

# MBS Analysis of a Free Flying Helicopter with Fully Articulated Rotor

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## Abstract

Since Multi Body System (MBS) codes have been proved to be potentially powerful simulation tools in the whole range of helicopter rotor dynamics, here the question of modelling the free flying helicopter in a pure MBS as well as in a hybrid FEMBS dynamical simulation model is highlighted. The objectives of this research work are modelling techniques for describing the dynamical behaviour and the structural interaction between articulated helicopter rotors and the nacelle of a free flying helicopter. Here the focus lies on the coupling of the rotating structure of the fully elastic, hinged main rotor with the non-rotating parts of the body structure via a flexible rotor-nacelle interface. As simulation platform the 9 [to] generic model "Helicopter H9" has been used. Representing the research object for this investigation it serves as a demonstrator model and as dynamic reference configuration for both the MBS and the FEM calculations.

For reasons of a better clarification of the rotor-fuselage coupling effects the center of gravity of the helicopter fuselage exhibits large offsets in all three coordinate directions. As a consequence we get a highly non-symmetrical dynamical system w.r.t. the main rotor axis and a rotated principal axes system. Concerning the specific dynamic coupling effects between rotor and nacelle a survey study with topics like the main rotor suspension (lateral and vertical) or the elasticity of the drive train had been conducted. In systematic variation of the respective stiffness values (over four decades) the results of different parameter studies are presented as numerical results for single constant rotor speeds as well as in frequency fan diagrams for the overall dynamical behaviour under the change of rotor speed. By applying different blade pitch and  $\delta_3$  angles the influence of the blade pitch position on the rotor eigenbehaviour has been tested. Even cases of stability loss of the free flying helicopter concerning elastic eigenmodes of the coupled rotor-nacelle-system — an air resonance type — could be detected in this work.

The validation of the models and the computational procedure had been done by comparing the eigenmodes and the eigenvalue results produced with the two elasto-mechanical methods MBS and FEM. Thus different algorithms and independent tools have been used in the examination. It has been shown that for the non-rotating as well as for the rotating test cases the coupling effects caused by the blade hinges and pitch angles will be reproduced without any restriction in both approaches. The FEMBS modification of the pure MBS model renders a hybrid formulation which can combine advantages of both sides. Thus the potential of a sophisticated MBS code like SIMPACK as a powerful simulation tool for helicopter dynamics of the free flying system has been demonstrated with respect to the characteristics of an articulated rotor.

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## 1 Introduction

In recent time Multi Body System (MBS) codes have found their way into structural analysis within the helicopter industry, and the use of commercial MBS tools in the general design and development process seems to become fruitful. These MBS codes combine their inherent property of describing large deflections of the (rigid) structure including full geometric non-linearities with in general high performance time integration algorithms. In combination with special algorithmic features Finite Element Model (FEM) substructures can be incorporated into the MBS model thus replacing one or several rigid body components. By applying these so called FEMBS techniques consistent elastic properties can be introduced into the structure to any desired amount. Together with these modal FEMBS structures and additional degrees of freedom (DOF) added to the — now — hybrid MBS model the total dynamic model can be subjected to any kind of numerical simulation. Thus with MBS and FEM two fundamentally different approaches in structural dynamics can be combined with their respective advantages to potentially high power CSD tools.

Since the most MBS codes have not primarily been designed for describing elastic helicopter



Figure 1: The five-bladed dynamic helicopter demonstrator H9 with articulated rotor — *side view*

rotors with their numerous potentially coupling mechanisms, here the question of modelling the free flying helicopter in a MBS dynamical simulation model is highlighted. In order to take into account also the characteristics of the flexible rotor-nacelle interface, the specific modelling features of the MBS code have been used. In this paper these characteristic features have been subjected to a systematic investigation to verify their performance, correctness and reliability. It could be shown that one potential drawback of the (nonlinear) MBS approach — the numerical composition of the system matrices in a linearised equation of motion for the consecutive eigenvalue analysis — is in general successfully tackled due to numerical high performance differentiating algorithms.

In this paper the structural model of the investigated helicopter demonstrator has been built up parallelly with the three methods MBS, FEM

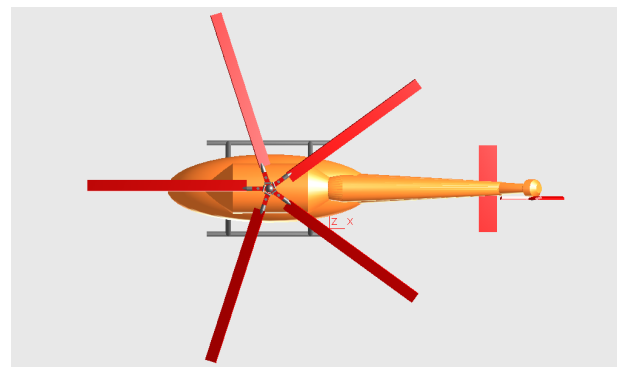


Figure 2: The five-bladed dynamic helicopter demonstrator H9 with articulated rotor — *top view*

and FEMBS, thus resulting in system features independent from each other, but with a physical agreement as good as possible. The commercial MBS code SIMPACK has been basically validated by comparisons to the scientific FEM code GYRBLAD, designed for the solution of rotordynamic problems. Additional comparisons have been done to the commercial FEM tool NASTRAN, which also had been used to supply the elastic substructures for the FEMBS models.

The effort to demonstrate a low error margin in the results to be compared proved to be successful. Most of the eigenvalue results show a relative error of around 0.1%. To reach values further below this margin would have needed an additional numerical effort in higher model resolution. On the other hand error values approaching or passing the 1% margin are to be considered a hint for wrong or physically incomplete modelling on either of the both sides to be compared and has — at least — to be explained thoroughly.

## 2 Build-up of the articulated “Helicopter H9” dynamic demonstrator

The objectives of this research work are the application and the validation of MBS modelling techniques for describing on one hand the general dynamical behaviour of a free flying helicopter and on the other hand the structural interaction between the helicopter rotors — main and tail rotor — and the nacelle of this free helicopter. In this context the focus lies on the dynamic coupling of the rotating structure of the fully elastic main rotor and the — non-rotating — parts of the body structure. For this purpose a generic model, the 9 [to] “Helicopter H9” (see Fig. 1 and 2), had been developed as a simulation platform. Representing the basic research object in this investigation, it serves as a structural demonstrator model and as dynamic reference configuration for both the MBS and the FEM calculations. Since in this investigation the focus was put on the dynamical behaviour of the rotating, elastic structure the influence of the aerodynamic forces had been neglected. Nevertheless one can consider it justified to address the “free” system also as a “free flying” helicopter since in contrast to a ground fixed system it contains the essential dynamic features like the influence of the fuselage mass or the boundary

