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EXPERIMENTAL MODAL ANALYSIS

by

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INTRODUCTION

Helicopter vibrations are highly particular phenomena; the rotor is a powerful vibration generator causing helicopter specific (mainly forced vibrations) problems. Such vibrations generate fatigue problems resulting from alternate stresses and comfort problems resulting from cabin vibrations.

To solve these various problems, vibration phenomena observed must be correctly identified; the development of an experimental modal analysis software program applied to the dynamic behavior characterization of a structure by identification of its vibration modes seemed to serve this purpose.

The technique applied is based on transfer function measurement. Subsequently, the results are processed with a multi-degre-of-freedom identification program (error minimization with least squares technique is applied in this case) and a mode shape animation program, a genuine tool for understanding better and describing comprehensive structure motions.

The purpose of this document is to describe the application of this procedure to three different structures

- MGB component (AS 332 SUPER PUMA planet carrier)

- Complex fluid/structure interaction in AS 355 ECUREUIL tanks

- ECUREUIL structure including forward bottom structure, tail boom and fin assembly.

Finally, the various options envisaged for the development of modal applications shall also be described.

I - GENERAL

HYPOTHESES

The two principles exposed below are assumed to be observed throughout this exposé.

Linearity

The structure is assumed to have a linear response i.e. proportional to excitation.

Reciprocity (Maxwell's theorem)

In linear behavior structures, it is assumed that point I response to a unit excitation at point J is identical with Point J response to a unit excitation at point I

INTRODUCTION TO MODES OF VIBRATION

A natural mode is a general property of an elastic structure. This mode is characterized by a natural frequency and damping factor that may be identified from any point of the structure and also by a mode shape indicating a comprehensive motion of the system under study.

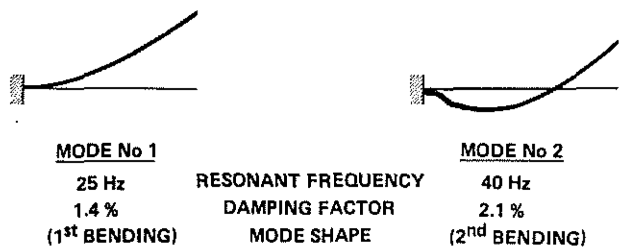

FIG. 1 : NOTION OF VIBRATIONS MODES - CASE OF A CANTILEVERED/FREE BEAM (BENDING)

MATHEMATIC FORMULATION

Let us consider a structure partitioned into n points, the equation of movement is a second order differential system with n equations that may be formulated as follows:

(1)

$$Md^2x(t)/dt^2(t) + Cdx(t)/dt + K\,x(t) = f(t)$$

Where M is the mass matrix (dimension n x n)

K is the rigidity matrix

C is the damping matrix

x(t) is the displacement vector

f(t) is the external load vector

If we apply Laplace's transform of equation (1) (supposing nil initial conditions) we obtain the following over the range of frequencies:

(2)

$B(s)\ X(s) = F\ (s)$
or
$X(s) = B^{-1}\ (s)\ F\ (s)$
with $B(s) = M\ s^{2} + Cs + K$

Where s is Laplace's complex variable
X(s) is Laplace's transform of x(t) (Generic term Xi(s))
F(s) is Laplace's transform of f(t) (Generic term Fi(s))

The final aim is to identify matrix $H(s)=B^{-1}(s)$ designated transfer function matrix as each of its Hij(s) components represents the Xi(s) / Fj(s) ratio i.e. the mechanical transfer function between a load input at point j and a displacement output at point i

1- The natural modes of the structure appear as complex solutions σ_k and U_k of equation:

$B(\sigma_k)\ U_k = 0$ (therefore $\det B(\sigma_k) = 0$)

The σ_k 's constitute therefore the set of complex poles of each transfer function hij(s). Decomposition into simple elements can be expressed as follows:

(3)

$$h_{ij}(s) = \sum_{k=1}^{n} \left(\frac{a_{kij}}{s-P_k} + \frac{a_{kij}^{*}}{s-P_k^{*}} \right)$$

Where $P_k = \sigma_k + j\ \omega_k$, σ_k = Natural damping coefficient of mode k
ω_k = Natural angular velocity of mode k
a_{kij} = Complex residue proportional to modal mode shape of mode k at point i
* represents the complex conjugate

Therefore each elementary transfer function may be formulated from modal characteristics.

