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STABILITY AND CONTROL MODELLING

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ABSTRACT

This paper discusses the influence of rotor dynamics and dynamic inflow on the stability and control characteristics of single rotor helicopters in near hovering flight. Body attitude and rate feedback gain limitations which arise due to rotor dynamics and dynamic inflow are discussed. It is shown that attitude feedback gain is limited primarily by body-flap coupling and rate gain is limited by the lag degrees of freedom. Dynamic inflow is shown to produce significant changes in the modes of motion.

1. Introduction

This paper considers aspects of mathematical modelling helicopters with a focus on stability and control characteristics. In particular it emphasizes the high frequency characteristics of the transfer functions describing the response of helicopters associated with the rotor degrees of freedom and also examines the impact of these on the design of automatic control systems. As technology moves forward and helicopter control systems change from mechanical to electronic or fly-by-wire systems, authority limits have less significance and it is important to examine the limits on automatic control design imposed by the rotor dynamics. In order to obtain some physical insight into this problem only the hovering trim condition is discussed in the paper, and consequently the vertical and yaw degrees of freedom are neglected. While it is realized that lags associated with the control system and flight control computers may be equally as important as rotor dynamics in limiting the design of automatic flight controls, these limits are not considered in this paper.

It has been recognized for quite some time that the flapping dynamics of an articulated rotor system imposes limitations in the design of automatic control systems for rotorcraft. In particular, Ellis [1] showed that including the flapping dynamics in the helicopter dynamic model produces limitations in rate and attitude feedback gains. Hall has shown that if an optimal control system devised based on the quasi-static flapping assumption is applied to a model with flap dynamics included, instabilities result [2]. Chen [3] has recently conducted a thorough investigation of the limitations in gain encountered when the flapping dynamics are included and presents experimental verification of these trends. His results were influenced by other delays in the control system of the helicopter and in a more recent study some of these effects have been removed [4]. In general, the gain limits which occur due to flapping modes are quite high. Similar results are obtained by Landis [5] who finds that flapping dynamics produce gain limitation, if the control system specification includes a minimum damping ratio, and in fact in the design study of this reference the damping ratio of the regressing flap mode was a critical element in the gain selection. Hanson [6] has noted the importance of the flapping dynamics in parameter identification studies.

Little attention however has been given to the lag degrees of freedom and their influence on the automatic control system design. The lag modes are characteristically lightly damped and for a typical fully articulated rotor design occur on the complex plane right in the region where the flapping roots cross into the right half plane as rate feedback gain is increased. Thus it would be expected that the lag dynamics may be of importance in producing gain limitations and this paper considers the influence of the lag dynamics in some detail. A related investigation is that of Straub and Warmbrodt [7] who have examined stabilization of the lag mode by feedback focussing on ground resonance rather than flight dynamics.

In recent years, there have been many investigations of the influence of dynamic inflow on the flap-lag and flap-lag-body stability of rotorcraft [8,9,10]. Reference 11 shows that it is necessary to include dynamic inflow in order to obtain good correlation with model data on ground resonance. However there appears to have been little investigation of the impact of dynamic inflow on the predicted stability and control characteristics of rotorcraft with the possible exception of References 12, 13, and 14. Reference 12 shows that dynamic inflow effects are significant in the thrust response to rapid collective inputs. Referencs 13 and 14 use quasi-steady inflow models. Of primary interest are the harmonic components of the inflow which are expected to be of significance in the pitch and roll motions of a rotorcraft. It will be shown that the time constants associated with the inflow dynamics are of the same order as the low frequency flapping dynamics, and thus if flapping dynamics are included it seems likely that the inflow dynamics should be included as well.

This paper will consider the following topics: body-flap coupling, the influence of dynamic inflow on the body-flap motion, lag dynamics and its influence on rotorcraft response, and the limitations which occur in rate and attitude feedback due to these higher order effects.

2. Discussion

A linearized model of the helicopter is used in this study to promote physical insight. The body motion is described by four degrees of freedom: fore and aft translation; lateral translation; roll and pitch. Uniform inflow as given by momentum theory is assumed for the constant part of the inflow. The model used for dynamic inflow is described in some detail below. Blade motion is treated in terms of multi-blade coordinates describing the shaft referenced tip path plane motion. Similarly, lag motion is described in terms of multi-blade coordinates. The blades are assumed to be rigid and fully articulated. Hingeless rotor characteristics are modelled by springs on the hinges. The coning angle is assumed constant and the steady lag angle does not appear in the equations of motion as a result of the multi-blade transformation.

First, the interaction between flapping modes and the body are examined and then the influence of dynamic inflow on these modes are considered before examining the more complex case including the lag degrees of freedom.

First, the simplest case of body-flap coupling is studied as it serves as a useful introduction to the more complex cases.

3. Body-Flap Coupling

For the conventional articulated rotor helicopter, the flapping modes are rather weakly coupled to the body modes due to their faster time scale. *Figure 1 shows a typical example of the modes of motion* for an articulated rotor helicopter. The helicopter characteristics are the same as in References 2 and 3 so the results of this study can be compared to those results. The uncoupled flapping modes are very close to the results of a complete coupled analysis.

The classical quasi-static approximation predicts the low frequency modes which are primarily associated with body motion quite well, i.e., the rotor plane dynamics and the body dynamics are well separated in time. However if the flapping hinge offset or equivalently the flapping stiffness is increased which in effect speeds up the body motion, significant coupling occurs between the rotation of the body and the flapping modes. Figure 2 shows the predicted characteristic roots for a small hingeless rotor helicopter. The uncoupled flapping mode is now quite different from the coupled body-flap modes. In particular, the body roll mode, which is faster than the body pitch due to the lower inertia, combines with the flap regressing mode to produce an oscillatory mode as shown in Figure 2. The characteristics of this coupled body flap mode can be estimated quite well by a simple analytical model. If we assume that the roll moment applied to the helicopter is proportional to lateral flapping and represent the low frequency flap mode by a time constant equal to the real part of the flapping mode then the equations of motion are, with time nondimensionalized by rotor RPM,

$$\begin{aligned} p' - k_H b_{1s} &= 0 \\ -\frac{\partial b_1}{\partial p} p + \tau_B b'_{1s} + b_{1s} &= A_{1s} \end{aligned} \qquad (1)$$