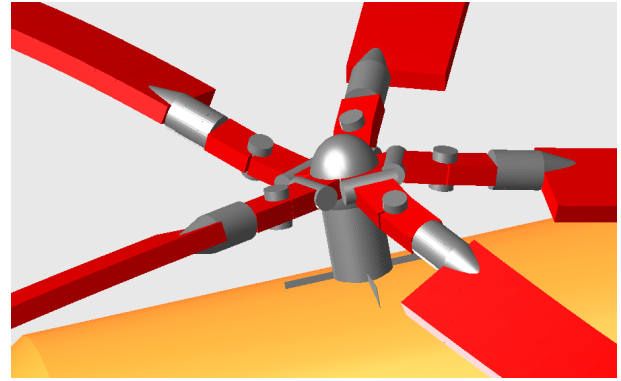


Figure 3: The five-bladed H9 rotor head deformation of the 19. articulated *rotor* eigenmode,  $f = 15.3843$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

conditions (BC). The investigation whose results are presented in this paper was done in analogy to the one conducted in [9]. The main difference between the investigated H/C systems lies in the introduction of hinges and control pitch angles into the blades and thus the general build-up of the rotor head (see Fig. 3 and 4). Formal MBS code modelling questions like where to begin the imported elastic beam structure along the blade axis, i.e. where to place the blade joints in the model had been answered: All kinematic boundaries with potentially large deflections were established in the MBS part outside from the imported FEM part in order to keep the MBS intrinsic modelling capability of large, non-linear joint states. Beside the formal aspects, further rotor dynamics

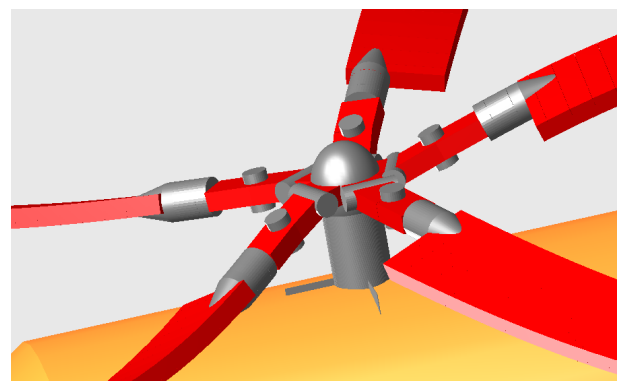


Figure 4: The five-bladed H9 rotor head deformation of the 29. articulated *rotor* eigenmode,  $f = 26.7526$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

|                                                                    |                                               |                                              |                                               |                     |
|--------------------------------------------------------------------|-----------------------------------------------|----------------------------------------------|-----------------------------------------------|---------------------|
| total mass                                                         | $m_{tot} = 9118.4$                            |                                              |                                               | [kg]                |
| position of total CG                                               | $x_{cg} = 2.6320$                             | $y_{cg} = -1.7547$                           | $z_{cg} = -2.6320$                            | [m]                 |
| total inertia (principal axes)                                     | $\Theta_x = 28724.780$                        | $\Theta_y = 64820.231$                       | $\Theta_z = 76555.202$                        | [kgm <sup>2</sup> ] |
| total inertia ( $\alpha_0 = 15.^{\circ}$ )                         | $\Theta_x = 28724.880$                        | $\Theta_y = 64820.298$                       | $\Theta_z = 76555.034$                        | [kgm <sup>2</sup> ] |
| Kardan angles (principal axes)                                     | $\alpha = 22.710$                             | $\beta = 12.584$                             | $\gamma = -11.090$                            | [ $^{\circ}$ ]      |
| Euler angles (principal axes)                                      | $\psi = 16.411$                               | $\vartheta = 25.801$                         | $\varphi = -30.037$                           | [ $^{\circ}$ ]      |
| fuselage mass                                                      | $m_{fus} = 8000.0$                            |                                              |                                               | [kg]                |
| position of fuselage CG                                            | $x_{fus} = 3.0$                               | $y_{fus} = -2.0$                             | $z_{fus} = -3.0$                              | [m]                 |
| fuselage inertia (basic axes)                                      | $\Theta_{xfu} = 7200.0$                       | $\Theta_{yfu} = 36000.0$                     | $\Theta_{zfu} = 36000.0$                      | [kgm <sup>2</sup> ] |
| 5-bladed main rotor mass                                           | $m_{mr} = 1118.4$                             |                                              |                                               | [kg]                |
| diameter main rotor                                                | $D_{mr} = 16.0$                               |                                              |                                               | [m]                 |
| position of main rotor hub                                         | $x_{mr} = 0.0$                                | $y_{mr} = 0.0$                               | $z_{mr} = 0.0$                                | [m]                 |
| main rotor inertia (basic axes)                                    | $\Theta_{xmr/ymr} = 11931.697$                | $\Theta_{zmr} = 23862.928$                   |                                               | [kgm <sup>2</sup> ] |
| main rotor inertia ( $\alpha_0 = 15.^{\circ}$ )                    | $\Theta_{xmr/ymr} = 11931.814$                | $\Theta_{zmr} = 23862.694$                   |                                               | [kgm <sup>2</sup> ] |
| 4-bladed tail rotor mass                                           | $m_{tr} = 156.58$                             |                                              |                                               | [kg]                |
| diameter tail rotor                                                | $D_{tr} = 2.80$                               |                                              |                                               | [m]                 |
| position of tail rotor hub                                         | $x_{tr} = 11.540$                             | $y_{tr} = -0.400$                            | $z_{tr} = -0.300$                             | [m]                 |
| tail rotor inertia (basic axes)                                    | $\Theta_{xtr} = 51.442$                       | $\Theta_{ytr} = 102.82$                      | $\Theta_{ztr} = 51.442$                       | [kgm <sup>2</sup> ] |
| rotor transmission ratio                                           | $n_{mr}/n_{tr} = \frac{40}{7} = 5.7143$       |                                              |                                               | [-]                 |
| main rotor suspension point                                        | $x_{smr} = 0.0$                               | $y_{smr} = 0.0$                              | $z_{smr} = -2.0$                              | [m]                 |
| norm. <i>isotropic</i> main rotor<br>suspension stiffness $/f_s$   | $\hat{c}_u = 2.0$<br>$\hat{c}_{\alpha} = 4.0$ | $\hat{c}_v = 2.0$<br>$\hat{c}_{\beta} = 4.0$ | $\hat{c}_w = 3.0$<br>$\hat{c}_{\gamma} = 5.0$ | [Hz]                |
| norm. <i>orthotropic</i> main rotor<br>suspension stiffness $/f_s$ | $\hat{c}_u = 1.0$<br>$\hat{c}_{\alpha} = 4.0$ | $\hat{c}_v = 2.0$<br>$\hat{c}_{\beta} = 5.0$ | $\hat{c}_w = 3.0$<br>$\hat{c}_{\gamma} = 6.0$ | [Hz]                |
| soft suspension case factor                                        | $f_s = 1.0$                                   |                                              |                                               | [-]                 |
| medium suspension case factor                                      | $f_s = 10.0$                                  |                                              |                                               | [-]                 |
| stiff suspension case factor                                       | $f_s = 100.0$                                 |                                              |                                               | [-]                 |

