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FEASIBILITY STUDY OF HELICOPTER BLADE ROOT SHEAR REDUCTION BY MEANS OF GEOMETRIC STIFFNESS ALTERATION

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ABSTRACT

The paper sets out to investigate the feasibility of reducing the root shear force acting on an articulated helicopter blade by means of superimposing varying degrees of pre-stressing at separate sections along the span of the blade. The study is restricted to the analysis of the second mode of vibration (one nodal point) set up as a consequence of the three-per-rev excitation frequency, and the aerodynamic load function acting over the surface of the blade is assumed to be represented by a five term polynomial. From the results computed it was found that compressive pre-stressing at sections either near to the hub, or, in the vicinity of the blade tip resulted in large reduction of root shear force. Practical aspects of this reduction in stiffness are discussed.

1. INTRODUCTION

The complex problem of blade vibration reduction is currently one of the principal means of helicopter research. Much of this work is concerned with the transmission of oscillatory blade loads through the rotor hub to the fuselage where excessive levels of vibration can impair the comfort and efficiency of the crew, as well as providing a poor environment for complex avionics. A valuable survey of recent and current work concerned with optimising structural design to achieve blade vibration reduction is given by Friedmann in Ref. 1. His broad conclusion is that a 15-42% reduction in transmitted loads can be obtained by optimally distributing blade mass and stiffness.

A primary objective is to keep the rotating blade natural frequencies as far away from the blade passing frequencies as possible but, in the case of the first and second flatwise modes, this is strictly limited by the fact that blade stiffness is dominated by the blade tension and elasticity contributes very little. The excitation of modes is caused by aerodynamic forces which necessarily comprise integer multiples of the blade passing frequency harmonics. There will also be aerodynamic damping present which, if positive (preferably), can result in a wide resonance peak in a neighbouring blade mode, which is consequently excited. An account of the mechanisms included is given by Gupta in Ref. 2.

A relatively simple design technique limiting the level of the hub shears transmitted to the airframe is put forward by Taylor in Ref. 3. His contention is that the blade modal shape can be altered by judicious distribution of blade mass and stiffness as a means of reducing the hub shears attributable to the dynamic inertia loads. Aerodynamic loads are not included in his summation of the hub shears. He concentrates upon the flatwise bending modes and by assuming a polynomial form for the spanwise aerodynamic distribution, defines a modal shaping parameter which links a particular aerodynamic polynomial term with a given blade mode shape and mass distribution. The significance of the modal shaping parameter is that it is independent of both the natural and the forcing frequencies. Taylor shows in Ref. 3 that the addition of a relatively small mass at the blade tip can reduce the level of the MSP considerably with a consequent reduction in hub shear.

