CONCEPTUAL MODELING AND ANALYSIS OF A NONLINEAR DYNAMIC VIBRATION ABSORBER FOR HELICOPTER APPLICATIONS

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ABSTRACT

This paper presents the conceptual modelling and numerical analysis of a nonlinear dynamic vibration absorber designed for integration into helicopter fuselage structures. While conventional dynamic vibration absorbers are typically tuned to a narrow frequency range and target a single excitation frequency, the proposed system introduces a nonlinear stiffness element, modeled as a polynomial function to enhance performance across multiple frequencies. The analysis focuses on vibration mitigation in the vertical (z-axis) direction, which is particularly critical for pilot comfort and structural durability. A performance comparison is conducted between the linear and nonlinear dynamic vibration absorbers under multiple harmonic excitation forces representative of rotor-induced vibrations. The nonlinear spring coefficients and absorber mass are optimized using a genetic algorithm to minimize the vibration amplitude of the main body. Simulation results show that the nonlinear vibration absorber can improve vibration suppression over a broader frequency range, making it a promising solution for enhancing vibration control in rotorcraft fuselage applications.

NOTATION

Symbols

$A(f_i)$	Displacement amplitude at frequency f_i (m)
a_1, a_3, a_5	Linear, cubic, and quintic nonlinear stiffness coefficients (N/m, N/m³, N/m⁵)
c_1	Damping coefficients of primary system (Ns/m)
c_2	Damping coefficient of linear dynamic vibration absorber (Ns/m)
c_n	Damping coefficients of nonlinear dynamic vibration absorber (Ns/m)
c_{eff}	Effective damping coefficient of nonlinear dynamic vibration absorber (Ns/m)
$F(t)$	External excitation force (N)
F_{0i}	Amplitude of the i-th harmonic component of force (N)
F_s	Spring force (N)
F_d	Damping force (N)
f_i	Frequency of the i-th harmonic component (Hz)
I	Fitness (objective) function value
k_1, k_2	Linear stiffness coefficients of primary system and absorber (N/m)
k_{non}, k_{eff}	Nonlinear stiffness and effective nonlinear stiffness coefficients of absorber (N/m)
m_1, m_2	Mass of primary system and absorber (kg)
$x_1(t)$	Displacement of primary mass (m)
$x_2(t)$	Displacement of absorber mass (m)
x_{rel}	Relative displacement between absorber and primary mass (m)
$\dot{x}_1, \dot{x}_2, \dot{x}_{rel}$	Velocity of primary, absorber, and relative displacement (m/s)
w_i	Weighting factors for frequency component f_i
ζ	Damping ratio
ω_n	Natural frequency of primary system (rad/s)
ω_{DVA}	Natural frequency of linear dynamic vibration absorber (rad/s)
Ω	Main rotor speed of rotation, rad/s

Acronyms:

DVA	Dynamic Vibration Absorber
FFT	Fast Fourier Transform
GA	Genetic Algorithm
NDVA	Nonlinear Dynamic Vibration Absorber
SDOF	Single Degree of Freedom

1. INTRODUCTION

Helicopter structures are particularly susceptible to complex vibratory loads due to interactions among rotor dynamics, aerodynamics, and structural responses. These vibrations can negatively impact passenger comfort, structural integrity, and the performance of onboard equipment. To mitigate such effects, dynamic vibration absorbers (DVAs) have been extensively implemented across various components of helicopters. DVAs are secondary systems designed to reduce vibration amplitudes by introducing an auxiliary mass-spring mechanism, which is tuned to the dominant excitation frequencies of the primary structure. First, it was introduced by Frahm (Ref. 1) and later systematically analyzed by Ormondroyd and Den Hartog (Ref. 2). DVAs have evolved into widely adopted solutions in aerospace and mechanical systems.

In helicopter applications, various types of DVAs, such as bifilar and pendulum absorbers, have been incorporated into rotor heads, blades, and fuselage structures. In particular, fuselage-integrated absorbers are commonly used to target vibrations in the vertical (z-axis) direction, which become significant during hover and forward flight. These solutions are passive, cost-effective, and do not require significant modifications to the primary system. Reviews by Reichert (Ref. 3) and Welsh (Ref. 4) provide comprehensive overviews of DVA implementations in modern rotorcraft. Figure 1 illustrates a typical commercial DVA used in rotorcraft applications (Ref. 5) and a conceptual illustration of its potential integration into the helicopter fuselage.

