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The Influence of the Induced Velocity Distribution and the Flapping-Lagging Coupling on the Derivatives of the Rotor and the Stability of the Helicopter

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ABSTRACT

The derivatives of the rotor are very important in the research of the stability and control. Generally, the models are considered in the research of the stability of the helicopter only including the flapping motion and the simple assumption of the induced velocity distribution.

In this paper, the first harmonic variation of induced velocity distribution derived by Wang's generalized vortex theory is taken in account in detail and the coupling of the first flapping-lagging bending mode of the blade elasticity is considered.

A sample calculation of a typical helicopter has been made. The influence of the induced velocity distribution and the coupling flapping-lagging on the derivatives of the rotor and the stability of the helicopter are described and drawn in detail, expecially the derivatives of the rotor with μ parameter. All the basic mechanics expressions of the hingeless rotor are presented.

NOTATION

δ_s ——forwards incidence angle of rotor shaft

ϕ ——azimuth angle of blade

ξ_e ——lagging angle of blade

β_e ——— flapping angle of blade

α_s ——— helicopter angle of attack

β_s ——— helicopter side slip angle

β_* ——— inflow angle

$\varnothing_*$ ——— incidence of the blade

$\varnothing_7$ ——— blade collective pitch (blade pitch angle at 0.7 radius)

$\Delta\varnothing$ ——— blade twist angle

θ_c, θ_s ——— lateral and longitudinal cyclic blade pitch

$$\varnothing_* = \theta_0 + \theta_c \cos\psi + \theta_s \sin\psi$$

$$\theta_0 = \varnothing_7 + \Delta\varnothing(z-0.7)$$

u_e, v_e ——— displacement of the blade in the X_3, Y_3 directions respectively

R ——— radius of the rotor

r ——— location of the blade element in the Z_3 directions $z = \frac{r}{R}$

b ——— rotor blade chord

$m(z)$ ——— mass of the blade per unit length

K ——— Number of blades

σ ——— solidity of rotor

V_0 ——— flight velocity of the helicopter

V_H ——— velocity of the hub center

V_i ——— induced velocity

$$v_i = \frac{V_i}{\Omega R} = v_0 + v_{ic} \cos\psi + v_{is} \sin\psi$$

v_{i0} ——— the average nondimensional induced velocity

μ ——— advanced ratio, $\mu = V_{Hx} / \Omega R = \mu_x$

$-\lambda_0$ ——— inflow ratio, $-\lambda_0 = V_{Hy} / \Omega R = \mu_y$

λ_z ——— $\lambda_z = \mu_z = V_{Hz} / \Omega R$

Ω ——— rotational speed of the rotor

W ——— resultant velocity of the blade element

W_{x3} ——— chordwise componets of relative wind, positive in negative X_3 axis

W_{y_3} —— component of relative wind normal to the disk plane, positive in negative y_3 axis

ω_x, ω_y, ω_z — angular velocity of the helicopter

N —— centrifugal tension in the blade, $N = \int_r^R m r \Omega^2 dr$

T, H, S, —— rotor thrust, drag, side force and coefficients of the rotor
C_T, C_H, C_s force respectively

C_y^{α}, a_∞ —— lift curve slope of the blade

C_{xp}, CC —— average blade profile drag coefficient

M_{Hx}, M_{Hz} —— rotor hub rolling and pitchingmoment respectively

M_{Hy}, M_K —— Mk is the rotor torgue $M_{Hy} = -M_K$ (profile)

Γ —— circulation

$\varkappa$ —— modified coefficient of lift for tip loss

ρ_A —— air density

EI_β, EI_ξ —— bending stiffness of blade section in flapwise, chordwise respectively

EC —— thrust factor, $EC = \frac{1}{2}\rho_A \pi R^2 (\Omega R)^2$

f_β, f_ξ —— the first mode shape of rotor blade for flapwise, chordwise respectively

f'_β, f'_ξ —— the derivatives of the mode shape

λ_β, λ_ξ —— nondimensional natural frequency of vibration of the first mode of flapwise, chordwise respectively $\lambda_\beta = \omega_\beta / \Omega$, $\lambda_\xi = \omega_\xi / \Omega$

Subcripts

0 —— Constant item

C —— Cosine item

S —— sine item

β —— flapwise (flapping)

ξ —— chordwise (lagging)

H —— hub

1. Introduction

The derivations of rotor forces, hub moments and aerodynamics derivatives are necessary and important in study the stability and control of helicopter with hingeless rotor. Generally, in study the stability and control, the simplified assumption of induced velocity and the flapping elastic bending are considered.

For helicopters the induced flow field is a quite important as well as a complicated problem. The effect of nonuniformity of induced velocity distribution on stability and control of the helicopter with hingeless rotor is more important and must be taken into account. In this paper, the induced velocity distribution derived by Wang's vortex theory Ref.(1) has been used, which has a closed form of equations for circulation and induced velocity.

Generally the lagging motopn is neglected for the studies of stability and control of the helicopter. One of the conclusions is that the lagging motion must be considered in study the stability and control for helicopter with hingeless rotor (2).

In this paper, considering the elastic coupling in flapping and lagging and the nonuniformity of induced velocity distribution, the derivatives of hingeless rotor forces and hub moments are derived and the stability roots of the helicopter are obtained. A sample calculation of a typical helicopter has been made. The influence of the induced velocity distribution and the coupling flapping-lagging on the derivatives of the rotor and the stability of the helicopter is described and drawn in detail, expecially the derivatives of the rotor with μ parameter.

The expressions of the induced velocity distribution included the factors of the coupling in flapping and lagging and the other basic expressions are presented. The original deta, equilibrium data and the basic expressions of derivatives of the helicopter body, horizontal plane and the tail rotor are according to the ref (3). The derivatives values simplified by the ref.(4) agree well with the ref.(4) at the speed of μ=0.2.

2. Expressions

2-1 The Aerodynamic Force of Blade Element

It is assumed the hingeless rotor blades have flexbility only in flapwise direction and lagwise direction and is infinitely in torsional direction.

The transformation matrices of axis system can be expressed as.

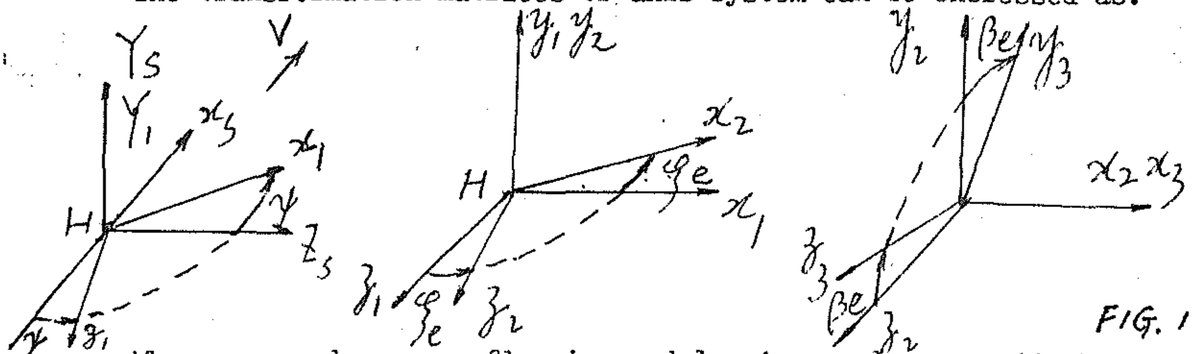


FIG. 1

Where and are flapping and lagging angle respectively,

$$\sin\beta_e = \frac{\partial v_e}{\partial r}, \quad \cos\beta_e = 1, \quad \sin\xi_e = \frac{\partial u_e}{\partial r}, \quad \cos\xi_e = 1$$

In this way, the following transformation matrix is given (5)

$$\begin{Bmatrix} i \\ j \\ k \end{Bmatrix} = \begin{Bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{Bmatrix} \begin{Bmatrix} i_3 \\ j_3 \\ k_3 \end{Bmatrix} \qquad (2-1-1)$$

$$D_{11} = \sin\psi + \frac{\partial u_e}{\partial r}\cos\psi, \quad D_{12} = \frac{\partial v_e}{\partial r}\cos\psi,$$

$$D_{13} = -\cos\psi + \frac{\partial v_e}{\partial r}\sin\psi, \quad D_{21} = 0, \quad D_{22} = 1, \quad D_{23} = \frac{\partial v_e}{\partial r}$$

$$D_{31} = \cos - \frac{\partial u_e}{\partial r}\sin\psi, \quad D_{32} = -\sin\frac{\partial v_e}{\partial r}, \quad D_{33} = \sin\psi + \frac{\partial v_e}{\partial r}\cos\psi$$

In calculated the airloads on the blades, the other following assumptions are made:

(a) Stall, compressibility and reversed flow effects are neglected. Thus, the aerodynamic loads are valid for $\mu \leqslant 0.3$

(b) The blades are untapered

(c) The induced velocity thru the rotor disc is assumed to have a frist harmonic variation around the rotor azimuth, i.e.

$$v_1 = v_0 + v_{1c}\cos\psi + v_{1s}\sin\psi$$

(d) The profile drag coefficient and lift curve slope are assumed constant.

(e) Tip losses are modified by introducing the coefficient

The deformation of flapping-lagging of the blade element can be written as:

$$u_e = Rf_\xi \left(\xi_0 + \xi_c \cos\psi + \xi_s \sin\psi\right)$$

$$v_e = Rf_\beta \left(\beta_0 + \beta_c \cos\psi + \beta_s \sin\psi\right)$$

The expressions of the velocity in cross section of blade are following: (7)

$$W_{x_5} = \Omega r + u_e + \left(\mu_x \frac{\partial u_e}{\partial r} + \mu_y + \omega_x v_e\right)\cos\psi + \left(\mu_x - \mu_z \frac{\partial u_c}{\partial r} - \omega_z v_c\right)\sin\psi$$

$$W_{y_5} = v_0 + v_{1c}\cos\psi + v_{1s}\sin\psi + \mu_y + v_e + \left(\mu_x \frac{\partial v_e}{\partial r} - \omega_x u_e - \omega_z r\right)\cos\psi + \left(-\mu_z \frac{\partial v_c}{\partial r} - \omega_x r + \omega_z u_e\right)\sin\psi \quad (2\text{—}1\text{—}2)$$

$$\begin{aligned} \mu_x &= \overline{V}_{ox} + \overline{\omega}_y z_H - \overline{\omega}_z y_H \\ \mu_y &= \overline{V}_{oy} + \overline{\omega}_z x_H - \overline{\omega}_x z_H \\ \mu_z &= \overline{V}_{oz} + \overline{\omega}_x y_H - \overline{\omega}_y x_H \end{aligned} \quad (2\text{—}1\text{—}3) \qquad \begin{aligned} \overline{V}_{ox} &= \overline{V}_o \cos\beta_s \cos\alpha_s \\ \overline{V}_{oy} &= \overline{V}_o \cos\beta_s \sin\alpha_s \\ \overline{V}_{oz} &= \overline{V}_o \sin\beta_s \end{aligned} \quad (2\text{—}1\text{—}4)$$

