ANALYSIS OF HELICOPTER MISSION TASK ELEMENTS BY USING NONLINEAR OPTIMAL CONTROL METHOD

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Abstract: The paper deals with a nonlinear optimal control approach to analyze flight characteristics of helicopter. Two points boundary value problem, formulated as the necessary condition of optimal control, has been solved by a multiple shooting algorithm. The history of reference trajectory is given in prior as a function of time and the deviation from this trajectory is added to the integral part of the cost function, which converts the system optimality to an unconstraint optimal control problem. This paper focuses on the numerical methodology of solving optimal control equations, which is the most time consuming routine in the resolution of optimal control problems using the indirect method. The solutions with various methods such as a standard algebraic equation solver, the nonlinear programming, and a differential algebraic equation solver, are compared with the number of function call (right hand side of system ODE) and the computation time in order to measure the relative advantage of each method. The most favorable numerical method selected by using linear models is applied to a nonlinear helicopter mathematical model. The applications cover various helicopter mission task elements such as bob up, side step, turn, slalom, and deceleration after acceleration. The results can provides valuable directions for using optimal control approach to the analysis of helicopter mission task elements.

INTRODUCTION

The Aeronautical Design Standard ADS-33[1] defines a series of mission task elements and specifies the related performance standards in order to provide a basis for an overall assessment of the rotorcraft¡s ability to perform certain critical tasks. The assessment of ADS33 MTEs usually needs extensive flight test maneuvers which can be carried out at the final step of vehicle design and are hazardous to both the pilot and the vehicle itself. Therefore, reliable numerical methods have an essential role in allowing quick and low-cost design iterations at early design stages and to provide benchmark results in order to increase the productivity of future flight tests.

Two usual methods for the analysis of helicopter maneuvers are the inverse simulation and dynamic optimization. The inverse simulation finds controls that enable the helicopter to follow in exact manner a prescribed flight path whose history is given in prior as a function of time. Various inverse simulation methods have been developed over the converse of last decade [2,3,4]. Among them, the integration inverse method, proposed by Hess et al. has popularly been used because of its lower dependency on model structure. But numerical instability or oscillatory behavior in its solutions causes some difficulties in its application with a high fidelity helicopter model.

Dynamic optimization [5,6,7] computes the controls, the state, and possibly the final time that minimize a cost function, subject to state equations and to various other constraints, as required by the problem at hand. The numerical solution of a dynamic optimization problem is usually solved via the direct method or the indirect method. This paper applies the indirect method where we first apply calculus of variation to find the necessary conditions which minimize a cost function. Then the problem is reduced to a TPBVP (two-point boundary value problem) in the infinite dimension and a suitable MSM (multiple-shooting method) can be used to resolve it in the finite dimension [7, 8, 9]

Since the applications of the indirect method to rotorcrafts are relatively scarce, we describe the approaches used for analyzing mission task elements from optimal control formulation to its numerical methods used in this paper. The solution of optimal control equations derived by applying Pontryagin minimum principle is the most time consuming routine in the indirect method. The related equations can be formulated in the form of a minimization problem or an algebraic equation if no control constraints are active. So, various numerical methods can be selectable, depending on its formulation. This paper carries out trade-off study among the standard algebraic equation solver, the nonlinear programming, and the differential algebraic equation solver with linear helicopter model. The results using each method are compared with the number of function call (right hand side of system ODE) and the computation time in order to measure the relative advantage of each method and to select the most favorable one. The selected method is applied to the analysis of various helicopter mission task elements such as bob up, slalom, turn, side step, deceleration after acceleration maneuvers using both linear and nonlinear models.

