x} \in R(n \times 1)$ gives an estimate of the state vector x. The vector $\hat{y} \in R(m \times 1)$ is an estimate of the output. The difference between the measured and the estimated outputs acts as a corrective term into the first equation of the observer to make the estimated state to converge toward the true state. The command of the system may now be defined as a function of the estimated state :

$$u_i = -K\,\hat{x}_i \qquad (3)$$

Replacing u and $\hat{y}$ by their values, the equation of the observer becomes:

$$\hat{x}_{i+1} = (A - BK - DC)\,\hat{x}_i + D\,y_i$$

by taking $A' = A - BK - DC$, one obtains :

$$\hat{x}_{i+1} = A'\,\hat{x}_i + D\,y_i$$

In this equation the observed variables y_i are acting as the command of the observer system. Let us suppose that we are able to find a stable and converging observer (for example by the LQG method). In this case the eigenvalues of the matrix A' have absolute values less than 1. Moreover, let us suppose, that in case of an eigenvalue with multiplicity p, it is possible to find p independent eigenvectors. Then the spectral norm of matrix A' is less than 1.

$$\|A'\| \leq \rho < 1$$

Replacing the estimated state $\hat{x}$ by its equation in the command relation gives :

$$u_i = -K\,D\,y_{i-1} - K\,A'\,\hat{x}_{i-1}$$

The process may be repeated up to the point where $\hat{x}_0$ appears. It is possible to chose $\hat{x}_0 = 0$. One thus obtains for the command the relation :

$$u_i = -K(D\,y_{i-1} + A'D\,y_{i-2} + A'^2 D\,y_{i-3} + \ldots\ldots\ldots\ldots + A'^{(i-1)}\,D\,y_0\,)$$

Defining : $K'_q = -K\,A'^{q-1}D$, the above equation becomes :

$$(4) \qquad u_i = \sum_{q=1}^{N_q=i} K'_q\; y_{i-q}$$

In relation (4), the command is given as the sum of terms with decreasing importance.

$$\left\|K'_q\right\| \;\leq\; \|K\|\,\|D\|\,\rho^{q-1}$$

If $K'_q(i,j)$ is the term on row i and column j of matrix K'_q , we have :

$$\max_{i,j}\left|K'_q(i,j)\right| \leq \left\|K'_q\right\| \leq \|K\|\,\|D\|\,\rho^{q-1}$$

So, if it is possible to find both a good active control and a rapidly converging observer with a small spectral norm ;

Then, it is possible to consider only a limited number of terms in relation (4).

Let us look now at the sizes of the matrices in relations (3) and (4). We have : $K'_q \in R(l,m)$ and $K \in R(l,n)$. Even if n, the order of the system, is large, we have always a limited number of commands l and of measurements m. In our case $l = m = 8$ and the number n is 38400. It is then very interesting to use relation (4) instead of relation (3) if the number N_q can be kept small enough. It is now necessary to detail how the regulator can actually be designed.

PRACTICAL DESIGN OF THE REGULATOR.

The regulators presented in this article are all constructed according to relation (4). The simplest regulator is obtained with $N_q = 1$. In this case, there is only one matrix, $K'_1 \in R(l,m)$ to be determined. The identification of the panel by Mäkilä's method does not allows the computation of the eigenfrequencies of the closed-loop system. One cannot either estimate a cost function over an infinite length of time or average an infinite number of input time functions. What is only possible is the numerical simulation over a reduced length of time of the response of the panel to a chosen input time function. This input can nevertheless be selected such as it will excite all the modes of the panel in the frequency range of interest. For that purpose, the input must be long enough in time and contain all the frequencies. The input sequence can be a limited part of the recorded experimental input with a power spectrum such as the one depicted in Figure 2. It can also be a sequence given by a numerical random generator. The length of the input sequence must be long enough for a good characterization of the regulator. For the panel, 1000 and sometimes 5000 steps at sampling frequency 4000Hz were computed. This corresponds to a very limited length of time, 0.25s or 1.25s but was enough to evaluate the proposed regulator. Each regulator is associated to a cost number J. This number is taken as the weighted sum of the RMS value of the outputs of the 8 PVDF films. A penalty term is also added to limit the command at an acceptable level. The cost function J is a very non linear function of the coefficients in the matrix K'_1. The optimization of matrix K'_1 is thus a uneasy task. The algorithm of optimization must be particularly robust and should not necessitate the computation of derivatives of J. This later condition comes from the fact that the derivatives of J can only be obtained by a numerical finite difference computation. In the present paper, the optimization of matrix K'_1 has been made by the Downhill Simplex method of Nelder and Mead [6]. This method has been preferred to Powell's method because of its superior robustness. Nevertheless the Downhill Simplex method does not converge very rapidly and the number of optimized parameters must be limited. For the present problem, 68 parameters, that is to say one single full matrix K'_q is the upper limit. For that number of parameters, the optimization of the cost function needs about 1000 evaluations of J. The above process allows only the characterization of the regulators over a finite length of time. It does not ensure long term stability. In fact, the computations have shown that some seemingly good solutions were weakly unstable. It has been found that the simplest and fastest way to test stability is the monitoring of the decay of the PVDF response when the excitation is cut off. When the decay is not rapid enough, the solution is not retained. This eliminates not only unstable solutions but also stable ones which are not sufficiently damped. More precise results could be obtained by the monitoring of the evolution of the Fourier transform of the signal. This necessitates a lot of computations and has not been done. For the panel, this analysis was not necessary because the structure was sufficiently well excited with 1000 steps of random input for the electro-mechanical shaker. Moreover, the chosen condition for a solution to be acceptable was that the RMS value of the responses decreases to less than 10^{-4} of the value at the start in less than 1000 steps. This decrease is sufficiently rapid to eliminate any instability. One must also note that the test for stability does not need to be done for each matrix K_1'. It is only necessary to make the test for the solutions which seem to be better than the best solution found so far. A very large penalty term is added to J in case of instability so an unstable solution is never retained.