The transfer function

$$\frac{p}{A_{1s}} = \frac{k_H}{(s)\ (\tau_B s + 1) - \frac{\partial b_1}{\partial p} k_H}$$

It can be seen that if the blade time constant is neglected then this is just the classical first order response and the equivalent quasi-static characteristics are:

$$\bar{L}_p = \frac{\partial b_1}{\partial p} k_H,\quad \bar{L}_{b_{1s}} = k_H,\quad k_H \cong \frac{b}{2}\frac{I_B}{I_F}(p^2 - 1)$$

As the hinge offset or stiffness is increased k_H increases and the quasi-static body roll time constant becomes comparable to the blade time constant and the initial roll motion becomes a coupled second order system producing an overshoot in the initial response. For the simple theory used it may be noted that the blade and body time constants are,

$$\tau_B = -\frac{\partial b_1}{\partial \bar{p}} = \frac{16}{\gamma} \quad ; \quad \tau_F = \frac{\gamma}{16 k_H}$$

and thus the primary parameters influencing the flap-body coupling are the hinge stiffness as represented by k_H and the Lock number. This simple model works quite well to predict the body-flap coupling modes. A similar description is possible for the pitch axis, however here the coupling will tend to be weaker due to the larger pitch inertia (smaller k_H). Thus when the time constant associated with the quasi-static body motion (τ_F) becomes comparable to the blade flapping time constant (τ_B) there will be significant coupling between these modes. A small Lock number also increases the coupling. Figure 3 shows the locus of roots as flapping stiffness decreased on the hingeless rotor helicopter and the decoupling of the modes becomes evident. This simple model also predicts an instability for roll attitude feedback which occurs at a gain, k_A = 1 deg/deg. This gain produces a neutral oscillation in which the rotor plane remains level and the oscillation frequency is $(k_H)^{\frac{1}{2}}$. However, it is too simple to predict the instability which arises due to rate feedback described later in the paper. The advancing flap mode is involved in this instability.

4. Harmonic Inflow

We now wish to examine the effects of harmonic inflow on these modes. It has been recognized for quite some time that in certain motions of a rotor such as a steady pitch rate [13] or in the transient motion of rotors and in steady motion of a hingeless rotor [14] that since the rotor can exert a first harmonic aerodynamic moment on the airstream it would be expected that a first harmonic component of inflow would be present. It was also recognized that there are time constants associated with the development of the harmonic inflow components. The time constant of the harmonic inflow response to a change in aerodynamic moment can be thought of as related to a mass of air which must be accelerated to the new inflow state [11,12]. The inflow model described in detail below has been verified experimentally in Reference 8 and a similar model is used in Reference 11 to show a considerable improvement in correlation between theory and experiment for model tests.

This theory can be formulated in the following way. Assume that the inflow consists of a constant and first harmonic components.

$$\bar{v} = \bar{v}_o + \bar{v}_c \cos\psi + \bar{v}_s \sin\psi$$

It is assumed that v_o is a constant due to the hovering trim condition, and independent of radius, and that v_c and v_s vary linearly with radius. Writing an angular momentum equation, assuming that a cylinder of air of a height H must be locally accelerated in this process, the results of momentum analysis can be expressed as

$$\begin{aligned} \bar{\tau}_o \bar{v}_c' + \bar{v}_c &= -K_M \hat{C}_{M_A} \\ \bar{\tau}_o \bar{v}_s' + \bar{v}_s &= -K_M \hat{C}_{L_A} \end{aligned} \qquad (3)$$

where the constant K_M can be thought of as a momentum theory inflow gain

$$K_M = -\left.\frac{\partial \bar{v}_c}{\partial \hat{C}_M}\right|_M = -\frac{a\sigma}{\bar{v}_o N}$$

and the nondimensional inflow time constant is

$$\bar{\tau}_o = \left(\frac{H}{R}\right) \frac{1}{2\bar{v}_o N}$$

N is a parameter that depends upon the linearization assumption in the development of the momentum theory and N = 1 for a rigid wake assumption and N = 2 for a non-rigid wake assumption [11]. In the examples here N = 1. C_{M_A} and C_{L_A} are the aerodynamic moment coefficients in a fixed reference frame acting on the rotor plane. Now to obtain a physical idea of the influence of this model we can write the flapping equations as,

$$a_1'' + \ldots\ldots\ldots = \gamma\, \hat{C}_{M_A}$$

$$b''_1 + \ldots\ldots\ldots = \gamma\, \hat{C}_{L_A}$$

where it has been assumed that the hinge offset is small. That is, precisely speaking the aerodynamic moments in Eqs. (3) are about the hub and the moments in Eqs. (4) are about the flapping hinge. For purposes of this discussion these two moments are assumed to be equal.

Now the inflow components v_c and v_s will influence C_{M_A} and C_{L_A} through changes in blade element angle of attack and thus

$$\hat{C}_{M_A} = \hat{C}_{M_{OA}} + K_B\, \bar{v}_c$$

$$\hat{C}_{L_A} = \hat{C}_{L_{OA}} + K_B\, \bar{v}_s \qquad (5)$$

where $C_{M_{OA}}$ and $C_{L_{OA}}$ are aerodynamic moments without the influence of inflow and K_B can be thought of as blade element theory gain. For the assumption of a linear distribution of the harmonic components of inflow assuming small offset,

$$K_B = \left.\frac{\partial \hat{C}_M}{\partial \bar{v}_c}\right|_B = \frac{1}{8}$$

Thus we can think of harmonic inflow as arising from a feedback process related to the aerodynamic moment applied to the blades. These relationships are illustrated in block diagram form in Figure 4. Substituting Eq. (5) into Eq. (3) we obtain

$$\left(\frac{\bar{\tau}_o}{1 + K_L}\right) \bar{v}_c' + \bar{v}_c = \frac{-K_M}{1 + K_L}\, \hat{C}_{M_{OA}} \qquad (6)$$

where $K_L = K_M K_B$ is the inflow loop gain and $\bar{\tau}_{oc} = \bar{\tau}_o/1+K_L$ can be considered a closed loop inflow time constant. Now the feedback loop shown in Figure 4 can be collapsed into a single block as shown and this theory can be considered as an operator on the Lock number, i.e., or the aerodynamic moment producing flapping, i.e.,

$$a_1'' + \ldots\ldots = \left(\frac{1}{1 + K_L}\right)\left(\frac{\bar{\tau}_o s + 1}{\bar{\tau}_c s + 1}\right) \gamma \hat{C}_{M_0}$$

If the dynamics of the inflow are neglected $\tau_O = \tau_C = 0$ then this theory simply results in an effective Lock number reduction as shown a number of years ago [14]. Note that this applies only to the harmonic relationships and not to the coning. Considering C_{M0} as an input say due to cyclic pitch the theory results in a moment jump and decay similar to the thrust response in Reference 12.