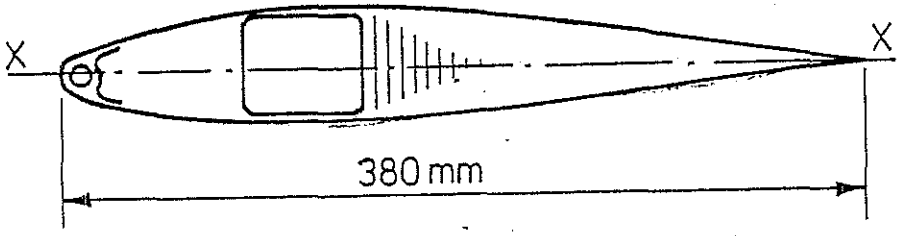


Figure 1 Blade section

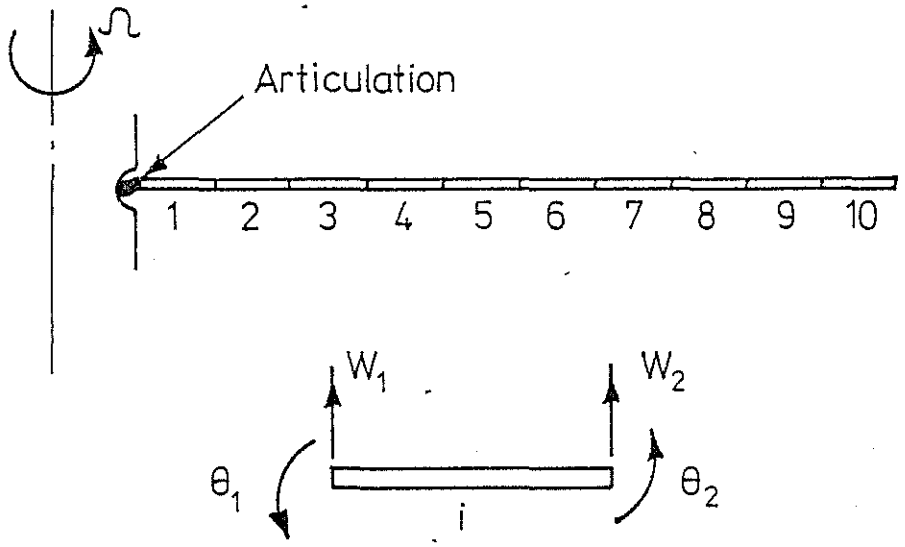


Figure 2 Finite element model of blade

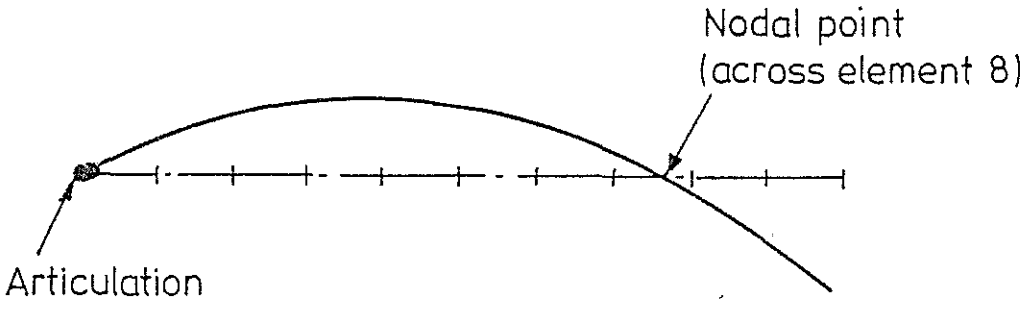


Figure 3 Vibratory shape of second mode of vibration

Some shortcomings of the MSP method are that the dynamic amplification factor is ignored (the proximity of the natural and forcing frequencies) and that the aerodynamic loads are excluded from the summation of hub shears. Nevertheless, the technique does provide a useful and simple tool for dealing with the transmitted load problem.

Taylor found that very little advantage could be gained by tailoring the blade elastic stiffness and this is not surprising because, when rotating, the stiffness is largely due to blade tension. The purpose of this paper is to investigate the effects of changing the geometric stiffness of the blades by pre-tensioning, or pre-compressing, a section of the blade. A development of Taylor's method is used to show the effects on hub shear of altering the tension in short lengths of blade. This technique can significantly alter the mode shape of the blades and so provides an alternative method to the addition of mass.

2. BLADE DETAILS

As shown in Figure 1, the blade cross section is that of a typical symmetric aerofoil of cord length 380 mm and thickness to cord ratio of 12%. The leading edge skin is 18 gauge stainless steel and the torsion box is of 20 gauge stainless steel. The trailing edge skins are assumed to be of $0^{o}/90^{o}$ GFRP, and the trailing edge core is NOMEX honeycomb which has not been included in the analysis.

Upon the basis of the above details, the section modulus about XX, and the mass per length of the wing were computed to be 37400 Nm and 4.81 kg/m respectively. The total length of the blade was taken as 5.4 m and articulated at 0.95 m from the central rotating axis. The speed of rotation was assumed to be 425 rpm. Upon the basis of private communication, Ref. 4, only the second flapwise mode of vibration (one nodal point) was considered (see Figure 3), and it was further assumed that this mode was predominantly excited by the three per rev. excitation. Upon the basis of the above wing data, the natural frequency of this mode was computed to be 1262.25 vibrations per minute, i.e. 2.97 times the rotational speed of 425 rpm. For the purpose of modal reduction, which will be discussed at a latter stage, only structural damping was assumed and to be represented by a value of $\epsilon = 0.15\%$.