According to Joukowskis theorem and blade element thery we can obtain the expression:

$$\overline{\Gamma} = \frac{C_y^\alpha \overline{b}}{2}\left(W_{x_5}\varnothing_* - W_{y_5}\right) = \overline{\Gamma}_0 + \overline{\Gamma}_c \cos\psi + \overline{\Gamma}_s \sin\psi \qquad (2\text{—}1\text{—}5)$$

$$\begin{aligned}
\overline{\Gamma}_0 = \tfrac{1}{2} C_y^\alpha \overline{b}\Big[&\theta_0\left(z + \tfrac{1}{2}\overline{\omega}_x \beta_s f_\beta - \tfrac{1}{2}\mu_z \xi_s f'_\xi - \tfrac{1}{2}\overline{\omega}_x \beta_c \beta_\beta + \tfrac{1}{2}\mu_x \xi_c f'_\xi\right) + \tfrac{1}{2}\theta_s\left(\mu_x - \overline{\omega}_z \beta_0 f_\beta + \mu_z \xi_0 f'_\xi - \xi_c f_\xi\right) \\
&+ \tfrac{1}{2}\theta_c\left(\mu_z + \overline{\omega}_x \beta_0 f_\beta - \mu_x \xi_0 f_\xi + \xi_s f_\xi\right) - \overline{v}_0 - \mu_y \\
&- \tfrac{1}{2}\left(\overline{\omega}_z \xi_s f_\xi + \overline{\omega}_z \beta_s f'_\beta - \overline{\omega}_x \xi_D f_\xi - \mu_x \beta_c f'_\beta\right)\Big] + \Delta\overline{\Gamma}_0
\end{aligned}$$

$$\begin{aligned}
\overline{\Gamma}_{1c} = \tfrac{1}{2} C_y^\alpha \overline{b}\Big[&\theta_0\left(\mu_z + \overline{\omega}_x \beta_0 f_\beta - \mu_x \xi_0 f'_\xi + \xi_s f_\xi\right) \\
&+ \tfrac{1}{4}\theta_s\left(\overline{\omega}_z \beta_c f_\beta - \mu_z \xi_0 f'_\xi - \overline{\omega}_x \beta_s f_\beta + \mu_x \xi_s f'_\xi\right) \\
&+ \frac{1}{4}\theta_c\left(4z + \overline{\omega}_z \beta_s f_\beta - \mu_z \xi_s f'_\xi - z\overline{\omega}_x \beta_c f_\beta + 3\mu_x \xi_c f'_\xi\right) \\
&- \overline{v}_{1c} - \left(-\overline{\omega}_z z + \overline{\omega}_x \xi_0 f_\xi + \mu_x \beta_0 f'_\beta - \beta_s f_\beta\right)\Big] + \Delta\overline{\Gamma}_{1c}
\end{aligned}$$

$$\begin{aligned}
\overline{\Gamma}_{1s} = \tfrac{1}{2} C_y^\alpha \overline{b}\Big[&\theta_0\left(\mu_x - \overline{\omega}_z \beta_0 f_\beta + \mu_z \xi_0 f'_\xi - \xi_c f_\xi\right) \\
&+ \tfrac{1}{4}\theta_s\left(4z + 3\overline{\omega}_z \beta_s f_\beta - 3\mu_z \xi_s f'_\xi - \overline{\omega}_x \beta_c f_\beta + \mu_x \xi_c f'_\xi\right) \\
&+ \tfrac{1}{4}\theta_c\left(\overline{\omega}_z \beta_c f_\beta - \mu_z \xi_0 f'_\xi - \overline{\omega}_x \beta_s f_\beta + \mu_x \xi_s f'_\xi\right) \\
&- \overline{v}_{1s} - \left(-\overline{\omega}_x z - \overline{\omega}_z \xi_0 f_\xi - \mu_z \beta_0 f'_\beta + \beta_0 f_\beta\right)\Big] + \Delta\overline{\Gamma}_{1s} \qquad (2\text{—}1\text{—}6)
\end{aligned}$$

which $\Delta\overline{\Gamma}_0$, $\Delta\overline{\Gamma}_{1c}$, and $\Delta\overline{\Gamma}_{1s}$ are nonlinear terms appendix(1)

Based on Wang's generalized vortex theory Ref.(5) and limited to first harmonics, the relation of induced velocity and circulation can be written as:

$$\overline{v}_0=\frac{k}{4\pi v_1}\overline{\Gamma}_0$$

$$\overline{v}_{1c}=k\left[\left(\frac{2\cos\alpha_1}{1+\sin\alpha_1}\right)\left(-\frac{2}{3}+z+\frac{1}{2}z^2\right)\overline{\Gamma}_0+\left(\frac{2\sin\alpha_1}{1+\sin\alpha_1}\right)\overline{\Gamma}_{1c}\right]/4\pi v_1$$

$$\overline{v}_{1s}=k\left[\left(\frac{2\cos\alpha_1}{1+\sin\alpha_1}\right)v_1(3-3z+z^2)\overline{\Gamma}_0+\frac{2}{1+\sin\alpha_1}\overline{\Gamma}_{1s}\right]/4\pi v_1 \qquad (2\text{-}1\text{-}7)$$

$v_1=(\mu_y^2+\mu_x^2)^{\frac{1}{2}}$; $\mu_y+\overline{v}_{10}=(-\lambda)$

$\alpha_1=(-\alpha_0)+\varepsilon$

$\overline{v}_{10}=C_T/4v_1\varkappa$

$v_1\sin\alpha_1=-\lambda$

$v_1\cos\alpha_1=\mu_x$

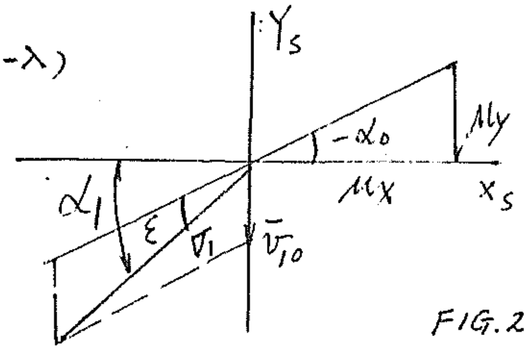


FIG.2

$$\overline{v}_1\doteq D_{0c}+D_{0\beta_0}\beta_0+D_{0\beta c}\beta_c+D_{0\beta s}\beta_s+D_{0\xi 0}\xi_0+D_{0\xi c}\xi_c+D_{0\xi}\xi_s+\Delta\overline{v}_0$$

$$\overline{v}_{1c}\doteq D_{1c0}+D_{1c\beta}\beta_0+D_{1c\beta c}\beta_c+D_{1c\beta s}\beta_s+D_{1c\xi 0}\xi_0+D_{1c\xi c}\xi_c+D_{1c\xi s}\xi_s+\Delta\overline{v}_{1c}$$

$$\overline{v}_{1s}=D_{1sc}+D_{1s\beta_0}\beta_0+D_{1s\beta c}\beta_c+D_{1s\beta s}\beta_s+D_{1s\xi 0}\xi_0+D_{1s\xi c}\xi_c+D_{1s\xi s}+\Delta\overline{v}_{1s} \qquad (2\text{-}1\text{-}8)$$

The expressions of D_{0C}, $D_{0\beta 0}$ etc and $\Delta\overline{v}_0$, $\Delta\overline{v}_{1c}$, $\Delta\overline{v}_{1s}$ are presented in the appendix(2)

According to the blade element theory, the aerodynamic forces acting on the blade are obtained as follows:

$$dY=\tfrac{1}{2}C_y^{\alpha}\rho_A b(\varnothing_* W_x^2-W_x W_y)dr \qquad (2\text{-}1\text{-}9)$$

$$dX=\tfrac{1}{2}\rho_A bC_{xp}W_x^2 dr$$

dx, dy : drag and lift of the blade respectively

$$dF_{Ax_3}=dx\cdot\cos\beta_*-dy\cdot\beta_* \qquad (2\text{-}1\text{-}10)$$

$$dF_{Ay_3}=dx\cdot\beta_*+dy\cos\beta_*$$

$$\frac{dF_{Ay_3}}{dr}=\frac{dF_{Ay_30}}{dr}+\frac{dF_{Ay_3c}}{dr}\cos\psi+\frac{dF_{Ay_3s}}{dr}\sin\psi$$

$$\frac{dF_{Ax_3}}{dr}=\frac{dF_{Ax_30}}{dr}+\frac{dF_{Ax_3c}}{dr}\cos\psi+\frac{dF_{Ax_3s}}{dr}\sin\psi \qquad (2\text{-}1\text{-}11)$$

2-2 The Inertia Force of Blade Element

Similarly, the acceleration of any point on the blades can be written as.

(1) V_{HUB} , (2) $\Omega \times (\Omega \times (x_1 i_1 + y_1 j_1 + z_1 k_1)$

(3) $\omega \times (x_1 i_1 + y_1 j_1 + z_1 k_1)$

(4) $2\omega \times (\Omega \times (x_1 i_1 + y_1 j_1 + z_1 k_1))$

(5) $2(\Omega + \omega) \times (x_1 i_1 + y_1 j_1 + z_1 k_1)$

(6) $x_1 i_1 + y_1 j_1 + z_1 k_1$ (7) $\omega \times (\omega \times (x_1 i_1 + y_1 j_1 + z_1 k_1))$

Finally inertial contribution to the external forces can be expressed as

$$\frac{dF_{Nx_1}}{dr} = \frac{dF_{Nx_{10}}}{dr} + \frac{dF_{Nx_{1c}}}{dr}\cos\psi + \frac{dF_{Nx_{1s}}}{dr}\sin\psi$$

$$\frac{dF_{Ny_1}}{dr} = \frac{dF_{Ny_{10}}}{dr} + \frac{dF_{Ny_{1c}}}{dr}\cos\psi + \frac{dF_{Ny_{1s}}}{dr}\sin\psi \qquad (2\text{-}2\text{-}1)$$

$$\frac{dF_{Nz_1}}{dr} = \frac{dF_{Nz_{10}}}{dr} + \frac{dF_{Nz_{1c}}}{dr}\cos\psi + \frac{dF_{Nz_{1s}}}{dr}\sin\psi$$

2-3 Derivation of Flapping Motion and Lagging Motion

The differential equation of the blade flapping bending is given by.

$$\frac{\partial^2}{\partial r^2}\left[EI\frac{\partial^2 v_e}{\partial r^2}\right] - \frac{\partial}{\partial r}\left[N\frac{\partial v_e}{\partial r}\right] + m\frac{\partial^2 v_e}{\partial t^2} = \frac{\partial F_y}{\partial r} \qquad (2\text{-}3\text{-}1)$$

The solution of the elastic flapwise bending equation (2-3-1) is

$$v_e = Rf_\beta(\beta_0 + \beta_c\cos\psi + \beta_s\sin\psi)$$

Utilizing the method of separation of variables and the property of orthogonality of the mode shape, then veilds the following equation:

$$\lambda_\beta^2\beta_0 - (\lambda_\beta^2 - 1)\beta_c\cos\psi - (\lambda_\beta^2 - 1)\beta_s\sin\psi$$

$$= \left(\Omega^2R^2\int_0^1 mf_\beta^2\,dz\right)^{-1}\int_0^1\frac{\partial F_y}{\partial z}f_\beta\,dz \qquad (2\text{-}3\text{-}2)$$

Similarly, the lagging equation is

$$\lambda_\xi^2\xi_0 - (\lambda_\xi^2 - 1)\xi_c\cos\psi - (\lambda_\xi^2 - 1)\xi_s\sin\psi$$

$$= \left(\Omega^2R^2\int_0^1 mf_\xi^2\,dz\right)^{-1}\int_0^1\frac{\partial F_x}{\partial z}f_\xi\,dz \qquad (2\text{-}3\text{-}3)$$

in which

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_{Ay_3}}{\partial z} + \frac{\partial F_{Ny_3}}{\partial z}, \quad \frac{\partial F_x}{\partial z} = \frac{\partial F_{Ax_3}}{\partial z} + \frac{\partial F_{Nx_3}}{\partial z} \qquad (2\text{-}3\text{-}4)$$

The equations yield a set of algebraic equations upon collecting constant and coefficients of sin ψ and cos ψ . Then the coefficients of motion $\beta_0, \beta_c, \beta_s, \varphi_0, \varphi_c$ and φ_s may be obtained.