The non-dimensional open loop inflow time constant associated with this process is given for N = 2,

$$\bar{\tau}_o = \frac{\bar{H}}{4\bar{v}_o}$$

Its nondimensional numerical value for the helicopter used in the example calculation is $\tau_O = 2.09$. The inflow loop gain is,

$$K_L = \frac{a\sigma}{16\bar{v}_o} = 0.51$$

and the closed loop inflow-time constant is

$$\bar{\tau}_{oc} = 1.38$$

For the quasi-steady inflow theory, the effective Lock number is approximately three-quarters the true Lock number. The harmonic inflow time constants are slower than the blade flapping time constant

$$\bar{\tau}_B = \frac{16}{\gamma} = 1.5$$

Similar numbers are obtained for the hingeless rotor helicopter with the blade time constant about twice as long due to the lower Lock number. Note that N has a significant influence on these values.

These results indicate that if the flapping dynamics are included in a dynamic analysis of a rotorcraft, the harmonic inflow dynamics should be included as well. Note that the quasi-steady inflow theory indicates that the blade flapping time constant is significantly increased and thus increased body-flap coupling would be expected. The effect of inflow dynamics on the coupled body flap dynamics is now examined.

The effect of dynamic inflow on the modes of motion of the example helicopter is quite significant. Figure 5 shows the modes of motion for the articulated rotor helicopter adding dynamic inflow. A small loss in damping can be noted for the advancing flap mode. There is little change in the low frequency modes involving body rotation and translation. There is a significant change in the regressing flap mode which couples with the body motion and produces real roots. The coupled inflow modes appear as well damped modes considerably speeded up by the coupling following the trend shown in Reference 10. The real part of the inflow mode is approximately -24. The regressing flap modes are two real roots with values less than would be indicated by the quasi-steady inflow theory, i.e., using a reduced Lock number. This reduction is due to the inflow time constant. As the inflow time constant is reduced ($H \to 0$) the values will approach those given by Lock number reduction. However there is now significant coupling of the flap mode with the body modes since the coupling effect of the harmonic inflow theory has been to slow down the flap mode while speeding up the inflow mode.

A simplified model can be used to predict quite well the modes associated with the coupling of the flap regressing mode, the body and the inflow, by adding the inflow equation and the harmonic inflow effects to the first order equations used to predict the effect of body flap coupling. Thus the simplified equations are

$$p' - k_H b_{1s} = 0$$

$$(b'_{1s} + p) = \frac{\gamma}{16} [-b_{1s} + A_{1s} + \bar{v}_c]$$

$$\tau_o \bar{v}'_c + \bar{v}_c = - \frac{K_M}{8} [-b_{1s} + A_{1s} + \bar{v}_c]$$

These three coupled equations give approximate modes very close to the modes of the complete system. The roll rate to lateral cyclic transfer fuction is

$$\frac{p}{A_{1s}} = \frac{(\tau_o s + 1)\ \eta^* k_H}{s[(\tau_{oc} s + 1)\ s + \eta^* (\tau_o s + 1)] + k_H (\tau_{oc} s + 1)} \qquad (8)$$

where $\eta^* = \frac{1}{1 + K_L} \frac{\gamma}{16}$

The transient response characteristics show less change than would be expected based on the changes in the modal characteristics.

Now consider these effects on a hingeless rotor helicopter. The modes of motion with dynamic inflow are shown in Figure 6 where we note that for this helicopter with an equivalent offset of about 11 percent increased coupling between the lower flap and the body roll mode, compared in Figure 2.

Addition of the dynamic inflow model causes a significant change in the transient roll response to a step cyclic input as indicated in Figure 7. These results strongly suggest that stability and control investigations should include dynamic inflow. Limited comparison with flight test data tend to suggest that this dynamic inflow theory may overestimate the effects. Recall however that the theory used here has been correlated with experiment in various model tests. Note the large change in the response predicted if $N = 1$ is used in the inflow theory [11].

Before considering the effect of lag motion, one last topic is of interest in helping to understand the complex interaction of the rotor and body dynamics. It was suggested many years ago [1] that for an articulated rotor helicopter it should be possible to cascade the rotor dynamics with the body dynamics since the flapping takes place relatively rapidly compared to the body motion. In the above discussion it has been seen that this assumption is likely to be valid when the quasi-static body modes are well separated from the flap modes. We can approximately view the problem in the following way. If the complete transfer function of the articulated rotor helicopter for roll rate to lateral cyclic pitch is grouped by high frequency and low frequency modes, then it can be noted then that the high frequency character (s large), or short time behavior, of this transfer function shows that the low frequency pole-zero configuration can be approximated by 1/s indicating that the remaining poles and zeros can be interpreted as the transfer function of the moment applied to the helicopter to cyclic pitch. Shown in Figure 8 is the response of the moment applied to the helicopter without body motion given by the high-frequency pole-zero configuration for a step input in cyclic. It can be noticed that there is an initial jump in the moment due to direct production of an inplane force due to cyclic pitch deflection and then a response taking the order of one revolution or so for the full moment due to control to develop. There are an equal number of poles and zeros in the moment transfer function. The final value corresponds in the case of the centrally hinged rotor to thrust tilt proportional to cyclic pitch. In looking at the shape of this response it can be approximately viewed as a short time delay associated with the flap advancing modes (note the approximate Padê configuration of the pole-zero combination associated with the transfer function) and the first order blade time constant associated with the flap regressing mode response. This response will be reexamined later when the lag modes are included.

