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"DAMAGE TOLERANCE ANALYSIS FOR ROTORCRAFT, WHAT THE ISSUES ARE"

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DAMAGE TOLERANCE ANALYSIS FOR ROTORCRAFT: WHAT THE ISSUES ARE

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ABSTRACT

Although safe-life design concepts have served the rotorcraft industry and its users well, since the invention of the helicopter, the need for dmage tolerant design is ever increasing. This paper discusses the issues that have impared such concepts to date with emphasis on improved crack growth data base for small cracks, understanding crack growth near threshold together with retardation effects, and characterization of composites under delamination. Of equal importance is the development of simple but accurate inspection techniques for small cracks at field maintenance level. An initiative comparable to those for ballistic tolerance of the 70's to establish and implement simple damage tolerant criteria warrants consideration.

1. WHY DAMAGE TOLERANCE ANALYSIS IS IMPORTANT

The "safe-life" approach used to define component retirement times for rotorcraft parts that are critical in fatigue strength has served the industry and rotorcraft users well, in terms of maximizing safety. Few accidents of rotorcraft, as we know them today, have resulted solely from a fatigue failure of primary structure or a dynamic component. This has been made possible through the use of tremendous conservatisms in the test and analysis involved in convincing airworthiness authorities that published component retirement times were safe.

These conservatisms spanned the full gambit of considerations, including the rotorcraft planned usage, the use of laboratory failure data from tests conducted on full scale parts, and the analysis of measured flight loads. Some manufacturers have taken no credit for fatigue resistant improvements incorporated in the manufacturing process such as shot pinning, cold working, etc. Thompson and Adams in Ref. 1 document that the results of these conservatisms can result in a part with a mean life of 200,000 flight hours being assigned a retirement time as low as 5,000 flight hours.

Previous investigations by the industry as a whole, through organizations like the American Helicopter Society, have documented the wide disparity in potential component retirement times resulting from different conservatisms applied to identical sets of strength, usage, and loads raw data. Product liability considerations have precluded standardization of techniques across the industry. The economic burden placed on both military and commercial operators due to extensive conservatism in component retirement time determination process is not practical to calculate. However, the significance of this burden is becoming of increased importance. Planned activities of the American Helicopter Society, in conjunction with its corporate membership and military affiliates, were well documented in two papers presented at the 47th Annual American Helicopter Society Forum last May in Phoenix, AZ. See Refs 2 & 3.

As the rotary wing technology advances and these air items become more capable for a wider range of missions with one basic design, and wider flight envelopes through conservatism techniques such as with tilt-rotor configurations, the fatigue issues become more important. The H-60 series is an excellent example of the extremely wide range of mission applications. These aircraft serve to (a) transport troops, (b) as electronic intelligence/warfare platforms, (c) for search and rescue, (d) anti-submarine warfare operating totally from ships at sea, and (e) for deep strike/covert action as special operations weapons systems. These missions encompass the widest gambit of environmental conditions: corrosive salt water, extremely hot and dry deserts, high mountain elevations, and the cold weather of Alaska, as well as in extreme humidity. Operating gross weight ranges from original design missions at gross weights of approximately 16,500 pounds to new missions approaching 25,000 pounds gross weights. Experience has shown that military training base aircraft are subjected to substantially different fatigue spectrum than aircraft in tactical units, with the higher gross weight operation of Special Operations Forces being even worse. A few examples of this impact on component retirement times can be seen in the following table. While all components are intended to utilize the same conservatism approaches, the results can vary by an order of magnitude due to usage alone.

H-60 Component Retirement Times Comparison

COMPONENT (Sample)	UH-60A	UH-60L/ESSS	MH-60K
Rotating Swashplate	Unlim	11000 hrs.	6500 hrs.
M/R Cuff/Attach Bolts	6900	1900	500
Expandable Blade Pin	11000	4700	1100
T/R Gear Box Shaft	6200	5000	4400
Servo Beam Railings	4600	3200	780

***NOTE:** Many components ot impacted by -60A to -60K usage difference, others were redesigned to enhance retirement times.*

The above differences in component retirement times has resulted in the U.S. Army assigning *different part numbers* to *identical parts* to distinguish between their installation on the UH-60A and the UH-60K.

Keep in mind that over the years, where "safe-life" design was protecting investments and lives, the cost of rotorcraft was increasing at a rate far greater than economic escalation. For example: UH-1 helicopters fully equipped (fly-away cost) were procured near the end of its production for approximate $750,000 then year $ in 1985. Between 1956 and 1985 a total of 16,200 units were built. UH-60's average fly-away cost even after delivery of over 1000 units is on the order of $6,000,000. The recent MH-60K aircraft are expected to cost up to $15,000,000 are due to their extensive mission equipment package. The days of corporate rotorcraft for executive transportation being available at less than $2,000,000 each have vanished. While these cost increases are the result of much mission related equipment, which is far more expensive than the basic air vehicle, the magnitude of the investment warrants full consideration to reduction in operating costs as long as safety is not impacted. Toward this end, component retirement times individual air items vs. force managment is advocated in light of a tremendous variation in safe component life with usage. Here damage tolerance analysis will pay its way and ultimately reduce operating costs.

2. ROTORCRAFT DIFFERENCES FROM FIXED-WING AIRCRAFT

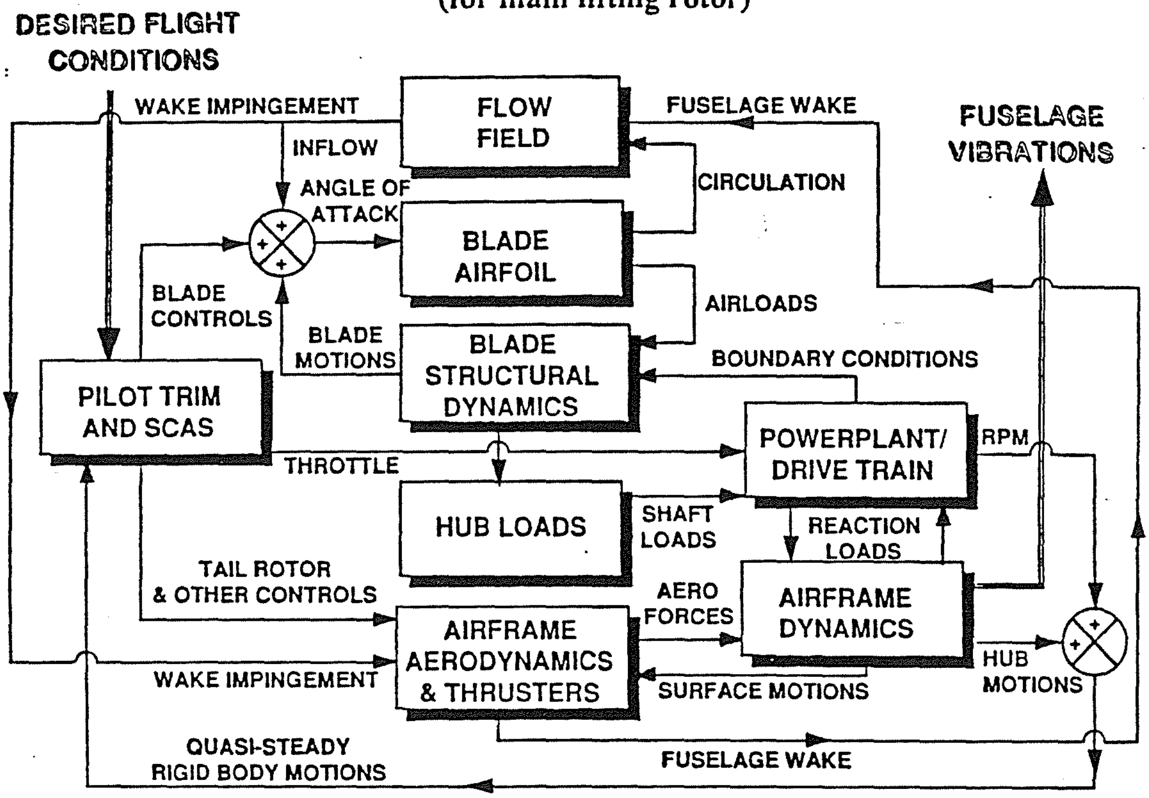
The success of damage tolerance with the fixed wing industry and users of its air items is well known, however this does not necessarily cinch the rotorcraft case. In general, helicopters are subjected to much greater vibratory loading conditions than fixed wing aircraft. There are several sources for these loads, such as aerodynamic gusts and transmission systems, but the primary source is the rotor system. Having the rotor tip path plane oriented horizontally gives the helicopter its ability to hover. By tilting the disk of the rotor through proper control phasing, forward, sideward, and rearward flight is attainable. The added flight regimes over fixed wing aircraft come with a price, however, due to the structural complexity of the rotor system and the inherent severe fatigue environment.