2-4 Rotor Forces and Hub Moments

Rotor thrust, drag and side force are the components along the rotor shaft axis Ys, -Xs and Zs respectively.

Their differential forces are as:

$$dT = \frac{\partial F_{Ays}}{\partial r} dr$$

$$-dH = (\sin\psi + \frac{\partial u_e}{\partial r}\cos\psi)\frac{\partial F_{Axs}}{\partial r}dr + (\frac{\partial v_e}{\partial r}\cos\psi)\frac{\partial F_{Ays}}{\partial r}dr \qquad (2\text{-}4\text{-}1)$$

$$ds = (\cos\psi - \frac{\partial u_e}{\partial r}\sin\psi)\frac{\partial F_{Axs}}{\partial r}dr + (-\frac{\partial v_e}{\partial r}\sin\psi)\frac{\partial F_{Ays}}{\partial r}dr$$

Rotor hub moments are very important parameters for hingeless rotor. Their differential moments are:

$$d\vec{M}_0 = \vec{r} \times d\vec{F} = dM_{Hx}\,\vec{i} + dM_H\,\vec{j} + dM_{Hz}\,\vec{k} \qquad (2\text{-}4\text{-}2)$$

in which

$$dM_{Hx} = [v_e dF_{Nx1} - r(dF_{Ays} + dF_{ny1})]\sin\psi - [u_e(dF_{Ays} + dF_{ny1}) - v_e(-dF_{Axs} + dF_{nx1})]\cos\psi$$

$$dM_{Hy} = r(-dF_{Axs} + dF_{nx1}) - u_e dF_{nx1}$$

$$dM_{Hz} = [v_e dF_{nx1} - r(dF_{Ays} + dF_{ny1})]\cos\psi + [u_e(dF_{Ags} + dF_{ny1}) - v_e(-dF_{Axs} + dF_{nx1})]\sin\psi$$

2-5 The Derivatives of the Rotor Forces and Hub Moments

In the calculation process, whether to take account of the terms of the small quantity is an important problem. In this paper, including all the terms of the small quantity and utilizing the method of the numeral value derivation, the derivatives of the rotor forces and hub moments are obtained.

The advantage of this method is avoiding the complex derivation of the forces, moments and motion coefficients derivatives.

Let the function f(x) can be derived, the derivative of f(x) may be obtained according to the following expression:

$$f'(x) = \frac{4T_2 - T_1}{3} + [\frac{8}{5!}(\frac{h}{4})^4 - \frac{2}{5!}(\frac{h}{2})^4]f^{(5)}(x)$$

$$T_1 = [f(x + \frac{h}{2}) - f(x - \frac{h}{2})]/h \qquad (2\text{-}5\text{-}1)$$

$$T_2 = [f(x + \frac{h}{4}) - f(x - \frac{h}{4})]/\frac{h}{2}$$

The value of h can be determined in terms of precision. In this paper, h value is about 10^{-2} to 10^{-4}.

3. Sample calculation

A Sample calculation of a typical helicopter has been done. The original data of an example helecopter are obtained from various references and measured results. The trim data are obtained from Ref(3). The derivatives of the helicopter with hingeless rotor in hovering and forward flight with different speed have been calculated. The influence of nonuniformity of induced flow field and elastic coupling in flapwise and chordwise on the derivatives of the rotor and the stability are taken into account simultaneously. Four cases have been calculated in the example.

First case: The induced velocity distribution and coupling in flapping-lagging are neglected.

Second case: The coupling is neglected, the induced velocity distribution is considered.

Third case: The elastic coupling is considered and the induced velocity distribution is neglecled.

Fourth case: Both are considered.

The diagrams of the aerodynamic derivatives of the rotor forces and hub moments and the table of the roots of the helicoptef dynamic stability are presented in the appendix(3).

4. Conclusions

(1) The induced velocity distribution has a great effect on the coefficients of the flapping and lagging (respect to the μ_x, μ_y and μ_z), expecially the low speed range.

(2) The lagging motion has a great effect on the aerodynamic derivatives of the coefficients of the flapping-lagging motion at the low speed range.

(3) In the flow field of the induced velocity, the lagging has a moderate effect on the derivatives of the flapping coefficient, but the lagging has a great effect on the aerodynamic derivatives of the rotor forces and hub moments.

(4) If the flow field of the induced velocity is neglected, the lagging motion has a moderate effect on the aerodynamic derivatives. But if the flow field of the induced velocity is considered, the lagging has a great effect on the derivatives.

(5) As far as the roots of the dynamic stability, whether the coupling is considered or not, the induced velocity distribution has a great effect on the roots of the dynamic stability.

(6) The influences of the induced velocity distribution on the roots of the stability are greater than the influences of the lagging motion.

(7) Studyning the problems of the aerodynamic character and derivatives, it is necessary to consider the induced velocity distribution field and the lagging motion simultaneously.

But studing only the stability for the engineering the induced velocity distribution should be considered. the lagging motion whether to he considered is deponded on the precision required.

(8) In this paper the antitorque is calculated using two methods, one is the projection of the hub moment on the y axis, another method is used according to the general experiment formula of antitorque.

The values of the antitorque calculated by two methods respectively are equivalent, but the derivatives of the antitorques are different in values.

(9) Simplifying the derivatives according to the (ref.4), i.e., the second case, and comparing with the (ref.4), a good agreement is obtained.

$$\Delta\overline{\Gamma}_0=\frac{1}{2}C_y^{\alpha}\bar{b}[DF(F_0+\frac{1}{2}(FD_3+FD_4))+\theta_a(\frac{1}{2}FD_2+\frac{3}{8}FG_2+\frac{1}{8}FG_4)$$

$$+\theta_s(\frac{1}{2}FD_1+\frac{3}{8}FG_1+\frac{1}{8}FG_3)-FG_5-\frac{1}{2}(FD_6+FD_7)]$$

$$\Delta\overline{\Gamma}_a=\frac{1}{2}C_y^{\alpha}\bar{b}[DF(FD_2+\frac{3}{4}FG_2+\frac{1}{4}FG_4)+\theta_a(F_0+\frac{1}{4}FD_3+\frac{3}{4}FD_4)$$

$$+\frac{1}{4}FD_5\theta_s-FD_8-\frac{3}{4}FG_7-\frac{1}{4}FG_9]$$

$$\Delta\overline{\Gamma}_s=\frac{1}{2}C_y^{\alpha}\bar{b}[DF(FD_1+\frac{3}{4}FG_1+\frac{3}{4}FG_3)+\theta_a\frac{1}{4}FD_5+\theta_s(F_0+\frac{3}{4}FD_3$$

$$+\frac{1}{4}FD_4)-FD_9-\frac{3}{4}FG_6-\frac{1}{4}FG_8]$$

其中 $DF=\varnothing_7+\Delta\varnothing(z-0.7)$

$$FD_1=-2f_\xi f_\xi'\xi_0\xi_s-2f_\beta'^2 z\beta_0\beta_s-\omega_x f_\beta f_\xi'\xi_0\beta_0$$

$$FD_2=-2f_\xi f_\xi'\xi_0\xi_a-2f_\beta'^2 z\beta_0\beta_a-\omega_x(f_\beta f_\xi'\xi_0\beta_0-zf_\beta' f_\xi'\xi_0\beta_0)$$

$$FD_3=f_\xi f_\xi'\xi_s^2+f_\beta'^2 z\beta_s^2+\omega_x f_\beta f_\xi'(\xi_0\beta_s+\beta_0\xi_s)$$

$$FD_4=f_\xi f_\xi'\xi_a^2+zf_\beta'^2\beta_a^2+\omega_x(f_\beta f_\xi'-zf_\beta' f_\xi')(\xi_0\beta_a+\beta_0\xi_a)$$

$$FD_5=2f_\xi f_\xi'\xi_a\xi_s+2zf_\beta'^2\beta_a\beta_s-\omega_x(zf_\beta' f_\xi'-f_\beta f_\xi')(\beta_0\xi_s+\xi_0\beta_s)$$

$$+\omega_x f_\beta f_\xi'(\xi_0\beta_a+\beta_0\xi_a)$$

$$FD_6=\omega_x(2f_\beta f_\beta'\beta_0\beta_s+zf_\xi'^2 2\xi_0\xi_s)+(f_\xi f_\beta'-zf_\beta' f_\xi')\beta_s\xi_s$$

$$FD_7=\omega_x(f_\beta f_\beta' 2\beta_0\beta_a+2zf_\xi'^2\xi_0\xi_a)+(f_\xi f_\beta'-zf_\beta' f_\xi')\beta_a\xi_a$$

$$FD_8=-\omega_x(zf_\xi'^2\xi_0^2+f_\beta f_\beta'\beta_0^2)+(zf_\beta' f_\xi'-f_\xi f_\beta')(\beta_0\xi_a+\xi_0\beta_a)$$

$$FD_9=-\omega_x(f_\beta f_\beta'\beta_0^2+zf_\xi'^2\xi_0^2)+(zf_\beta' f_\xi'-f_\xi f_\beta')(\xi_0\beta_s+\beta_0\xi_s)$$

$$F_0=f_\xi f_\xi'\xi_0^2+zf_\beta'^2\beta_0^2,\qquad FG_1=-\omega_x f_\beta f_\xi'\xi_s\beta_s,$$

$$FG_2=-\omega_x(f_\beta f_\xi'-zf_\beta' f_\xi')\xi_a\xi_a,$$

$$FG_3=-\omega_x(f_\beta f_\xi'-zf_\beta' f_\xi')(\xi_a\beta_s+\xi_s\beta_a)-\omega_x f_\beta f_\xi'\xi_a\beta_a$$

$$FG_4=-\omega_x(f_\beta f_\xi'-zf_\beta' f_\xi')\xi_s\beta_s-\omega_x f_\beta f_\xi'(\xi_a\beta_s+\xi_s\beta_a)$$

$$FG_5=(f_\xi f_\beta'-zf_\beta' f_\xi')\xi_0\beta_0,\qquad FG_6=-\omega_x(f_\beta f_\beta'\beta_s^2+zf_\xi'\xi_s^2)$$

$$FG_7=-\omega_x(f_\beta f_\beta'\beta_a^2+zf_\xi'^2\xi_a^2),$$

$$FG_8=-\omega_x(f_\beta f_\beta'\beta_a^2+zf_\xi'^2\xi_a^2)-\omega_x(2\beta_a\beta_s f_\beta f_\beta')$$

$$FG_9=-\omega_x(2f_\beta f_\beta'\beta_a\beta_s+2zf_\xi'^2\xi_a\xi_s)-\omega_x(f_\beta f_\beta'\beta_s^2+zf_\xi'^2\xi_s^2)$$

$$D_{oc}=(A)_0\,(DF\cdot z+\frac{1}{2}\theta_c\mu_s+\frac{1}{2}\theta_c\mu_x-\mu_y)$$

$$D_{o\beta^o}=\frac{1}{2}(A)_0\,(\theta_c\omega_x-\theta_c\omega_x-\theta_s\omega_s)f_\beta,$$

$$D_{o\beta^c}=\frac{1}{2}(A)_0\,(\mu_x f'_\beta-DF\omega_x\,f_\beta)$$

$$D_{o\beta_s}=\frac{1}{2}(A)_0\,(DFf_\beta\omega_s-\mu_s f'_\beta)$$

$$D_{o\xi^o}=\frac{1}{2}(A)_0\,(\theta_c\mu_x f'_\xi-\theta_s\mu_s f'_\xi)$$

$$D_{o\xi^c}=\frac{1}{2}(A)_0\,(-DF\,\mu_x f'_\xi+\theta_s f_\xi-\omega_x f'_\xi)$$