Let us now suppose that more than one matrix K_q' are considered. The first matrix can be determined as previously. Then the first matrix is kept constant and a second matrix is optimized. The first matrix is no longer optimum for a regulator with two matrices. Nevertheless, the first matrix can be readjusted and the process can be repeated for any number of matrices K_q'. In fact, for the panel, the matrices are rather weakly coupled. After a first successive determination of the matrices K_q', only a single readjustment of the matrices was needed. In any case, a truly optimal solution is not necessary. The computations may be halted when the cost function J ceases to improve sufficiently.

Up to now, only the case of matrices K_q' with all their coefficients has been considered. It may happen that some coefficients are always equal to zero. This is the case when a sensor does not act on some actuator. In the particular case of the panel, sensors and actuators are collocated. The main diagonal of the matrices K_q' is likely to be predominant. One possible way to simplify the problem is thus to look for a solution with diagonal matrices K_q'. The number of free parameters is then considerably reduced. All the parameters may be optimized at the same time if the number N_q is kept small enough.

So far, the concept of frequency range of interest has been overlooked. As it may be seen in Figure 3 , there is a very large number of modes with frequencies less than 2000 Hz. In its simplest form, in the case of diagonal matrices K_q', the controller may be considered as a collection of l single input/single output FIR filters with only N_q coefficients. As N_q must remain small, the controller is not able to improve a response where many modes participate. One is thus obliged to limit the frequency range of action of the controller. In a helicopter or aircraft cabin, the lowest part of the audio-frequencies spectrum is troublesome for the passengers. Still, these frequencies are not easily attenuated by passive absorbers. For the study of the honeycomb panel, the frequency band 0-1000 Hz has been retained. The controller must be optimized for this frequency range and still have no action that could possibly decrease the natural damping of the panel at higher frequencies. This is accomplished by the introduction of a low-pass filter as a constitutive part of the controller. An elliptic discrete Infinite Impulse Response (IIR) filter [7] has been chosen as a compromise between performance, complexity and ease of implementation. The transfer function of this filter is depicted in Figure 4 . As shown in the figure, there is an important phase shift introduced by the filter. This phase shift increases (in absolute value) with the frequency. Over 1200 Hz the phase is not important because the gain has a very small value. A linear variation of the phase is equivalent to a delay which decreases the performances of the controller. This is unavoidable because good filters have always an important phase shift. In our case, the controller must be optimized with the phase shift introduced by the elliptic filter. The actual command is not $u_i(l\ ,1)$ given by (4) but $u_i'(l\ ,1)$ given by the following relation :

(5)

$$u_i' = \frac{b(1)u_i + b(2)u_{i-1} + \ldots\ldots + b(r+1)u_{i-r}}{1 + a(2)u_{i-1}' + \ldots\ldots + a(r+1)u_{i-r}'}$$

In the case of Figure 4 , $r = 7$. In relation (5), the value of the vector u_i is still given by (4). This new formulation of the command is more complex than the one given by (4). However, the difficulty of the problem is not really increased. The number of free parameters, that is to say the numbers of coefficients which must be adjusted in the matrices K'_q , remains the same.

APPLICATION AND COMPARISON WITH THE EXPERIMENT.