3 ANALYSIS

3.1 Finite element model of helicopter blade

Consider the articulated helicopter blade exhibiting at a cross sectional form as shown in Figure 1. Furthermore, since in this study only flapwise motion about section XX is being considered, the complete blade is modelled by 10 simple 4 degrees of freedom beam elements as shown in Figure 2. For any element i bounded between η_1 and η_2 as shown, the flexural stiffness and mass matrices, $[k_f]$ and [m] respectfully, based upon a non-dimensionalised vibratory deflection form of the form:

$$q = w/b = (a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3)\, e^{j\omega t} \qquad (1)$$

become:

$$[k_f] = \frac{EI}{b} [B^T] [D] [B] \qquad (2)$$

$$[m] = \rho A b^3 [B^T] [C] [B] \qquad (3)$$

Furthermore, since all elements are subjected to in-plane stressing due to centrifugal loading and, where applicable, static pre-stressing, a geometric stiffness matrix for each element must be included. Based upon an in-plane stress distribution of the form:

$$\sigma = \frac{\rho \Omega^2 b^2}{2} (1 + C - \eta^2) \qquad (4)$$

the geometric stiffness of the i^{th} element can be derived as:

$$[k_g] = \frac{\rho A b^3 \Omega^2}{2} [B^T] [G] [B] \qquad (5)$$

where:

$$[D] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & & 4(\eta_2-\eta_1) & 6(\eta_2^2-\eta_1^2) \\ & \text{Symmetrical} & & 12(\eta_2^3-\eta_1^3) \end{bmatrix}$$

$$[C] \quad \begin{bmatrix} \eta_2-\eta_1 & \frac{1}{2}(\eta_2^2-\eta_1^2) & \frac{1}{3}(\eta_2^3-\eta_1^3) & \frac{1}{4}(\eta_2^4-\eta_1^4) \\ & \frac{1}{3}(\eta_2^3-\eta_1^3) & \frac{1}{4}(\eta_2^4-\eta_1^4) & \frac{1}{5}(\eta_2^5-\eta_1^5) \\ & & \frac{1}{5}(\eta_2^5-\eta_1^5) & \frac{1}{6}(\eta_2^6-\eta_1^6) \\ & \text{Symmetrical} & & \frac{1}{7}(\eta_2^7-\eta_1^7) \end{bmatrix}$$

and $[B]$

$$\begin{bmatrix} \frac{\eta_2^2(\eta_2-3\eta_1)}{h^3} & -\frac{\eta_1\eta_1^2}{h^2} & \frac{\eta_1^2(3\eta_2-\eta_1)}{h^3} & -\frac{\eta_1^2\eta_2^1}{h^2} \\ \frac{6\eta_1\eta_2}{h^3} & \frac{\eta_2(\eta_2+2\eta_1)}{h^2} & -\frac{6\eta_1\eta_2}{h^3} & \frac{\eta_1(\eta_1+2\eta_2)}{h_2} \\ -\frac{3(\eta_1+\eta_2)}{h^3} & -\frac{(\eta_1+2\eta_2)}{h^2} & \frac{3(\eta_1+\eta_2)}{h^3} & -\frac{(\eta_2+2\eta_1)}{h^2} \\ \frac{2}{h^3} & \frac{1}{h^2} & -\frac{2}{h^3} & \frac{1}{h^2} \end{bmatrix}$$

where $h = \eta_2 - \eta_1$

and

$$[G] = \int_{\eta_1}^{\eta_2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ & f & 2\eta f & 3\eta^2 f \\ & & 4\eta^2 f & 6\eta^3 f \\ & \text{Symmetric} & & 9\eta^4 f \end{bmatrix}$$

where $f = 1 + C - \eta^2$

Hence the complete stiffness matrix of the i^{th} element can be written as:

$$[k] = [k_f] + [k_g] \qquad (6)$$

3.2 Aerodynamic load vector.

For k per rev excitation (k=1, 2, 3, etc.), the aerodynamic force per non-dimensionalised length (flapwise only) over the i^{th} element will be assumed to be adequately represented by the general form

$$p = \sum_{N=0}^{4} p_N \qquad (7)$$

where $p_N = A_N \eta^N e^{jk\Omega t}$

giving the 4 term force vector for the i^{th} element, for the N^{th} term of equation (7) as:

$$\{f_N\} = [B^T] A_N b^{N+2} \int_{\eta_1}^{\eta_2} \eta^N \{1 \ \eta \ \eta^2 \ \eta^3\}^T d\eta \qquad (8)$$

By means of standard finite element techniques, the stiffness matrices, mass matrices, and force vectors of all elements are combined to form the structural stiffness and mass matrices and the total force vector corresponding to the relevant value of N in equation (7).