2- Moreover the mathematical study of this decomposition makes it possible to show more generally that H(s) can be expressed as follows:

(4)

$$H\ (s) = \sum_{k=1}^{n} \left(\frac{U_k U_k^{t}}{s-P_k} + \frac{U_k^{*} U_k^{*t}}{s-P_k^{*}} \right)$$

with Uk being the modal mode shape of mode k
On that basis, we can state that defining a single line or column of matrix B is sufficient to determine the entire matrix.

II - MEASURING AND PROCESSING

We use a Fourier analyser HP 5451C to measure transfer function H(s) designated FREQUENCY RESPONSE along imaginary axis
$(s = j\ \omega)$

An additional software program HP5477A specialized in modal analysis is used to process and formulate measured functions as in equation 3.
Angular velocity and natural damping values as well as mode shape coefficients of every important mode are then extracted from EACH function.

REMARKS

Since angular velocity and damping are structural characteristics, extracting data from EACH transfer function is not really necessary but allows processing these values statistically

A modal test does not always permit identifying every mode of vibration of a structure as some of these modes are not correctly excited. The purpose of the test is in fact to determine the major modes of the structure considered i.e the modes whose contribution to the dynamic properties of the structure might be significant in a given vibratory environment.

Application of the properties of matrix B allows a number n of measurements, these measurements can be divided as follows:

- One response point and n excitation points (transient technique)

- One excitation point and n response points

III - APPLICATIONS

Three practical illustrations of modal analysis are presented:

- A main gear box component (AS 332 SUPER PUMA planet carrier)

- A fluid/structure interaction in AS 355 ECUREUIL tanks

- ECUREUIL structure including forward bottom structure, tail boom and fin assembly.

In each case the problems shall be presented, a descriptive chart of test and modes of vibration shall be drafted and a characteristic transfer function as well as identified mode shapes shall be presented.

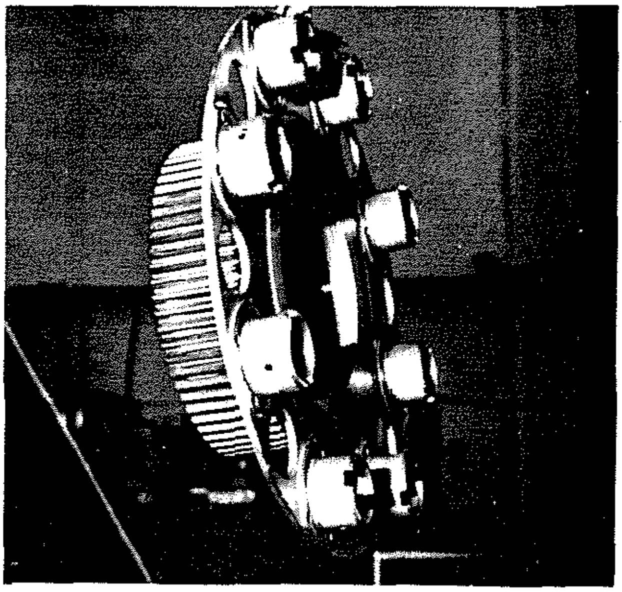

FIG. 2 : SUPER PUMA PLANET CARRIER

III.1 - Structure N° 1 : SUPER PUMA planet Carrier

Purpose of study: investigate experimentally the dynamic characteristics of the planet carrier in order to allow correlation with a finite-element model.

Choice of excitation technique: the rapidity, structure access, high frequential cover (1 kHz.) and low system damping criteria are typical of a transient excitation

Excitation means: a hammer fitted with an accelerometer.

Number of excitation points: 36 points.

Analysis parameters: there are five parameters.
maximum frequency (f MAX)
frequency resolution (Δf)
number of signal sampling points (N)
sampling rate (Δt)
signal total time (T)

These are related through the following three relations:

$T = 1/\Delta f$, $f_{MAX} = 1/2\Delta t$, $T = N\Delta t$
with $N \varepsilon \{64, 128, 256, 512, 1024, 2048\}$

According to the objectives sought, one can set two out of the five values, the other three being determined automatically.