Using the method described in the preceding paragraph, a regulator with 6 diagonal matrices K'_q has been optimized. The number of matrices determines the extent of the time history of the sensor outputs which are considered into the controller. Outside 6 sampling steps the sensor signals does not participate into the definition of the command. As the matrices are diagonal, another simplification is introduced. The controller is composed of $l = 8$ individual regulators computing the command of an actuator from the output of the collocated sensor. As an example, Figure 5 shows the transfer function of regulator n°8. In the figure, the amplitude of the transfer function decreases sharply after 1000 Hz and is negligible over 1200 Hz. The phase of the transfer function is also given. At frequency $f = 0$, the phase is +180° which simply means that the value of the Transfer Function is negative or equivalently that the input and output steady values have opposite signs. The controller has been realized using a Digital Signal Processor (DSP) TMS320C40 associated with a DEC alpha processor.

For a better definition of the controller, the highest sampling frequency of the DSP, 10000Hz has been selected. However, the DSP introduces a delay due to the time necessary to complete the computations. This time delay is a fraction of the sampling period. As shown on relation (4) a design delay of one time step (at 4000Hz) was appropriately introduced. The delay of the DSP is shorter and may be considered as part of this design delay. Thus the DSP computation time will not change the design of the controller. Using the MATLAB function « invfreqz » [7], it is possible to find the coefficients of an IIR filter working at the sampling frequency 10000Hz and having almost the same Transfer Function (time delay included) as the theoretical regulator defined for the sampling frequency 4000Hz. In our particular case, discrepancies between both Transfer Functions with 4000 and 10000 Hz as sampling frequencies are negligible.

Comparison between theory and experiment :

During the experiment, only the Transfer Functions (TF) between the electro-mechanical shaker and each of the 8 PVDF sensors were measured. One such TF, for sensor 2 , is given in Figure 6. Three curves are reported in this figure. One curve gives the TF without control. The two other curves give the theoretical and measured TF with control, respectively. One can note the almost perfect agreement between the theory and the experiment. This is also the case for the other sensors. Figures 6 and 7 show that the amplitude of the Transfer Functions is decreased almost everywhere excepted at the higher end of the spectrum. This can easily be explained. The cost function is the sum of the Root Mean Square value of the output signal of the sensors. The controller thus tries to decrease the amplitude of the signals independently of the frequency. The modes with the highest participation into the time signals are at the lower end of the spectrum. They show also the largest attenuation as it could be expected.

In Figure 7, the measured TFs have not been reported for clarity as they are almost identical to the theoretical ones.

If f is the frequency and $y_i(f)$ is the spectral density of sensor i , the function $y(f)$ given by the following relation:

$$y(f) \quad = \quad 20\log_{10}\left(\sum_{i=1}^{8} y_i(f)\right)$$

is the total spectral density in dB. The curves $y(f)$ with and without control are represented in Figure 8. This figure shows a substantial reduction (17 dB) on the first modal frequency (about 104 Hz) and a reduction of 15 dB on the second modal frequency (about 224 Hz). This apparent success must not hide the fact that the cost function (the sum of the RMS value of the sensor outputs) is only decreased by a relative factor of 24.55 % . The mean reduction of the vibratory level is thus low enough.

Another more complex controller has been designed but has not been experimented. This controller has 6 full matrices K'_q instead of 6 diagonal ones. This time, the input of a PZT actuator depends on the time history of the output of the 8 PVDF sensors. As the controller is more complex, the reduction of the vibrations is also more important. The cost function is reduced by a relative factor of 45.74 %. In Figure 9 , the curves $y(f)$ are represented. The first modal frequency shows a reduction of 23 dB and the second modal frequency a reduction of 19 dB . There is also some reduction for almost every frequency, even at the higher end of the spectrum. Because of its complexity, this last controller has not been realized.

One can also think to improve the controller by multiplying the number N_q of matrices in relation 4. This increases the length of the sensor outputs time histories participating into the definition of the commands. In fact, the controller becomes more complex to realize and more difficult to design for an improvement which may not be very large. In the case with 6 complete matrices, the last matrix improves only the results by about 4 % .

Another drawback of the method is the computer time necessary for the design of the controller. On a Sun Sparc Station 20, the first case with 6 diagonal matrices necessitates 39 hours of computation and the case with 6 full matrices, more than a week. The frequencies and modes shapes of the panel are strongly dependent of the tightening conditions of the screws fastening the panel on its support. The robustness of the regulator is thus very low. Any improvement of the robustness is likely to be accompanied with a decrease of the effectiveness of the control which is already not very high. One way to overcome this problem is to design an adaptive controller. The numerical method of optimization of the cost function J can be used for such a design. Most of the computer time is spent on the numerical simulation of the system. Still, when all the trials are added, the actual total length of time simulated is only a few minutes long. This may seem long enough but it must be noted that the cost function J decreases as new trials are performed. At the beginning the decrease is very fast and it becomes slower when the controller is close to optimal. One should then obtain rapidly an acceptable if not entirely optimal controller.