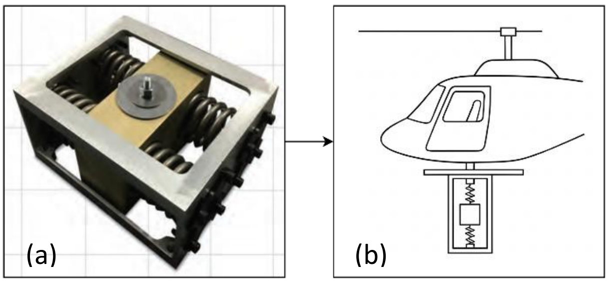

Figure 1: (a) A commercially available dynamic vibration absorber used in rotorcraft applications(Ref. 5). (b) Conceptual illustration of the dynamic vibration absorber integrated into a helicopter fuselage.

However, despite their effectiveness, conventional linear DVAs are limited in their operational bandwidth. They are typically tuned to a narrow frequency range and are therefore less effective under varying flight conditions, such as maneuvering or changes in rotor speed, where excitation frequencies may shift.

In rotorcraft applications, vibratory loads often occur not at a single frequency, but at multiple harmonic components of the main rotor rotational speed (e.g.,Ω,2Ω,3Ω, etc.). As a result, multiple excitation frequencies can act on the fuselage structure (Ref. 3,6). Since classical linear DVAs are generally tuned to a single target frequency, which is most commonly the blade-passing frequency. To address this challenge, the present study investigates a nonlinear dynamic vibration absorber (NDVA) integrated into the cockpit region of the fuselage, where reducing vibration levels is especially critical for both pilot comfort and airframe longevity. Unlike conventional designs, this approach aims to achieve effective performance across a broader frequency range by introducing nonlinearity into the system.

Most nonlinear dynamic vibration absorbers in the literature are modeled using various nonlinear stiffness formulations, such as polynomial, piecewise, hysteretic and geometric nonlinearities. These different modeling approaches are discussed in detail in studies by Zheng Lu (Ref. 7) and Balaji (Ref. 8). While many works, like Alsuwaiyan (Ref. 9), have adopted cubic stiffness systems due to their simplicity and tunability, this study defines the absorber's stiffness characteristics as a polynomial function, including linear, cubic, and quintic terms allowing for customizable force-displacement behavior.

Rather than focusing on a detailed physical design, this study emphasizes the mathematical formulation and assesses the potential dynamic performance of the proposed nonlinear dynamic vibration absorber. To this end, two separate simulation models have been developed: one incorporating the proposed nonlinear dynamic vibration absorber and the other based on a classical linear DVA. For a fair comparison, both models use the same absorber mass. The linear DVA is tuned to the blade-passing frequency, while the nonlinear DVA parameters, including the spring coefficients and absorber mass, are optimized using a Genetic Algorithm (GA) in MATLAB.

The remainder of this paper presents the mathematical formulation of the system, which details the numerical simulations and optimization procedures, and compares the dynamic responses of the linear and nonlinear configurations. To demonstrate broadband effectiveness, the nonlinear dynamic vibration absorber was tested under multiple excitation frequencies. The findings provide insights into the feasibility and advantages of integrating a nonlinear DVA into the helicopter fuselage structure.

2. DYNAMIC SYSTEM MODELING

In this section, the dynamic model of the primary system and the attached vibration absorbers is introduced. The analysis is restricted to the vertical (z-axis) direction. Two configurations are considered: a conventional linear dynamic vibration absorber (DVA) and a nonlinear dynamic vibration absorber (NDVA) with polynomial stiffness characteristics. Schematic representations of both configurations are shown in Figure 2.

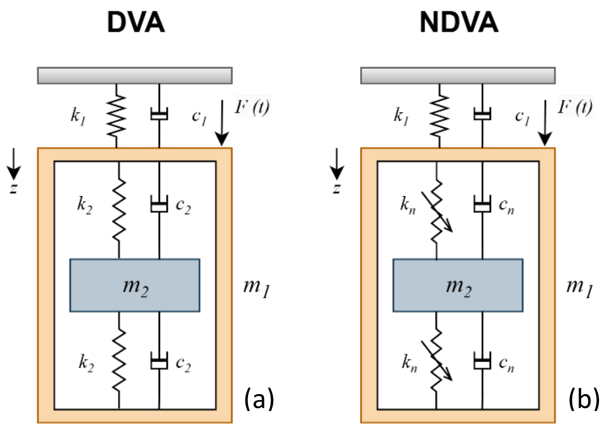

Figure 2: Schematic representation of two absorber types: (a) Conventional linear dynamic vibration absorber (DVA). (b) Nonlinear dynamic vibration absorber (NDVA) with polynomial stiffness characteristics.