$$D_{o\xi^s}=\frac{1}{2}(A)_0\,(-\theta_c f_\xi+DF\mu_s f'_\xi+\omega_s\,f_\xi)$$

$$D_{1\infty}=DZ(A)^0_{1c}(DFz+\frac{1}{2}\theta_s\mu_x+\frac{1}{2}\theta_c\mu_s-\mu_y)+(A)_{1c}(\theta_c z$$
$$+DF\mu_s+\omega_s\,z)+(A)^0_{1c}zz$$

$$D_{1c\beta_o}=DZ(A)^0_{1c}\frac{1}{2}\,f_\beta(\theta_c\omega_x-\theta_s\omega_s)+(A)_{1c}(DF\omega_x f_\beta-\mu_x f'_\beta)$$

$$D_{1c\beta_c}=DZ(A)^0_{1c}\cdot\frac{1}{2}\,(-DF\omega_x\,f_\beta+\mu_x f'_\beta)+(A)_{1c}(-\frac{3}{4}\theta_c\omega_x+\frac{1}{4}\theta_s\omega_s)f_\beta$$

$$D_{1c\beta^s}=DZ(A)^0_{1c}\frac{1}{2}(DF\omega_s f_\beta-\mu_s f'_\beta)+(A)_{1c}\frac{1}{4}(\theta_c\omega_s-\theta_s\omega_x+1)f_\beta$$

$$D_{1c\xi_o}=DZ(A)^0_{1c}\frac{1}{2}\,f'_\xi(\theta_c\mu_x-\theta_s\mu_s)+(A)_{1c}(DF\mu_x f'_\xi+\omega_x f_\xi)$$

$$D_{1c\xi^c}=DZ(A)^0_{1c}\frac{1}{2}(-DF\mu_x f'_\xi+\theta_s f_\xi-\omega_x f_\xi)+(A)_{1c}\,(-\frac{3}{4}\,\theta_c\mu_x$$
$$+\frac{1}{4}\theta_s\mu_s)f'_\xi$$

$$D_{1c\xi_s}=DZ(A)^0_{1c}\;\frac{1}{2}(DF\mu_s f'_\xi-\theta_c f_\xi+\omega_s f_\xi)$$
$$+(A)_{1c}(-DFf_\xi+\frac{1}{4}\theta_c\mu_s f'_\xi-\frac{1}{4}\theta_s\mu_x f'_\xi)$$

$$D_{1sc}=DE(A)^0_{1s}(DFz+\frac{1}{2}\theta_c\mu_s+\frac{1}{2}\theta_s\mu_x-\mu_y)+(A)_{1s}(\theta_s z+DF\mu_x+\omega_x z)$$

$$D_{1s\beta^o}=DE\cdot(A)^0_{1s}\frac{1}{2}f_\beta(\theta_c\omega_x-\theta_s\omega_s)+(A)_{1s}(-DF\omega_s f_\beta+\mu_s f'_\beta)$$

$$D_{1s\beta_c}=DE(A)^0_{1s}(-DF\,\omega_x f_\beta+\frac{1}{2}\mu_x f'_\beta)+(A)_{1s}(\frac{1}{4}\theta_c\omega_s-\frac{1}{4}\theta_s\omega_x-1)f_\beta$$

$$D_{1s\beta_s}=DE(A)^0_{1s}(DF\omega_s f_\beta-\frac{1}{2}\mu_s f'_\beta)+(A)_{1s}(\frac{3}{4}\theta_s\omega_s f_\beta-\frac{1}{4}\theta_c\omega_x f_\beta)$$

$$D_{1s\xi^o}=DE(A)^0_{1s}\frac{1}{2}\,f'_\xi(\theta_c\mu_x-\theta_s\mu_s)+(A)_{1s}(-DF\mu_s f'_\xi-\omega_s f_\xi)$$

$$D_{1s\xi c}=DE(A)^{0}_{1s}\frac{1}{2}(-DF\mu_x f'_\xi+\theta_s f_\xi-\omega_x f_\xi)$$

$$+(A)_{1s}(DFf_\xi+\frac{1}{4}\theta_c\mu_s f'_\xi-\frac{1}{4}\theta_s\mu_x f'_\xi)$$

$$D_{1s\xi s}=DE(A)^{0}_{1s}\frac{1}{2}(DF\mu_x f'_\xi-\theta_c f_\xi+\omega_x f_\xi)$$

$$+(A)_{1s}(\frac{3}{4}\theta_s\mu_s f'_\xi-\frac{1}{4}\theta_c\mu_x f'_\xi)$$

其中 $DF=\varnothing_7+\Delta\varnothing(z-0.7)$, $DE=3-3z+z^2$, $DZ=-\frac{2}{3}+z+\frac{1}{2}z^2$

又 $\Delta\overline{v_0}=(A)_0\,\Delta\overline{\Gamma_0}$

$$\Delta\overline{v_{1c}}=(A)^{0}_{1c}(-\frac{2}{3}+z+\frac{1}{2}z^2)\Delta\overline{\Gamma_0}+(A)_{1c}\Delta\overline{\Gamma_c}$$

$$\Delta\overline{v_{1s}}=(A)^{0}_{1s}(z-3z+z^2)\Delta\overline{\Gamma_0}+(A)_{1s}\Delta\overline{\Gamma_s}$$

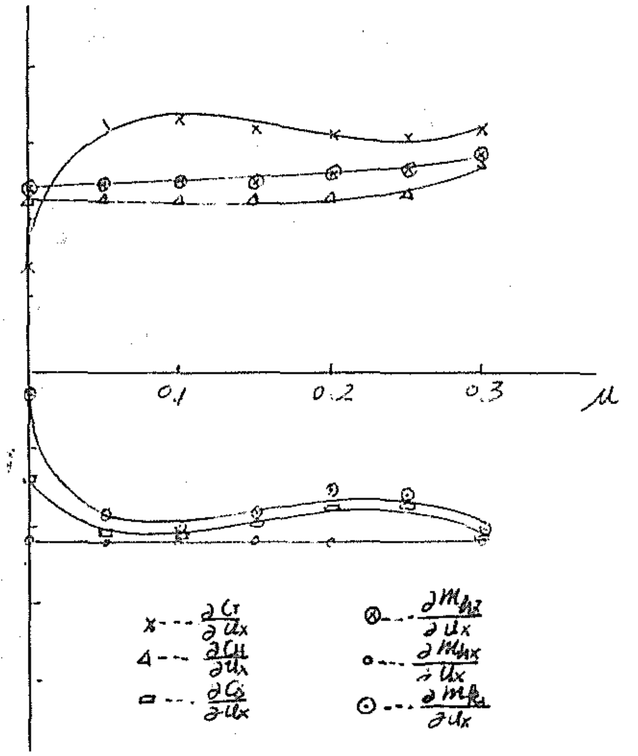
又 $A=\frac{C^{\alpha}_{y}bk}{4\pi}$, $(A)_0=\frac{A^{0}_{0}}{1+A^{0}_{0}}$, $A^{0}_{0}=A/2v_1$

$(A)_{1c}=\frac{A^{1c}_{1c}}{1+A^{1c}_{1c}}$, $(A)_{1s}=\frac{A^{1s}_{1s}}{1+A^{1s}_{1s}}$, $A^{1c}_{1c}=\frac{A\sin\alpha_1}{v(1+\sin\alpha_1)}$

$A^{1s}_{1s}=\frac{A}{v_1(1+\sin\alpha_1)}$ $\quad (A)^{0}_{1c}=\frac{1}{1+A^{1c}_{1c}}\,\frac{A^{0}_{1c}}{1+A^{0}_{0}}$

$A^{0}_{1c}=\frac{A}{v_1}\,\frac{\cos\alpha_1}{1+\sin\alpha_1}$ $\quad (A)^{0}_{1s}=\frac{v_1}{1+A^{1s}_{1s}}\,\frac{A^{0}_{1s}}{1+A^{0}_{0}}$, $A^{0}_{1s}=A^{0}_{1c}$

Appendix 3



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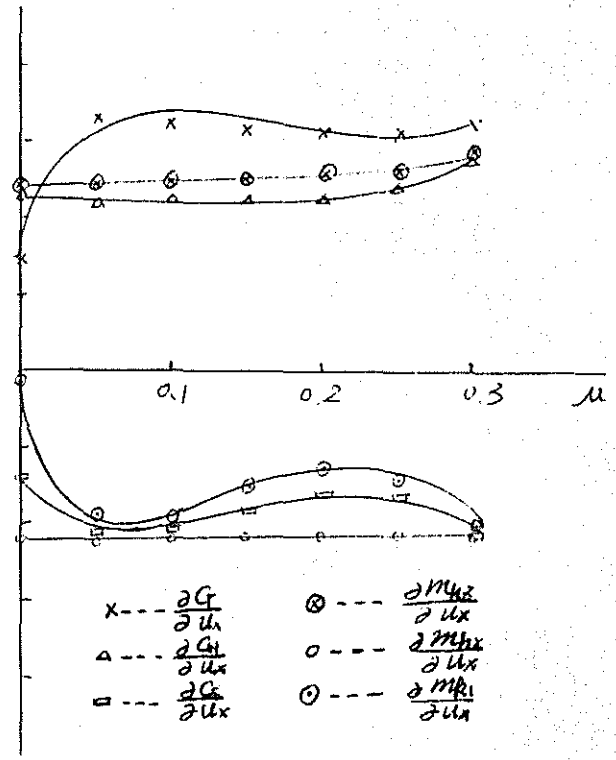

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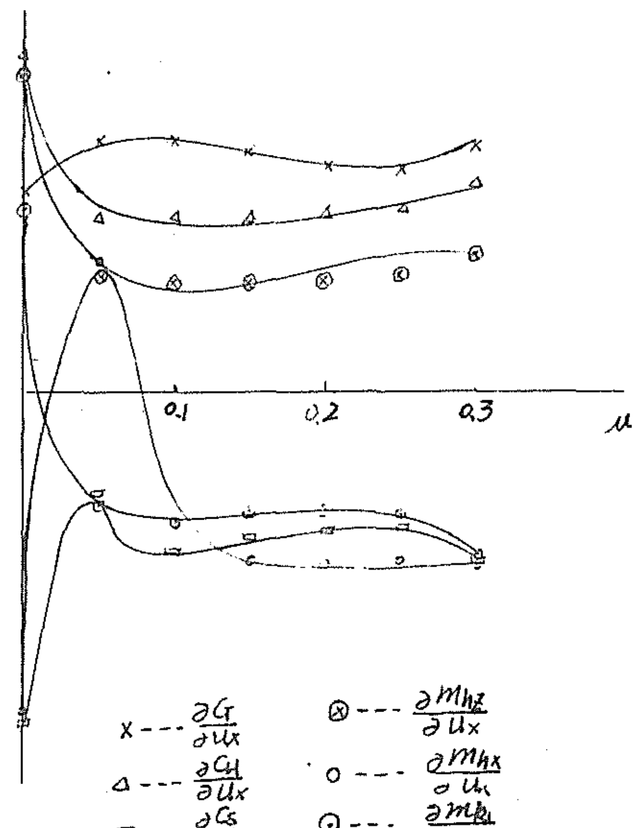