1. OPTIMAL CONTROL FORMULATION

The formulation of a general optimal control problem is covered in numerous textbooks, such as Bryson and Ho[10], Kirk[11], and papers. Here, key concepts related to the present study are reviewed. The optimal control problem, which takes the following standard Bolza form, is to find states $x^*(t)$, controls $u^*(t)$, and possibly final time t_f^* that minimize a cost function $J(\cdot)$:

$$\min \quad J(x,u,t) = \varphi(x(t_f)) + \int_{t_0}^{t_f} f_{OP}(x(t),u(t),t)dt \tag{1}$$

$$\text{s.t.} \quad \begin{aligned} &\dot{x}(t) = f(x(t),u(t),t) \\ &s(x(t),t) \le 0 \\ &x_i(t_0) = x_{i0} \qquad i \in I \subset \{1,\ldots,n\} \\ &\psi(x(t_f),t_f) = 0 \quad \psi \subseteq R^k \\ &u(t) \in U \subseteq R^m \\ &t \in [t_0,t_f] \end{aligned} \tag{2}$$

The operating cost function f_{OP} usually includes the quadratic function of controls to minimize control effort. If optimal controls calculated with this type of cost function are well within their limits, the control constraints can be deleted in the formulation. Also to avoid the difficulty of handling state inequality constraint $s(x(t),t) \le 0$ in the indirect method, a quadratic penalty function method is used in this paper, where trajectory deviations from a prescribed maneuver path are penalized and reflected in the cost function. With the penalty cost f_{SC} for state constraints, the original optimal control formulation can be rewritten as the following unconstrained optimal control problem in Mayer form:

$$\min \quad J(x,u,t) = \varphi(x(t_f),t_f) + x_{CO}(t_f) \tag{3}$$

$$\begin{aligned} \text{s.t.} \quad & \\ \dot{x}(t) &= f(x(t),u(t),t) \\ \dot{x}_{CO}(t) &= f_{CO}(x(t),u(t),t) \\ &= f_{OP}(x(t),u(t),t) + f_{SC}(x(t),t) \\ x_i(t_0) &= x_{i0} \\ x_{CO}(t_0) &= 0 \\ \psi(x(t_f),t_f) &= 0 \end{aligned} \tag{4}$$

The first-order necessary condition for system optimality can be derived from applying calculus of variation to the cost function. Based upon the formulation in Ref.10 and Ref.11, optimality conditions can be expressed as the following TPBVP with state variables $y(t) = [x(t), x_{CO}(t)]^T \in R^{n+1}$, costates $\lambda(t) \in R^{n+1}$, Lagrange multiplier $\mu(t) \in R^k$, and Hamiltonian $H(\cdot) = \lambda^T(t) g(y(t),u(t),t)$:

$$\dot{z}(t) = \begin{bmatrix} \dot{y}(t) \\ \dot{\lambda}(t) \\ \dot{\mu}(t) \end{bmatrix} = \begin{bmatrix} g(y(t),u(t),t) \\ -\{g_y(\cdot)\}^T \lambda(t) \\ 0 \end{bmatrix} \tag{5}$$

$$r(z(t_0), z(t_f)) = \begin{bmatrix} x_i(t_0) - x_{i0} \\ x_{CO}(t_0) \\ \psi(x(t_f),t_f) \\ \lambda(t_f) - \left(\dfrac{\partial(\varphi + x_{CO})}{\partial y} \right)^T_{t=t_f} - \left(\dfrac{\partial \psi}{\partial y} \right)^T_{t=t_f} \mu \end{bmatrix} = 0 \tag{6}$$

where the algebraic equations for optimal control are:

$$u^*(t) = \arg\min_u H(x^*, u, \lambda^*, t) \tag{7}$$

or

$$\frac{\partial H(x^*, u, \lambda^*, t)}{\partial u} = 0 \tag{8}$$

2. NUMERICAL METHODS

The shooting algorithm [7, 8, 9] is commonly used for resolving a TPBVP in ordinary differential equations. Since the application of a single-shooting method to unstable systems like helicopters is found to result in numerical divergence, the MSM(Multiple-Shooting Method) is a appropriate choice of a numerical method. The MSM refers to the discretization of a trajectory into a number of sub-intervals, each of which contains a separately defined initial value problem. Successive iterations follow until both the boundary conditions and the continuity of the state variables at each sub-interval node are satisfied. This study follows the procedure of the BNDSCO software[9], which has been developed by Oberle et al. and the detail description can be found in their report.