3.3 Modal reduction

Since, as previously mentioned, we are only concerned with the effect of the second mode of vibration of the blade, it is convenient to apply the standard procedure of modal reduction to the system, whereupon the system is reduced to a one-degree-of-freedom system describing the second mode, i.e.

$$\frac{d^2\gamma}{dt^2} + 2\zeta\omega_r \frac{d\gamma}{dt} + \omega_r^2 \lambda = K_N \gamma_N e^{jk\Omega t} \qquad (9)$$

where $k_N = A_N b^{N+2}/\rho A b^3$

and γ_N = modal force/modal mass

Expressing ω_r = natural frequency of the second mode

$= \alpha \Omega$

and solving for λ gives:

λ = principle co-ordinate of second mode

$$= K_N \gamma_N (1/\alpha\Omega)^2 / \{(1 - \beta^2) + j2\zeta\beta\} \quad (10)$$

where $\beta = k/\alpha$

Having solved for the principle co-ordinate λ and remembering that w = vibratory deflection = bq, one can solve for the root shear force SF from

SF = total inertial force over blade + aerodynamic load over blade

$$= (k\Omega)^2 \int_L \rho\, Aw\, dx + \int_L \text{aerod. force } dx \quad (11)$$

Performing the necessary algebraic manipulation, one arrives at an expression for the root shear force due to the k per rev excitation and the N term of the aerodynamic load of the form:

$$SF = A_N\, b^{N+1} \{SF^N_{REAL} + J\, SF^N_{IMAG}\} \quad (12)$$

4 RESULTS

For values of N in equation (7) equal to 0, 1, 2, 3 and 4, SF was plotted (in polar form) for varying values of c as contained in equation (4) acting separately at elements 2 to 10 in Figure 2. Two sets of results were computed, namely, when c ranging between 0 and 1 in increments of 0.2 were plotted and are shown in Figures 4a to 4e. For the cases of pre-compression, however, values of c ranging from 0 to the particular value of $C = C_{CRIT}$ corresponding to a state of buckling of the element under consideration, were plotted in increments of $0.2\ C_{CRIT}$. These latter results are presented in Figures 5a to 5e. The table below details the symbol key used in Figure sets 4 and 5.

Element	2	3	4	5	6	7	8	9	10
Symbol	△	▽	+	×	□	◇	○	*	⊕

5. DISCUSSION

Results presented in Figure groups 4 and 5 indicate that substantial changes to the root shear force can be effected by alteration to the geometric stiffness at certain sections of the blade length. Furthermore, as is evident from the results shown in Figure group 5, a decrease in geometric stiffness at any of the 10 elements considered will, in effect, reduce considerably the magnitude of the root shear force. With special reference to Figure group 5, it was observed that generally alterations to the stiffness of elements, 4, 5 and 6, by and large, was not quite so effective as alterations to all other elements. Upon reflection this can be justified when, for the particular mode of vibration under consideration in this study, one considers that is is in the vicinity of these elements that the vibratory slope is generally at a minimum, (see Figure 3), thus reducing in magnitude the terms of the geometric matrices of these particular elements. It would seem therefore that reduction of the root shear force by means of compressive pre-stressing at elements along the wing span is most effective when elements either near the hub or near the tip are subjected to this form of pre-stressing. In a practical context, such compressive pre-stressing could perhaps be effected by a pre-stressed wire running centrally through the section as shown in Figure 6. In such a situation, a reduction in the geometric stiffness matrix of the particular element could only be realised if the design was such as to allow the wire to dynamically perform as a finite "bar" element and the section of the wing to perform as a finite "beam" element.

As mentioned at an earlier stage of the report, the degree of compressive pre-stressing was a fraction of the critical compressive stress σ_c, which, if applied to the element would result in the onset of buckling. Upon calculating this critical compressive stress it was assumed that the natural stiffness of the section was the combination of the flexural stiffness and the geometric stiffness due to the centrifugal loading only. Furthermore, since the in-plane stress due to centrifugal loading rapidly towards the tip of the blade, then the value of $C_{CRIT}(= 2\sigma_c/p\Omega^2 b^2)$ decreases from 0.92 at the first element (at the hub), to 0.0823 at the tenth element (at the blade tip). This would suggest that if it were decided to reduce the stiffness at element(s) near to the hub, this could only be practically implemented by altering the geometric stiffness, which in this vicinity, is predominantly higher than the flexural stiffness. Conversely, if it were decided to investigate changes to the stiffness in the vicinity of the blade tip, then one may consider alteration to the flexural stiffness, since in this vicinity the flexural component of stiffness is more predominant due to the geometric component approaching its minimum level.