Table 1: The structural parameters of the dynamic demonstrator “Helicopter H9”

|                                       |                |                      |                                    |                  |                   |
|---------------------------------------|----------------|----------------------|------------------------------------|------------------|-------------------|
| length $l$                            | 8.0            | [m]                  | $l/b$                              | 40.              | [-]               |
| width $b$                             | 0.20           | [m]                  | $b/t$                              | 4.               | [-]               |
| thickness $t$                         | 0.05           | [m]                  | moment of inertia $J$              | $\eta b t^3 / 3$ | [m <sup>4</sup> ] |
| Prandtl’s torsional factor $\eta$     | 0.843          | [-]                  | flap joint posn.                   | 0.25             | [m]               |
| Young’s modulus $E$                   | $71.73 * 10^9$ | [N/m <sup>2</sup> ]  | lag joint posn.                    | 0.50             | [m]               |
| shear modulus $G$                     | $26.90 * 10^9$ | [N/m <sup>2</sup> ]  | pitch bearing posn.                | 0.75             | [m]               |
| Poisson ratio $\nu$                   | 0.33327        | [-]                  | pitch angle $\alpha_0$             | 0., 15., 30.     | [°]               |
| density $\rho$                        | 2796.0         | [kg/m <sup>3</sup> ] | $-\delta_3$ angle                  | 0., 15., 30.     | [°]               |
| stiffn. fact. ( $x = 0 \div .75$ [m]) | 100            | [-]                  | $\alpha_0$ ( $x = 0 \div .75$ [m]) | 0.               | [°]               |

Table 2: The structural parameters of the articulated “Helicopter H9” blade

related questions like the impact of the pitch control angle  $\alpha_0$  or the flap orientation  $\delta_3$  angle on the eigenbehaviour or the structural stability conditions of the free helicopter could be highlighted. Prior test calculations with the non-articulated rotor (which neither had a flex-beam like softened blade shaft area) showed significant system instabilities in the case of strong blade pitch control angles.

The “Helicopter H9” has a fully elastic five blade main rotor with a diameter of  $D = 16$  [m], a rigid four blade tail rotor with a diameter of  $D = 2.8$  [m] and a MTOW of 9118.4 [kg]. The rotor transmission ratio had been determined to such a value that the blade tip speed of main and tail rotor will result equal. The main rotor is rigidly mounted on a 2 [m] shaft whose lower end is elastically suspended. As suspension conditions two sets of stiffnesses are available, an isotropic and an orthotropic one. To the basic stiffness values a general factor is to be applied to get a stiffening or a softening effect. In both models (MBS and FEM) discrete springs have been applied in the mounting point with such stiffness values that would cause the desired frequency of a respective (rigid body) one degree of freedom oscillation. An increase of these basic suspension frequencies by the factor 10 e.g. would thus result in an stiffness increase by the factor 100. These main rotor suspension stiffness sets together with the complete set of the characteristic parameter values for the H9 helicopter are displayed in Tab. 1 and 2.

The helicopter nacelle is modelled as a six degree of freedom rigid body, while the elastic main rotor has up to  $\sim 1300$  degrees of freedom. For reasons of a better clarification and validation of the rotor-fuselage coupling effects — and to make the task more demanding — the center of gravity of the helicopter fuselage exhibits large offsets. Thus the overall center of gravity of the helicopter does not lie in the surroundings of the main rotor hub as usual but with distinct offsets in all three coordinate directions. As a consequence we get a highly non-symmetrical dynamical system w.r.t. the main rotor axis. It has been experienced that the geometric translation of the system matrices into the coordinate system of the overall center of gravity and the rotation of the coordinate axes about the adequate Euler angles onto the princi-

pal axes allow a better control of the rigid body modes which was useful in this investigation. Due to the fact that now the main rotor axis does not coincide with any of the three inertial axes any longer all three rigid body rotational modes will be coupled by the main rotor gyroscopic effect.

Beside the establishing of the principal axes as coordinate axes for the rigid body rotations — the rigid body transversal movement still is aligned along the original basic helicopter coordinate system — additional decoupling measures like very small but finite and disparate “free body” suspensions enable a clear distinction of the rotating and transversal rigid body eigenmodes. As long as the values of these free body suspension stiffnesses remain lower than one tenth of the smallest elastic rotor frequency there will be no significant interaction. (The quality of the rigid body modes can easily be controlled by a look at the modes of this “free” system.) Furthermore the advantage of special joint modelling techniques like the usage of pseudo bodies could be shown. This technique of introducing additional bodies and DOF into the “pure” MBS model could help to reach the aim of placing the required rotor mount positions without any spatial restrictions. (See also [8].)

The aspects of rotor suspension and the variation of the respective suspension stiffness parameters had been investigated here in analogy to [9]. Thus it is possible to compare the differences in the dynamical behaviour of also these two concepts caused by the introduction of the hinges into the blade (for example in the fan diagrams of the eigenfrequencies). It should be recognised that in the hinged rotor the blade shaft stiffness up to the pitch bearing had been raised by an overall factor of 100 (see Tab. 2). As a remark concerning the mass distribution and the data given in Tab. 1 it has to be stated that the whole H/C system is divided into two sub-groups, the rotor and the fuselage. Thus “fuselage” there comprises everything except the rotor, i.e. the tail rotor components mass values are included there.

### 3 The eigenbehaviour of the articulated single blade

The generic helicopter blade model was originally based on the “Princeton beam” (see also [1]), but had been changed in such a manner that makes

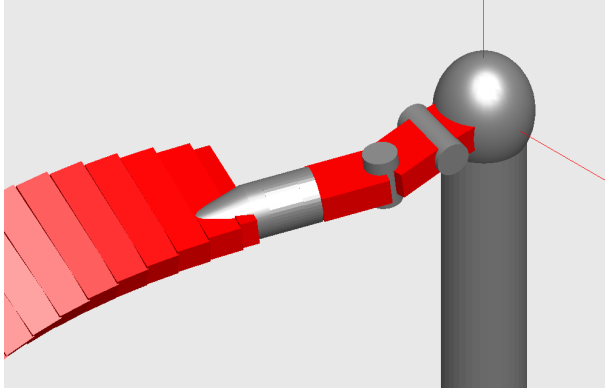


Figure 5: The single blade rotor hub deformation of the 8. articulated *blade* eigenmode — backward position,  $f = 48.5460$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

it more suitable for the present investigation concerning size, mass allocation and frequency distribution. With choosing MBS and FEM model build-ups two basic approaches for incorporating elastic properties into the rotor blade models are compared with each other. Here the way of mapping the continuously distributed elastic properties of the blade beam structure on dynamic equivalent discrete spring stiffnesses of a “pure” MBS model — in contrast to the strategy of importing separately built up elastic Finite Element models with modal substructure techniques (FEMBS), thus creating a hybrid MBS model — is demonstrated in order to achieve mechanical equivalent rotor models (see also [8]).

By applying different angle values

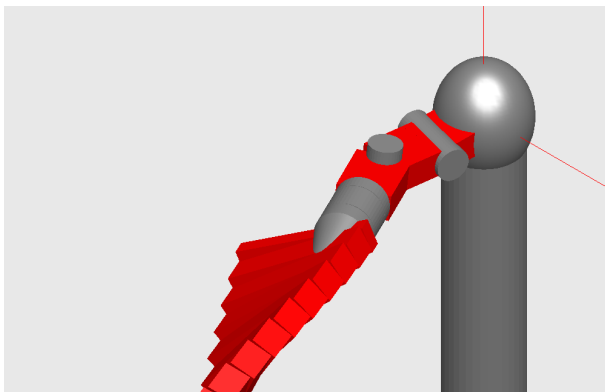


Figure 6: The single blade rotor hub deformation of the 8. articulated *blade* eigenmode — forward position,  $f = 48.5460$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

( $0^\circ, 15^\circ, 30^\circ$ ) the influence of the blade pitch position and the alignment of the blade flap hinge on the rotor eigenbehaviour has been tested in this work. The eigenfrequencies of the rotating and the non-rotating single blade computed with the three methods are displayed in Tab. 3 to 4 for one single rotor speed and different pitch angles and compared among each other. In Fig. 7 to 8 the frequency fan diagrams of the single blade are shown for these three different pitch angle cases. Again for the homogenous clamped blade the eigenfrequencies of some parameter constellations are displayed in Tab. 5 to 6 for one single rotor speed for reasons of comparison.