The most time consuming routine of the optimal control calculation is the evaluation of the Hessian matrix H_{uu}. If an optimal control is obtained by solving Eq. (7) with an efficient NLP approach using first-order methods, rather than Eq. (8) with the Newton method, it is possible to avoid the explicit calculation of the full Hessian matrix because the efficient NLP requires only the first-order gradient information. Also DAE(Differential Algebraic Equation) solvers can be used to simultaneously resolve both the ordinary differential equations, Eq. (5), and the algebraic equation, Eq. (8). Because a MSM numerically calculates an approximate Jacobean matrix by perturbing the system¡s initial states one at a time at each shooting node, The solution using DAE solvers needs the computation of consistent initial conditions at every time step. Finally, If system equations are affine in controls and a quadratic cost function is used for control effort with positive definite weighting matrix R, then from a modeling perspective, the Hessian, H_{uu}, can be approximated with $\lambda_{CO}R$, requiring no further function calls. It can also be a good approximation for the system nearly affine in the controls, which is the typical case even for nonlinear rotor dynamics equations.

Based upon the above consideration the following four numerical procedures to resolve the optimal control equations are investigated with suitable algorithms.

(1) Traditional Newton method to solve EQ.(8) with approximate Hessian, $H_{uu} \approx \lambda_{CO}R$
(2) Unconstrained optimization method to solve EQ.(7) with given states and costates
(3) DAE solver, RADAU5, to solve both EQ.(5) and EQ.(8) simultaneously
(4) Traditional Newton method to solve EQ.(8) with full Hessian matrix computation

3. HELICOPTER MODEL AND THE DEFINITION OF COST FUNCTION

The helicopter flight dynamic models in this paper are based on previous research results of Ref.13, where rotor dynamic equations for flap, lead-lag, and RPM dynamics have been formulated in fully implicit form. A rotor was modeled with rigid blades with spring and damper. Nonlinear quasi-steady aerodynamic theory has been applied through table look-up procedure. Trim calculations of the Bo-105 helicopter have been carried out using the harmonic balance method or a partial periodic trimming algorithm in which a DAE solver was used for time integration. The linear models used in this paper are derived by linearizing helicopter motion equations at a calculated trim condition. A state equation is approximated in time invariant form by averaging both state and control derivative matrices over one rotor revolution. The nonlinear model has been derived by the using linear aerodynamic theory rather than the quasi-linear aerodynamic theory in Ref.13.

The trajectory description with polynomials and trigonometric functions is a widely used method in studies on inverse simulation of the helicopter because of its analytical simplicity and smoothness [2,14,15]. This paper also uses similar analytical descriptions to those for inverse simulation. The trajectory can be expressed as the sum of states at maneuver entry and its variation during maneuver.

$$x(t) = x(t_{entry}) + \Delta x(t) \tag{9}$$

or $$x(t) = x(t_{entry}) + \int_{t_{entry}}^{t} \Delta \dot{x}(\tau) d\tau \tag{10}$$

Trigonometric functions can be good basis functions for trajectory generation because of their smoothness. In this paper the trajectories of bob up and side step maneuvers are generated with Eq.(11) and that of the slalom maneuver with Eq.(12), respectively. Also the polynomial function of Eq.(13) is used to define the variation of height for the vertical (bob down after bob up) maneuver and velocity for the deceleration after acceleration maneuver.

$$\Delta x(\bar{t}) = \frac{(\Delta x)_{\max}}{16.0}\left[8 + \cos(3\pi\,\bar{t}) - 9\cos(\pi\,\bar{t})\right] \tag{11}$$

$$\Delta x(\bar{t}) = \frac{(\Delta x)_{\max}}{46.8}\left[32 + \sin(2\pi\,\bar{t}) - 20\sin(4\pi\,\bar{t}) + 2\sin(8\pi\,\bar{t})\right] \tag{12}$$

$$\Delta x(\bar{t}) = 64(\Delta x)_{\max}(1.0 - 3.0\bar{t} + 3.0\bar{t}^{2} - \bar{t}^{3})\bar{t}^{3} \tag{13}$$

$$\text{where}\quad \bar{t} = (t - t_{entry})/(t_{finish} - t_{entry}) \quad 0 \le \bar{t} \le 1$$