6 NOMENCLATURE

A	cross sectional area of blade
A_N	constant relating to index N contained in aerodynamic loading expression
b	blade tip span (= 0.635 m)
C	pre-stress factor ($= 2\sigma/p\Omega^2 b^2$)
EI	section modulus through section XX of blade cross-section
k	constant indicating number of excitations per revolution of rotor
N	index in aerodynamic loading expression
q	non-dimensional vibratory deflection (= w/b)
x	distance from central rotating axis to a general point on the blade
w	flapwise deflection at a general point on the blade

η non-dimensional blade radius
σ constant pre-stress value
σ_c critical compressive pre-stress value to induce buckling
Ω rotational speed of blade
λ principle co-ordinate of second mode of vibration
ϵ non-dimensional damping factor (= 0.0015)

7. REFERENCES

1. Friedmann, P.P., 'Application of modern structural optimisation to vibration reduction on rotorcraft', Paper No. 62, Proceedings of the Tenth European Rotorcraft Forum, The Hague, The Netherlands, August 28-31, 1984.

2. Gupta, B.P., 'Blade design parameters which affect helicopter vibrations', Paper No. A-24-42-23-6222, Proceedings of the 48th Annual Forum of The American Helicopter Society, Arlington, Virginia, May 15-18, 1984.

3. Taylor, R.B., 'Helicopter vibration reduction by rotor blade modal shaping', Proceedings of the 38th Annual Forum of the American Helicopter Society, Anaheim, California, May 4-7, 1982.