The primary structure is modeled as a single-degree-of-freedom (SDOF) system, represented by a mass m_1 connected to ground via a linear spring and damper, denoted as k_1 and c_1, respectively. The absorber is modeled as a secondary mass m_2, attached to the primary mass m_1, which represents a section of the helicopter fuselage through a spring k_2 and a damper c_2.

Let $x_1(t)$ denote the displacement of the primary mass m_1, and $x_2(t)$ denote the displacement of the absorber mass m_2, both with respect to the inertial ground frame. The relative displacement and velocity between the absorber and the primary system are given by:

(1) $$x_{rel} = x_2 - x_1, \quad \dot{x}_{rel} = \dot{x}_2 - \dot{x}_1$$

In the absence of the dynamic vibration absorber, the natural frequency of the primary structure is given by:

(2) $$\omega_n = \sqrt{\frac{k_1}{m_1}}$$

For the linear dynamic vibration absorber, the natural frequency is:

(3) $$\omega_{DVA} = \sqrt{\frac{k_2}{m_2}}$$

and the spring and damping forces acting on the linear dynamic vibration absorber are:

(4) $$F_s(x_{rel}) = k_2 x_{rel}, \quad F_d(\dot{x}_{rel}) = c_2 \dot{x}_{rel}$$

For the nonlinear dynamic vibration absorber, the stiffness and damping are displacement-dependent and denoted as k_n and c_n, respectively. These coefficients are not constant but vary with displacement and velocity:

(5) $$F_s(x_{rel}) = a_1 x_{rel} + a_3 {x_{rel}}^3 + a_5 {x_{rel}}^5$$

(6) $$F_d(\dot{x}_{rel}) = c_{eff} \dot{x}_{rel}$$

Here, c_{eff} is the effective damping coefficient, calculated based on the instantaneous stiffness k_{eff} and a prescribed damping ratio ζ, assumed to be 0.005 (representing structural damping):

(7) $$c_{eff} = 2\zeta \sqrt{m_2 \cdot k_{eff}}$$

(8) $$k_{eff} = a_1 + 3a_3 {x_{rel}}^2 + 5a_5 {x_{rel}}^4$$

The system is subjected to an external excitation force $F(t)$, modeled as a summation of harmonic components due to rotor-induced periodic loads. These forces are primarily due to the rotor rotation frequency and its higher harmonics:

(9) $$F(t) = \sum_{i=1}^{11} F_{0i} cos(2\pi f_i t)$$

Where F_{0i} denotes the amplitude and f_i the frequency of the i-th harmonic component. The f_i values represent the harmonics of the rotor's rotational frequency, which is the dominant source of periodic excitation in helicopter structures. Applying Newton's second law to the primary and absorber masses yields the equations of motion for both configurations. The general form of the coupled equations is:

(10) $m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 - F_d(\dot{x}_{rel}) - F_s(x_{rel}) = F(t)$

(11) $m_2\ddot{x}_2 + F_d(\dot{x}_{rel}) + F_s(x_{rel}) = 0$

These coupled differential equations govern both absorber configurations. For the linear case, equations (4) apply; for the nonlinear case, equations (5) and (6) are used. The parameters used in the simulation are normalized to simplify the analysis and enable generalization of the results. Normalized values and model-specific coefficients are listed in Table 1 below.

Table 1: Normalized mass, frequency, and damping ratios along with nonlinear stiffness coefficients of the system.

Parameter	Symbol	Value
Mass ratio	m_2/m_1	0.055
Frequency ratio	ω_{DVA}/ω_n	0.5
Damping ratio (primary & DVA)	ζ	0.005
Optimized linear stiffness coefficient (NDVA)	a_1	2.21×10^5 N/m
Optimized cubic stiffness coefficient (NDVA)	a_3	-8.1×10^9 N/m^3
Optimized quintic stiffness coefficient (NDVA)	a_5	2.45×10^{14} N/m^5

3. SOLUTION METHODOLOGY

To evaluate the performance of both linear and nonlinear dynamic vibration absorbers, numerical simulations were conducted using MATLAB R2025a. The coupled second-order differential equations presented in the previous section were solved using the built-in ode45 solver, which implements an explicit Runge-Kutta method. This solver was chosen due to its reliability and efficiency in handling stiff and non-stiff ordinary differential equations in mechanical systems.