A typical maneuver can be described by its different maneuver phases such as entry stage, steady maneuver phase, and exit phase. To describe these different maneuver phases Thompson et al[15] proposed a piecewise polynomial method. By changing the time required for the entry phase and the exit phase, we can control the level of maneuver aggressiveness. A turning flight with a constant turn rate is an example of the steady maneuver phase. Here we apply this method to trajectory generation for a turn maneuver with following mathematical description:

$$\Delta\dot{x}(t) = \begin{cases} (\Delta\dot{x})_{\max}(-2\bar{t}^{3} + 3\bar{t}^{2}), & t_{entry} \le t \le t_{steady} \\ (\Delta\dot{x})_{\max}, & t_{steady} \le t \le t_{exit} \\ (\Delta\dot{x})_{\max}(-2\tilde{t}^{3} + 3\tilde{t}^{2}), & t_{exit} \le t \le t_{finish} \end{cases} \tag{14}$$

Where

$$\begin{aligned} (\Delta\dot{x})_{\max} &= (\Delta x)_{\max}/(t_{finish} + t_{exit} - t_{steady} - t_{entry}) \\ \bar{t} &= (t - t_{entry})/(t_{steady} - t_{entry}) \\ \tilde{t} &= 1 - (t - t_{exit})/(t_{finish} - t_{exit}) \end{aligned}$$

The requirements for maneuver accuracy are commonly specified with the boundary deviation from a constant reference value in flight speed, altitude, sideslip, heading, and positions, etc. In the same way, this paper uses these quantities to define the reference trajectory for other axes.

The quadratic cost function for control is generally used in dynamic optimization. As previously mentioned, this paper penalizes state constraints in the same manner as control constraints to avoid the difficulty of its numerical implementation in the indirect method. The resulting cost function can be expressed as:

$$f_{CO}(\bar{x}_R(t), u(t), t) = 0.5(\bar{x}_R - \bar{x}_{target})^T Q(\bar{x}_R - \bar{x}_{target}) + 0.5(u - u_{TRIM})^T R(u - u_{TRIM}) \tag{15}$$

Where

$\bar{x}_R$: reduced rigid body states

$\bar{x}_{target}$: target states

$$
\begin{aligned}
\bar{x}_R(t) &= [u, v, w, p, q, \dot{\psi}, \phi, \theta, \psi, x_E, y_N, h]^T \\
R &= dig(r_{\delta_0},\ r_{\delta_{1C}},\ r_{\delta_{1S}},\ r_{\delta_{TR}}) \\
Q &= diag(\ q_u,\ q_v,\ q_w,\ q_p,\ q_q,\ q_{\dot{\psi}},\ q_\phi, \\
&\qquad q_\theta,\ q_\psi,\ q_{xE},\ q_{yN},\ q_H\ )
\end{aligned}
$$

The target states $\bar{x}_{target}(t)$ are set to be trim states $(x_R)_{TRIM}$ except those which need the description of their time variation for a specific maneuver. The same control weight matrix R is used throughout this paper with its diagonal components as:

$$
r_{\delta_0} = 3600,\ r_{\delta_{1C}} = 900,\ r_{\delta_{1S}} = 900,\ r_{\delta_{TR}} = 600
$$

The positive semi-definite weight matrix Q has its components as listed in Table.1. The components show small variation in their value, depending on the required tracking accuracy for a specific maneuver.

Table.1 Diagonal components of the state weight matrix

q_u	q_v	q_w	q_p	q_q	$q_{\dot{\psi}}$
200	7.5	4.5	7.5	15	200
q_ϕ	q_θ	q_ψ	q_{xE}	q_{yN}	q_H
100	500	35	0	100	30
Note: The components are the same for all MTEs except 1. q_q = 40 for side step 2. $q_{\dot{\psi}}$ =400 and q_ψ =350 for turn 3. q_{yN} = 0 for turn					