CONCLUSION

In this article, a method for the modeling of a composite panel has been presented. The modeling has been used for the design of suboptimal controllers. One of them has been experimentally tried. In the frequency range 0 - 1000 Hz, it appears that a reduction of 25 % and even up to 45 % of the vibrations is possible. This has been accomplished with piezo-electrical patches acting as sensors and actuators. If the relationships between radiated noise and vibrations are available, the same method is applicable to noise reduction.

The present method can also be modified to define adaptive controllers acting on a large frequency range.

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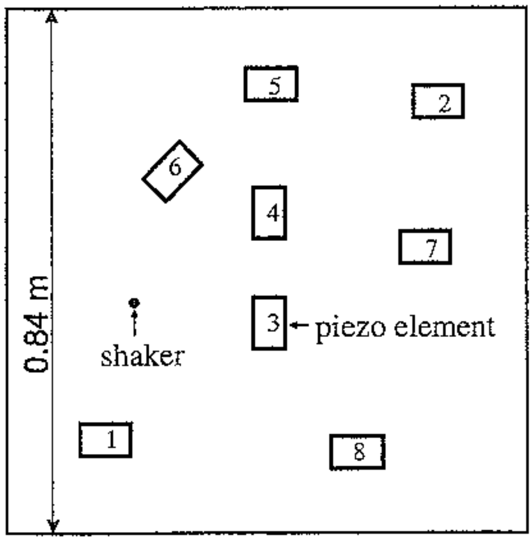

Figure 1. Planform of the plaque showing the position of the 8 actuators and sensors and of the shaker.

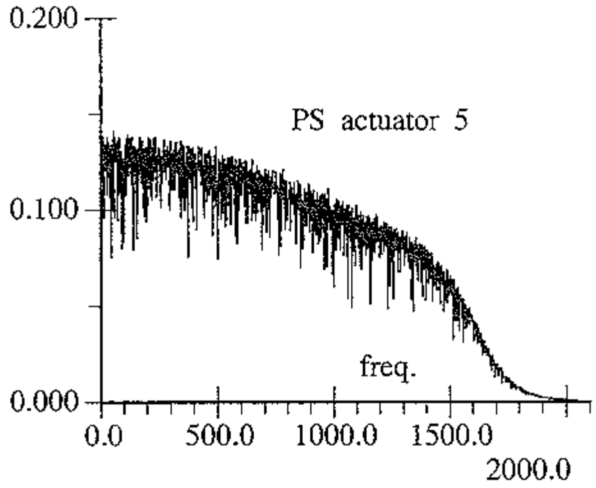

Figure 2. Power Spectrum (PS) of the input of actuator 5 for openloop identification.