5. Influence of Feedback

We now consider briefly the influence of body roll rate and attitude on the helicopter stability with the flapping modes included. The pole zero configuration and effect of rate feedback and attitude are shown in Figure 9 emphasizing the effects on the high frequency or blade modes. The trends are obtained here are similar to those obtained by Chen and Ellis.Note that the gains which produce instability ($k_R \cong 2$ deg/deg/sec, $k_A = 1$ deg/deg) are quite high compared to those conventionally used in rotorcraft. The limiting attitude gain is that predicted by the simple theory presented earlier. It may be noted that the lag frequencies for this helicopter are at approximately 0.75 and 1.25 per rev, thus one lightly damped mode occurs near where this root locus crosses the imaginary axis tending to indicate that consideration of the lag modes is highly desirable. The effects of adding the lag degrees-of-freedom are discussed in the next section.

6. Lag Degrees Of Freedom

The lag motion is now added in terms of two multi-blade coordinates γ_1 and γ_2 defined in an analogous way to the flapping coordinates, a_1 and b_1. These two coordinates can be thought of in terms of the motion of the center of mass of the rotor system, γ_1 corresponding to lateral motion, and γ_2 corresponding to fore and aft translation. For the typical articulated rotor helicopter used in this example, the lag frequency is nominally $.25\Omega$ and thus in the stationary frame the lag frequencies appear at $.75\Omega$ and 1.25Ω. It was noted in the derivation of the lag motion equations that a number of inconsistancies are found in the flap-lag equations of motion in the literature, due to inconsistant retention of terms in the linearized equations. None of the published equations in the literature appears to give the physically correct result that application of cyclic pitch in a hover trim condition should result in no off-axis coupling in the limiting case of no hinge offset. That is, in the notation employed here, if a constant cyclic pitch input A_{1s} is applied to the rotor, the steady state solution to the flap-lag equations with no body motion should yield:

$$b_{1s} = A_{1s}$$

$$\gamma_1 = \beta_o b_{1s}$$

$$a_{1s} = 0$$

$$\gamma_2 = 0$$

That this is the physically correct result can be seen by adopting a change of reference frame from the shaft to the control axis, or by noting how the lag angle changes with a change in reference frame. This result indicates that the harmonic lag motion produced by flapping is of the order of the coning angle, β_o, times the flap angle, and thus in equations of motion which include lag motion if terms such as $2\beta_o\zeta$, are retained in the flap equation, then terms of the order of $\beta^2{}_o\beta$ should also be included for consistency. If the equations are formulated carefully then in the rotating frame, the aerodynamic forces depend upon the quantity $(\Omega + \zeta)\cos\beta$, linearizing this expression, the aerodynamic forces depend upon the quantity $(\zeta + \beta_o\beta)$ which is invariant with a change in reference frame and thus the aerodynamic forces on a rotor blade are only a function of $(\zeta + \beta_o\beta)$ in addition to $(\theta + \beta)$. It will be found that if the inertial terms in the flap lag equations of motion are linearized retaining $\beta^2{}_o\beta$ terms then, the first harmonic motion of the rotor blade is only a function of the above two quantities. In the literature this result is not found for a variety of reasons. In Reference 14 for example, the inertial or kinetic energy terms are not linearized consistent with the treatment given the aerodynamic terms. Furthermore it is assumed that the induced velocity is normal to the shaft rather than normal to the blade or the tip path plane resulting in equations which do not properly transform with a change in reference frame.

Addition of the lag degrees of freedom produces little change in the basic modes of motion of the articulated rotor helicopter (without dynamic inflow) however the zero configuration for the roll rate to lateral cyclic pitch transfer function is now quite different. Adding the lag modes adds four poles, and essentially only two zeros. These zeros are located in the right half plane near the lag regressing mode. Now there are six high frequency zeros and eight poles. This configuration which may be considered as discussed above to describe the moment response to cyclic pitch applied to the body prior to body motion now has an initial zero value. That is, with the lag degree of freedom included, the inplane force due to cyclic which produced an instantaneous force on the helicopter when the lag is ignored as shown in Figure 8 now produces an acceleration of the rotor blade and no net initial inplane shear force is produced on the helicopter. In other words the high frequency behavior of the transfer function is altered significantly. Figure 10 compares the frequency response characteristics of the moment applied to the helicopter, with and without lag degrees of freedom. This indicates that the lag degree of freedom is important in any parameter identification studies due to its attenuation of inplane forces from the rotor blade and also that the lag degree of freedom will produce increased phase lag at high frequency.

Now consider the influence of body feedbacks on the stability of the system. Only the roll axis is considered although similar results will be obtained for the pitch axis. A root locus is shown for roll rate feedback to lateral cyclic pitch in Figure 11. The characteristic roots are shown normalized by rotor RPM. The character of the locus is now quite different from that obtained with only the flapping modes included. Both the advancing and regressing flap modes are destablized by rate feedback. The advancing lag mode is destabilized at quite a low value of rate gain approximately $k_R = 0.2$ deg/deg/sec. This behavior appears quite typical of rotorcraft. The behavior of the root locus for the small hingeless rotor helicopter is very similar to this one.

Figure 12 shows the stability boundary obtained for the articulated rotor helicopter in terms of attitude and rate gains. The attitude gain limit with no rate feedback occurs at $k_A = 1$ deg/deg the value given by the simple theory desribed above. The upper stability boundary on this graph which originates at the value is a body-flap mode stability limit and is essentially independent of the lag mode. The other boundary which is approximately a rate gain limit is due to addition of the lag degrees of freedom and differs by a factor of 10 from the rate limit boundary without lag indicated on Figure 9 and obtained in Reference 3. In fact the limiting level of rate gain obtained from this analysis is quite typical of actual helicopter automatic flight control systems [15,16] and thus appears to be a real practical limit. Addition of dynamic inflow had little influence on these boundaries. The precise value of the gain obtained for this boundary is of course sensitive to the estimations of the lag damper characteristics. Doubling the value of the mechanical lag damping in this example results in approximately a fifty percent increase in the allowable rate gain before instability is encountered. It should also be noted that mechanical lag dampers have non-linear damping characteristics which will also impact the precise value of the gain which produces instability as well as how close to the linear stability boundary it is desirable to be. The typical lag damper is designed to produce a maximum damping force and thus the effective lag damping decreases with increasing amplitude.