In the MBS model of the blade it had to be accounted for the right consideration of two torsion related and inertia based special internal beam moments: one is the s.c. *Propeller moment*,

$$M_x^I(x) = \Omega^2 \int_x^l \cos(\alpha_0 + \alpha_t + \alpha_e) \sin(\alpha_0 + \alpha_t + \alpha_e) (I_y - I_z) \frac{\mu}{A} dx, \quad (1)$$

an internal moment counter-acting on the pitch angle, and the other one the s.c. Trapeze effect or *Trapeze moment*,

$$M_x^{II}(x) = \Omega^2 (\alpha_0 + \alpha_t + \alpha_e)' (I_y + I_z) \frac{1}{A} \int_x^l \mu r dx, \quad (2)$$

which both render contributions to the equilibrium state and the torsional stiffness properties, where  $\alpha_0$  stands for the pitch control angle,  $\alpha_t$  the pre-twist and  $\alpha_e$  the elastic deformation content of the total declination angle of the blade cross section with respect to the rotor plane. In the rotor systems investigated in this paper all calculations were done around an undeformed equilibrium state (but with no restriction to the general validity of the chosen approach). This means for the MBS model that there were no pre-deformations (zero joint states) and the equilibrium had to be guaranteed for by the Propeller moments in the shape of s.c. Nominal force parameters, straight in analogy to the normal forces in blade axis direction caused by the centrifugal effects. The Trapeze moment is introduced into the stiffness properties of the blade model as a backing moment.

The impact of the two hub parameters  $\alpha_0$  (pitch control angle) and  $\delta_3$  (alignment of the flap

joint) on the eigenbehaviour of the rotating blade can be studied in the eigenfrequencies as well as in the eigenmodes. The most obvious changes in the eigenfrequencies occur in the branch of the first torsion and the neighbouring modes at a rotor speed of around 4 [Hz]. When the adjacent branches of the third lag mode and the fifth flap mode tend to approach and finally cross the torsion branch a strong interaction between these three modes, which results more intensive with rising angle values, can be perceived (see Fig. 7 to 8). This phenomenon of coupling of the deformation components (DOF) can be seen even clearer by a look at the respective eigenmodes. In Fig. 9 to 16 (and also in Fig. 5 to 6) the components *flap*, *lag* and *torsion* occur to a varying but not at all deniable extend in the affected eigenmodes. These two blade root angles combine already kinematically the deformation components *flapping* with *lagging* ( $\alpha_0$ ) and *flapping* with *torsion* ( $\delta_3$ ). To this boundary condition kind of interaction goes the gyroscopic coupling to result in the observed total impact.

#### 4 The eigenbehaviour of the free system with articulated rotor

Investigated are modelling techniques for simulating the dynamics of the structural behaviour of the free flying helicopter in the *frequency domain*. The results presented are produced by eigenvalue analyses and contain the eigen characteristics like eigenfrequency, natural damping and the eigenmodes. In the case of the MBS simulation code the dynamical model requires a numerical linearisation prior to the eigenvalue analysis. On the MBS side the commercial tool SIMPACK is tested while on the FEM side the scientific tool GYRBLAD is used, which is working on a linear formulation, as well as the imported NASTRAN blade models are linear FEM models.

A topographic and apparently important difference between the FEM and the MBS/FEMBS formulation had been introduced into the simulation models of the complete helicopter model. In the FEM model the rotor blade No. 1 had been aligned backward into the positive  $x$ -axis, while in the MBS model of the rotor the blade No. 1 was looking forward, i.e. into the negative  $x$ -axis, thus resulting in a different rotor blade placement rel-

ative to the helicopter fuselage: The rotor of the MBS model was turned about  $72^\circ/2 = 36^\circ$  in the positive (or negative) sense of rotation. Along with the orthotropic character of both the stiffness conditions of the suspension point and (as well important) the inertia relationship of the H/C fuselage two apparently different systems had to be compared — if there had not been the isometry (rotational symmetry) of the rotor. The phase angle shift in the alignment of the rotor blades was introduced into the models by purpose in order to show by the results the invariance of the (rotational symmetric) H/C main rotor with respect to the strongly non-isotropic boundary conditions in the coupling point between the rotating structure of the H/C rotor and the non-rotating structure of the H/C fuselage. As it can be seen e.g. by comparison of the eigenvalues in Fig. 18 (and also in the Tab. 7 and 8) the “proof” resulted positive both for the rotating and the non-rotating rotor case.

The validity of the rotor-body coupling in general has been proven also in advance by several separate examinations of subsystems like the comparison of the flexible and the rigid isolated rotor. For example the invariance of the *symmetrical* rotor towards *orthogonal* suspension conditions (fuselage mass and hub stiffness effect on boundary conditions) had been verified by comparing the eigenvalues of our reference system with results of a second rotor system which was built up identical except for the blades being mounted to the hub by an arbitrary offset (here:  $\sim 25.0^\circ$ ). Both systems rendered (numerically) equal eigenfrequencies. In other precedingly examined control cases like rotor systems with a rigid rotor also analytical solutions could be used for verification.

Results of the eigenvalue analyses of the complete helicopter model with soft orthotropic suspension are presented in Fig. 18 as frequency fan and stability diagrams, and in Tab. 7 and 8 as eigenvalues (damping and frequency) for the non-rotating case and a single rotor speed. In Fig. 17 and Tab. 9 again the values of the soft case are displayed together with the ones for the medium and the stiff suspension cases. The rotation boundary condition is body-fixed in the sense that the position of the rotor axis is freely moving in space with the rotating rotor shaft. Eigenmodes for some selected frequencies are displayed in Fig. 19 to 26.

To get an impression of the “mere” rotor-nacelle interaction without any suspension stiffness influence one has to look at the results for the stiff main rotor suspension case, because there the influence of the suspension springs is — at least for the displayed frequency range — almost negligible. In Tab. 9 the influence of the rotation of the tail rotor on the dynamic behaviour of the free system had been looked at by the comparison with the tail rotor case being put “off”.

By lining up the eigenfrequencies in ascending order a clear structure in the results can be perceived: Groups of five very closely neighboured frequencies follow each other where the identical pairs belong to the reactionless modes. These quintuples are based on the eigenfrequencies of the single blade and are eventually disturbed only by rigid body or elastic suspension effects. Obviously the stiffer the rotor suspension is the more pronounced the grouping occurs (and vice versa). Comparing also Fig. 8 with the stiff case in Fig. 17 illustrates the influence of the eigenbehaviour of the underlying rotor blade. Although frequencies of zero belong only to pure rigid body modes a small suspension stiffness does not interfere with the elastic mode shapes. By hitting the (small) presumable eigenfrequency number as the result value in the simulation this can be used as a control criterion for the quality of the eigensolution (compare e.g. the rigid body values for the non-rotating case, Tab. 7).