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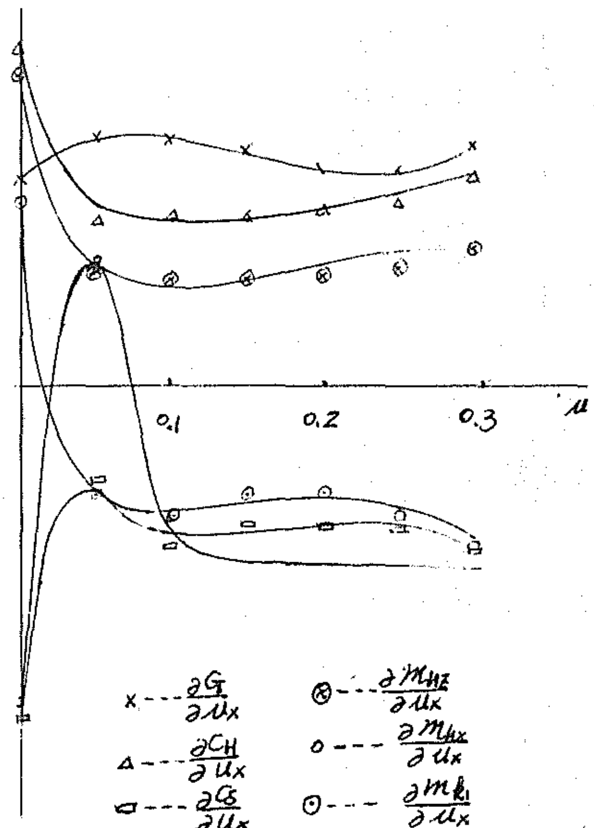

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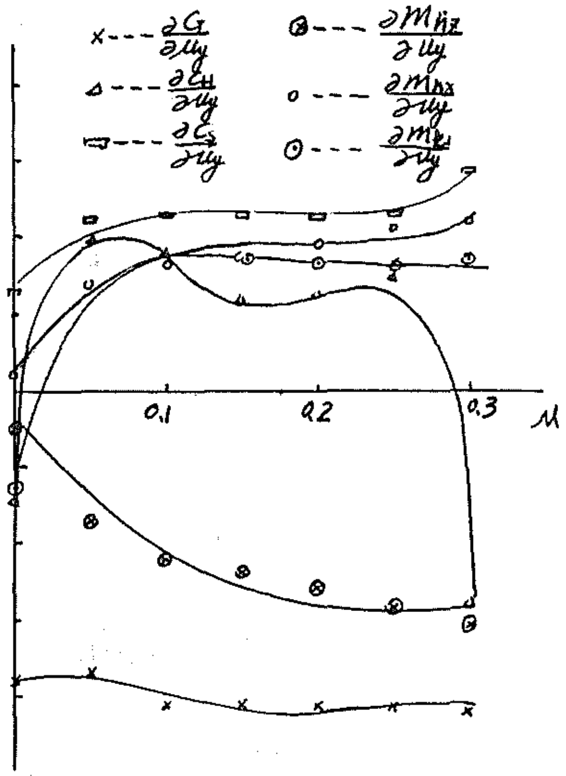

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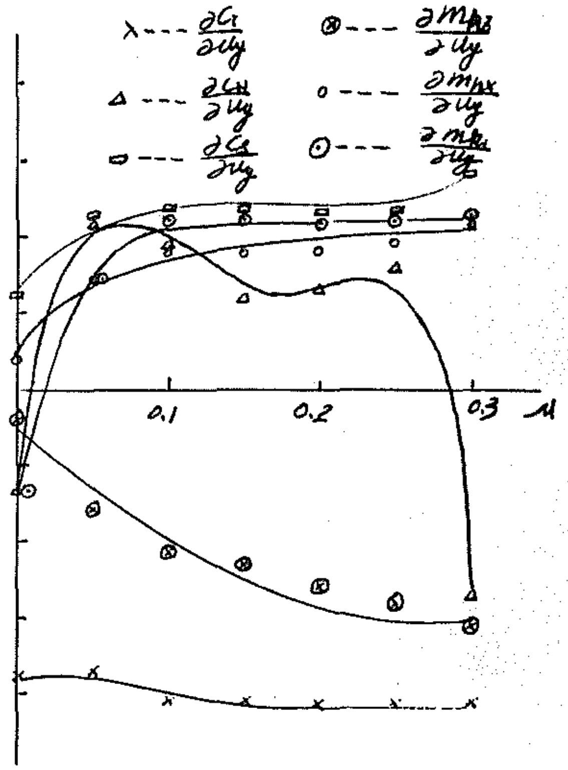

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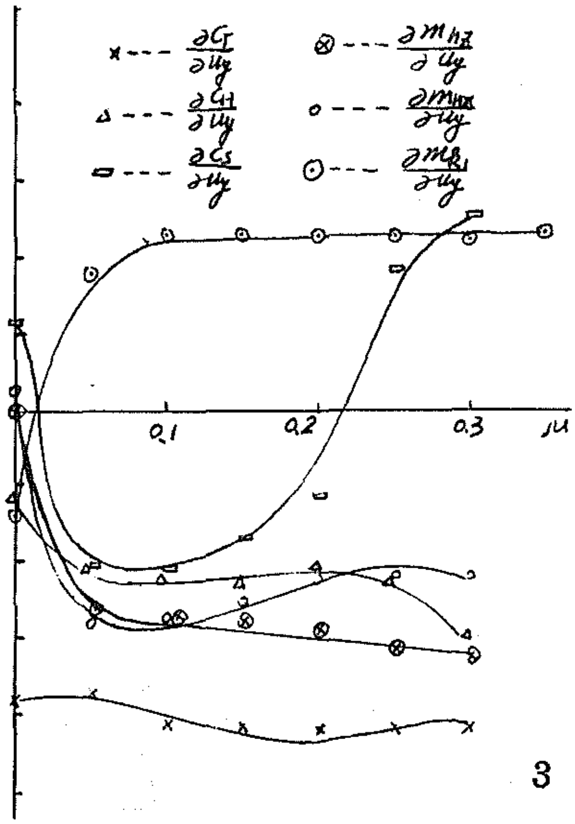

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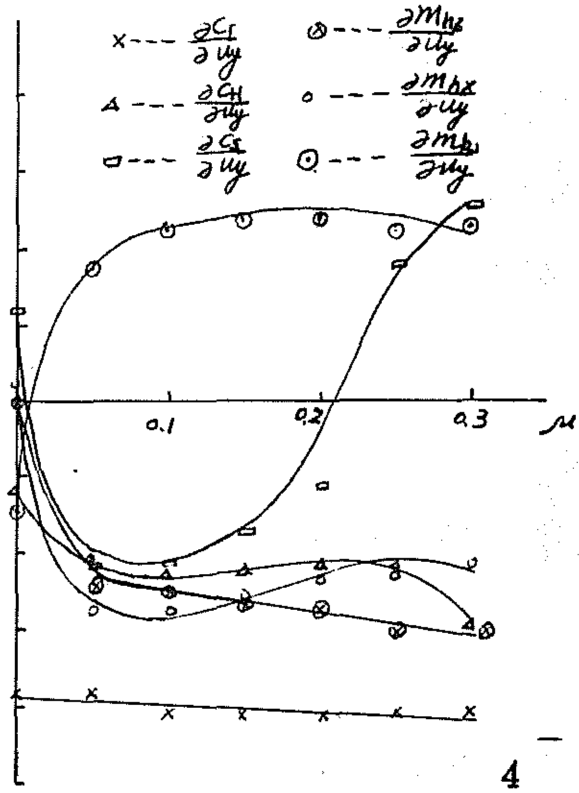

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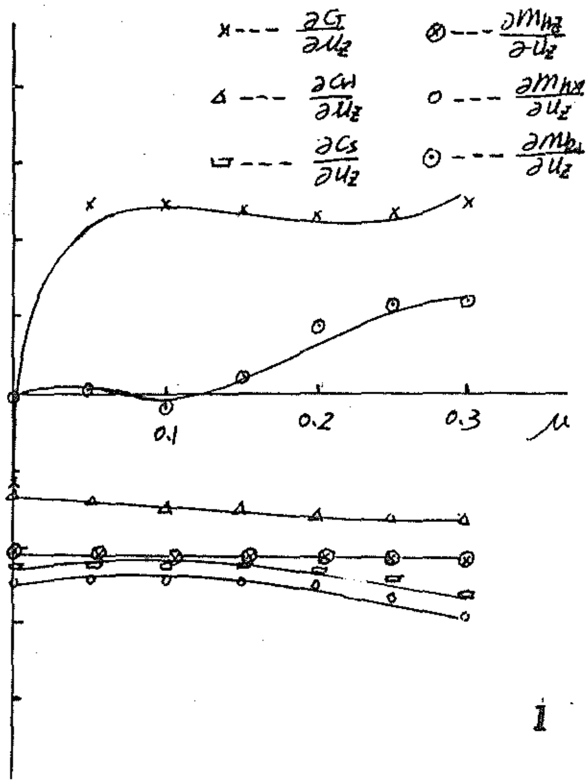
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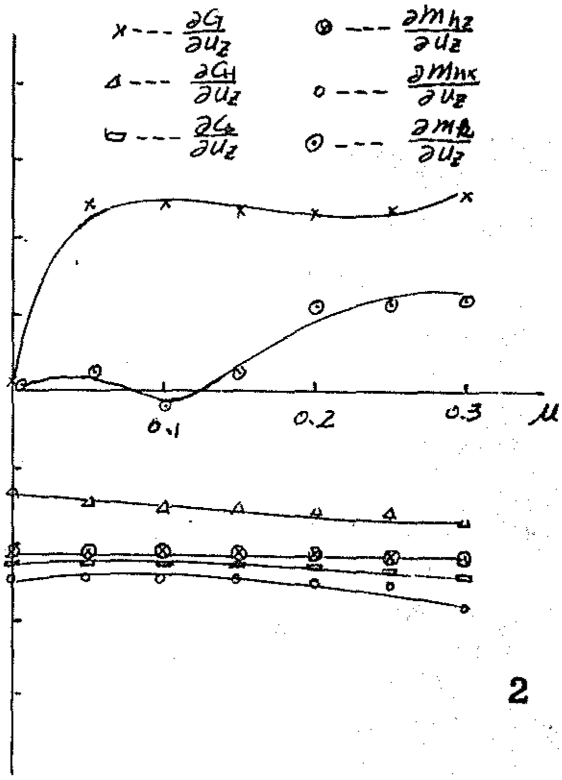
△ --- ∂CH/∂uz
o --- ∂mhx/∂uz
▭ --- ∂CS/∂uz
⊙ --- ∂mh1/∂uz
0.1
0.2
0.3
μ

1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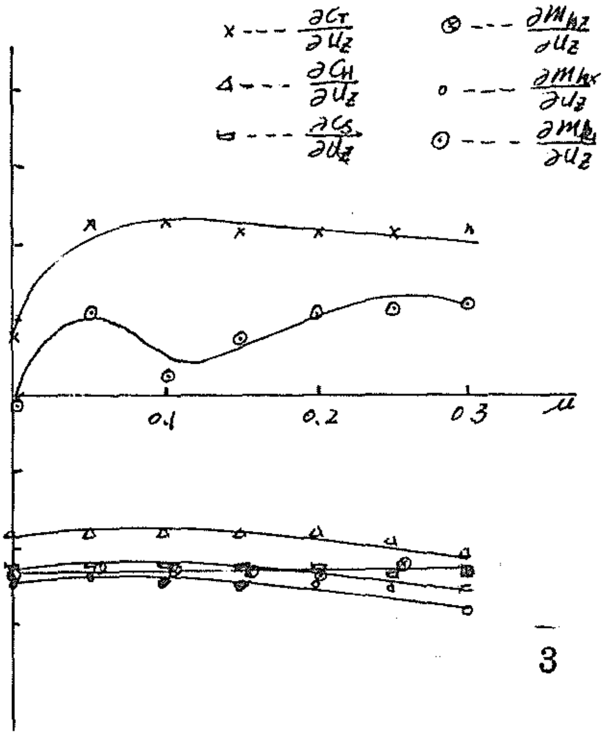
⊙ --- ∂mh1/∂uz
0.1
0.2
0.3
μ