Thus it has been shown that the lag degrees of freedom produce a marked decrase in the allowable body rate feedback.

7. Acknowledgement

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8. Conclusions

1. Hingless rotor helicopters or rotors with high offset and low Lock number have significant levels of body flap coupling.

2. Current dynamic inflow models indicate time constants associated with the dynamic inflow are of the same size as the time constants associated with flapping motion and consequently produce significant changes in the modes of motion associated with body-flap coupling.

3. Body attitude gain limitations arise primarily from stability limits associated with coupled body-flap modes.

4. Body rate gain limitations arise primarily from stability limits associated with the lag modes. This study indicates rate gain limitations with the lag modes included are about a factor of ten smaller than those obtained including only the flap modes.

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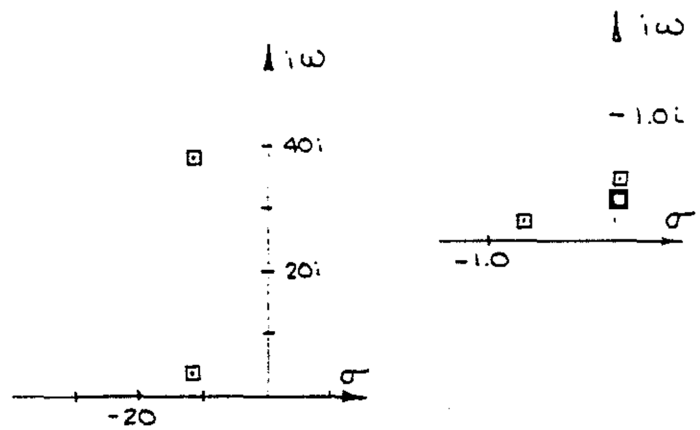

Fig. 1. Modes of Motion Articulated Rotor Helicopter, Without Lag Degrees of Freedom and and Dynamics Inflow.

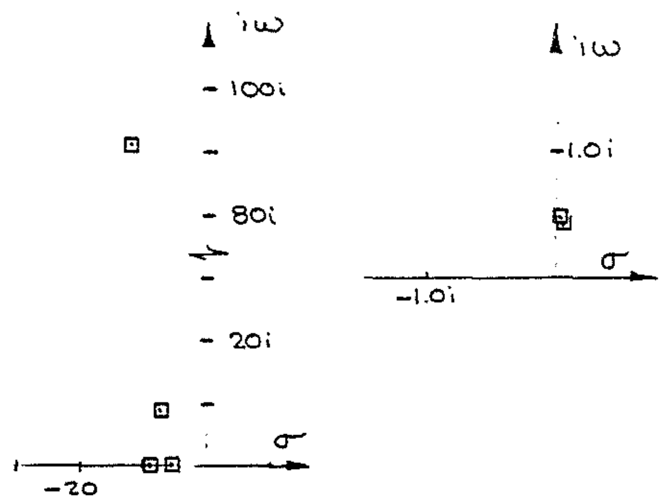

Fig. 2. Modes of Motion Hingeless Rotor Helicopter Without Lag Degrees of Freedom and Dynamic Inflow.

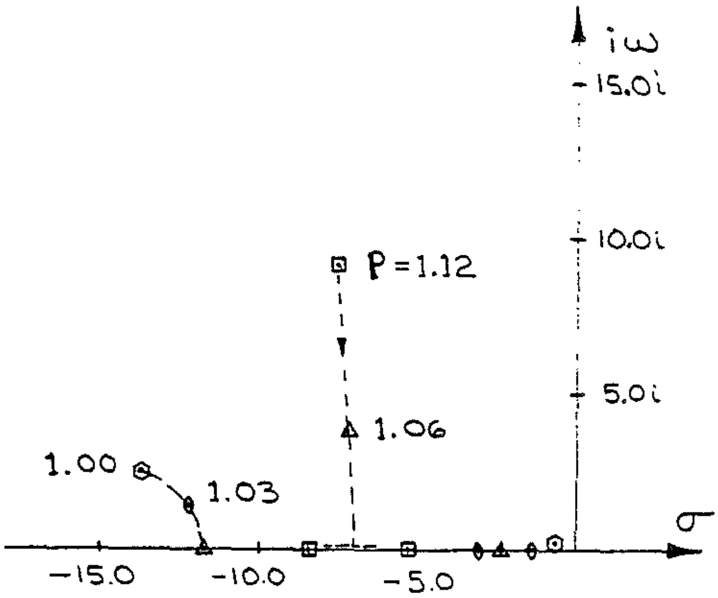

Fig. 3. Influence of Flapping Stiffness on Body-Flap Regressing Modes (γ = 5.0).

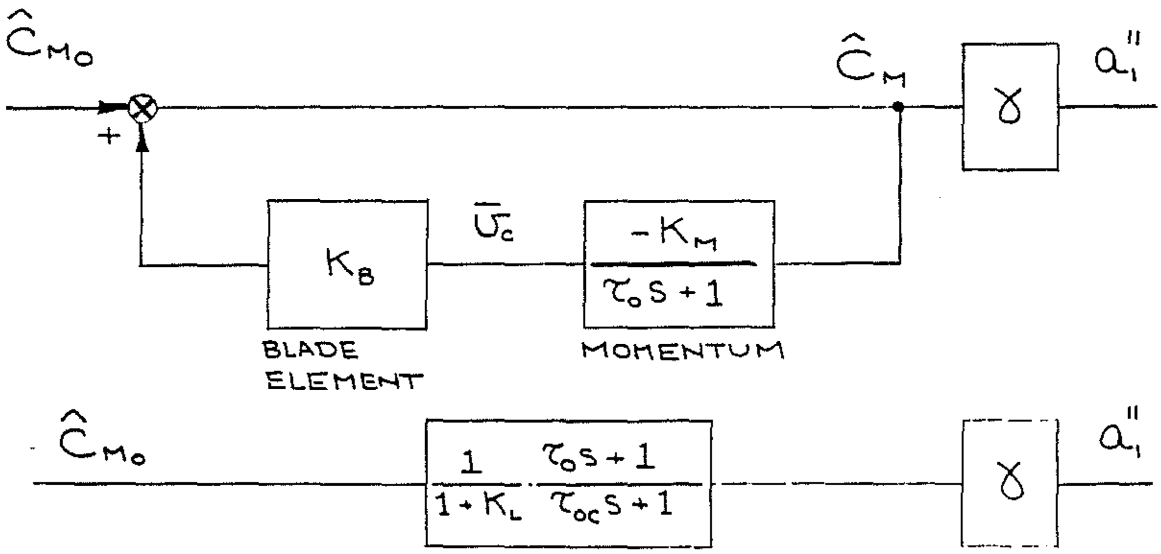

Fig. 4. Dynamic Inflow as a Feedback Process.