Helicopter rotors are essentially rotating beams subjected to dynamic airloads resulting from control inputs Ref. 4. As opposed to a fixed wing with a relatively constant airload distribution from root to tip, the rotor blade in hover has a much greater variation with the highest loads concentrated at the tip of the blade and the least at the root. Lift is increased through the application of collective pitch, which simultaneously changes the angle of attack on each blade equally. In translational flight, the advancing blade experiences more lift than the other blades, while the retreating blade experiences less. The blades must be individually controlled through cyclic pitch to reduce angle of attack on the advancing blade and increase it on the retreating blade, thus balancing the system. These control inputs occur once every revolution and cause high frequency changes in pressure at a rate on the order of three hertz or higher. These loads are transmitted into the control system, transmission, and airframe via mechanical attachments and aerodynamic impingement on the fuselage. Natural frequencies of the individual rotor blades can couple with fixed system components and applied loads to create flight instabilities or ground resonance.

Unlike fixed wing aircraft, helicopters must make use of several gearboxes and shafts to reduce the high engine output shaft speeds to a much lower rate for the rotor systems. The main transmission also serves as a primary load carrying structural member by transmitting all forces and moments from the rotor hub into the fuselage. The presence of a tail rotor requires one or more gearboxes to change driveshaft direction and speed. Transmissions themselves are made up of many rotating gears which create a vibratory environment. The torsional natural frequencies of the drive shafts can cause problems if they couple with other loading or structural frequencies.

Many papers have been written dealing with the interdisciplinary aspects of rotorcraft design. I believe the picture has been best portrayed in a single chart by Dr. David Peters of Georgia Tech. The following figure contains some minor rearrangements of his work. It was used to discuss the potential inaccuracies in rotorcraft design and optimization in Ref. 5.

ROTORCRAFT INTERDISCIPLINARY INTERACTIONS
(for main lifting rotor)

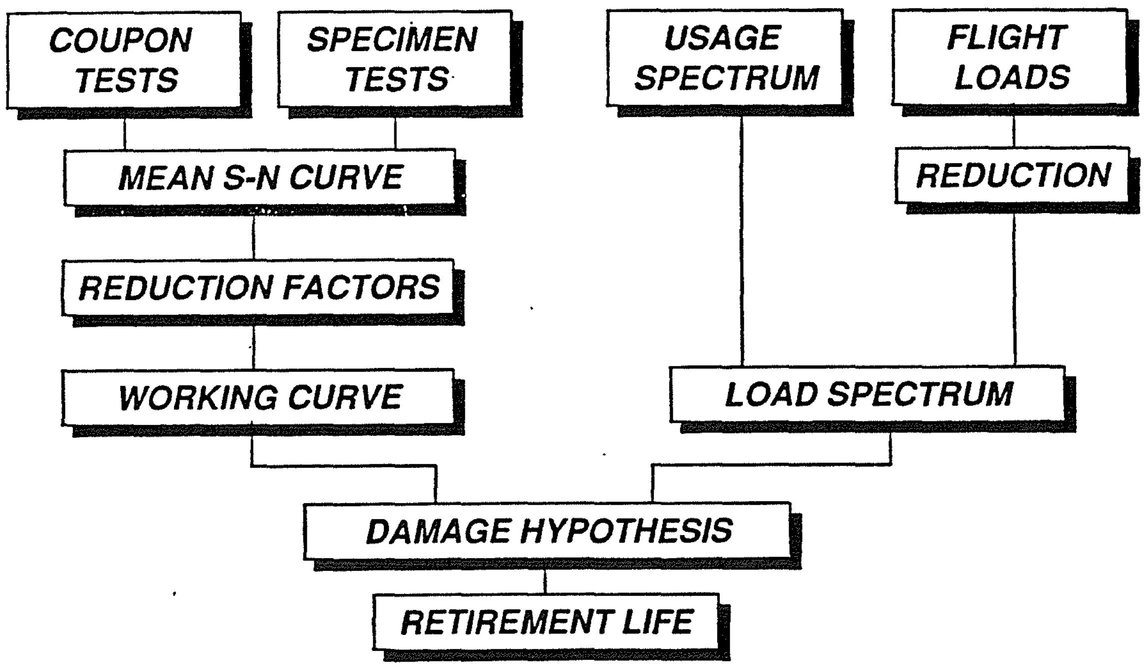


The basic dynamic components of a helicopter and their relationship to each other are shown graphically in this figure. It displays the complex interdisciplinary interactions that the rotorcraft technical community must accommodate. It considers only a single lifting rotor, but would be more complicated for multi-rotored configurations and anti-torque rotor systems. The upper three blocks represent the main lifting rotor. The rotor is divided into three basic theoretical models: blade aerodynamics, blade structural dynamics, and external flow field. These three theoretical models are not only highly coupled to each other, but are also coupled with the other areas of the dynamic analysis, airframe dynamics, airframe aerodynamics, powerplant/drive system dynamics; and, of course, the flight control system. The complex interaction between these disciplines makes dynamic analysis and in turn the production of fatigue producint vibratory loads, one of the most difficult rotorcraft problems.

2.1 Safe-Life vs. Fail-Safe

Much has been written about these two fatigue substantiating philosophies, however, the rotorcraft industry has relied exclusively on save life concepts. An excellent summary of this philosophy is contained in Ref. 4, therefore, only a brief synopsis of major points is contained herein. To ensure that the probability of failure is remote, fatigue critical components are retired from service at or before specified times. The specification of these times contained much conservatism, however, the process is not capable of anticipating failures due to errors in design, manufacture flaws, or in-service damage. Only predictable failures that are easily defined in the laboratory are protected. Safe-life estimates are the result of many conservative assumptions and generally use the Palmgren-Miner life prediction method. In the presence of these conservatisms, safe-life methodology has been very successful particularly in light of the complex fatigue loadings on rotating components resulting from the earlier discussion. Safe-life calculations methodology is diagramed below.