4. Sobey, A.J., RAE Farnborough, Private Communications, 1985.

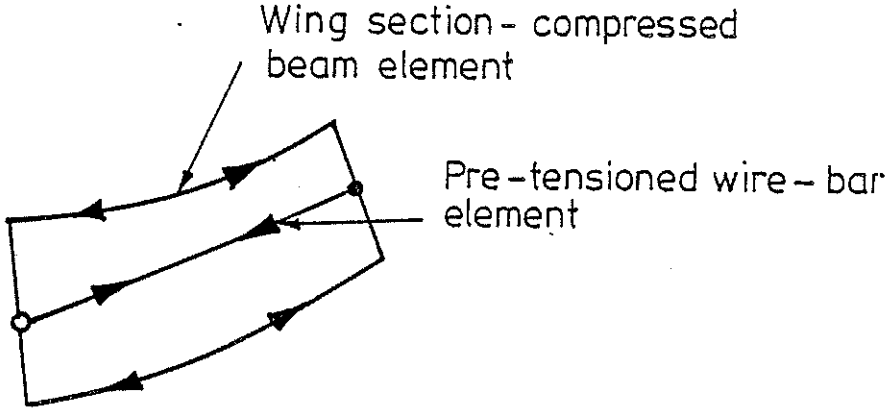


Figure 6

Figure 4a

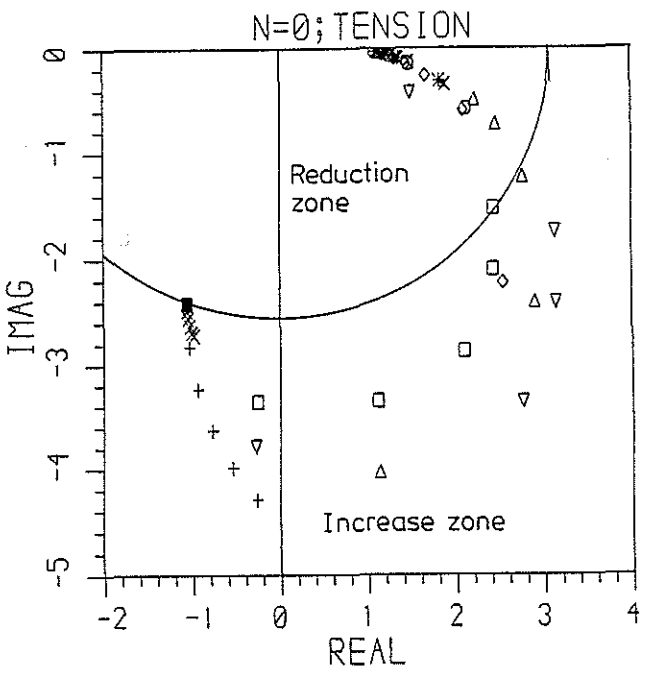


Figure 4b

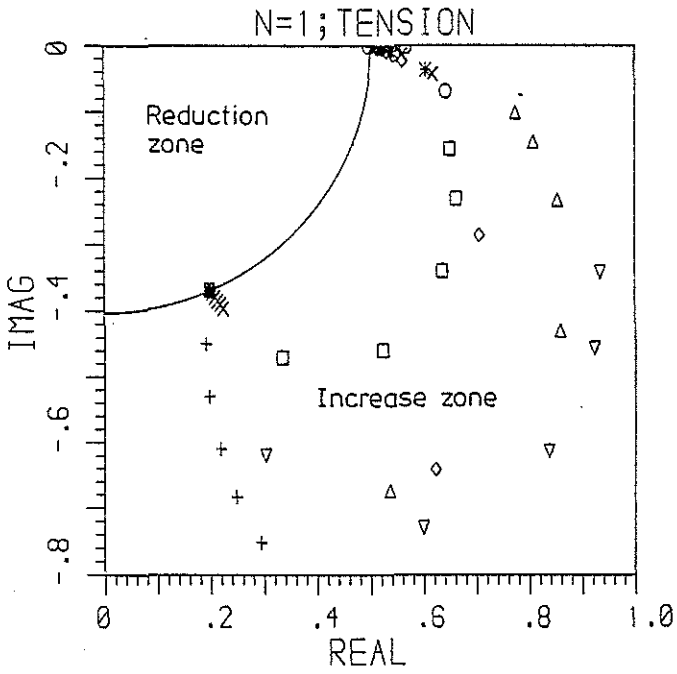