In the present case we want good frequency resolution associated with the best frequency cover, hence the following options:

$\Delta f = 1$ Hz
f MAX = 1024 Hz
$\Delta t = 1/2048$ s
T = 1 s
N = 2048 pts

Number of identified modes: 4

	Frequency	Remarks
Mode 1	391 Hz	"Umbrella" mode
Mode 2	396 Hz	2-diameter mode
Mode 3	505 Hz	1-circle&1-dia. mode
Mode 4	569 Hz	3-diameter mode

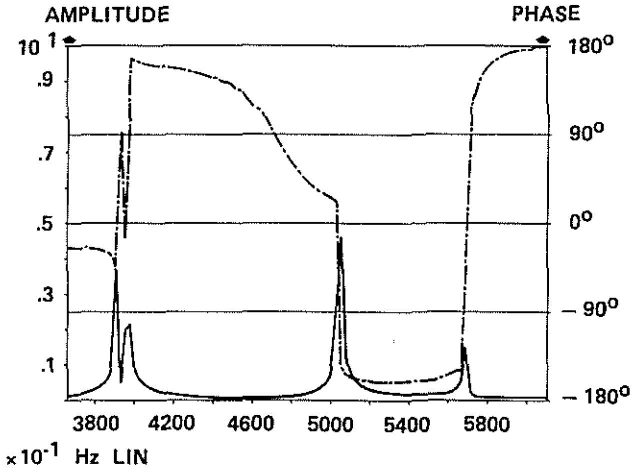

FIG. 3 : TRANSFER FUNCTION

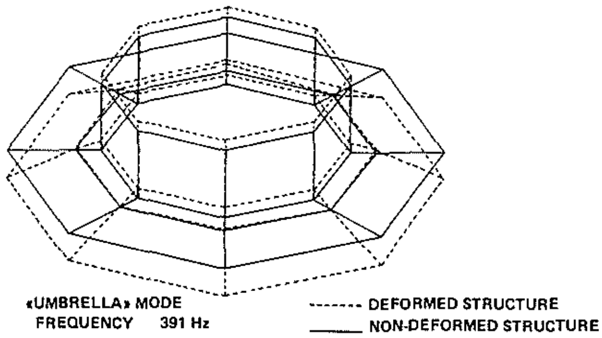

FIG. 4 : MODE SHAPE No. 1

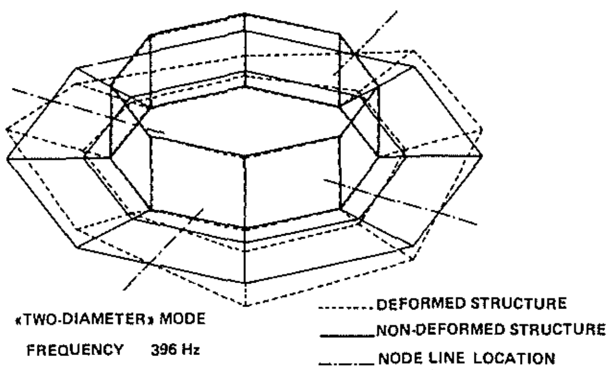

FIG. 5 : MODE SHAPE No. 2

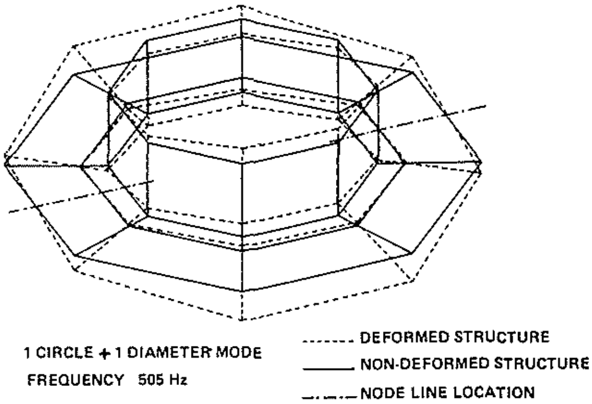

FIG. 6 : MODE SHAPE No. 3

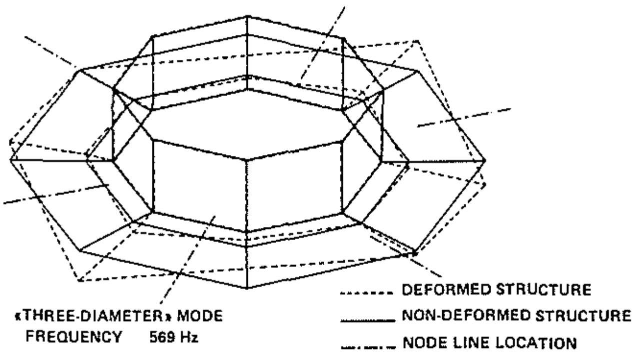

FIG. 7 : MODE SHAPE No. 4

III.2. - Structure No.2 : Ecureuil tanks

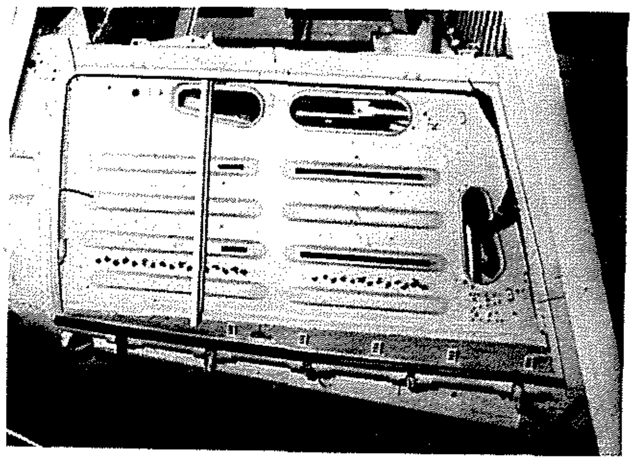

FIG. 8 : ECUREUIL TANKS

Purpose of study: investigate experimentally the dynamic characteristics of tanks (mainly frequencies and mode shapes) in several configurations knowing however that a "calculation" modelization is hard to achieve given the difficulty for setting up a hydrodynamic model of the fluid.

Choice of excitation technique: the presence of fluid within the tanks - generating substantial damping in the system hence to excitation energy problems that must be solved - and the small frequency cover sought (5 Hz) make us select sinusoidal excitation.

Excitation means: suspended flapping mass.

Excitation point: this point has been chosen on the LH rear spar which corresponds to the load input plane.

Number of measuring points: 46

Analysis parameters:
F MIN = 18 Hz
F MAX = 23 Hz
Δ f = 0.1 Hz

Modes identified (tank 100% full)

Frequency	Characteristics
Initial configuration	
19 Hz	tank bottoms:180° phase-lag
22.1 Hz	tank bottoms in phase
Final configuration	
20 Hz	tank bottoms:180° phase-lag
21.4 Hz	tank bottoms in phase