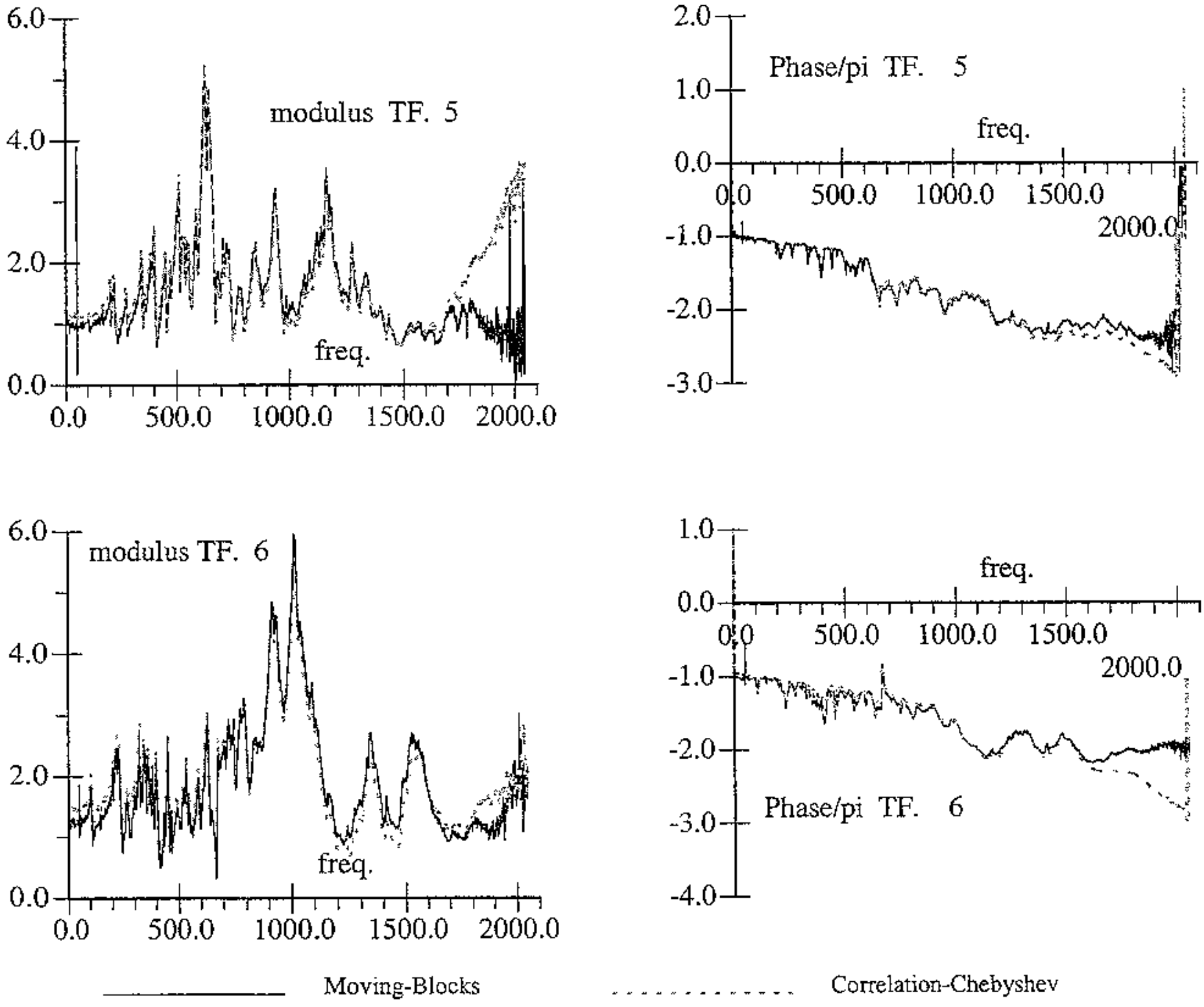

Figure 3. Transfert Functions (modulus and phase) between actuator 5 and sensor 5 and between actuator and sensor 6.

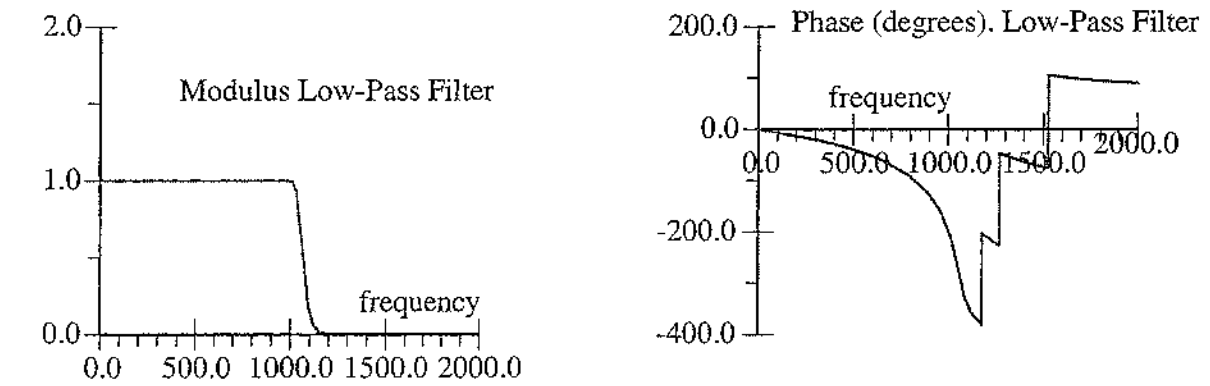

Figure 4. Gain of the low-pass filter , amplitude and phase.

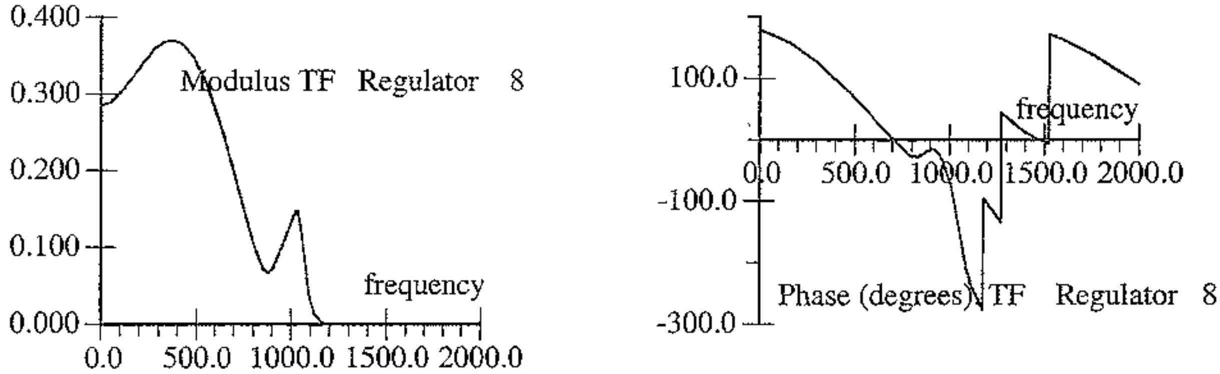

Figure 5 Gain of regulator 8, amplitude and phase.