For the free flying helicopter several instability regions occur over the whole rotor speed range. Especially in terms of structural (and aeroelastic) stability the physical “completeness” of the rotor dynamic model plays a crucial role. By introducing different kinematical and dynamical boundary conditions into it, cases of stability loss due to ground resonance can be produced. Even for the free flying helicopter cases of stability loss concerning elastic eigenmodes of the coupled rotor-nacelle-system — an air resonance type — could be detected. Beside the dynamic properties of the elastic rotor blade the design of the main rotor suspension has an essential influence here. In the soft rotor suspension case large areas with negative damping values occur (see negative damping values in Fig. 18) while the stiffer rotor suspension cases proved to be stable over the whole range of rotor speed. As a general tendency it can be

stated that the softer the main rotor suspension is the more numerous and the more severe instability cases are to be recognised.

## 5 Parameter variation of the main rotor suspension and the tail rotor

Concerning the specific dynamic coupling effects between rotor and nacelle a survey study with topics like the main rotor suspension (lateral and vertical) and the elasticity of the drive train (torsional) had been conducted. Therefore six degrees of freedom in the elastic suspension point of the main rotor can be collocated and addressed at the lower end of the rotor shaft. The pairs of translational and rotational displacement around the  $x$ — and the  $y$ —axis can be used to model the dynamic and kinematic conditions of the rotor mount while the third rotational DOF around the rotor  $z$ —axis stands for the elastic torsional stiffness of the drive train. A spatial spreading of these six DOF to individual locations is optional. (The transversal vertical DOF in  $z$ —direction had been included just for the sake of completeness.) In systematic variation of the respective stiffness values (in three steps over four decades) the results of different parameter studies are presented as numerical results for distinct constant rotor speeds. In fan diagrams (Campbell diagrams) the overall dynamical behaviour with the change of rotor speed is illustrated. The results of the eigenvalue analyses are shown in the figures and tables already mentioned above for the complete free helicopter.

In the course of the investigation H/C systems with three different main rotor suspension stiffness sets (referring to the normalised stiffnesses in Tab. 1) have been tested. The values of the set of 6 DOF for the basic (the soft) version are given in Tab. 1. The normalised suspension stiffnesses are defined as

$$\hat{c}_i = \sqrt{\frac{c_i}{m_i}} / (2\pi) \quad [\text{Hz}], \quad (3)$$

representing the frequency of the respective one DOF rigid body movement in the principal coordinate system. To get to the values for the medium and the stiff rotor suspension cases, the  $\hat{c}_i$  values are enlarged by the factors 10 and 100, which corresponds with a rise of the physical stiffness values in the six DOF of the suspension point about the

factors 100 and 10000. In Tab. 7 to 9 numerical values of the eigenfrequencies (and damping coefficients) of the complete H/C system are presented for some parameter constellations. The Tab. 7 and 8 show the results for the soft rotor suspension case according to the three independent simulation models and their corresponding discretisation method FEM, MBS and FEMBS, with Tab. 7 for the H/C with non-rotating rotors and Tab. 8 for the case with rotating rotors (both main and tail rotor). Tab. 9 finally gives the eigenvalues for the varying rotor suspension stiffnesses, the *stiff*, *medium* and *soft* case, according to one single modelling method and solution algorithm, here FEM (using code GYRBLAD).

Also the effect of the tail rotor on the eigenfrequencies can be studied in the Tab. 9. The displayed relative difference values give a nice impression of the influence of the rotating tail rotor on the frequency of each (lower) eigenmode. The third sub-column in each of the three meta-columns there shows as percentage the difference of the displayed frequencies — the cases with both the main and the tail rotor in action — to the respective cases with a non-rotating tail rotor (referred to the displayed values). That means that for the cases with negative deviation values setting the tail rotor in rotation will give a lift to the eigenfrequencies. This comparison was included into the investigation to get a clue for the magnitude of the influence of the rotation of the tail rotor. At least for our test case H9 with a rigid fuselage it can be stated that the impact of the tail rotor is quite low, i.e. almost zero for all the higher modes and only for a few yaw or roll dominated lower modes with either positive or negative deviations of around 0.5[%] in the eigenfrequency values.

During the investigation of the single blade and the complete helicopter model it could be experienced that the reachable numerical model and eigenvalue resolution was around 0.1[%], so that any physical effect contributing at such a magnitude could not be resolved. On the other hand it could be proved in general that in test calculation cases with the frequency differences approaching or crossing the 1.0[%] margin always inconsistent or defective modelling of e.g. the rotatory effects were underlying. Looking at Tab. 8 the differences in the results according to the three methods lie

around 0.1[%], thus showing a very good agreement between the respective models. Only for two frequencies (the 6<sup>th</sup> and the 7<sup>th</sup>) an error margin of around 2.0[%] was rendered. It could be found out that the value of this difference depends on and rises with the quotient between the stiffness factors  $f_s$  of the rotor suspension and the (very soft) suspension in the CG of the complete system. Consequentially, setting this ratio to 1 — which means identically normalised stiffnesses in the respective 6 suspension DOF — an equally good deviation level of 0.1[%] or less could be obtained in all eigenmodes. Of course this elastic strongly mounted system does not represent the free system anymore, but by this comparison it could be shown that the involved systems are *physically equivalent* and that the occurring 2.0[%] deviation stems (presumably) from the numerical linearisation of the MBS model. In this context a program control parameter of SIMPACK was tested in order to influence and improve the numerical accuracy of the solution. Setting the overall factor *vepar*(11) (offered to improve the numerical quality of the linearisation by affecting for example the difference quotients) equal 1.e+5, all the *rigid body* frequencies from MBS and FEMBS (SIMPACK) of the rotating system (see Tab. 8) could be highly improved and brought to convergence with the FEM (GYRBLAD) solutions. (The highest, the gyro rigid body eigenmode No. 6, is already interacting with the adjacent elastic modes.)

## 6 Conclusion

In this study the dynamic demonstrator of a generic helicopter had been modelled and the structural behaviour of the free flying system investigated. The focus of the investigation lied on the computational and modelling facilities the MBS code SIMPACK offers for describing the interface between the rotating, fully elastic blades of the rotor and the fuselage of the H/C. The MBS joints as main elements of the MBS code for defining system DOF with their intrinsic properties of describing potentially large deformations have been used in building up the blade hinges, the rotor hub and finally the elastic rotor suspension. The introduction of further physical model characteristics, like a swashplate, pitch links with

double load paths or a more complex drive train model (and the respective DOF) into the system, seems both feasible and promising.

The validation of the models and the proceeding had been done by comparing the eigenmodes and the eigenvalue results produced with the two elasto-mechanical methods MBS and FEM. Thus different algorithms and independent tools have been used in the examination. It has been shown that for the non-rotating as well as for the rotating test cases the coupling effects caused by the blade hinges and pitch angles will be reproduced without any restriction in both approaches. The FEMBS modulation of the pure MBS model renders a hybrid formulation which can combine advantages of both constituents. Thus the potential of a sophisticated MBS code like SIMPACK as a powerful simulation tool for helicopter dynamics of the free flying system has been demonstrated with respect to the characteristics of an articulated rotor coupled to the non-rotating fuselage.