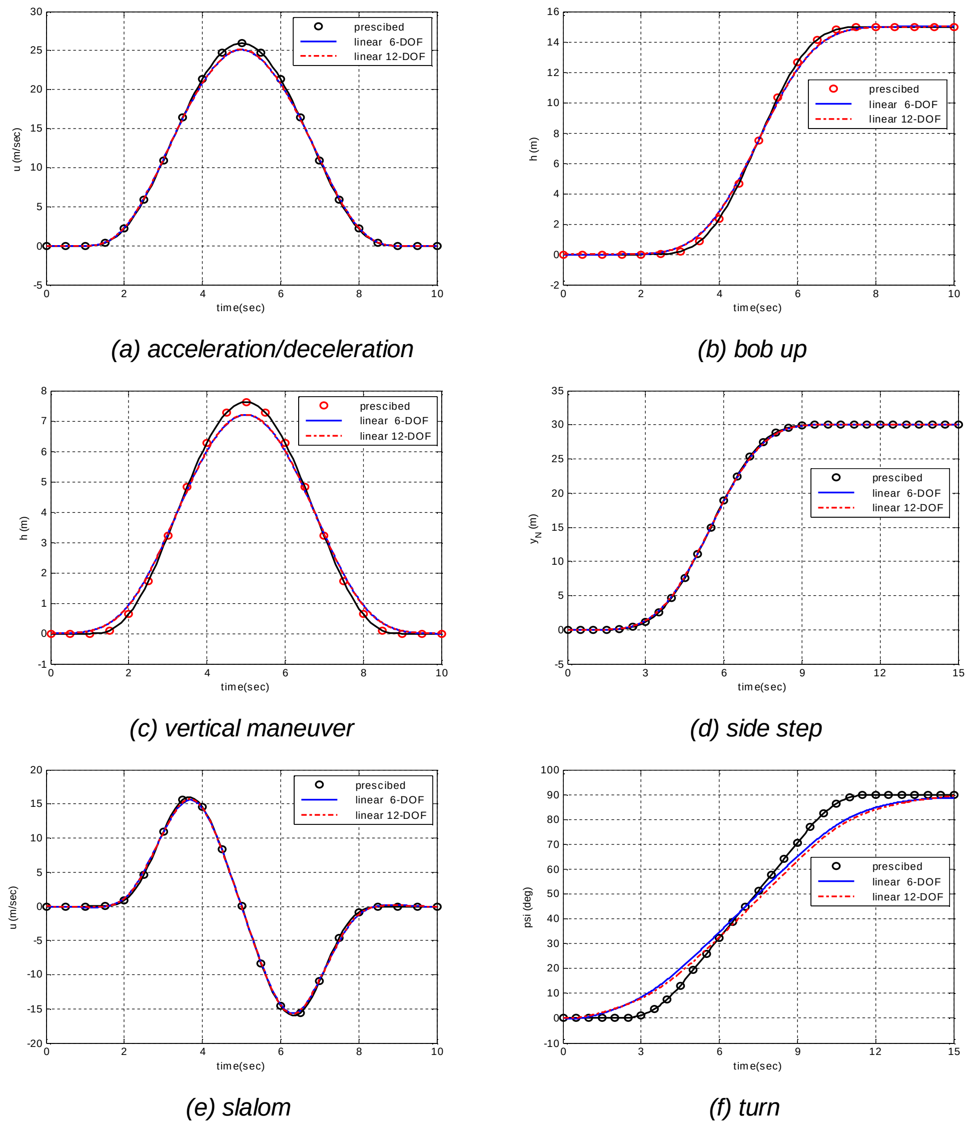
The initial conditions for state variables can be specified with the results of trim analysis because maneuvers in this study are started from a steady trim condition and their terminal conditions can be defined with target states at the end of a maneuver.

$$
\begin{aligned}
x_i(t_0) &= x_{i,\,trim}, \quad i = 1, \ldots, n \\
\bar{x}(t_f) &= \bar{x}_{target}(t_f)
\end{aligned}
$$

4. APPLICATIONS

The present approaches have been applied to representative flight maneuvers for Bo-105 helicopter configuration. The bob up, bob up and bob down, side step, and deceleration after acceleration maneuvers start its maneuver from hover and the slalom and turn maneuvers from steady level flight at a forward speed of 60 knots and 120 knots, respectively. Time integration is carried out using the 4-stage Runge-Kutta method with fixed time step size. The number of shooting nodes is determined to guarantee the numerical stability of MSM. Because the state and costate variables for flight trajectory are not available, those variables at each shooting node are initialized with the same values. For linear models, they are simply set to zero except a costate variable corresponding to cost equation with 1.0 and trim states are used as initial guesses for the analysis using the nonlinear model.