Figure 4c

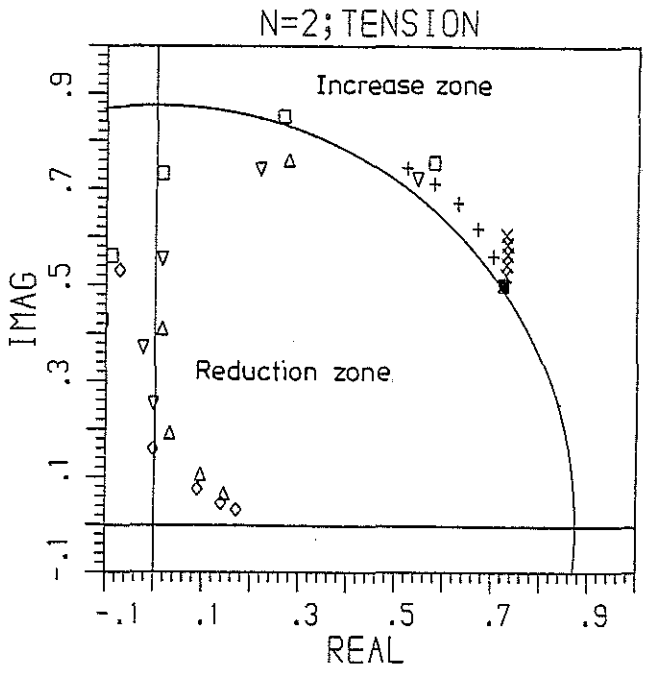


Figure 4d

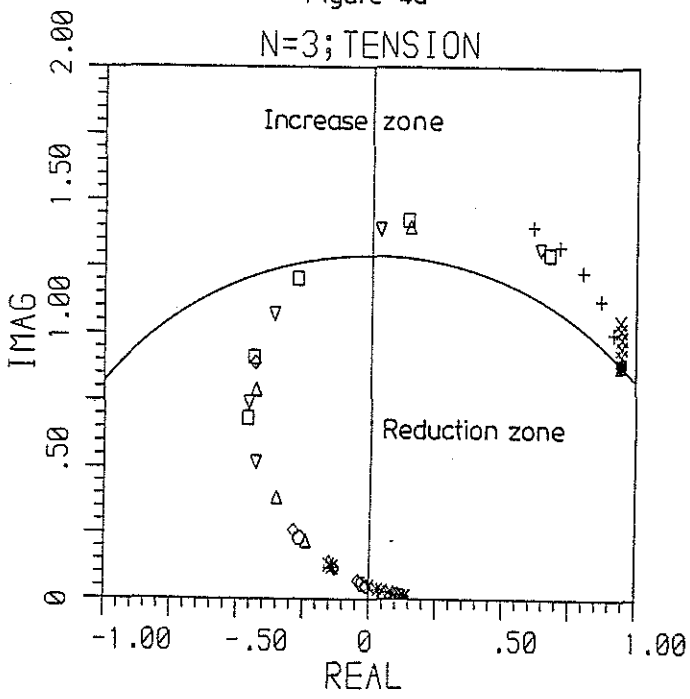


Figure 4e

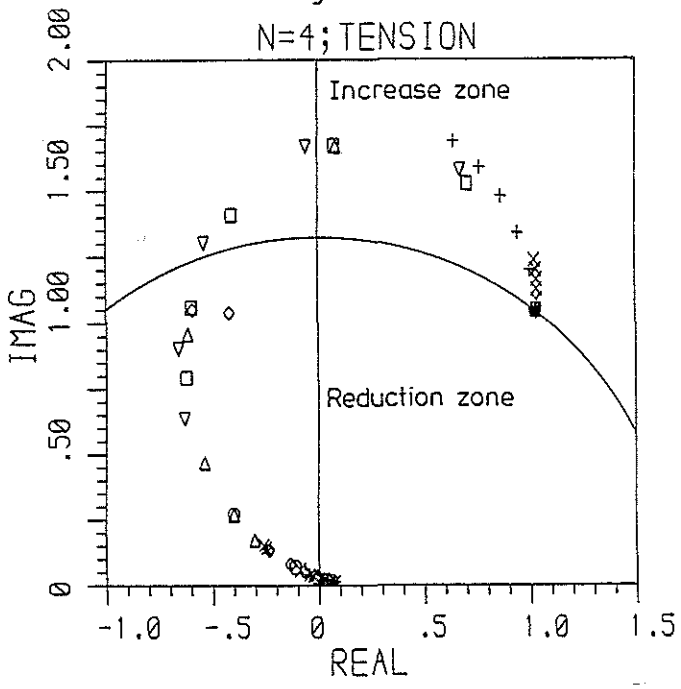


Figure 5a