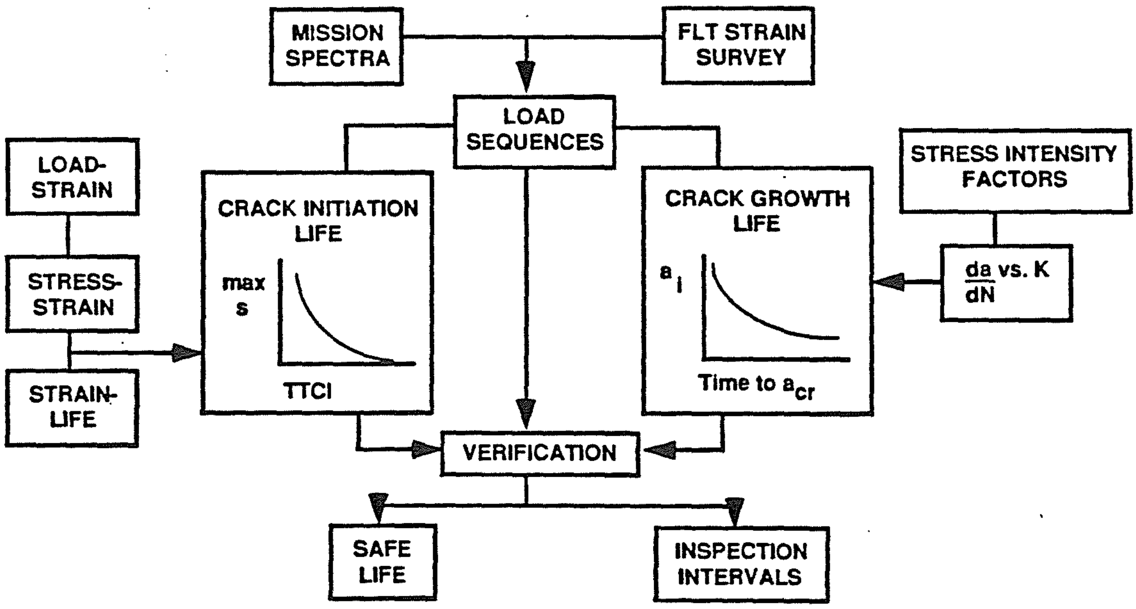
Safe-Life Calculation Methodology



Past damage tolerance design approaches incorporated fail-safe techniques, with redundant load points and crack arresting capability. They have been successfully used on semi-monocoque fuselage construction. A few slow crack growth techniques have been used on components such as rotor blades and main rotor hubs. The record of additional safety features such as cruise guide indicators (displaying high loads generally encountered with the onset of blade stall), and internal pressure indicators demonstrating blade flaws. Currently, the concern of overall reliable goals is being considered to fit structural integrity requirements. This allows for the manufacturer to design systems to a safe-life criteria, damage tolerant criteria or some methodology that combines the benefits of both.

For the future, a total life approach which in essence marries safe-life / damage tolerant design appears feasible but will require more reliance on statistical methods. Future alliance of crack initiation and guaranteed durability and the requirements for crack growth and ensure damage tolerant design approaches. All expect the increased use of component materials to aid implementation. The U.S. Army's model for this future total life methodology is diagramed below. More detail is contained in Ref. 6.

Flow Diagram of the Total Life Methodology



For the latest U.S. military helicopter programs, the AH-66 Comanche, a reliability based approach is being sought which will provide a 99.9999% reliability for dynamic components. (See Ref. 7) The "six nines" methodology requires a rigorous statistical understanding of the three areas of component fatigue strength, flight loads, and aircraft usage. The Army has also recently initiated plans for a Helicopter Structural Integrity Program (HSIP) described in Ref. 8, patterned after MIL-STD-1530A, Aircraft Structural Integrity Program, airplane requirements (ASIP), Ref. 9 which was a USAF initiative the early 1970's. It will take into consideration the differences between rotary wing and fixed wing aircraft. The current draft version of the Army HSIP will allow the establishment of structural integrity requirements for Army rotorcraft by specifying an overall reliability goal and will, at the same time, provide the Army with a means to assess and substantiate the structural integrity throughout the life of the aircraft; this will require a fundamental re-evaluation of the way rotorcraft are designed, manufactured, and fielded. Thus, the Army's new HSIP will be applicable to new rotorcraft designs but not necessarily applicable to current helicopters. Structural integrity assessments for currently fielded helicopters must implement a hybrid approach of existing structural integrity programs, with attempts to apply damage tolerance approaches where applicable.

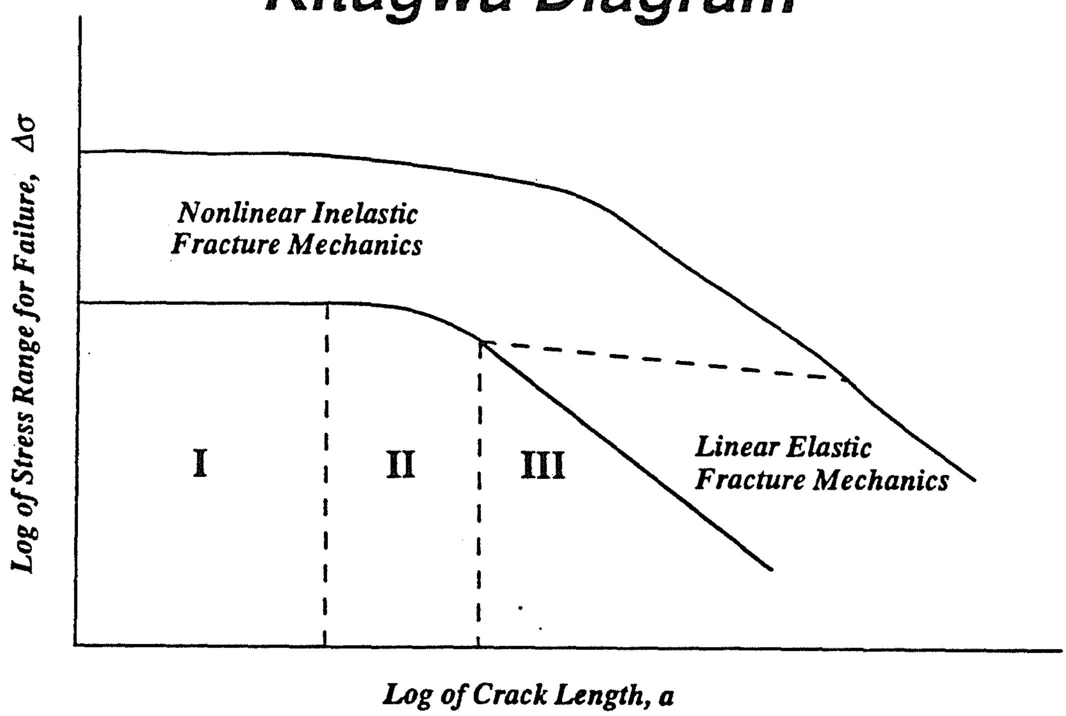
2.2 The Origin of Aircraft Damage Tolerance Analysis

Reference 10 is an excellent summary of the USAF experience with damage tolerance. It was prepared by Dr. John W. Lincoln of the U. S. Air Force Systems Command who has been the "godfather" for fixed wing damage tolerance analysis within the military. Dr. Lincoln pointed out that of the catastrophic events leading to the establishment of the USAF ASIP, the B-47 fatigue failures stand out as the most significant because these problems crippled the main strike force of the Strategic Air Command at a time of extreme world tension. The B-47 and other airplane provided the hard lesson that airplane designs based on static strength alone would not likely reach their planned service life. Initially airplane operating times were limited to a safe-life that was demonstrated in the laboratory in terms of equivalent flight time to failure divided by a scatter factor (usually four). Other significant operational problems included in the presence of laboratory durability testing of the F-111 which indicated that there should be adequate operational safety for at least 4,000 hours, an F-111 crashed with approximately 100 flight hours due to a manufacturing defect in the wing pivot fitting that was undetected during inspections. The KC-135 based on laboratory testing was judged to have a safe-life of 13,000 hours, yet, 14 cases of unstable cracking in the aluminum lower wing skin were discovered at flight times ranging from 1,800 to 5,000 hours. Fortunately, there was adequate fail safety in the wing design to preclude catastrophic failures. The last example to be mentioned here is the wing of the F-5 aircraft. Believed to be safe for 4,000 hours using the safe-life technology, an aircraft was lost at approximately 1,900 hours due to a failure in the lower wing skin root region. This area was prone to in-service damage from routine maintenance.