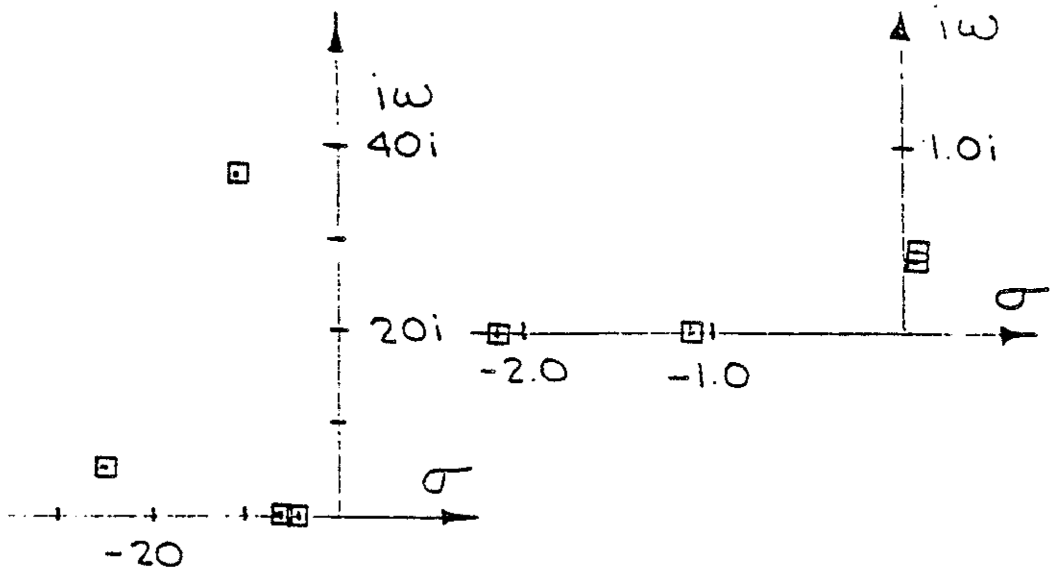

Fig. 5. Modes of Motion Articulated Rotor Helicopter Without Lag Degrees of Freedom, With Dynamic Inflow, ($\bar{H}$ = 0.46, N = 2).

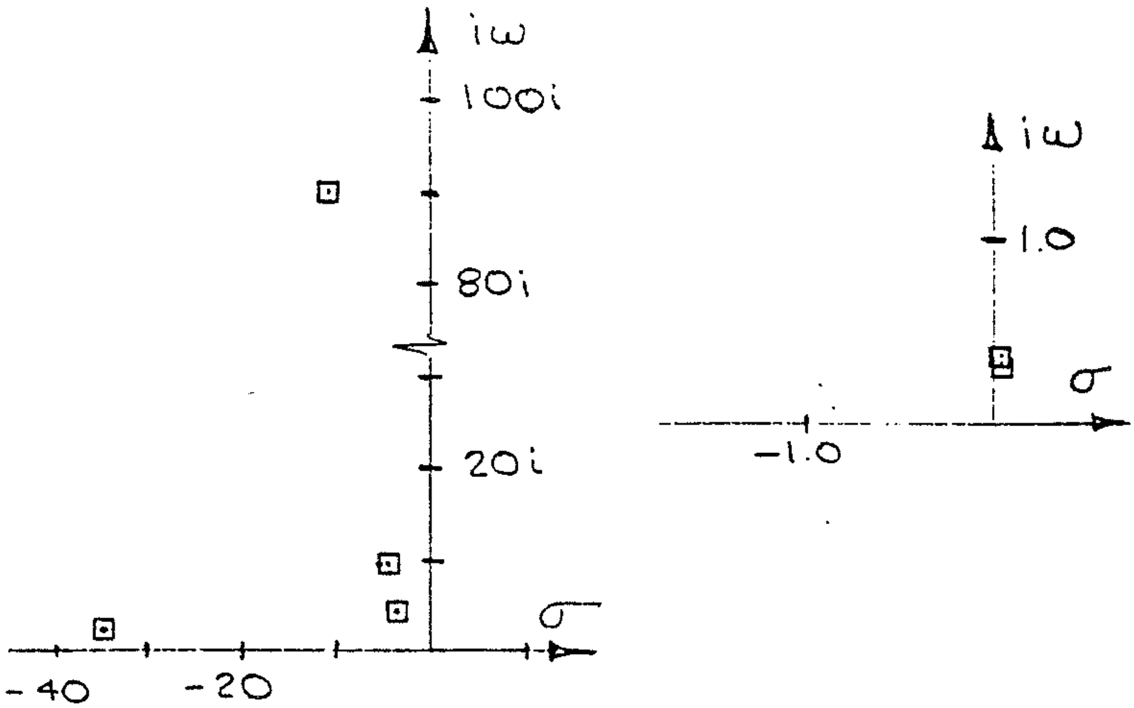

Fig. 6. Modes of Motion Hingless Rotor Helicopter Without Lag Degrees of Freedom, With Dynamic Inflow ($\bar{H}$ = 0.46, N = 2).