The simulation workflow is divided into two main parts:

- Dynamic analysis, in which the system response is computed for a given set of parameters under multi-frequency harmonic excitation.
- Parameter optimization, applied only for the nonlinear dynamic vibration absorber case, where the nonlinear stiffness coefficients a_1, a_3, a_5 and absorber mass m_2 are optimized to minimize the steady-state vibration amplitude of the primary mass.

Genetic Algorithm (GA) is a population-based heuristic optimization method inspired by natural evolution, including selection, crossover, and mutation operators (Ref. 10–12). It is particularly effective in solving complex, multi-modal problems where classical gradient-based methods may struggle.

The general operation of GA involves creating an initial population of candidate solutions and improving them through successive generations. At each step, individuals are evaluated using a fitness function, and new candidates are generated through selection, crossover, and mutation. Figure 3 shows the overall flow of the genetic algorithm process used in this study.

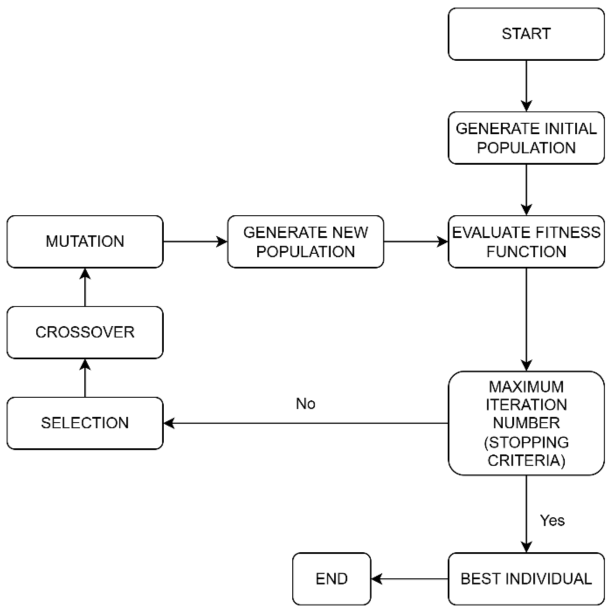

Figure 3: General flowchart of the Genetic Algorithm (GA) optimization process employed in this study.

The optimization objective is to minimize the vibration amplitudes of the primary system at multiple dominant excitation frequencies. To achieve this, a cost (fitness) function is formulated as the weighted sum of the displacement amplitudes corresponding to each excitation frequency component:

(12) $$I = \sum_{i=1}^{n=11} w_i \cdot A(f_i)$$

Here,

- I denotes the fitness function to be minimized,
- $A(f_i)$ is the displacement amplitude of the system at frequency f_i,
- w_i is the weighting factor for the corresponding frequency.

Equal weights are assigned to all frequency components, though these weights can be adjusted to prioritize certain frequencies based on their physical importance:

$$w_1 = w_2 = w_3 = \cdots = w_{11} = \frac{1}{n} = \frac{1}{11} \tag{13}$$

The design variables optimized using the genetic algorithm include the absorber mass and the nonlinear stiffness coefficients. Their allowable ranges are set according to physical constraints and insights from existing literature.

4. NUMERICAL SIMULATION RESULTS

In this section, the dynamic responses of the linear dynamic vibration absorber (DVA) and a nonlinear dynamic vibration absorber (NDVA) are evaluated and compared. The results are derived from time-domain simulations using the parameters and solution methodology presented earlier. Key performance metrics such as displacement amplitude, frequency content, and reduction percentage are considered. All displacement results are normalized with respect to the maximum response of the baseline DVA configuration. Time and frequency axes are also normalized by the natural frequency of the primary system.

Figure 4 illustrates the force-displacement characteristics of the absorber spring components for both the linear and nonlinear configurations.