The USAF's ASIP has created an active mode for problem identifications. This in itself may well be the most significant benefit. A few examples are discussed here, again from Ref. 10. In the F-4 damage tolerance analysis, which was one of the earliest assessments, the pylon attachment fitting showed that 15 F-4's were over the safe limit of the fitting. Subsequent inspection of these aircraft revealed that two aircraft, in fact, had the indicated cracks. Corrective action was taken before readiness was impacted. In the area of the outer wing lower skin on the C-130 B/E, a large number of aircraft were determined by analysis to be over safe limits. Although the durability test article, which had been exercised far in excess of the safe limit, showed neglible cracking at the focal point of the analysis, an inspection of the fleet found 60% of these models to have detectable cracks. The C-5A damage tolerance analysis set a precedence for future programs in that it allowed aircraft to be operated through inspections prior to planned modifications rather than the loss of readiness through fleet grounding. Damage tolerant analysis programs for the C-130, C-141, and T-38 permitted the use of different usage spectrums which reduced operating costs by adjusting maintenance actions in the structural area to actual usage. For example, the load spectrum on the T-38 is substantially different when operating for Air Training Command usage compared to Thunderbird demonstrations, and usage in lead to fighter training. The success of the USAF ASIP can be measured by the structural safety record of that service. Although no mishaps is an admirable goal, damage tolerance analysis can and should contribute much to increased economy of rotorcraft operation as well as their fixed wing counterparts.

The development of issues related to successful use of damage tolerance analysis in conjunction with and/or in addition to the "safe-life" approach presented in this paper are largely drawn from the analysis performed on a major helicopter weapons system and extensive research efforts in the laboratory testing of coupons and less than full scale components. The latter will be referenced when it is discussed in detail, however, the work related to the H-53 systems is contained in Refs. 11, 12, & 13 published by Sikorsky under contract with the Warner Robins Air Logistics Center and Ref. 14 published by the Georgia Tech Research Institute under contract with the same sponsor.

Kitagwa Diagram



3. FATIGUE FAILURE CRITERIA

The issues which must be considered in the application of damage tolerance analyses can be identified by recognizing that in general, design procedures must be based on a fundamental understanding of pertinent failure mechanisms. In the case of fatigue, crack nucleation and growth pass through distinct regimes which can be characterized by crack length. The relation of the fatigue failure loading boundary to crack length has been well illustrated with the Kitagwa diagram from Ref.15 shown in the preceding figure. In the plot of stress range versus crack length, the boundary is divided into three regimes which are depicted as two straight lines in regimes I and III and a curve in regime II. Stress range values below the boundary correspond to cases in which cracks are arrested. Above the boundary, crack growth occurs.

The ordinate value of the boundary in regime I corresponds to the endurance limit. The line in regime III represents the value of stress range, $\Delta\sigma$, corresponding to the threshold value of the stress intensity range, ΔK, in the relation

$$\Delta K = Y \Delta\sigma \sqrt{\pi a}.$$

Y is constant for the given crack configuation and the crack length = a. If the dashed lines were extended and used as the boundary in regime II, predictions would be nonconservative because cracks are observed to grow below these lines. The boundary in regime II is, therefore, represented by the solid curve connecting the two straight lines. The primary emphasis here will be on regimes II and III. Before proceeding to these topics, however, a few general remarks about the regimes in the Katagawa diagram may be worthwhile.

Crack initiation and initial growth are dependent on microstructural features such as inclusions, grain boundaries and texture Refs.16, 17, 18, 19, 20. Also, the transition from regime II to regime III occurs at different values of crack length for different alloys. Refs. 21, 22 suggest that transition length depends on grain size and yield strength .

It has also been observed that in comparing two alloys, the one with the greater endurance limit can have the lower threshold for crack growth Refs. 19, 23. This reversal in resistance behavior may create difficulties in developing correlations between crack initiation and crack growth behavior Refs. 24, 25. Efforts to use fracture mechanics as a basis for extrapolating back to the initial phase of fatigue damage appear to involve issues requiring further study.

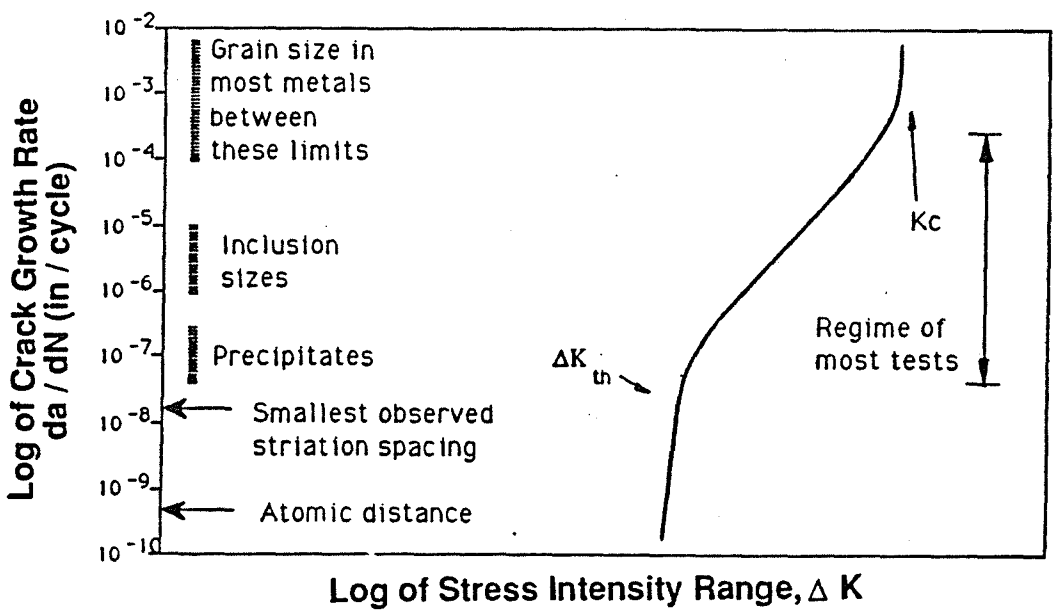
An extended representation of fatigue crack growth behavior may be developed by considering the Kitagawa diagram for the case in which the minimum stress is zero. Then the stress range is equal to the maximum stress, and an upper bound boundary corresponding to the fracture toughness can be included as depicted by the upper, dashed curve. The map between the boundaries may, in turn, be divided into two regions corresponding to a linear elastic fracture mechanics (LEFM) response and a nonlinear inelastic fracture mechanics response (NIFM).

In the sections which follow emphasis will be placed on the growth of cracks for which the lengths correspond to regimes II and III. These often are described, respectively, as physically short and physically long cracks. Current issues for the latter, which can be analyzed by use of LEFM, are discussed in the next section.

3.1 Long Crack Issues

Although it is generally accepted that physically long cracks can be analyzed by methods of LEFM, a number of issues need to be carefully scrutinized. These include the acquisition of a data base in the near threshold region, the effect of variable amplitude loading, the effect of negative stress ratio ($\sigma_{min}/\sigma_{max} < 0$), and mixed mode behavior. Each of these topics will be discussed in this section.