2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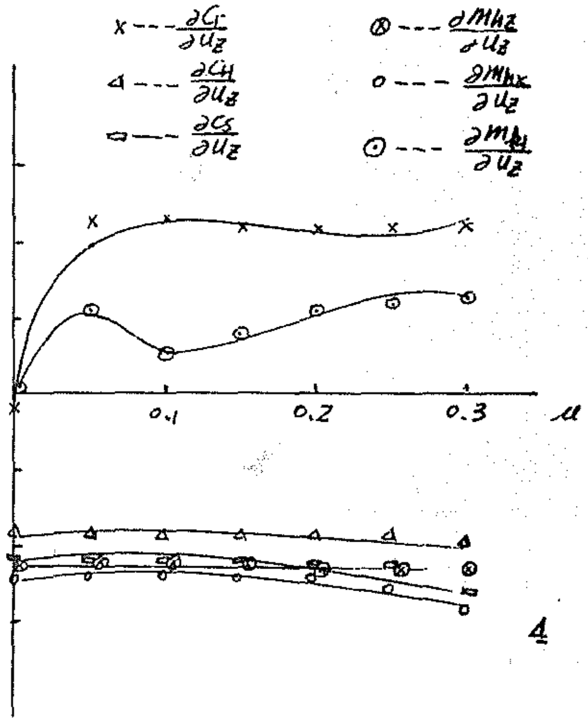
0.1
0.2
0.3
μ

3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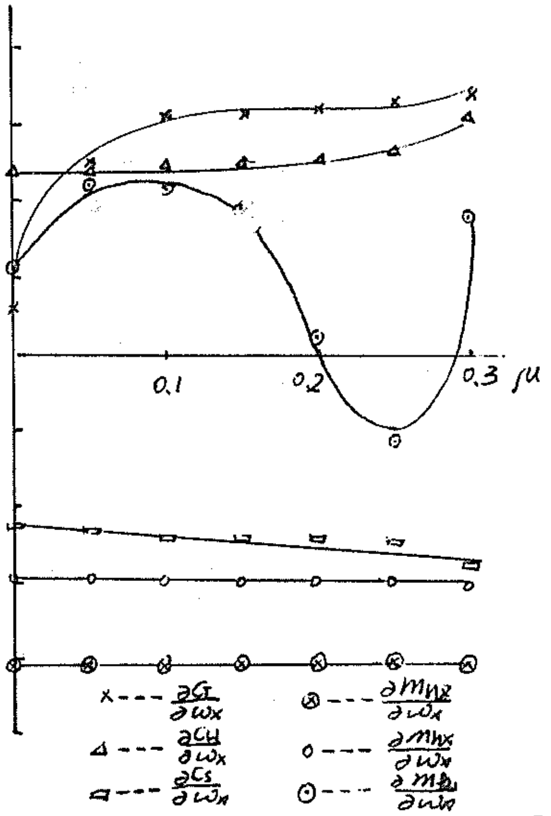
0.1
0.2
0.3
μ

4


0.1
0.2
0.3
μ
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$\square$ --- $\frac{\partial C_S}{\partial \omega_x}$
$\otimes$ --- $\frac{\partial m_{hz}}{\partial \omega_x}$
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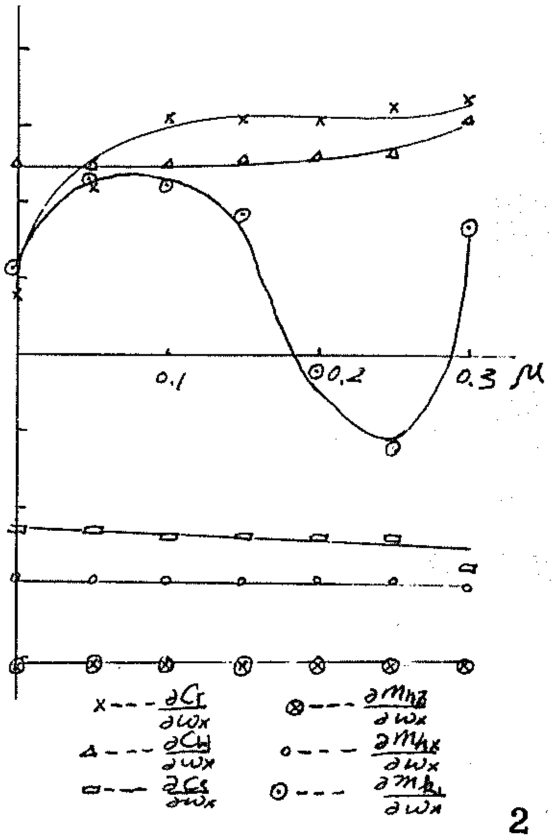
1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μ
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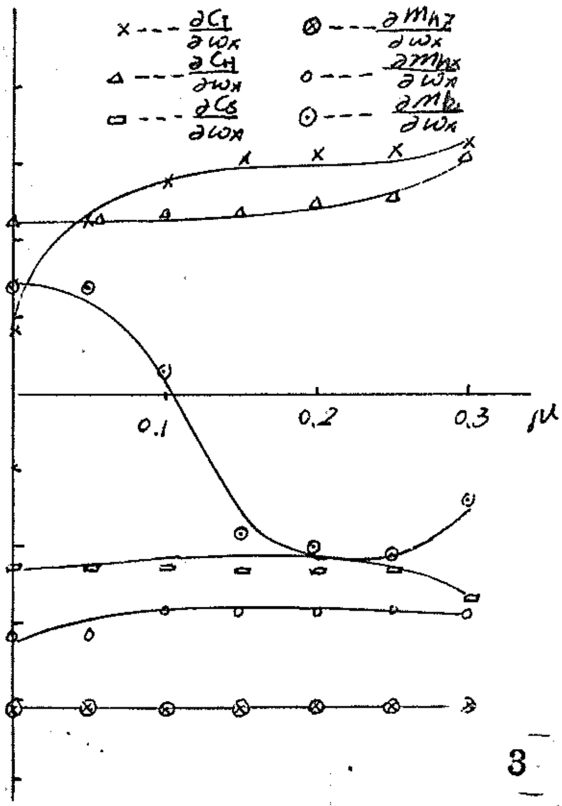
$\square$ --- $\frac{\partial C_S}{\partial \omega_x}$
$\otimes$ --- $\frac{\partial m_{hz}}{\partial \omega_x}$
$\circ$ --- $\frac{\partial m_{hx}}{\partial \omega_x}$
$\odot$ --- $\frac{\partial m_{b_1}}{\partial \omega_x}$

2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$\circ$ --- $\frac{\partial m_{hx}}{\partial \omega_x}$
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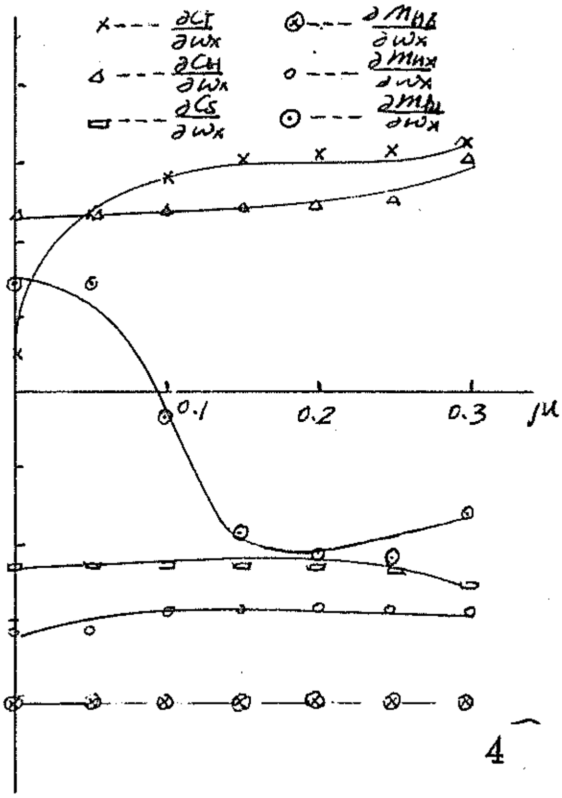
0.1
0.2
0.3
μ

3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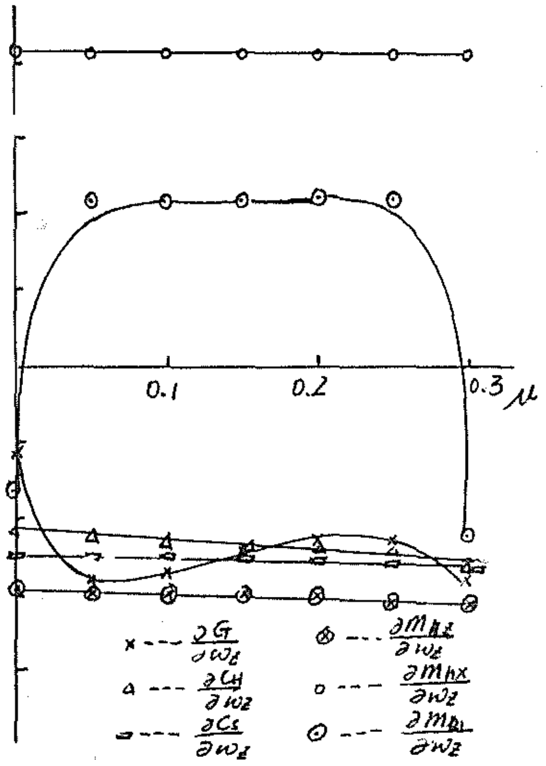
0.2
0.3
μ

4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$\partial m_{hz}/\partial \omega_z$
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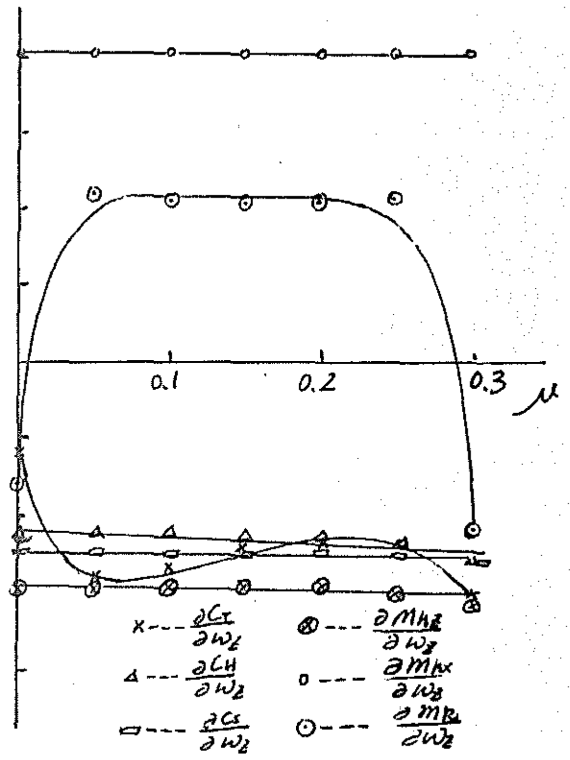
$\partial m_{b1}/\partial \omega_z$