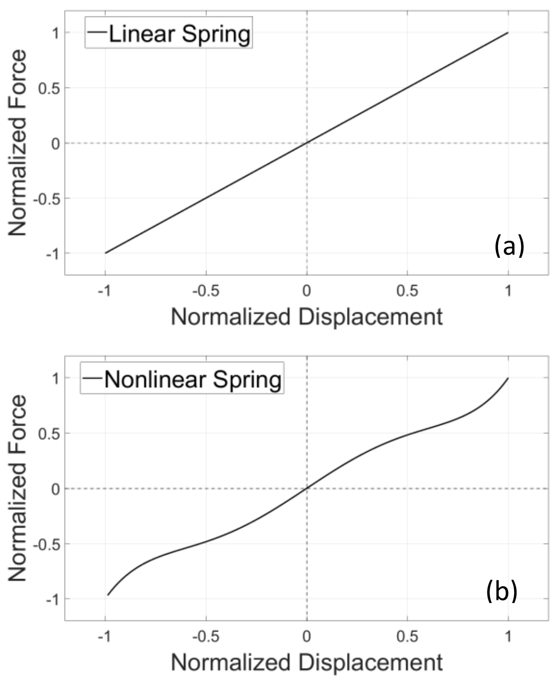

Figure 4: Normalized force–displacement characteristics of the absorber springs. (a) Linear spring, (b) Nonlinear spring.

While the linear DVA exhibits a purely linear stiffness profile, the NDVA follows a nonlinear stiffness curve due to the presence of higher-order terms. This results in increased stiffness at larger displacements, allowing for enhanced vibration suppression under high-amplitude excitation. This nonlinear behavior plays a significant role in the overall system dynamics, especially when the system operates under multi-frequency loading conditions.

Figure 5 presents the time-domain displacement response of the primary mass for both absorber types. Although there is no significant reduction in the overall vibration amplitude observable directly from the time history, certain peaks appear to be damped more effectively by the NDVA. To better understand this behavior and identify which frequency components are attenuated, a frequency-domain analysis (FFT) is performed.

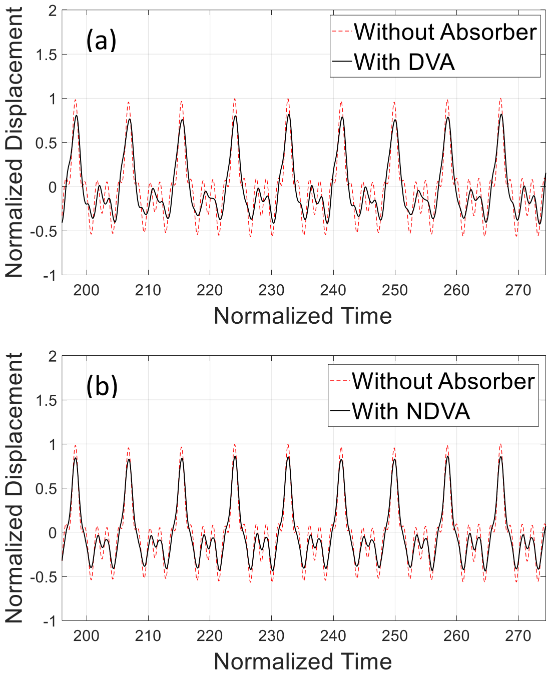

Figure 5: Normalized time-domain displacement response of the primary mass under harmonic excitation. (a) With linear dynamic vibration absorber, (b) With nonlinear dynamic vibration absorber.

Figure 6 presents the frequency-domain (FFT) response of the primary mass for both DVA and NDVA configurations. As seen in the plots, the linear dynamic vibration absorber which is tuned to the blade-passing frequency effectively attenuates vibrations at that specific frequency. However, its performance is limited to this narrow band. In contrast, the NDVA not only reduces the response at the tuned frequency, but also demonstrates significant suppression at a neighboring frequency, indicating a broader attenuation bandwidth.

Nevertheless, a closer inspection of the NDVA's spectrum reveals an unexpected increase in amplitude just beside the blade-passing frequency. This behavior suggests that while NDVA improves overall frequency coverage, it may also introduce local amplification in certain spectral regions. Such outcomes could potentially be mitigated through a more refined optimization process, tailored to prioritize specific frequency bands depending on their physical significance.