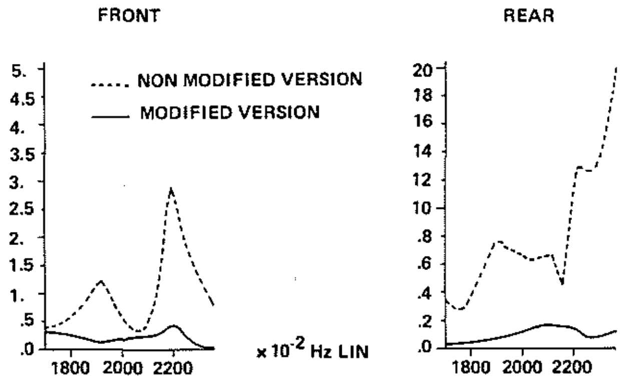

FIG. 9 : TANKS No. 3

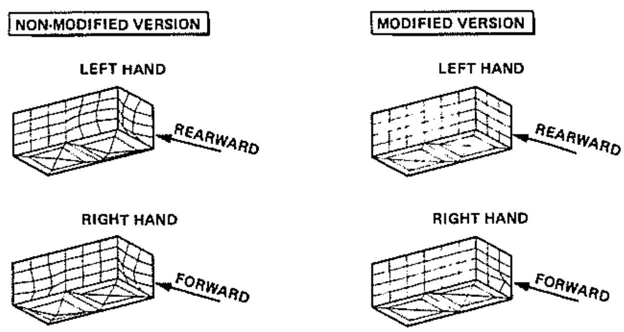

FIG. 10 : TANK No. 1

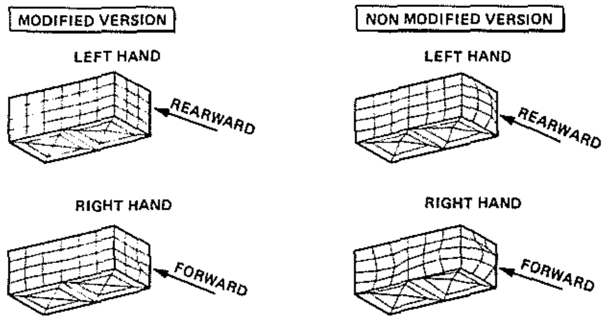

FIG. 11 : TANK No. 2

III.3 - Structure No.3 : Ecureuil structure

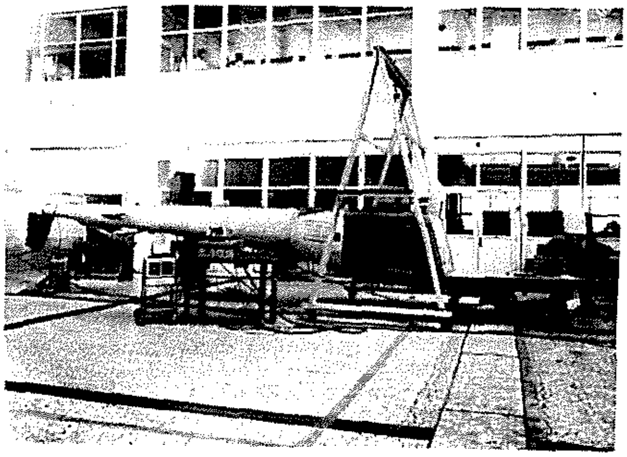

FIG. 12 : ECUREUIL STRUCTURE

Purpose of study: identify a geometrically complex system. This identification serves as a basis for a calculation-experimentation correlation.

Choice of excitation technique: given the great number of measuring points, the low structural damping and especially the high frequency resolution (i.e. a long acquisition time) the best compromise is a random excitation.

Excitation means: an electro-dynamic vibrator.

Analysis parameters:

Δ f = 0.1 Hz
f MAX = 102.4 Hz
Δ t = 10/2048 s
T = 10 s
N = 2048 pts

Main modes identified:

	Frequency	Remarks
Mode 1 "	22.8 Hz "	Fwd bottom struct. torsion
Mode 2	26.1 Hz	"Two-node" mode
Mode 3	42.8 Hz	"Three-node" mode