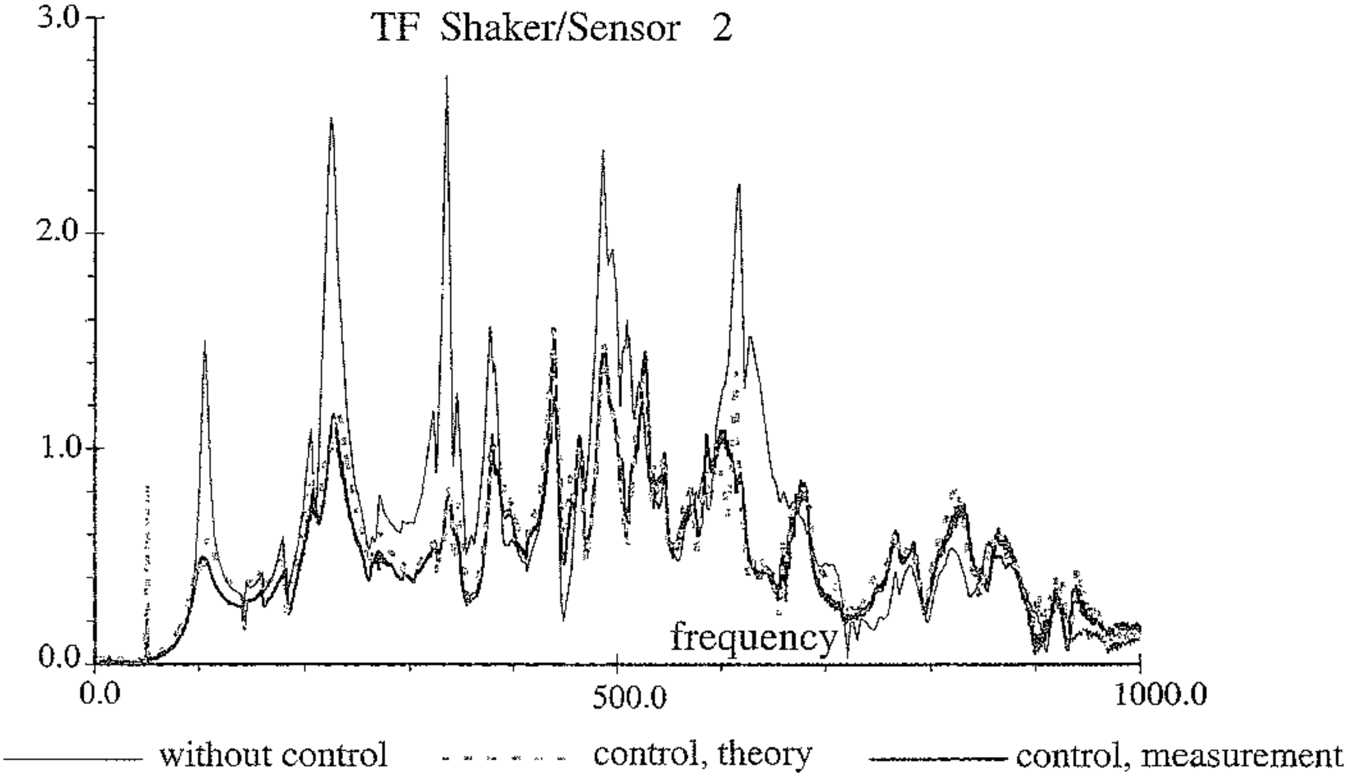

Figure 6 . Transfer Functions (TF) with and without control between the electro-mechanical shaker and sensor 2.