FATIGUE CRACK GROWTH RATE BEHAVIOR



The application of LEFM to fatigue crack growth requires a data base which is derived from tests which provide graphs of the type shown in the above figure. At present, data for the alloys of most interest are available for the upper part of the representative curve shown. Data in the near threshold region, which is of considerable importance for the length of service life that is desired, are often not sufficient. Also, it is clear from the intersection of the growth rate curve and the dashed constant value of ΔK line that a very small change in ΔK can result in a very large change in crack growth rate. This indicates that the computed value of ΔK must be very accurate. In addition, scatter of crack growth rate data in this region can introduce considerable uncertainty in the precise location of the curve. Thus, the data obtained must be sufficient to provide a measure of reproducibility. Also, the sensitivity to scatter should include a sampling from different heats of each alloy, an examination of orientation effects, and in components for which processing, such as forging is involved, a survey of location effects.

Computational methods for computing stress intensity factors for corner and thumbnail cracks in complex components are available. Since these involve elaborate numerical schemes and various approximations, it would, for sake of confidence, be desirable to validate analytical procedures by conducting tests on critical, full scale components. If predictions cannot be verified under controlled laboratory testing, predictions for whole vehicles nust be suspect.

The tests conducted to produce data curves of the type shown in Kitagwa diagram are carefully controlled and not representative of service conditions which involve variable amplitude loading (VAL). Recently, Schijve Ref. 26 upon reviewing the predictive capability for service type loading, concluded that "it cannot be said that a generally applicable and reliable crack growth prediction model is now available." The difficulties associated with developing a crack growth prediction model can be recognized by considering the transient effects which have been observed after a single overload.

Most studies on the application of overloads have focused on the transient retardation effect which follows a tensile overload. Of the mechanisms which have been proposed to describe retardation effects, two have received the most extensive examination and application. These are the effect of a compressive residual stress in front of the crack tip and the effect of the development of an obstruction to closure. Willenborg Ref. 27 proposed a model which accounts for the residual stress by reducing the stress ratio. It successfully exhibits a retardation effect and modified versions have been used in crack growth codes.

The models for closure obstruction result in a reduction in the range of the stress intensity factor, and also, therefore, lead to retardation. Two distinct types of models have been proposed to describe closure effects. Newman Ref. 28 has proposed a modified Dugdale strip model which has been applied by Ward-Close et al Ref. 29 to overload experiments. They have demonstrated that the model has features which exhibit the response observed after an overload. A discrete asperities model has also been proposed Ref. 30 and it has been demonstrated that it can be used as a diagnostic tool for analyzing overload data Ref. 31.

In the near threshold region clear evidence of closure obstruction has been presented, and closure models successfully describe the behavior observed. When the crack faces do not impinge on one another during a cycle, the residual stress mechanism provides a rational explanation for retardation. The possibility of coupled contributions from each mechanism for intermdeiate loading conditions has been discussed Ref. 31. This topic requires further examination. Since behavior in the near threshold region is of primary concern, however, an acceptable adaptation of a closure model to VAL would appear to have a higher priority.

The objective of both the Dugdale strip models and the discrete asperity model is the same; i.e., to determine, quantitatively, the magnitude of the obstruction misfit which prevents closure. The strip models proceed by continuously computing the thickness of a perfectly plastic layer, which is of the order of microns, and introducing a crack extension criterion. The issue of the degree of plane strain versus plane stress present is incorporated in the analysis by the use of a constraint factor.

The discrete asperity model makes use of test measurements, so the experimentally integrated misfit parameter which is measured includes thickness effects. To date, however, this model has been used primarily to diagnose overload test results.

The form of closure obstructions has been a topic for some conjecture. Recent experimental results obtained by McEvily and Yang Ref. 32 have provided new insight into this issue. They presented convincing evidence that indicated that even though crack closure obstruction prior to an overload was of the plane strain form, obstruction during the transient retardation interval occurred at the specimen faces; i.e., under plane stress conditions. This suggests that the contraint factor would have been a variable in their overload tests. This issue needs to be clarified by additional experiments and analysis.

In the experiments just discussed, the tests involved the simplest possible VAL test conditions. Results are also available for loading conditions which more closely resemble those encountered in service. The results of a round robin program in which various methods were used to analyze several loading spectrums are presented Ref. 33. The results of these analyses are summarized in the paper by Newman Ref. 28. Five basic aircraft load spectra were simulated on center-crack tension specimens with different scale factors to represent different load levels. A brief summary of results for the ratio of the predicted life, N_P, to the experimental life, N_T, for two values of constraint factor, α, is presented in the following table. The ratios for predictions based on a linear cumulative damage, LCD, method are also given. The LCD method does not include either retardation or acceleration effects.

SUMMARY OF N_P/N_T VALUES FOR SPECTRUM LOADING

	*Closure model**	*Closure model**	*LCD*
	a = 2.37	*a = 1.0*	
Smallest value	*0.66*	*0.64*	*0.62*
Largest value	*1.48*	*2.52*	*1.81*
Mean value	*0.98*	*1.12*	*1.12*
Standard Deviation	*0.28*	*0.50*	*0.63*

**Dugdale strip model [n+13]*

In general it appears that the results for the simple LCD method are not substantially inferior to those for the closure model. The closure model results for $\alpha = 2.3$ do have a superior standard deviation value, but the data also reveal the sensitivity of the closure method results to the selection of a constraint factor value. If, as noted earlier, the constraint factor varies during loading, the selection of suitable values could be a problem.

Partl and Schijve Ref. 26 point out that comparisons can be accomplished in different ways; i.e., by crack growth lives, by crack growth curves, by crack length increments for each flight, by increments per cycle. This recognition led them to conduct studies in which they reconstructed crack growth from fractographic observations after simulated loading. This type of investigation is very time consumming, but it does focus attention on the need for studies in which the responses developed are more closely tied to specific loading conditions. In this regard, experiments involving controlled single and multiple overloads Refs. 29, 32 may provide a sounder basis for evaluating candidate prediction methods than experiments with complex load spectra.

In analyzing load spectrum data, it is common practice to neglect the compression portion of a cycle; i.e., the load range analyzed includes only tensile parts of the excursions. Recently obtained experimental results suggest, however, that this practice may produce nonconservative predictions. It has been found that plots using the "adjusted" ΔK values exhibit increasing crack growth rate with decreasing (more negative) R values Refs. 34,35 Although some proposals for explaining this behavior have been offered, they are tentative and are at the conjectural stages. Clearly, however, this is an issue which needs to be studied more extensively.

A discussion of long crack issues must recognize that some of the service problems involve crack and loading configurations that produce mixed mode conditions at the crack tip. Analytical methods for determining the stress intensity factors for modes I, II and III are available, so that is not a restriction on the development of methods for predicting fatigue crack growth. Another element of such methods involves the use of a criterion for crack extension; i.e., what is the orientation of the crack plane extension relative to the initial crack surface.

Tohgo, et al Ref. 36 have recently presented data which suggest that crack extension orientation may be material dependent. They conducted two distinct fatigue crack experiments in which the ratio of K_{II} and K_{III} varied differently along elliptical crack fronts. In each of the experiments, the orientations of the cracks relative to the loading differed for the two types of experiments. Tests were conducted on two alloys: SM41A structural steel and the 2017-T4 aluminum alloy. Each material was subjected to each of the two tests; i.e., a total of four tests were conducted. When the steel and aluminum results were compared for each type of test, it was found that different crack extension surface orientations were produced. For the steel, cracks were observed to grow on planes upon which a maximum tensile stress was acting. For the aluminum, growth was on planes upon which the shear stress was a maximum. The shapes of the fracture surfaces in the aluminum alloy and the steel for the experiments were completely different. This indicates that criteria for crack extension orientation must consider not only the stress state at the crack tip, but also the character of the preferred mode of response of the given material.