1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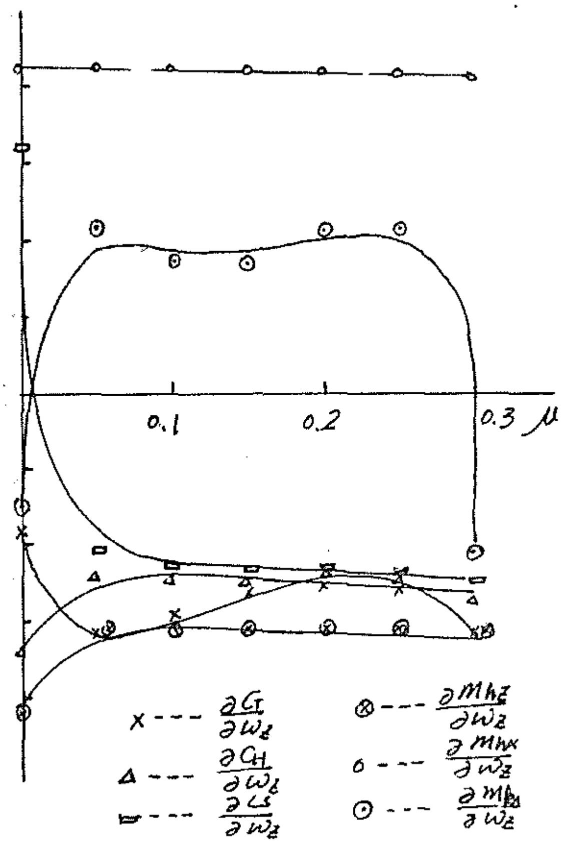
$\partial m_{b1}/\partial \omega_z$

2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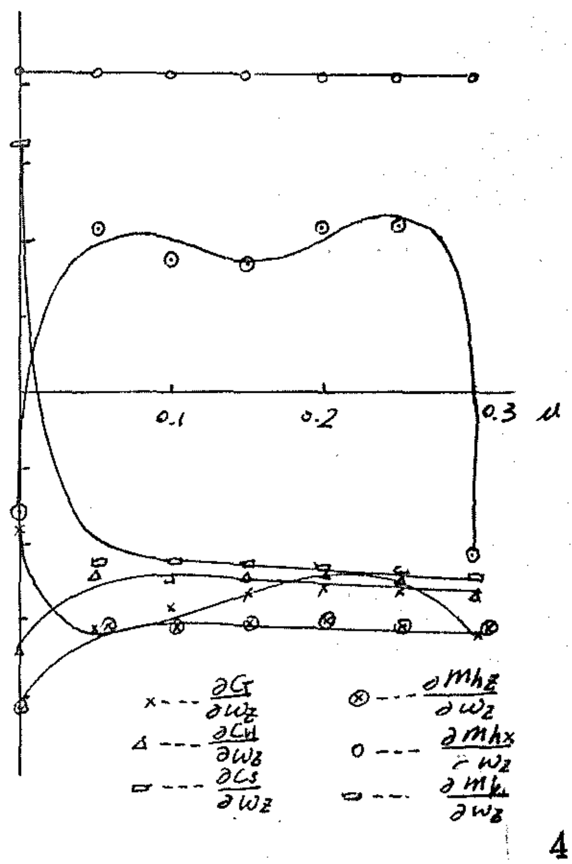
$\partial m_{b1}/\partial \omega_z$

3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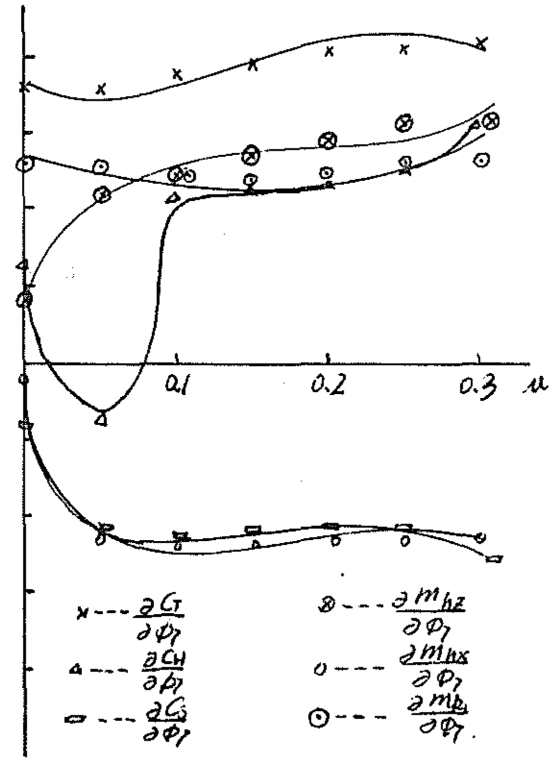
4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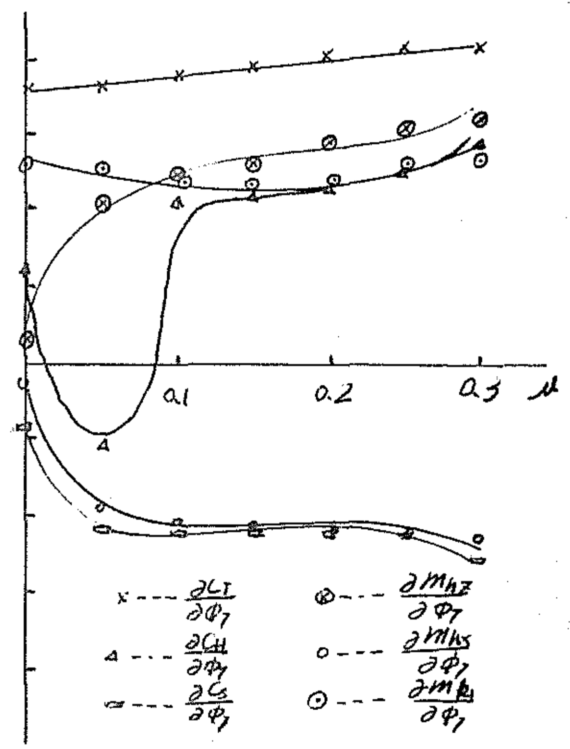
1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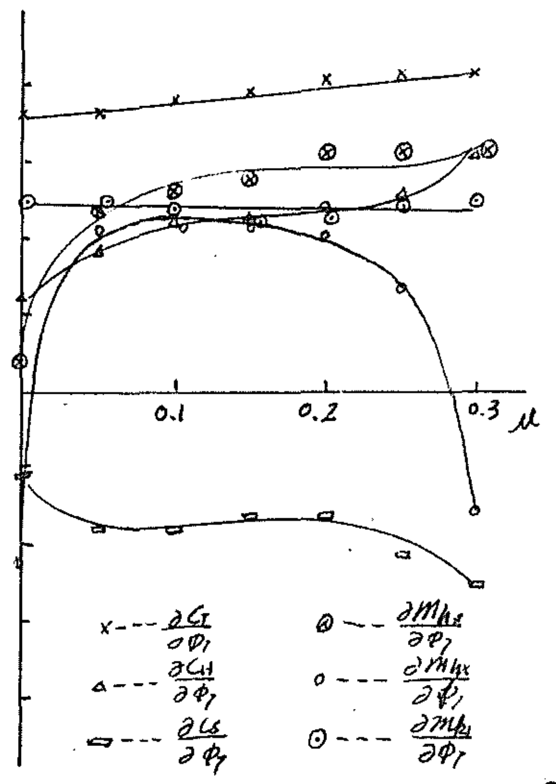
2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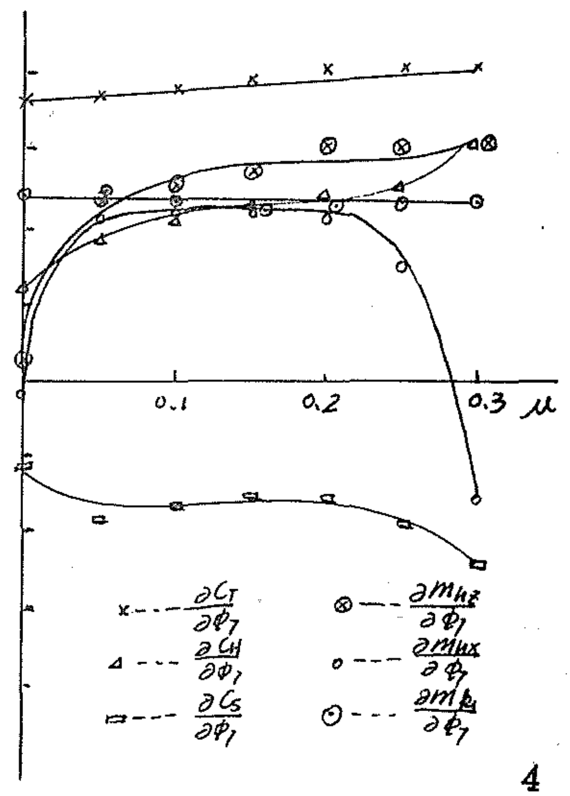
3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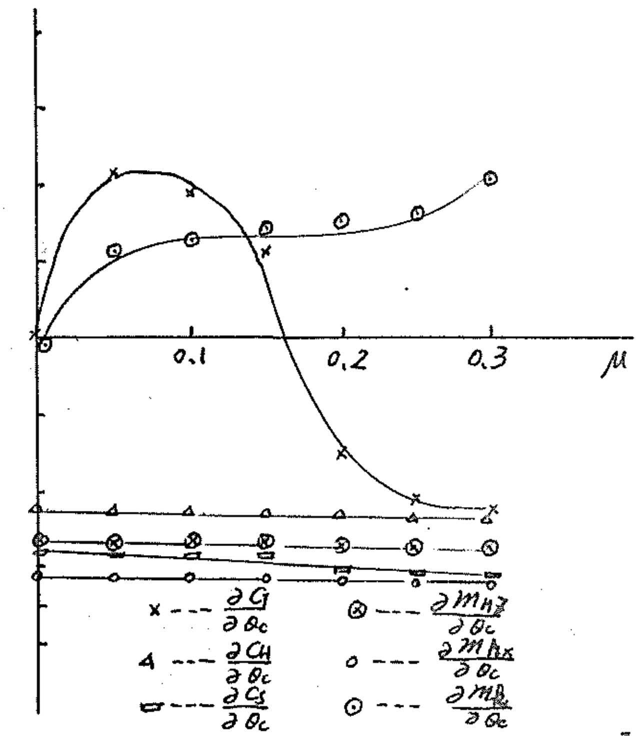
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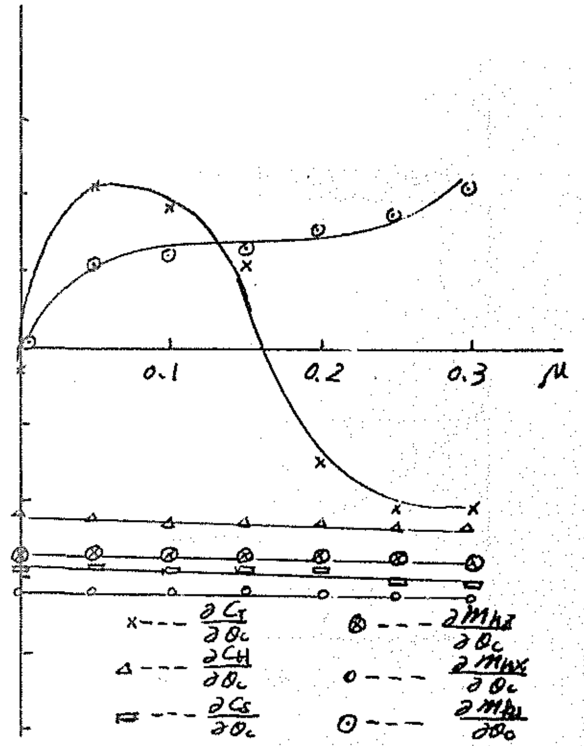
$\odot$ --- $\frac{\partial m_{b1}}{\partial \theta_c}$