## 7 Copyright statement

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## 8 Appendix: Additional figures and tables

- Fig. 7 to 8: Frequency fan diagrams of the *single blade*
- Tab. 3 to 6: Eigenvalues of the *single blade*
- Fig. 9 to 16: Eigenmodes of the *single blade*
- Fig. 17 to 18: Frequency fan diagrams of the *free helicopter H9*
- Tab. 7 to 9: Eigenvalues of the *free H/C H9*
- Fig. 19 to 26: Eigenmodes of the *free helicopter H9*

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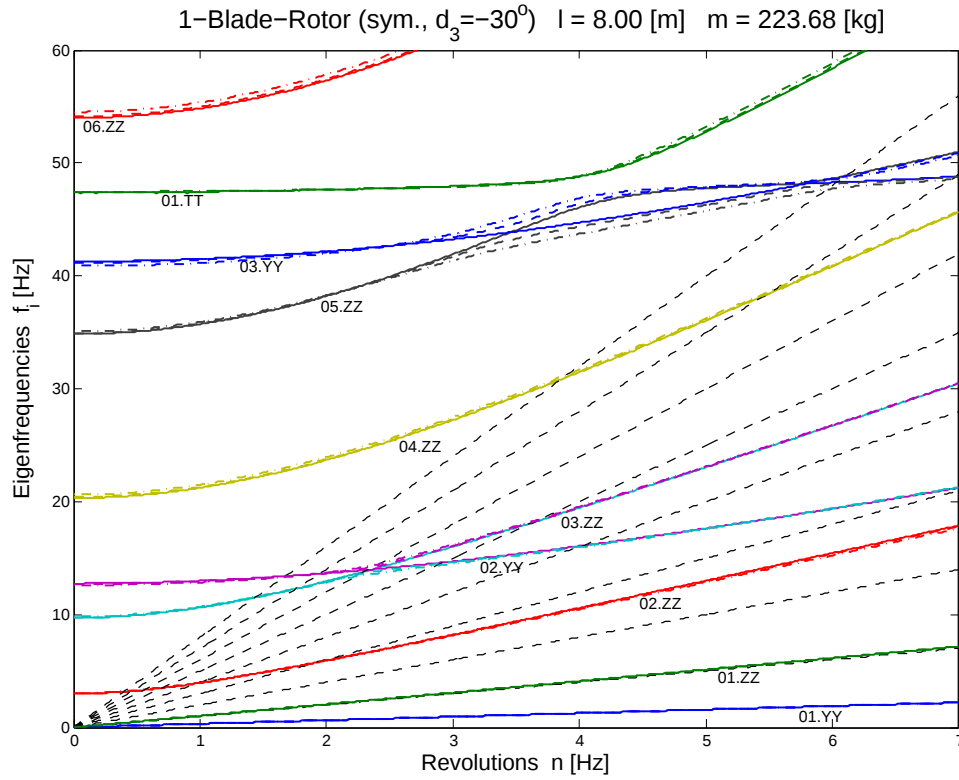


Figure 7: The articulated single blade ( $\delta_3 = -30^\circ$ ): The *eigenfrequencies* with respect to the pitch control angle ( $\alpha_0 = 0^\circ$ : — ;  $\alpha_0 = 15^\circ$ : - - - ;  $\alpha_0 = 30^\circ$ : - . - . -)

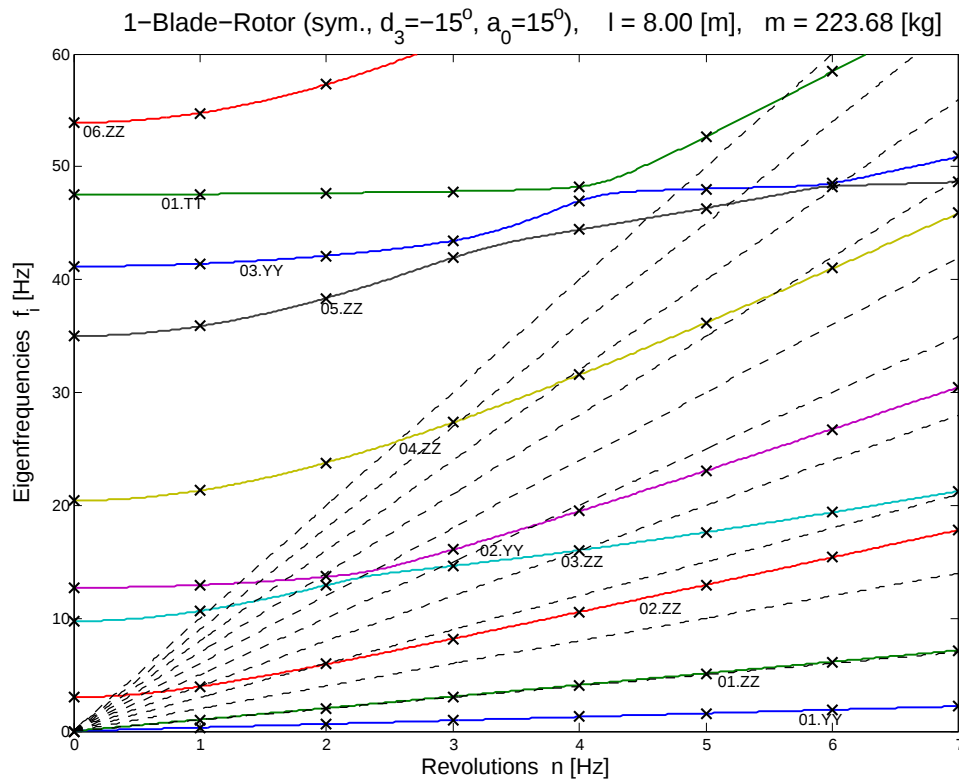


Figure 8: The articulated single blade ( $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$ ): Comparison of the *eigenfrequency* results ( $\times$  = MBS SIMPACK, — = FEM GYRBLAD)