Another complicating factor in mixed mode loading is featured in results obtained by Linnig, et al Ref. 37. They conducted experiments in which superimposed mode I and mode II loading were produced. They found that friction between fracture surfaces had a profound affect on the crack rate growth. Although accounting for friction effects poses difficult analytical problems, the possibility of their occurrence in service cannot be excluded.

In addition to "simple" friction effects, the consequences of fracture surface roughness must also be recognized. This will result in closure obstruction of a type which has, for example, been observed in tests on circumferential cracks on rods in torsion Ref. 38. Although such tests were designed to examine mode III fatigue crack growth, mode I is introduced as fracture surfaces "ride up" relative to one another. This type of event is not of purely theoretical interest. Rods subject to torsion are common in service applications.

Finally, the effects of variable amplitude, mixed mode loading can be expected to further complicate predictive capability. At present, the understanding of mixed mode loading effects is elementary. Many issues need to be examined.

3.2 Short Crack Issues

Short crack behavior is characterized by regime II of the Kitagwa diagram. Serious consideration of the behavior of short fatigue cracks began with results published in 1975 by Pearson . The behavior observed has been described as "anomalous," because crack growth for short cracks was observed to occur at values of stress intensity factor range below the threshold value.

Recent results indicate that the anomalous designation is inappropriate, and is a consequence of the use of correlation procedures which are not valid. A clarification of short fatigue crack growth behavior appears to require that two mechanisms must be considered. One concerns the role of obstruction to closure upon unloading. For a given range of stress intensity factor, obstruction to closure is generally less for short cracks than for long cracks. A second mechanism involves the fact that the requirement of small scale yielding for the use of stress intensity factor range as a correlation parameter is often not satisfied for short cracks.

It is not cited in the literature, but it is of interest to note that Rice predicted the observed behavior on theoretical grounds in 1967 Ref. 40. Rice predicted that for the same value of ΔK, short cracks should grow more rapidly than long cracks. This can be illustrated by observing that for an edge crack in a wide plate

$$\Delta K = 1.1\Delta\sigma\sqrt{\pi a},$$

where ΔK is the stress intensity factor range, $\Delta\sigma$ is the applied stress range and a is the crack length. When plots of short and long crack data are made, this is the type of equation used. If a short crack is one tenth the length of a long crack, it follows that the ratio of the applied stress ranges for the same ΔK is

$$\frac{\text{short } \Delta\sigma}{\text{long } \Delta\sigma} = \sqrt{10} = 3.16.$$

Rice presents results of plasticity analyses that indicate that when the ratio of applied stress to the yield stress reaches a value of 0.4, solutions based on small scale yielding begin to underestimate the size of the plastic zone. Blom, et al Ref. 19 have recently presented results of a comparison of finite element elastic-plastic analyses for long and short cracks which call attention to this same effect; i.e., the plastic zone size for the short crack is much larger than that for the long crack for the same $\Delta K = K_{MAX}$. These results indicate that comparisons based on plots of da/dN versus ΔK for long and short cracks are invalid.

Nisitani, et al Ref. 26 have recognized the importance of the limitations of LEFM and have proposed that the crack growth rate is proportional to the cyclic plastic zone size. They then assume a relation between the plastic zone size and stress and the crack size to obtain an equation for crack growth rate. Empirical constants must be determined for the final relationship. Although this equation may be used to correlate crack growth data, it would not appear to be adaptable to service type loading conditions.

Most of the recent work on short crack growth has focused on the effect of closure obstruction. It is concluded that closure obstruction is less for short cracks and that all that is required is to adopt an effective ΔK to account for the observed behavior. Blom, et al Ref. 19 present numerical results which support the reduction in closure obstruction conjecture. A number of researchers have adopted this approach, and have presented results of correlations which show agreement with experimental data Refs. 19, 25, 42. It would appear, however, that this procedure evades the question of the validity of the use of the stress intensity factor when small scale yielding requirements are not satisfied. The rationale for the use of an effective ΔK may, however, be viewed as being analogous to Irvin's plastic zone size correction to the crack length Ref. 43

It was noted earlier that the level of load applied in the short crack experiments may result in the net stress being a large proportion of the yield strength. Components in aircraft which are subjected to this level of loading on a sustained basis could be expected to have very short lives for longer cracks. Such loading is not likely to be experienced, however, as such high loads would occur on only an intermittent basis. This reasoning leads to the conclusion that the effects of intermittant loading should be studied. Some spectrum loading tests on short cracks have been conducted recently Refs. 25, 44. Some future tests should also, however, be conducted for simpler loading conditions. Considerable insight has been gained, for example, from tests in which ΔK is maintained constant except for a single overload. Tests of this type could reveal the development of retardation, or possible arrest effects which could have an impact on service applications involving short cracks.

Understanding of the elements of the short fatigue crack growth problem has advanced considerably in the last few years. More progress is needed, however, to provide a basis for incorporating a predictive capability into damage tolerance analyses.

3.3 Crack Growth Technology Summary

From the review of research (29 references, 15 thru 44) performed on both long and short cracks in light of the Kitaqua diagram regions, it can be concluded that:

(a) Since the mechanisms of fatigue crack nucleation, short crack growth and long crack growth are different, attempts to extrapolate back from long crack growth behavior to the earlier regimes is not believed possible.

(b) Values of da/dN in the near threshold region are extremely sensitive to small changes in ΔK. The data base for this region must, therefore, be thoroughly validated.

(c) Fatigue crack growth under variable amplitude loading are found to be complicated by a number of effects. These include: coupling of residual stress in front of the crack tip and closure obstruction, changes in the form of closure obstruction before and after overloads, evidence that compressive load excursions can affect crack growth rate.

(d) The preferred orientation of crack extension under mixed mode loading can be not only stress state dependent, but also material dependent.

(e) Fatigue crack growth under mixed mode loading can be influenced by both friction between crack faces and by crack face surface roughness.

(f) Correlation of short fatigue crack growth through use of the range of the stress intensity factor is generally invalid, because small scale yielding restrictions are not satisfied.

(g) Use of an effective range of stress intensity factor provides a basis for correlating short crack growth data. This approach may empirically incorporate a correction for the violation of the small scale yielding requirement. Effects of compressive load excursions have not been included in the procedures being used.

(h) The high levels of loading used in short fatigue crack tests in the laboratory are not likely to be encountered on a sustained basis in service. The behavior, therefore, of short cracks under variable amplitude loading conditions needs thorough investigation.