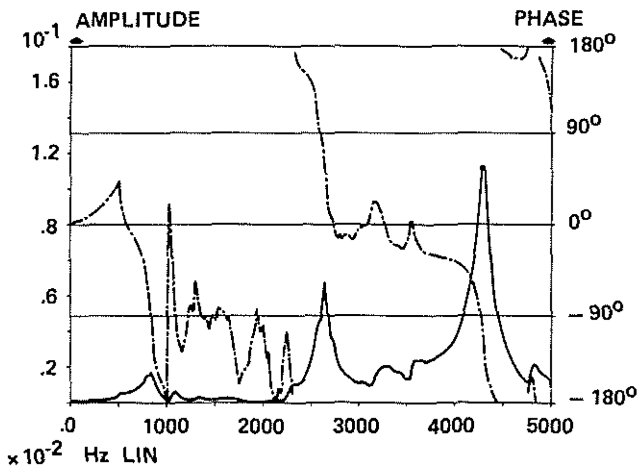

FIG. 13 : ECUREUIL STRUCTURE

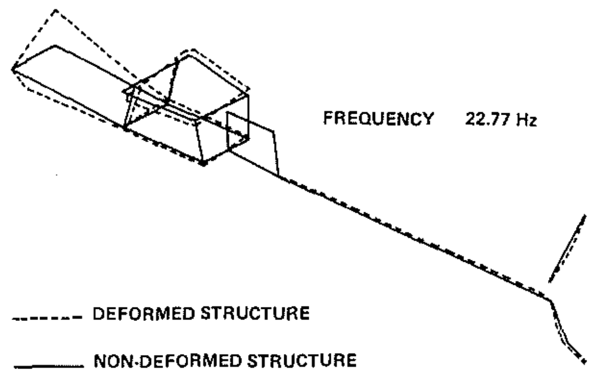

FIG. 14 : ECUREUIL MODE No. 1

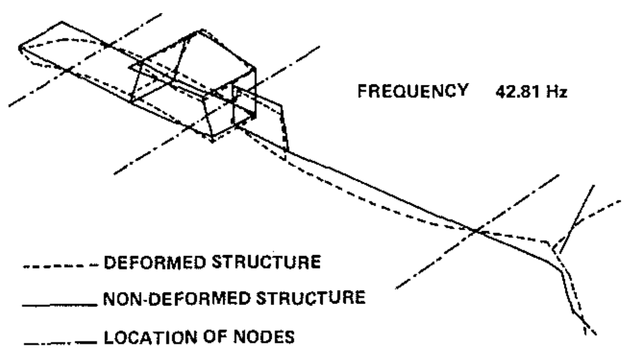

FIG. 15 : ECUREUIL MODE No. 2

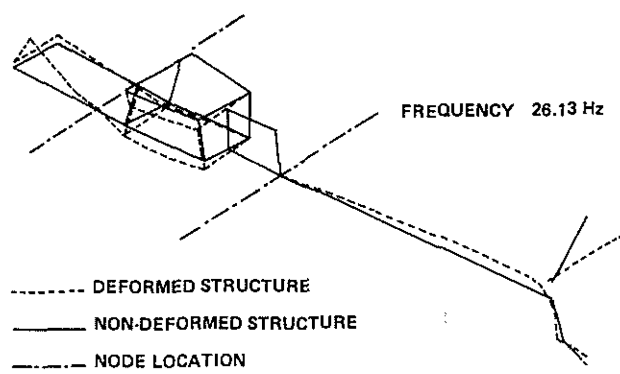

FIG. 16 : ECUREUIL MODE No. 3

CONCLUSION AND DEVELOPMENTS

- The development phase of experimental modal analysis is now over and the structures studied evidenced its rapidity (as concerns measurements), accuracy (in natural frequency evaluation) and results synthetizing capabilities (mode shape animation)

- The identification technique will be developed in three main directions:

  - Multiexcitation approach (closer to physical reality)
  - Development of new methods increasing coherence between experimental and designed modal vectors
  - Allowance for a more accurate approach to modal damping

However, modal analysis remains nothing but a BASIC TOOL and large efforts are devoted to the study of its derivatives, sensitivity analysis and modal synthesis * which are DECISION MAKING TOOLS.

The experience gained shows that, if it is applied to helicopter production problems, modal analysis turns out to be an indispensable tool for:

1. evaluating rapidly the importance of a dynamic problem thanks to the ease of processing and presenting the results obtained from impulse or random excitations (time saving on structural tests as compared to appropriation methods can be 1 to 10).

2. obtaining more easily a rapid estimate of modal characteristics (generalized mass, damping). It must be reminded that a very careful appropriation and micro-scanning are necessary requirements for obtaining such results in conventional cases.

3. assisting opinion-forming and decision-making through the process of identification between theoretical models (finite elements) and test results: the model is becoming richer thanks to the easier interpretation of test results and engineers can test smoothing models on test data compatible with their approach to the problem to be solved.

However, special attention must be granted to non-linearity aspects of dynamic problems as impulse or random excitations are always weak and do not easily evidence threshold problems. Preference will then be given to those simulation techniques which bring engineers closer to the actual problem conditions.
Once they have been identified, helicopter problems often boil down to a single-frequency problem which may reduce the interest of using this tool. In that case engineers may very well prefer using conventional appropriation methods.

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*Sensitivity analysis: sensitivity analysis consists of a program processing modal parameters (natural frequency, natural damping and mode shape); considering the objective defined (critical frequencies displacement for example), this analysis is used to optimize location of punctual weight, rigidity or damping modifications by calculation.
Modal synthesis: modal synthesis allows synthesizing dynamic behavior of an assembly when modal characteristics (frequency, damping, natural mode shape) of sub-assemblies from initial structure are known.