Fig.1 presents the calculated trajectory using linear models for various mission task elements and the present results shows a good comparison with the prescribed trajectories. The trajectory deviation from the prescribed one can be reduced by adjusting the components of state weighting matrix. In this case the heuristic tuning is required because of the sensitivity of its parameters to numerical convergence of the indirect method.



(a) acceleration/deceleration (b) bob up

(c) vertical maneuver (d) side step

(e) slalom (f) turn

Figure 1. The calculated trajectories with different linear models

The most time consuming routine using the indirect method is that of calculating optimal controls, which requires the numerical approximation for the Hessian matrix. The following four different approaches are studied in order to select a candidate for its applications to the MTE analyses using nonlinear helicopter models.

Method 1: affine approximation in controls, $H_{uu} \approx \lambda_{CO} R$
Method 2: optimization based on DFP (Davidon-Fletcher-Powell) method [13]
Method 3: DAE (Differential Algebraic Equation) solver, RADAU5[14]
Method 4: full Hessian matrix calculation

Fig.2 and Fig.3 compare the computation efficiency of each method for the linear 6-DOF and 12-DOF models. The number of function calls and the computational time are the average value for one iterative step of MSM. Even though the DFP method needs no explicit calculation of Hessian matrix, the efficiency is nearly the same as Method 4. Whereas the design optimization problems generally allow relatively low numerical precision enough to guarantee the design improvement, the optimal controls should be calculated with high precision for its use in succeeding forward simulation. If the controls calculated with low precision are used in the indirect method, the succeeding analysis generally suffers from numerical divergence, which degrades the numerical efficiency of DFP method in design optimization. The method 3 shows the worst performance among four methods. The DAE approach has an advantage of simultaneously solving both the ordinary differential equations, Eq.(6), and the algebraic equation, Eq.(8). But the computation of consistent initial conditions at every shooting node can reduce the efficiency of DAE solvers. Moreover, a tight step-size control adopted in this application is likely to increase the number of function calls and fails to continue the time integration when a calculated step size is less than the specified step-size tolerance.

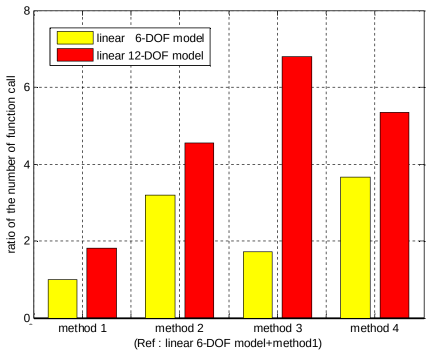


Figure 2. Relative number of function calls

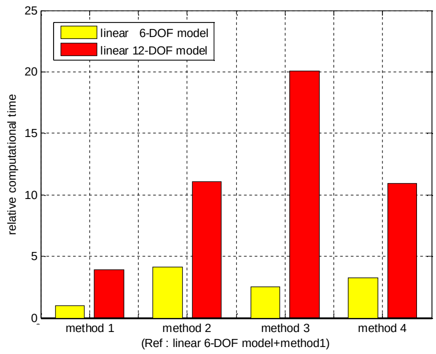


Figure 3. Relative computational time

Fig.4 and Fig.5 represent the convergence history of MSM with different helicopter model. The bob up maneuver is analyzed with 500 shooting nodes. The method 1 and method 4 denote the same convergence history and show better convergence than the method 2 and method 3. Especially, the method 4 is difficult to get a converged solution for the 12-DOF linear model. Because the efficiency of each method depends on the problems at hand and there are so many parameters to set in each solver, the direct comparison is not so simple. But regarding above comparison, the method 1 which shows the best efficiency for the present application is chosen for the later applications.