MH-53J Damage Tolerance Assessments

STRUCTURE	SIKORSKY			GEORGIA TECH		
Initial Crack Size	0.005 in	0.010 in	0.050 in	0.005 in	0.010 in	0.050 in
MAIN ROTOR						
Upper Hub Plate	555/380	62/50	NA			
Lower Hub Plate	HIGH	727/558	–			
Hoz. Hinge Pin	221/212	16/14	NA			
Spindle:						
Lug	HIGH	HIGH	100/90	–/982	–/811	–/473
Shank Radius	HIGH	250/195	NA			–/5270
Sleeve	1120/150	512/42	NA			
Push Rod	HIGH	HIGH	HIGH	–/12340	–/11620	–/1971
TAIL ROTOR						
Spindle						
Lug				–/20200	–/16800	–/1630
Shank Radius	518/433	70/56	NA	–/272	–/183	–/35
Threads	10/10	7/6	NA	–/5140	–/4125	–/254
AIRFRAME						
Left Upper Tail Pylon Hinge Fitting (-101)	610/410	230/170	NA	795/717	217/169	36/27
Accessory Gear Box Support Fitting	58/50	31/29	NA	646/96	22/39	31/14

Crack Propagation times with / without retardation in hours

3.4 Some Analytical Comparisons

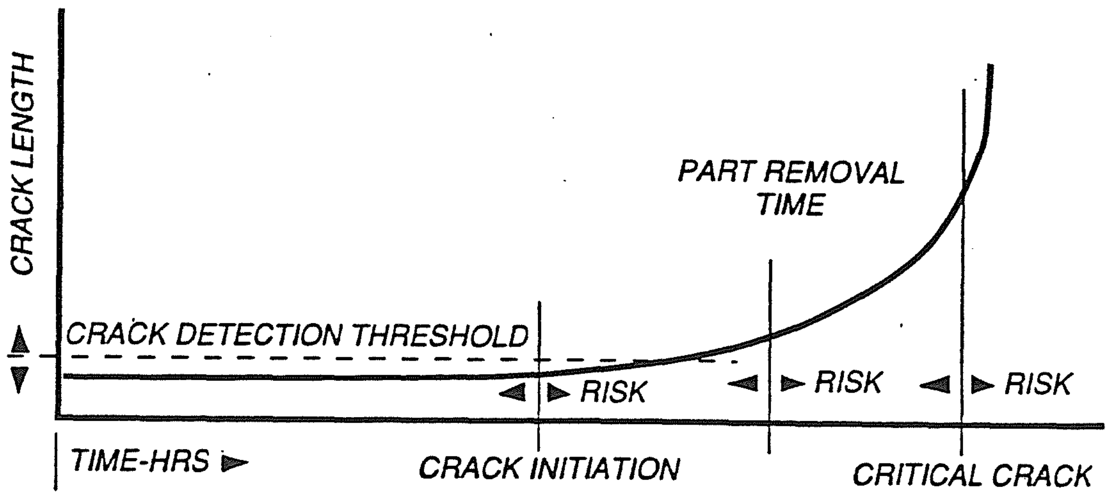
As mentioned earlier, Refs.11 through 14 document specific damage tolerance analyses formed by Sikorsky Aircraft Division of United Technologies and the Georgia Tech Research Institute for the USAF MH-53J. The purpose here is to show the wide variation that can result from these analyses due to small variations in the data base and the tremendous impact of the initial crack size. The preceding table includes data that considers crack propagation times with and without retardation, in hours.

4. FORCE MANAGEMENT

The force management goal is to provide structural integrity procedures for increasing aircraft operational safety, reliability, availability, and maintainability with minimum increase in operation and support costs and with minimum impact on mission accomplishment, Ref. 9.

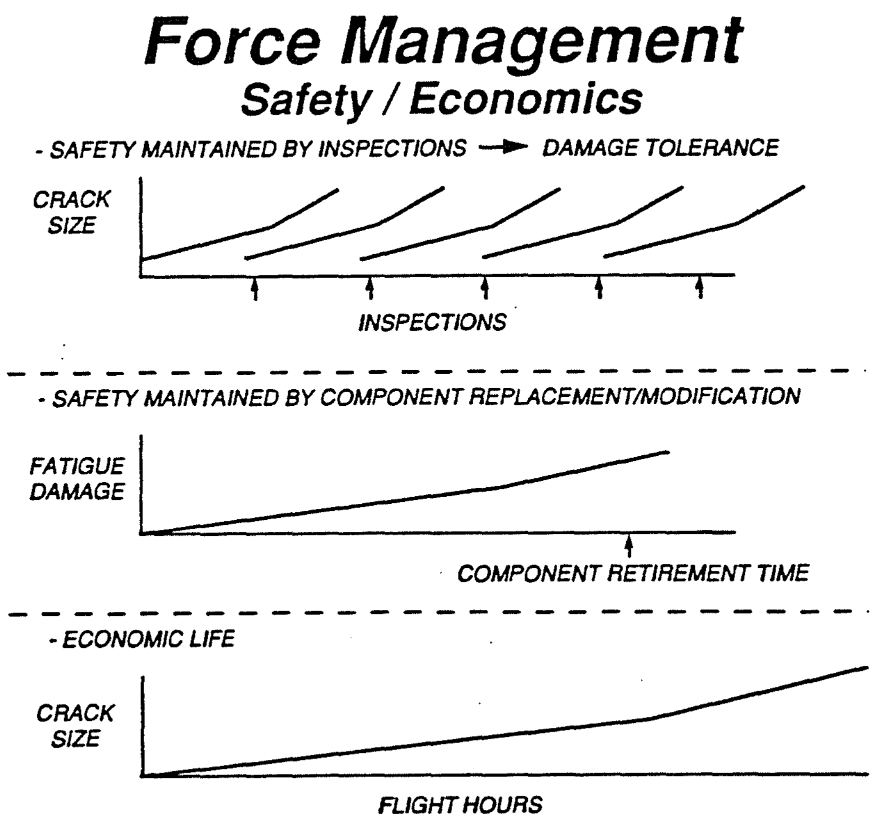
The crack growth tracking approach to force management is strongly influenced by the inspection capability of the field organizations and the depot facility. The following figure from Ref. 14, shows the impact of crack detection capability on flight hours available for DTA technique application. Crack initiation may occur after some number of flight hours depending upon the particular part and usage. The critical crack length is also part and usage dependent. The flight hours available between detection and critical crack length represent the opportunity to apply component crack growth prediction technology to aid in force management decisions.

Crack Detection Capability vs Flight Hours Remaining



The initial inspection time for damage tolerance based force management is set to one half of the flight time from crack detection to critical crack length as shown in the figure. If a very small crack could be detected in the field or depot by non-destructive inspection (NDI) technicians, then maximum flight hour availability could be achieved. This impact of crack detection capability upon force management decisions must be considered for each candidate part. In may happen that the flight hours available are too short for credible application of damage tolerance techniques to force management. Reducing the crack detection threshold significantly below current field capability may not be cost-effective. It may be less costly to replace parts than to improve reliable NDI crack detection levels. Removing parts for retrograde to depot level (or laboratory) NDI equipment is an option that must be weighed against cost, operational impacts, and the ability to track individual parts.

The following figure, also from Ref. 14, compares the various options for force management, as previously provided by Dr. John W. Lincoln, USAF Systems Command.



Force management concepts must consider both safety and economic factors to be viable options for consideration. The upper graph in the figure describes a damage tolerance approach wherein a structural part inspection is established based upon the predicted crack growth. The crack growth curve results from analysis, testing, or inspection. At each inspection interval, the part is non-destructively inspected and reworked or modified to enhance the part life. The crack growth curve is reset at each inspection interval (NDI offset) to allow the part to accumulate additional flight hours before an eventual part safety limit is reached. This approach is the ultimate goal of a damage tolerance program; however, the determination of the crack growth curves and confidence in them is driven by the crack growth modeling technology and material data bases available.

The middle graph in the figure describes a force management concept based upon component removal and replacement, based upon accumulated fatigue damage. Rotating helicopter components, as ell as engine rotating components, are subject to this force management approach. The lower graph describes a force management approach based upon the economics of part inspection and replacement. Some parts, particularly those that are restricted to depot repair, could be more cost effective to remove and replace in conjunction with other depot activities since the part may be readily accessible and tooling and jigs available. Economic replacement should not compromise operational safety in that safety limits are always top priority.