| SINGLE BLADE : w/hinges $n = 0$ [Hz] $d_3 = -30$ [°] $pitch = 00$ [°] |                          |          |      |                              |          |       |                                         |          |       |
|-----------------------------------------------------------------------|--------------------------|----------|------|------------------------------|----------|-------|-----------------------------------------|----------|-------|
|                                                                       | GYRBLAD<br>FEM (32 Elm.) |          |      | SIMPACT 895<br>MBS (64 Bod.) |          |       | NASTRAN-SIMPACT 895<br>FEMBS (120 Mod.) |          |       |
|                                                                       | Nat.Dmp.                 | Frequen. | Dif. | Nat.Dmp.                     | Frequen. | Dif.  | Nat.Dmp.                                | Frequen. | Dif.  |
|                                                                       | [-]                      | [Hz]     | [%]  | [-]                          | [Hz]     | [%]   | [-]                                     | [Hz]     | [%]   |
| 1.F                                                                   | 1.00                     | 0.0000   | 0.00 | 1.00                         | 0.0000   | 0.00  | .000                                    | 0.0000   | 0.00  |
| 1.L                                                                   | -1.00                    | 0.0000   | 0.00 | 1.00                         | 0.0000   | 0.00  | .000                                    | 0.0000   | 0.00  |
| 2.F                                                                   | .000                     | 2.9901   | 0.00 | .000                         | 2.9901   | 0.00  | .000                                    | 2.9896   | -0.02 |
| 3.F                                                                   | .000                     | 9.7106   | 0.00 | .000                         | 9.7100   | -0.01 | .000                                    | 9.7095   | -0.01 |
| 2.L                                                                   | .000                     | 12.7492  | 0.00 | .000                         | 12.7490  | -0.00 | .000                                    | 12.7480  | -0.01 |
| 4.F                                                                   | -.000                    | 20.3250  | 0.00 | .000                         | 20.3227  | -0.01 | .000                                    | 20.3210  | -0.02 |
| 5.F                                                                   | .000                     | 34.8505  | 0.00 | .000                         | 34.8446  | -0.02 | .000                                    | 34.8382  | -0.04 |
| 3.L                                                                   | -.000                    | 41.2772  | 0.00 | .000                         | 41.2756  | -0.00 | .000                                    | 41.2309  | -0.11 |
| 1.T                                                                   | -.000                    | 47.3815  | 0.00 | .000                         | 47.3764  | -0.01 | .000                                    | 47.3791  | -0.01 |
| 6.F                                                                   | -.000                    | 54.0034  | 0.00 | .000                         | 53.9904  | -0.02 | .000                                    | 53.9736  | -0.06 |
| 7.F                                                                   | .000                     | 76.5246  | 0.00 | .000                         | 76.5015  | -0.03 | .000                                    | 76.4588  | -0.09 |
| 4.L                                                                   | -.000                    | 86.0111  | 0.00 | .000                         | 86.0035  | -0.01 | .000                                    | 85.7497  | -0.30 |
| 8.F                                                                   | .000                     | 103.4177 | 0.00 | .000                         | 103.3783 | -0.04 | .000                                    | 103.2936 | -0.12 |
| 9.F                                                                   | .000                     | 134.3235 | 0.00 | .000                         | 134.2576 | -0.05 | .000                                    | 134.1164 | -0.15 |
| 2.T                                                                   | -.000                    | 143.2767 | 0.00 | .000                         | 143.0483 | -0.16 | .000                                    | 143.2296 | -0.03 |
| 5.L                                                                   | .000                     | 146.8399 | 0.00 | .000                         | 146.8156 | -0.02 | .000                                    | 145.9900 | -0.58 |
| 10.F                                                                  | -.000                    | 170.1376 | 0.00 | .000                         | 170.0215 | -0.07 | .000                                    | 169.7925 | -0.20 |
| 1.S                                                                   | -.000                    | 174.4969 | 0.00 | .000                         | 174.4706 | -0.02 | .000                                    | 174.4969 | 0.00  |
| 11.F                                                                  | .000                     | 209.7081 | 0.00 | .000                         | 209.5174 | -0.09 | .000                                    | 209.1804 | -0.25 |
| 6.L                                                                   | .000                     | 223.6121 | 0.00 | .000                         | 223.5496 | -0.03 | .000                                    | 221.5317 | -0.93 |
| 3.T                                                                   | -.000                    | 238.6027 | 0.00 | .000                         | 237.7216 | -0.37 | .000                                    | 238.5974 | -0.00 |

Table 3: The articulated *non-rotating* single blade ( $n = 0$  [Hz],  $\delta_3 = -30^\circ$ ,  $\alpha_0 = 0^\circ$ ): *Eigenfrequencies* and *damping* according to the three methods in comparison to the FEM solution

| SINGLE BLADE : w/hinges $n = 6$ [Hz] $d_3 = -30$ [°] $pitch = 30$ [°] |                          |          |      |                              |          |       |                                         |          |       |
|-----------------------------------------------------------------------|--------------------------|----------|------|------------------------------|----------|-------|-----------------------------------------|----------|-------|
|                                                                       | GYRBLAD<br>FEM (32 Elm.) |          |      | SIMPACT 895<br>MBS (64 Bod.) |          |       | NASTRAN-SIMPACT 895<br>FEMBS (120 Mod.) |          |       |
|                                                                       | Nat.Dmp.                 | Frequen. | Dif. | Nat.Dmp.                     | Frequen. | Dif.  | Nat.Dmp.                                | Frequen. | Dif.  |
|                                                                       | [-]                      | [Hz]     | [%]  | [-]                          | [Hz]     | [%]   | [-]                                     | [Hz]     | [%]   |
| 1.L                                                                   | .000                     | 1.8918   | 0.00 | .000                         | 1.8904   | -0.07 | .000                                    | 1.8905   | -0.07 |
| 1.F                                                                   | .000                     | 6.1430   | 0.00 | .000                         | 6.1426   | -0.01 | .000                                    | 6.1424   | -0.01 |
| 2.F                                                                   | .000                     | 15.2351  | 0.00 | .000                         | 15.2270  | -0.05 | .000                                    | 15.2391  | 0.03  |
| 2.L                                                                   | -.000                    | 19.4009  | 0.00 | .000                         | 19.3974  | -0.02 | .000                                    | 19.4121  | 0.06  |
| 3.F                                                                   | -.000                    | 26.7837  | 0.00 | .000                         | 26.7747  | -0.03 | .000                                    | 26.7711  | -0.05 |
| 4.F                                                                   | -.000                    | 41.0100  | 0.00 | .000                         | 41.0187  | 0.02  | .002                                    | 41.0371  | 0.07  |
| 1.T                                                                   | .000                     | 47.7234  | 0.00 | .000                         | 47.7113  | -0.03 | .001                                    | 47.6124  | -0.23 |
| 2.T                                                                   | .000                     | 48.5583  | 0.00 | .000                         | 48.5480  | -0.02 | -.001                                   | 48.4649  | -0.19 |
| 5.F                                                                   | .000                     | 58.8573  | 0.00 | .000                         | 58.8971  | 0.07  | -.001                                   | 58.8387  | -0.03 |
| 6.F                                                                   | -.000                    | 79.1859  | 0.00 | .000                         | 79.2738  | 0.11  | -.000                                   | 79.1971  | 0.01  |
| 3.L                                                                   | -.000                    | 92.4806  | 0.00 | .000                         | 92.4946  | 0.02  | .000                                    | 92.3221  | -0.17 |
| 7.F                                                                   | -.000                    | 103.8918 | 0.00 | .000                         | 104.0361 | 0.14  | .000                                    | 103.8572 | -0.03 |
| 8.F                                                                   | -.000                    | 131.1403 | 0.00 | .000                         | 131.3567 | 0.17  | .001                                    | 131.1305 | -0.01 |
| 3.T                                                                   | .000                     | 144.1190 | 0.00 | .000                         | 143.8809 | -0.17 | -.001                                   | 144.0416 | -0.05 |
| 4.L                                                                   | -.000                    | 152.6096 | 0.00 | .000                         | 152.6431 | 0.02  | -.001                                   | 151.9617 | -0.42 |
| 9.F                                                                   | -.000                    | 164.3455 | 0.00 | .000                         | 164.6174 | 0.17  | -.001                                   | 164.1159 | -0.14 |
| 1.S                                                                   | .000                     | 174.8063 | 0.00 | .000                         | 174.7801 | -0.01 | .000                                    | 174.8062 | -0.00 |
| 10.F                                                                  | -.000                    | 199.2224 | 0.00 | .000                         | 199.5974 | 0.19  | -.000                                   | 198.9601 | -0.13 |
| 5.L                                                                   | -.000                    | 228.3455 | 0.00 | .000                         | 228.4076 | 0.03  | .000                                    | 226.7277 | -0.71 |
| 4.T                                                                   | .000                     | 239.3658 | 0.00 | .000                         | 238.8297 | -0.22 | .001                                    | 238.8560 | -0.21 |
| 5.T                                                                   | .000                     | 242.0847 | 0.00 | .000                         | 241.9984 | -0.04 | -.001                                   | 242.0347 | -0.02 |