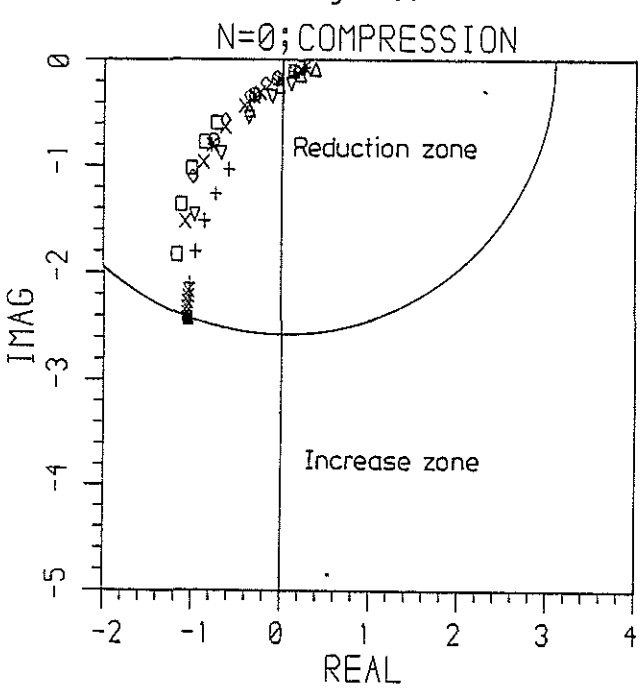


Figure 5b

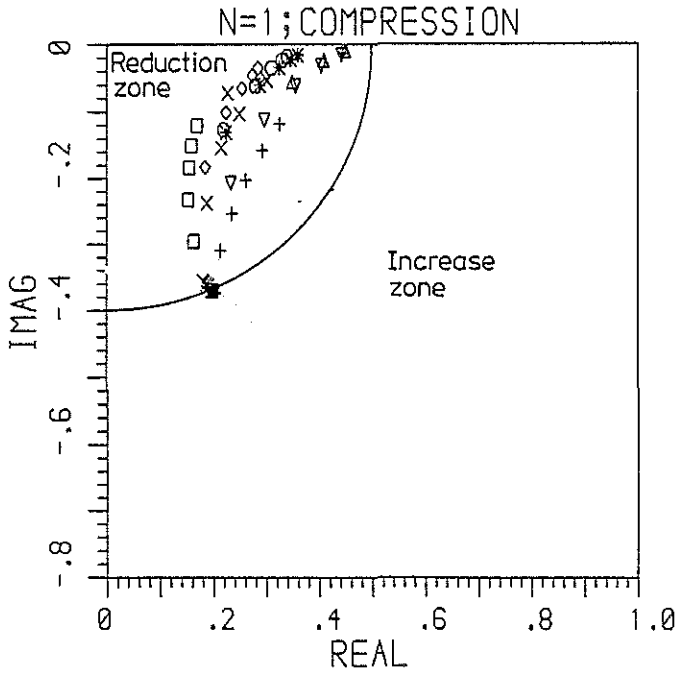


Figure 5c

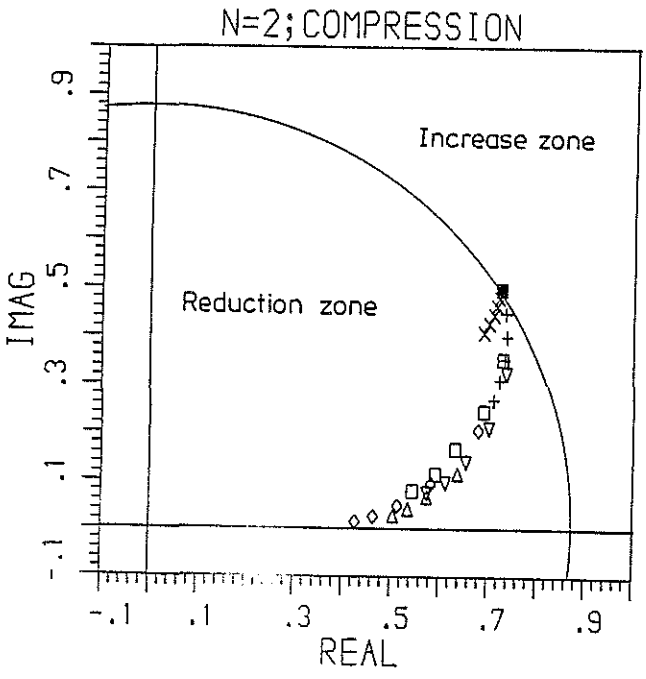


Figure 5d

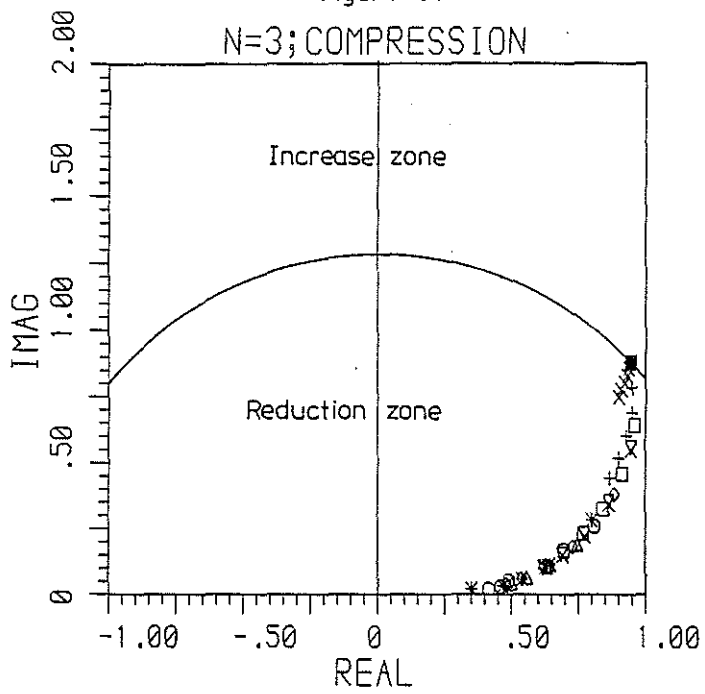


Figure 5e