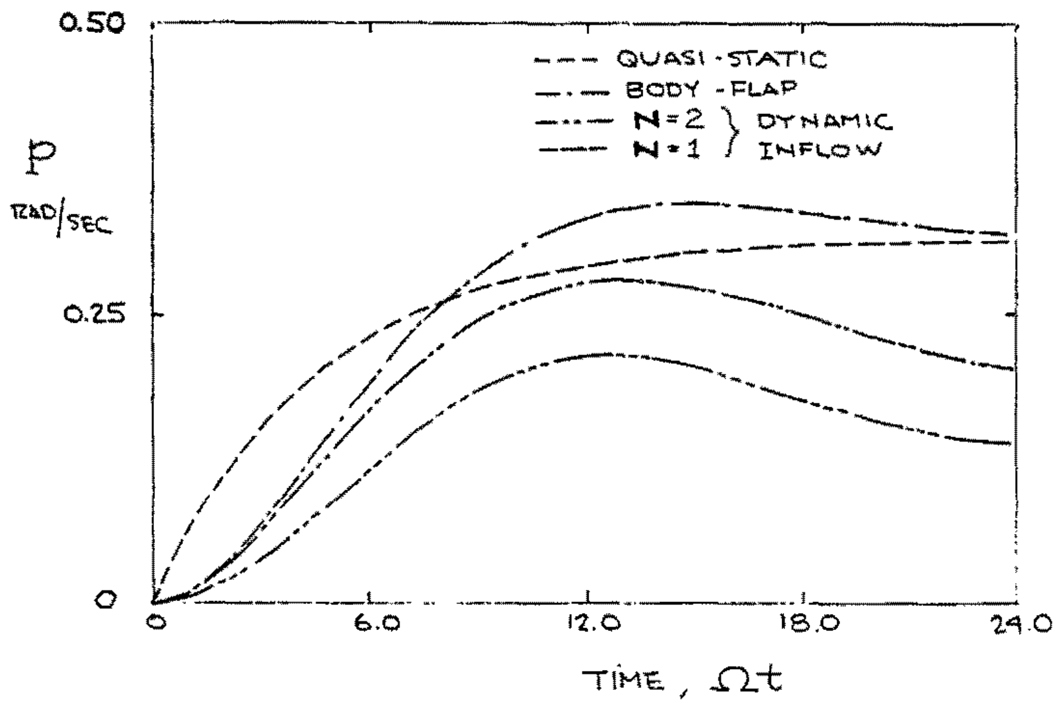

Fig. 7. Hingeless Rotor Helicopter. Roll Rate Response to Lateral Cyclic. Comparison of Various Models. Without Lag Degrees of Freedom (Ω = 44.4 rad/sec).

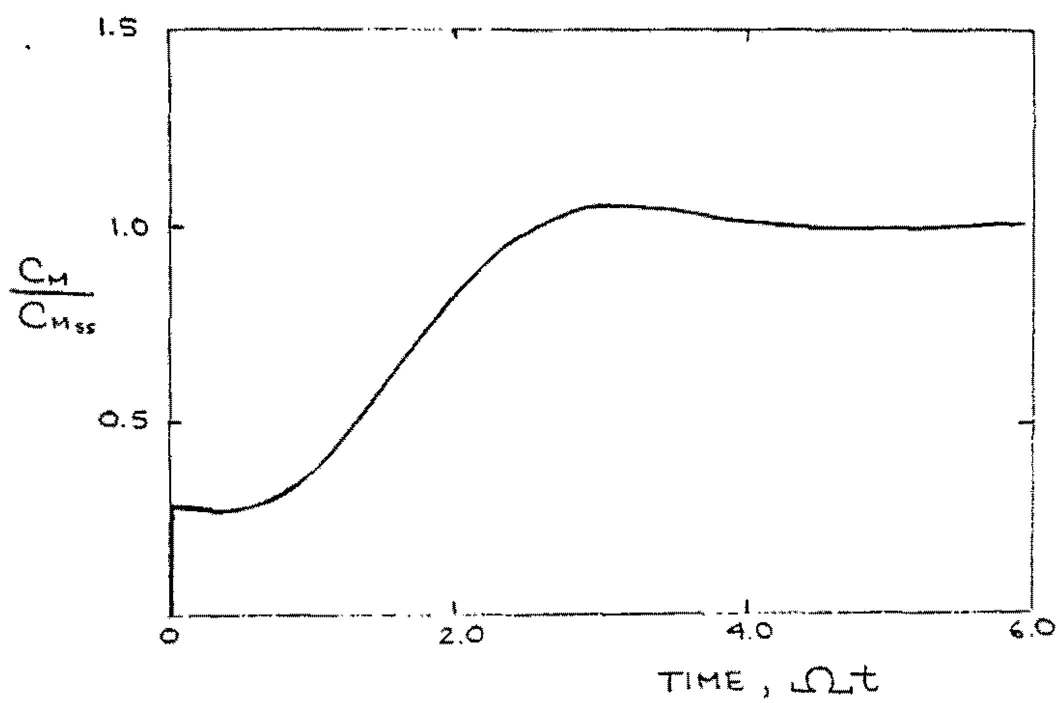

Fig. 8. Moment Response to Cyclic Step Input. Without Body Motion, Lag Degrees of Freedom and Dynamic Inflow. (Ω = 21.3 rad/sec.).

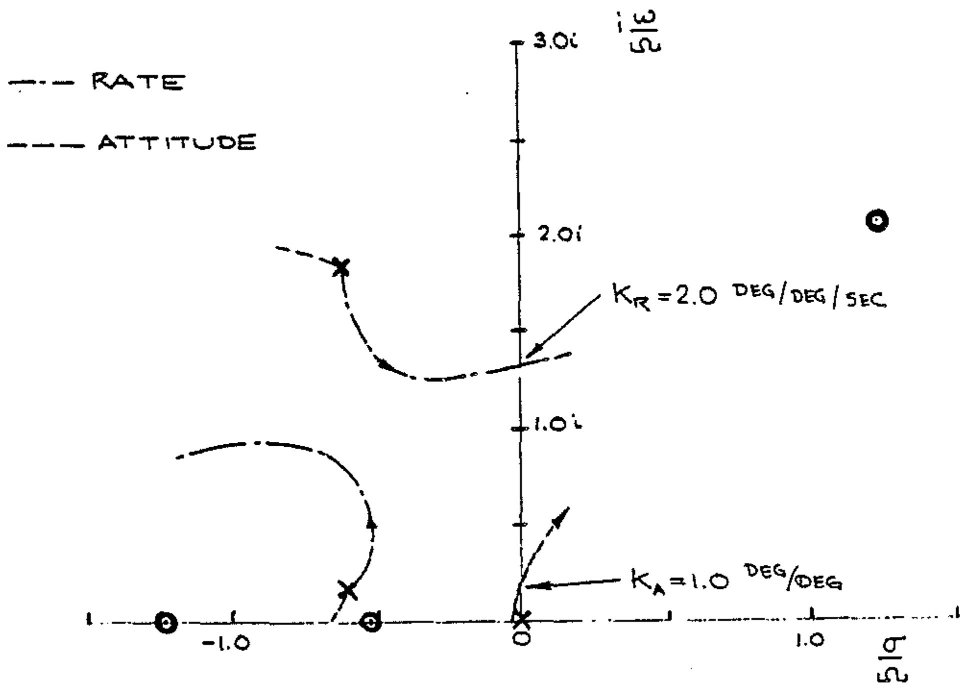

Fig. 9. Root Locus for Roll Rate and Roll Attitude Feedback. Articulated Rotor Helicopter Without Lag Degrees of Freedom Without Dynamic Inflow.