Several NDI methods are in widespread use today, and newer improved techniques are constantly being developed. Some of the most popular methods include low cost timely inspections which can be done in the field or at the depot. Visual techniques include those with the naked eye as well as optically assisted magnifying systems with the ability to inspect difficult regions using fiber optic cable. Radiographic methods basically use X-rays to perform inspections in areas where safety precautions can reliably be adhered to. Ultrasonics provide a safe way to field inspect easy to reach components where cracks are prone to occur. Eddy current techniques can be performed in the field under certain conditions and part geometries, but the most reliable are done in the depot or laboratory. Dye penetrant and magnetic particle suspension methods both provide a visual indication of cracks using florescent light and magnetic fields respectively. These methods are as sensitive as eddy current surface scans but require depot level or laboratory facilities. Table NDI Comparison summarizes these methods and their approximate flaw size detection capabilitie

NDI Comparison

NDI METHOD	DETECTABLE CRACK (ins.)
VISUAL	.500 - 1.000
RADIOGRAPHIC	.500 - 1.000
ULTRASONIC SURFACE SCAN	.020 - 0.090
EDDY CURRENT SURFACE SCAN	.020 - 0.060
DYE PENETRANT	.025 - 0.060
MAGNETIC PARTICLE	.025 - 0.060
EDDY CURRENT (AUTOMATED)	.010 - 0.030

Other more exotic (and expensive) NDI methods include photo and acoustic emission detection which can detect a crack as it grows rather than afterwards. These methods are somewhat degraded in field conditions where they would find the best use, but perform well in the laboratory. The engine community has developed improved eddy current methods which can reliably detect flaws as small as .005 inches in the laboratory. Adapting these techniques to helicopter rotor systems would provide enormous benefits. Cost and risk would have to be weighed for opting to use such high powered systems for long duration inspections as opposed to more frequent but cheaper methods.

An essential ingredient of force management is an individual helicopter tracking program (IHTP). The large variation in component retirement time for an identical part used in rotorcraft with different missions (UH-60A vs. MH-60K) was illustrated early in this paper. This is justification in itself. IHTP status is illustrated as follows:

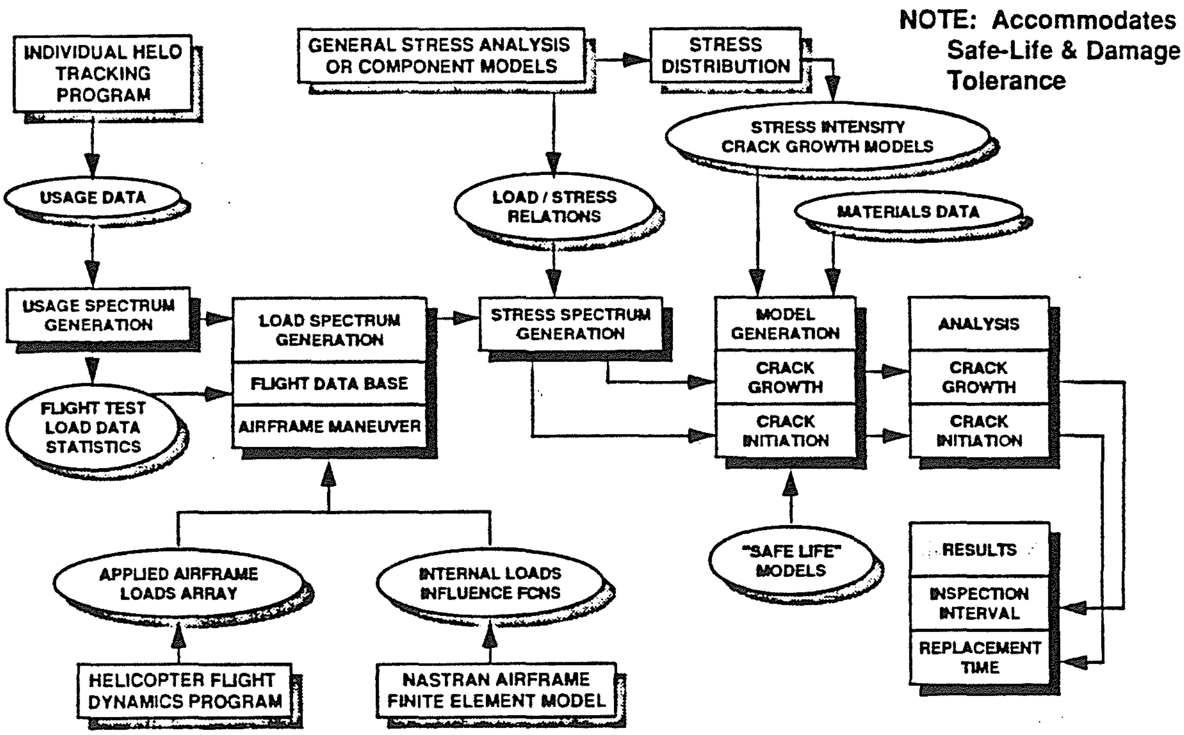
Individual Helo Track Prog

- ***EACH SERVICE HAS SOME TRACKING PROG IN WORK***
  - *Army concentrates on crash survivable memory units (accident investigation)*
  - *Navy prototype SH-60B digital recorder*
  - *USAF prototype MH-53J demo program*
- ***FOR ECONOMY, STANDARDIZATION IS NEEDED***
  - *Software languages not same*
  - *Regime reorganization essential*
  - *Automated data analysis essential*

DO NOT BURDEN FIELD MAINTENANCE UNITS

The master tool for force management is likely to be some form of a structural integrity program illustrated below which accommodates both Safe-Life and Damage Tolerance.

Structural Integrity Computer Program



All scheduled tasks are brought together graphically to provide a clear interaction picture. There are six basic steps to the processor, as labeled by the shaded boxes in the previous figure. First, usage spectrum generation is either based on a generalized spectrum for the entire fleet, or tailored for an individual aircraft through the use of the Individual Helicopter Tracking Program (IHTP). This usage data is combined with flight loads from either flight test or analytical methods. Loads are then converted to stresses for individual structural components. Crack growth and crack initiation models are subsequently generated for critical crack locations on individual parts. Damage tolerance and safe-life analyses are performed, investigating desired parametric effects on fatigue, such as sensitivity to applied stress, or component geometry. Finally the results are used to determine appropriate inspection intervals and component retirement times.

5. SUMMARY

Cost effective force management of rotorcraft using damage tolerance analysis without jeopardizing safety depends upon *reduced conservatism* in the analytical models used. This reduced conservatism can safely be accomplished only through:

a. *Better definition of the threshold region*, which requires emphasis on small crack detection and propagation.

b. *Incorporation of retardation effects* which have been validated through full scale laboratory component testing

c. The conduct of *full scale component strain surveys* to more accurately define load paths and locate test instrumentation for use in flight load measurements.

d. The use of automated *cycle counting* of measured flight loads during data reduction.

More accurate knowledge of *individual aircraft usage* is also essential to cost effective force management. The key is the use of flight data recorders and techniques for critical flight regime recognition with limited flight data.

Simple but accurate inspection techniques for small cracks at the field maintenance level. Where depot maintenance is available, the application of engine non-destructive inspection techniques should be considered.

Innovative design initiatives for damage tolerance analysis that are comparable to those that facilitated ballistic tolerance of the 1970's, represents a useful challenge. This must be accompanied with a major campaign to get *user acceptance* of damage tolerance analysis in order to reap its cost benefits vis-a-vis a safe-life history of rotorcraft operations.

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