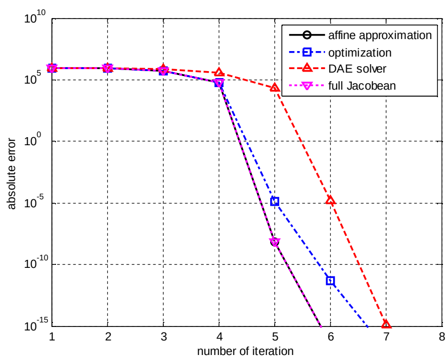


Figure 4. Convergence history (6-DOF linear model)

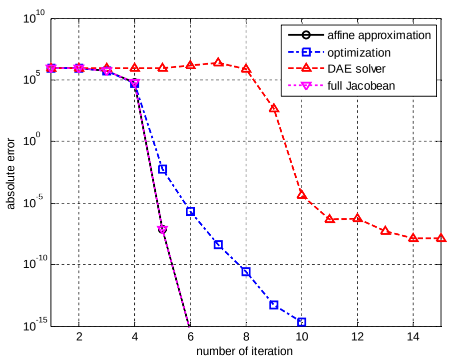


Figure 5. Convergence history (12-DOF linear model)

Fig.6, Fig.7, and Fig.8 present the history of linear velocity components, and the corresponding costates, and controls for the bob up maneuver. The analyses are carried out with 1500 shooting nodes, 6 integration nodes and with variation of helicopter models. The results using linear models are summed up with trim states and trim controls in order to give direct comparisons.

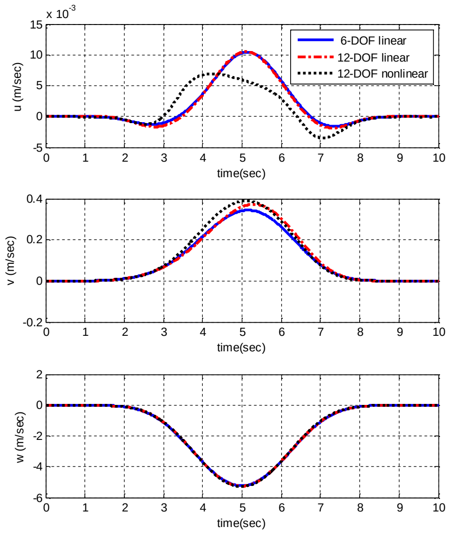


Figure 6.Linear velocity with different models (bob up maneuver)

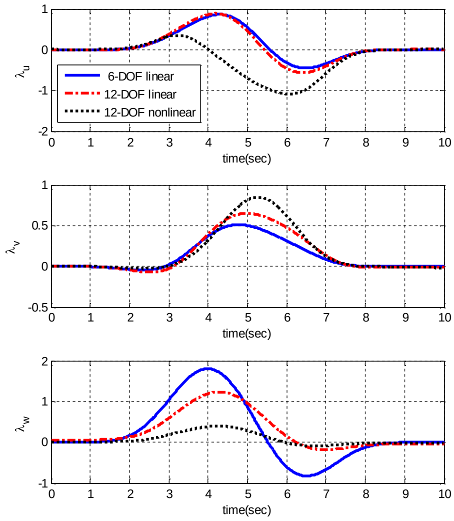


Figure 7.Costates with different models (bob up maneuver)

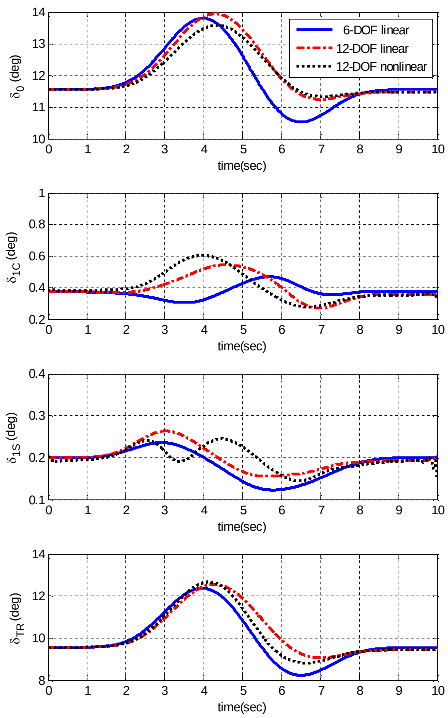


Figure 8. Controls with different models (bob up maneuver)