1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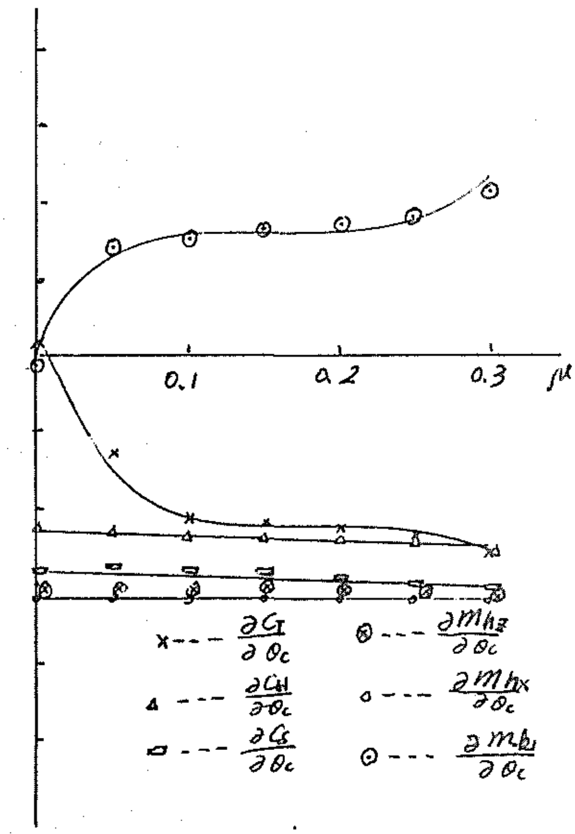
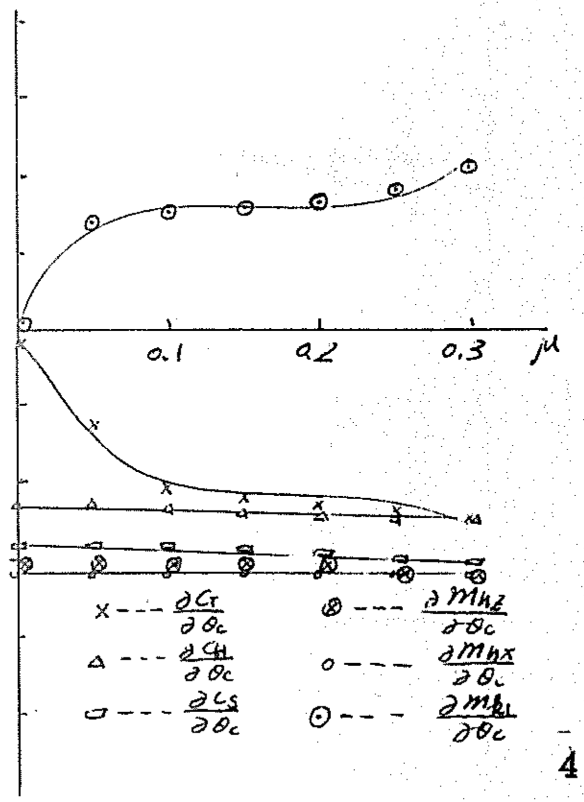
$\circ$ --- $\frac{\partial m_{hx}}{\partial \theta_c}$
$\square$ --- $\frac{\partial C_S}{\partial \theta_c}$
$\odot$ --- $\frac{\partial m_{b1}}{\partial \theta_0}$

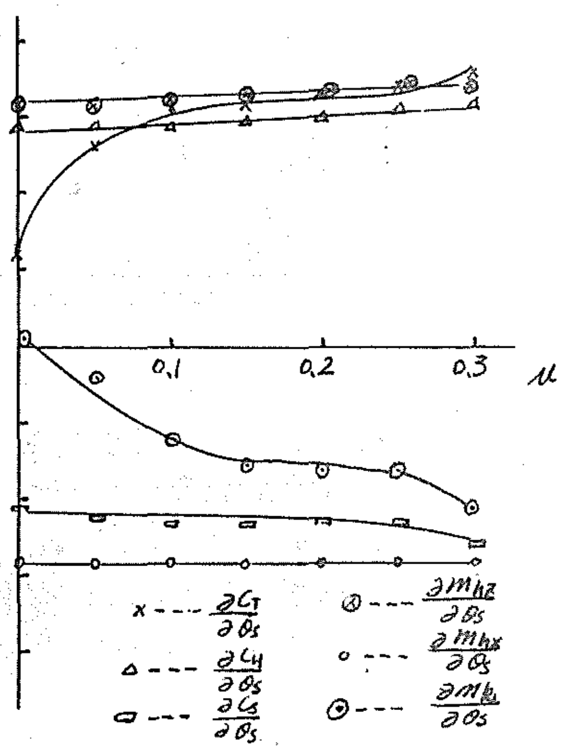
2


0.1
0.2
0.3
μ
$\times$ --- $\frac{\partial C_T}{\partial \theta_c}$
$\otimes$ --- $\frac{\partial m_{hz}}{\partial \theta_c}$
$\triangle$ --- $\frac{\partial C_H}{\partial \theta_c}$
$\circ$ --- $\frac{\partial m_{hx}}{\partial \theta_c}$
$\square$ --- $\frac{\partial C_S}{\partial \theta_c}$
$\odot$ --- $\frac{\partial m_{b1}}{\partial \theta_c}$

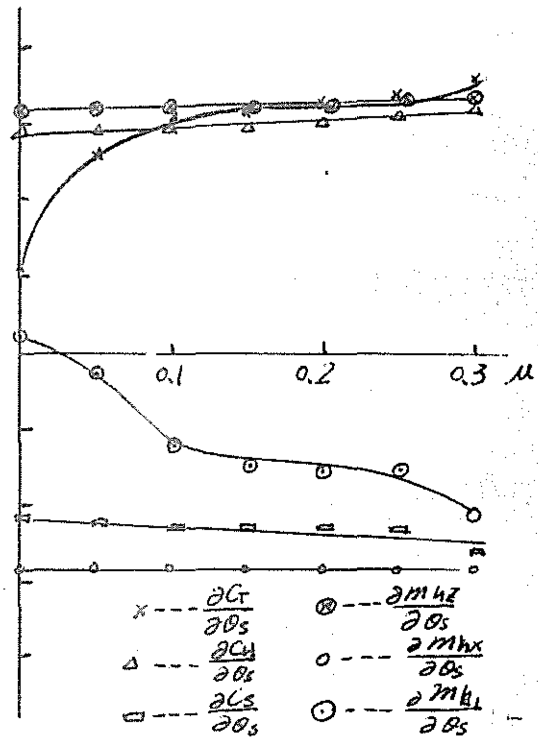
3


0.1
0.2
0.3
μ
$\times$ --- $\frac{\partial C_T}{\partial \theta_c}$
$\otimes$ --- $\frac{\partial m_{hz}}{\partial \theta_c}$
$\triangle$ --- $\frac{\partial C_H}{\partial \theta_c}$
$\circ$ --- $\frac{\partial m_{hx}}{\partial \theta_c}$
$\square$ --- $\frac{\partial C_S}{\partial \theta_c}$
$\odot$ --- $\frac{\partial m_{b1}}{\partial \theta_c}$

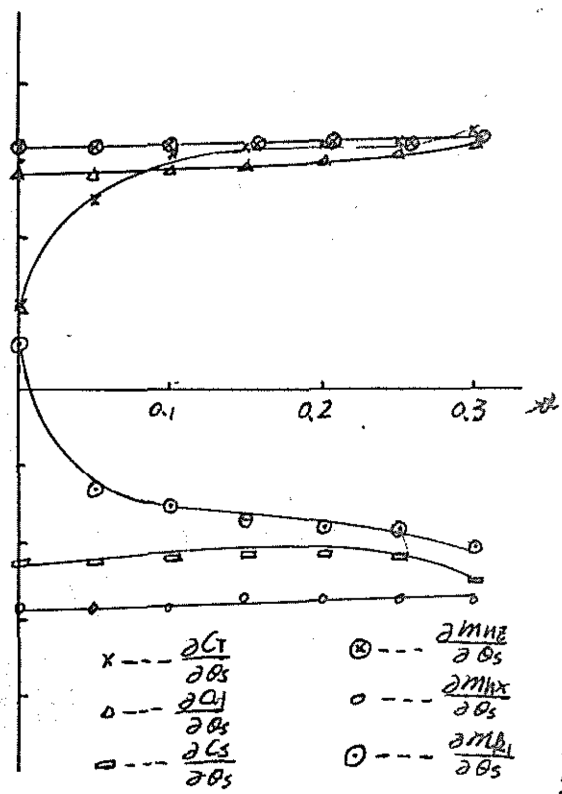
4


0.1
0.2
0.3
μ
× --- ∂C_T/∂θ_s
△ --- ∂C_y/∂θ_s
▭ --- ∂C_s/∂θ_s
⊗ --- ∂m_hz/∂θ_s
○ --- ∂m_hx/∂θ_s
⊙ --- ∂m_h1/∂θ_s

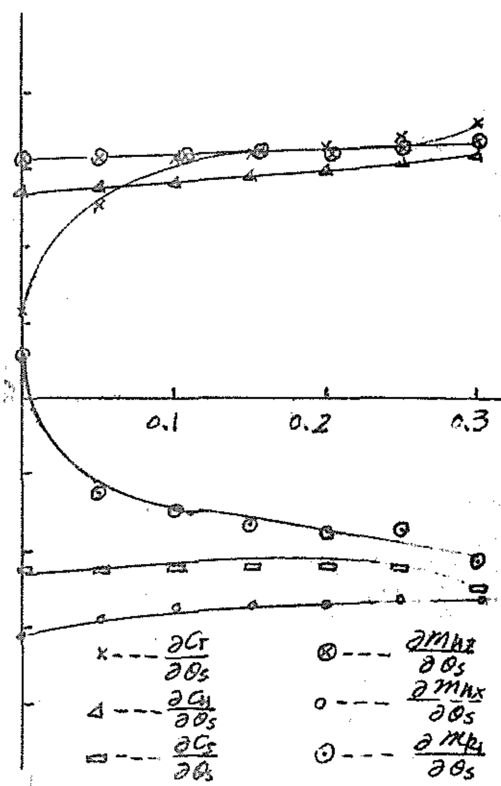
1


0.1
0.2
0.3
μ
× --- ∂C_T/∂θ_s
△ --- ∂C_y/∂θ_s
▭ --- ∂C_s/∂θ_s
⊗ --- ∂m_hz/∂θ_s
○ --- ∂m_hx/∂θ_s
⊙ --- ∂m_h1/∂θ_s

2


0.1
0.2
0.3
μ
× --- ∂C_T/∂θ_s
△ --- ∂C_y/∂θ_s
▭ --- ∂C_s/∂θ_s
⊗ --- ∂m_hz/∂θ_s
○ --- ∂m_hx/∂θ_s
⊙ --- ∂m_h1/∂θ_s

3


0.1
0.2
0.3
μ
× --- ∂C_T/∂θ_s
△ --- ∂C_y/∂θ_s
▭ --- ∂C_s/∂θ_s
⊗ --- ∂m_hz/∂θ_s
○ --- ∂m_hx/∂θ_s
⊙ --- ∂m_h1/∂θ_s

(4)

root	case 1	case 2	case 3	case 4
λ_1	0.0335	0.00839	0.03144	0.007643
	±	±	±	±
λ_2	0.2321i	0.2251i	0.2322i	0.2250i
λ_3	−0.01830	0.01376	−0.01819	0.0095
	±		±	
λ_4	0.2321i	−0.03396	0.0112i	−0.2999
λ_5	−0.6020	−0.8570	−0.6197	−0.8527
		±		±
λ_6	−0.8336	2.161i	−0.8313	2.159i
	±		±	
λ_7	2.167i	−0.864	2.168i	−0.8823
λ_8	−1.935	−1.7336	−1.942	−1.744
	±		±	±
λ_9	13.07i	13.181i	13.08i	13.19i

Table 1:

The Roots of Stability at $\mu = 0.2$

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