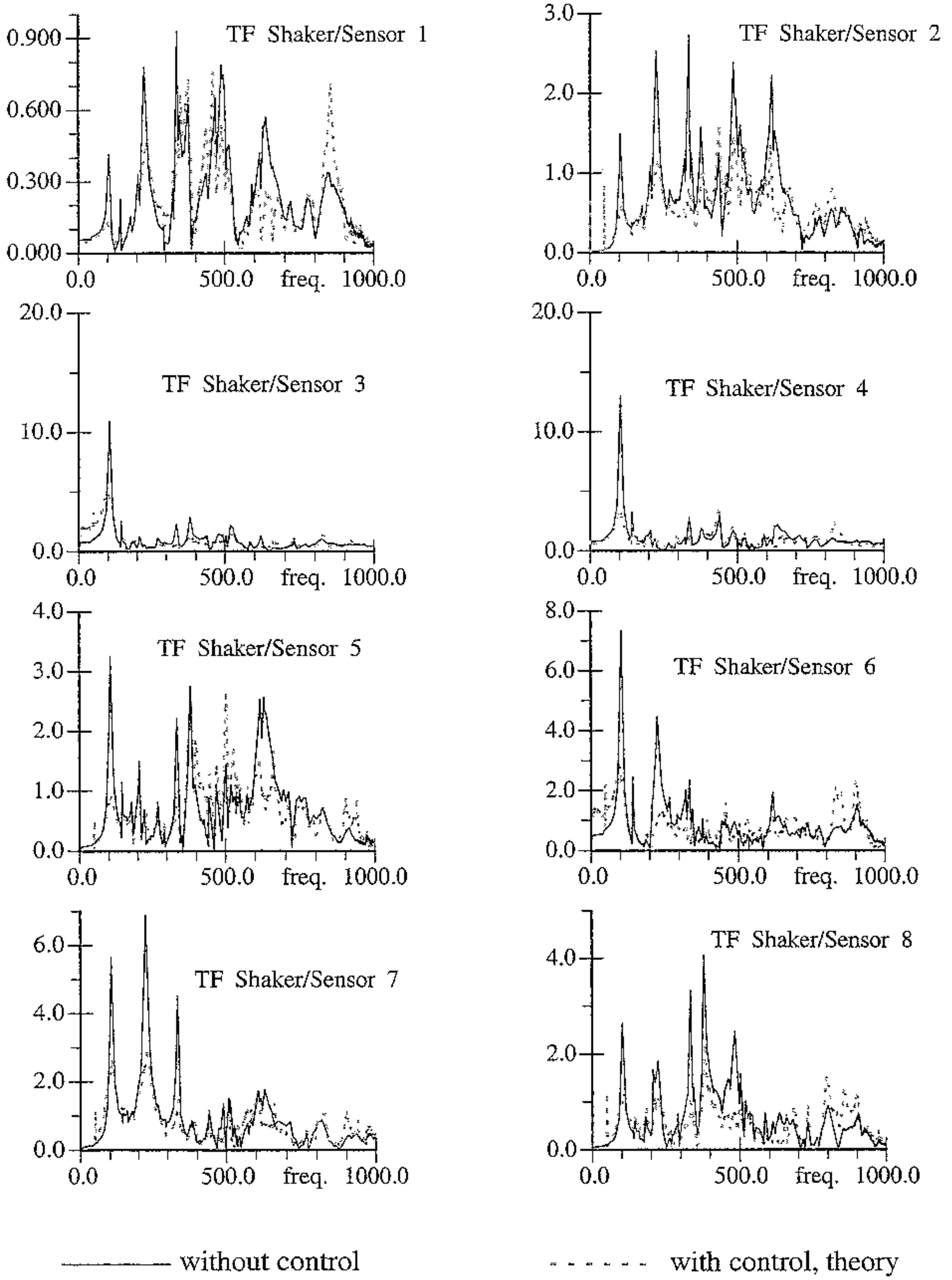

Figure 7. Tranfer Function (TF) with and without control between the electro-mechanical shaker and the 8 sensors.