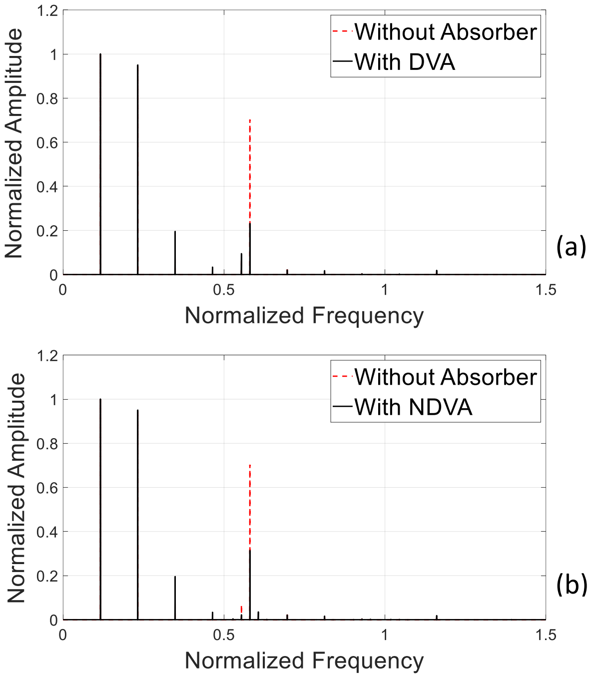

Figure 6: Normalized frequency-domain response (FFT) of the primary mass under harmonic excitation. (a) With linear dynamic vibration absorber, (b) With nonlinear dynamic vibration absorber.

To better quantify the dynamic vibration absorber performance across the full range of excitation frequencies, Figure 7 presents the vibration attenuation ratios at each harmonic component. The results show that while the DVA achieves a high attenuation ratio of 67.1% at the blade-passing frequency, this value drops to 55.4% with the NDVA. This confirms the superior narrowband performance of the linear dynamic vibration absorber at its tuned frequency.

However, the NDVA exhibits a clear advantage at adjacent frequencies—especially at the harmonic immediately neighboring the blade-passing frequency—achieving up to 67% reduction. Additionally, modest improvements are observed at other mid-range frequencies. It is worth noting that the NDVA provides little to no enhancement in low-frequency regions; however, this behavior is specific to the current system configuration, as the linear dynamic vibration absorber also shows negligible effectiveness in these frequency ranges.

These findings highlight a fundamental trade-off: while the DVA is effective in targeting a single critical frequency, the NDVA offers a broader suppression range at the cost of slightly reduced peak performance. To fully capitalize on the NDVA's potential, future work could focus on optimization strategies that balance performance across multiple target frequencies. Identifying which frequencies are most critical to suppress is essential for guiding such an optimization.

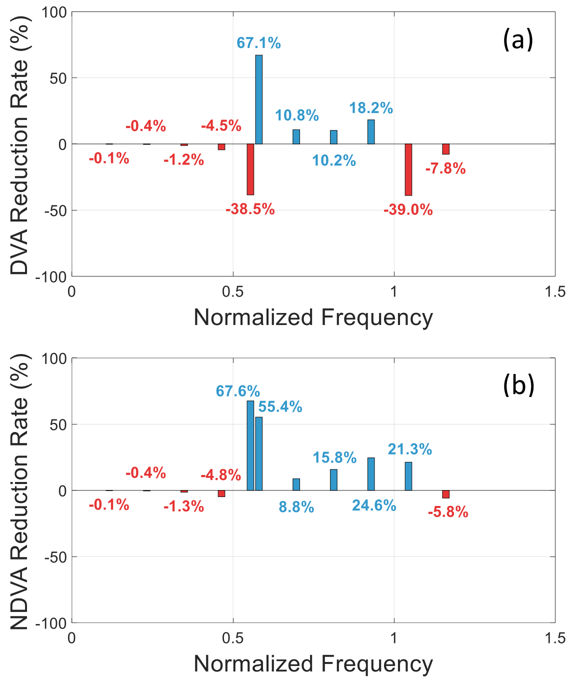

Figure 7: Vibration attenuation ratios of the dynamic vibration absorber systems across all excitation frequencies. (a) With linear absorber, (b) With nonlinear absorber.

5. CONCLUSIONS

The following conclusions can be drawn based on the numerical findings:

1. The nonlinear dynamic vibration absorber (NDVA) provides improved performance over the linear DVA, especially near and around its target frequency.
2. While the linear DVA is effective only at its tuned frequency, the NDVA attenuates vibrations across a wider frequency range.
3. At low frequencies, neither absorber yields significant improvement, which is due to the system configuration rather than the absorber type.
4. The presence of secondary peaks in the frequency response suggests the need for careful optimization, balancing performance across multiple frequencies.

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