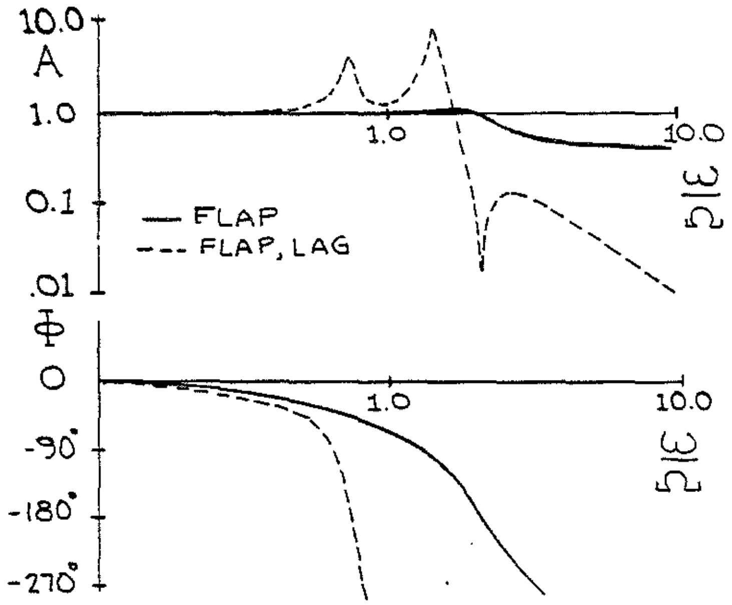

Fig. 10. Frequency Response Roll Moment to Cyclic Pitch Without Body Motion for Articulated Rotor Helicopter Showing Influence of Lag Degrees of Freedom.

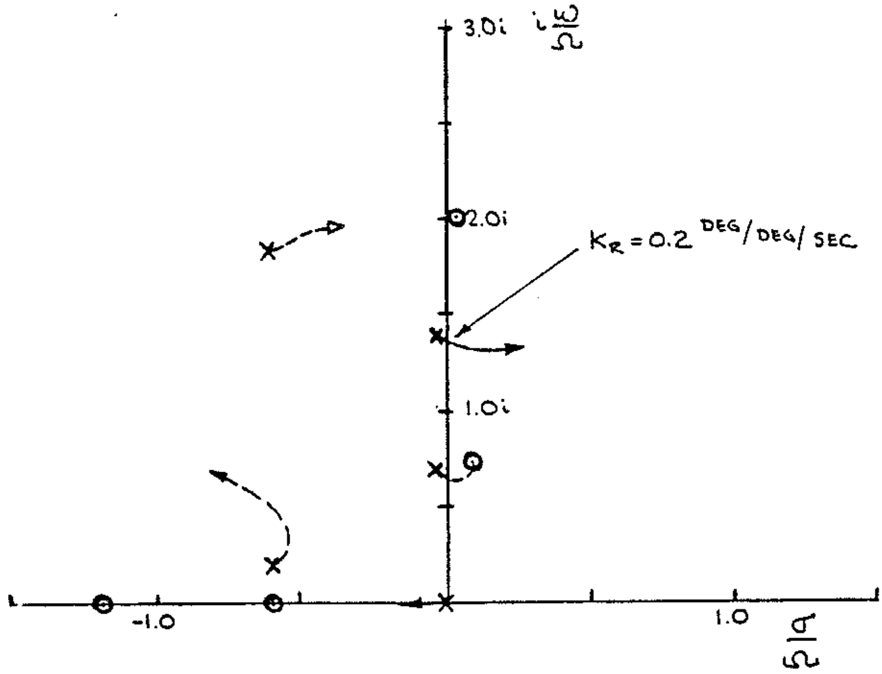

Fig. 11. Root Locus for Roll Rate Feedback. Articulated Rotor Helicopter With Lag Degrees of Freedom. Without Dynamic Inflow.

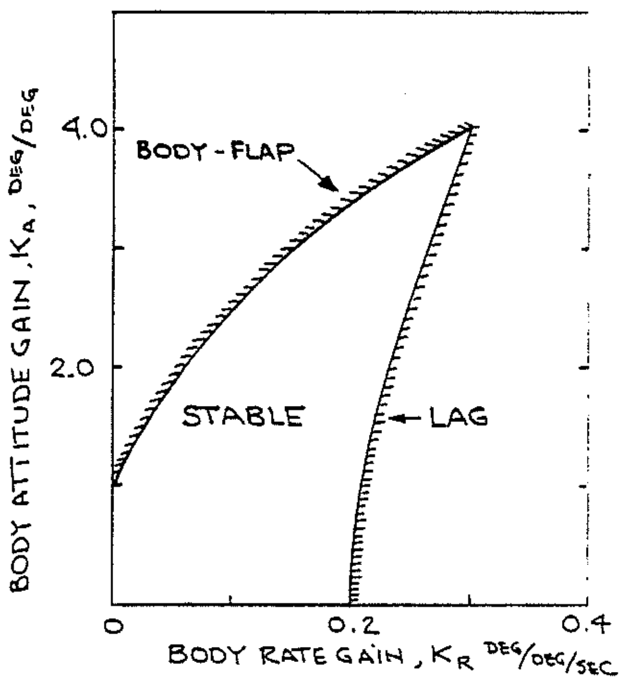

Fig. 12. Roll Rate and Roll Attitude Gain Stability Limits Without Dynamic Inflow.