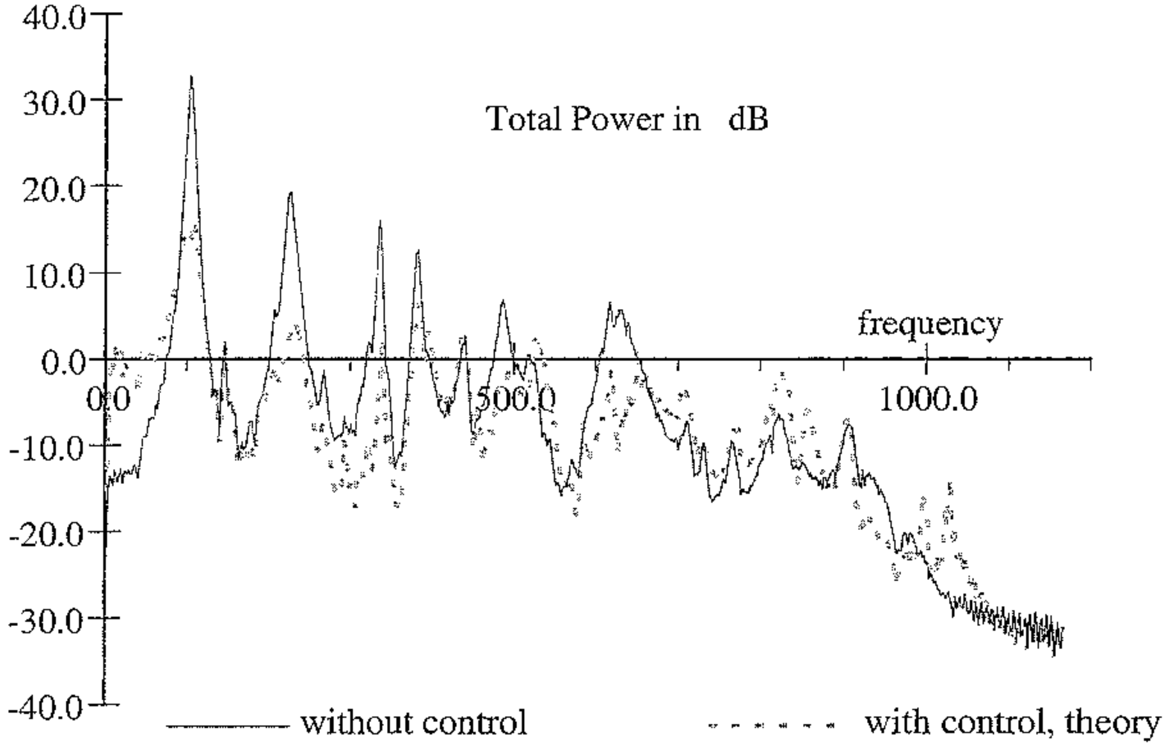

Figure 8. Sum of the power spectrums of the 8 sensors, case with 6 diagonal matrices for the controller, gain RMS = 24.55 % .

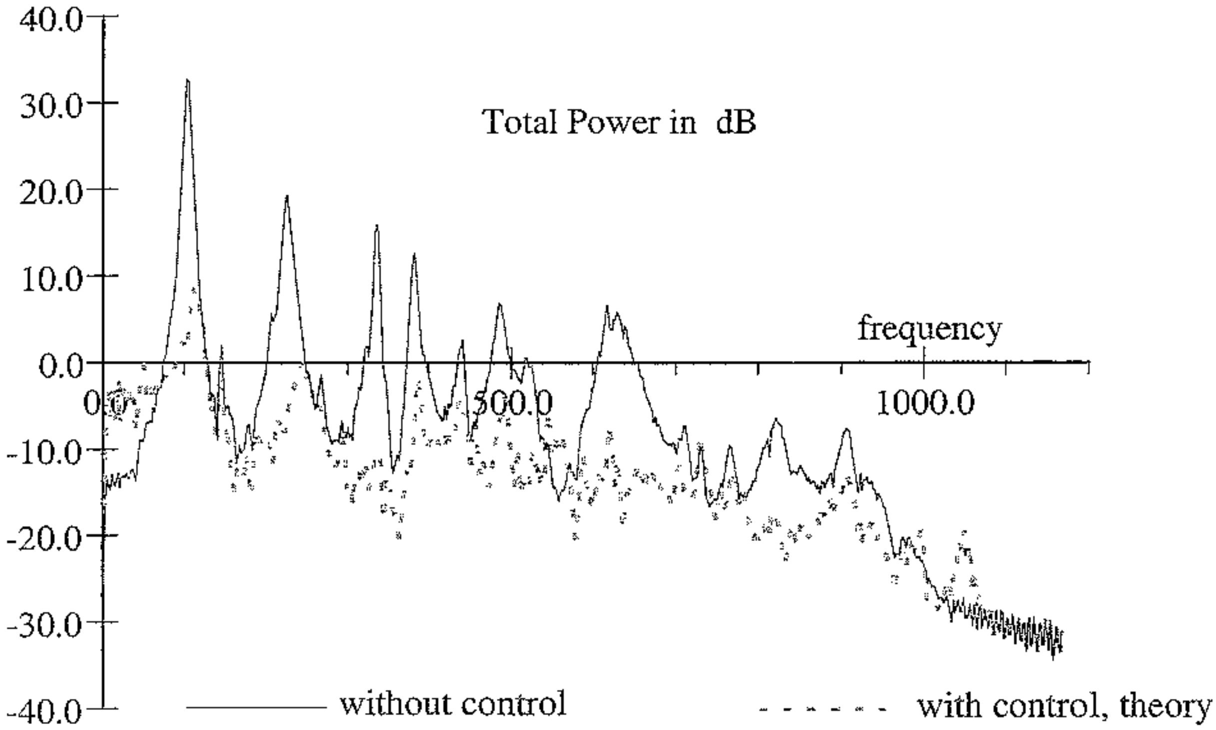

Figure 9. Sum of the power spectrums of the 8 sensors, case with 6 full matrices for the controller, gain RMS = 45.74 % .