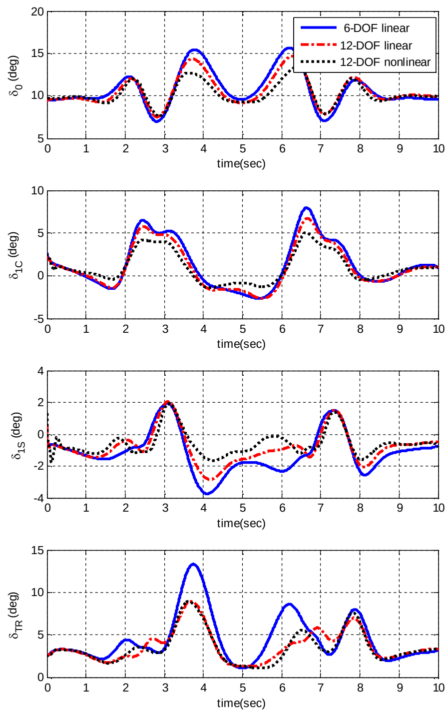


Figure 9. Controls with different models (slalom maneuver)

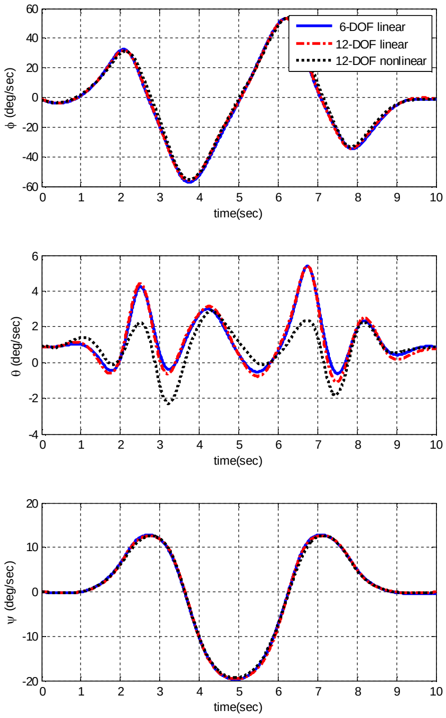


Figure 10.Attitude and heading with different models(slalom maneuver)

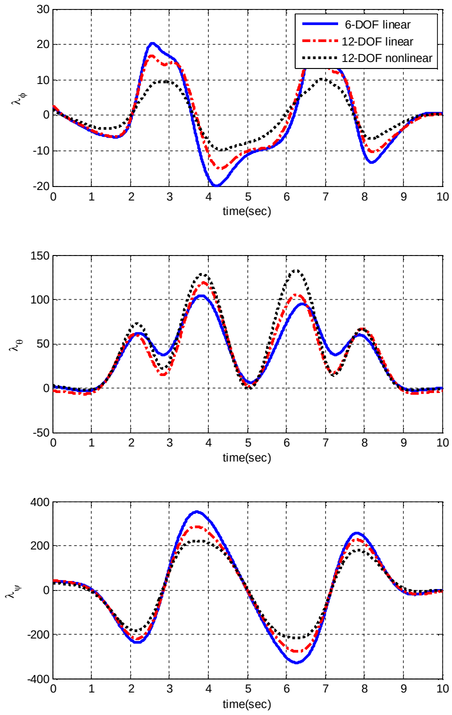


Figure 11.Costates with different models (slalom maneuver)

Fig.9, Fig.10 and Fig.11 show the results of slalom maneuver analysis with the same analysis condition as above. These comparisons enable us to determine the applicable flight range of linear models.

5. CONCLUSIONS

This paper has studied nonlinear optimal control approaches to the analysis of helicopter MTEs. The results showed the predicted optimal trajectory well tracked the prescribed one, which proved the validity of the penalty function approach. The comparative studies on the four different methods of resolving the optimal control equation have been carried out to select the most efficient one for the present applications. The approximation of Hessian matrix under the assumption that the system at hand is affine in control has shown the superior efficiency to the other methods. The same approach is used for the analyses using the nonlinear helicopter model. The optimal solution for each MTE presented big differences depending on the model used. So the further studies related to the high fidelity helicopter models seem to be required.

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