Table 4: The articulated *rotating* single blade ( $n = 6.0$  [Hz],  $\delta_3 = -30^\circ$ ,  $\alpha_0 = 30^\circ$ ): *Eigenfrequencies* and *damping* according to the three methods in comparison to the FEM solution

| SINGLE BLADE : clamped n = 6 [Hz] pitch = 00. [o] |                          |          |      |                              |          |       |                                         |          |       |
|---------------------------------------------------|--------------------------|----------|------|------------------------------|----------|-------|-----------------------------------------|----------|-------|
|                                                   | GYRBLAD<br>FEM (32 Elm.) |          |      | SIMPACT 895<br>MBS (64 Bod.) |          |       | NASTRAN-SIMPACT 895<br>FEMBS (120 Mod.) |          |       |
|                                                   | Nat.Dmp.                 | Frequen. | Dif. | Nat.Dmp.                     | Frequen. | Dif.  | Nat.Dmp.                                | Frequen. | Dif.  |
|                                                   | [-]                      | [Hz]     | [%]  | [-]                          | [Hz]     | [%]   | [-]                                     | [Hz]     | [%]   |
| 1.L                                               | -.000                    | 3.4146   | 0.00 | .000                         | 3.4153   | 0.02  | .000                                    | 3.4087   | -0.17 |
| 1.F                                               | -.000                    | 6.1988   | 0.00 | .000                         | 6.2017   | 0.05  | .000                                    | 6.1976   | -0.02 |
| 2.F                                               | .000                     | 15.5739  | 0.00 | .000                         | 15.5734  | -0.00 | .000                                    | 15.5695  | -0.03 |
| 2.L                                               | .000                     | 21.2790  | 0.00 | .000                         | 21.2811  | 0.01  | .000                                    | 21.2654  | -0.06 |
| 3.F                                               | .000                     | 26.9831  | 0.00 | .000                         | 26.9904  | 0.03  | .000                                    | 26.9779  | -0.02 |
| 4.F                                               | .000                     | 41.2738  | 0.00 | .000                         | 41.3115  | 0.09  | .000                                    | 41.2705  | -0.01 |
| 1.T                                               | -.000                    | 43.9811  | 0.00 | .000                         | 43.9596  | -0.05 | .000                                    | 43.5800  | -0.91 |
| 3.L                                               | -.000                    | 51.0314  | 0.00 | .000                         | 51.0508  | 0.04  | .000                                    | 50.9205  | -0.22 |
| 5.F                                               | .000                     | 58.3910  | 0.00 | .000                         | 58.4789  | 0.15  | .000                                    | 58.3787  | -0.02 |
| 6.F                                               | -.000                    | 78.4173  | 0.00 | .000                         | 78.5755  | 0.20  | .000                                    | 78.3930  | -0.03 |
| 4.L                                               | -.000                    | 94.4371  | 0.00 | .000                         | 94.4902  | 0.06  | .000                                    | 93.9896  | -0.47 |
| 7.F                                               | .000                     | 101.5047 | 0.00 | .000                         | 101.7531 | 0.24  | .000                                    | 101.4554 | -0.05 |
| 8.F                                               | -.000                    | 127.7888 | 0.00 | .000                         | 128.1458 | 0.28  | .000                                    | 127.6975 | -0.07 |
| 2.T                                               | -.000                    | 130.8052 | 0.00 | .000                         | 130.6095 | -0.15 | .000                                    | 130.6701 | -0.10 |
| 5.L                                               | .000                     | 151.6450 | 0.00 | .000                         | 151.7493 | 0.07  | .000                                    | 150.4141 | -0.81 |
| 9.F                                               | -.000                    | 157.3732 | 0.00 | .000                         | 157.8535 | 0.31  | .000                                    | 157.2099 | -0.10 |
| 1.S                                               | -.000                    | 158.6410 | 0.00 | .000                         | 158.6220 | -0.01 | .000                                    | 158.6410 | 0.00  |
| 10.F                                              | -.000                    | 190.3332 | 0.00 | .000                         | 190.9452 | 0.32  | .000                                    | 190.0669 | -0.14 |
| 3.T                                               | .000                     | 218.1916 | 0.00 | .000                         | 217.4285 | -0.35 | .000                                    | 218.1136 | -0.04 |
| 6.L                                               | .000                     | 222.6136 | 0.00 | .000                         | 222.7861 | 0.08  | .000                                    | 219.8991 | -1.22 |
| 11.F                                              | -.000                    | 226.7246 | 0.00 | .000                         | 227.4678 | 0.33  | .000                                    | 226.3136 | -0.18 |

Table 5: The clamped homogeneous *rotating* single blade ( $n = 6.0$  [Hz],  $\alpha_0 = 0.^\circ$ ): *Eigenfrequencies* and *damping* according to the three methods in comparison to the FEM solution

| SINGLE BLADE : clamped n = 6 [Hz] pitch = 30. [o] |                          |          |      |                              |          |       |                                         |          |       |
|---------------------------------------------------|--------------------------|----------|------|------------------------------|----------|-------|-----------------------------------------|----------|-------|
|                                                   | GYRBLAD<br>FEM (32 Elm.) |          |      | SIMPACT 895<br>MBS (64 Bod.) |          |       | NASTRAN-SIMPACT 895<br>FEMBS (120 Mod.) |          |       |
|                                                   | Nat.Dmp.                 | Frequen. | Dif. | Nat.Dmp.                     | Frequen. | Dif.  | Nat.Dmp.                                | Frequen. | Dif.  |
|                                                   | [-]                      | [Hz]     | [%]  | [-]                          | [Hz]     | [%]   | [-]                                     | [Hz]     | [%]   |
| 1.L                                               | -.000                    | 2.9811   | 0.00 | .000                         | 2.9837   | 0.09  | .000                                    | 2.9757   | -0.18 |
| 1.F                                               | -.000                    | 6.4185   | 0.00 | .000                         | 6.4205   | 0.03  | .000                                    | 6.4170   | -0.02 |
| 2.F                                               | -.000                    | 15.2462  | 0.00 | .000                         | 15.2462  | 0.00  | .000                                    | 15.2432  | -0.02 |
| 2.L                                               | -.000                    | 21.5158  | 0.00 | .000                         | 21.5173  | 0.01  | .000                                    | 21.5019  | -0.06 |
| 3.F                                               | -.000                    | 26.8120  | 0.00 | .000                         | 26.8200  | 0.03  | .000                                    | 26.8094  | -0.01 |
| 4.F                                               | .000                     | 41.1625  | 0.00 | .000                         | 41.2014  | 0.09  | .000                                    | 41.1619  | -0.00 |
| 1.T                                               | -.000                    | 43.8007  | 0.00 | .000                         | 43.7787  | -0.05 | .000                                    | 43.5813  | -0.50 |
| 3.L                                               | -.000                    | 51.1221  | 0.00 | .000                         | 51.1415  | 0.04  | .000                                    | 51.0105  | -0.22 |
| 5.F                                               | .000                     | 58.3126  | 0.00 | .000                         | 58.4013  | 0.15  | .000                                    | 58.3041  | -0.01 |
| 6.F                                               | .000                     | 78.3587  | 0.00 | .000                         | 78.5178  | 0.20  | .000                                    | 78.3386  | -0.03 |
| 4.L                                               | .000                     | 94.4861  | 0.00 | .000                         | 94.5389  | 0.06  | .000                                    | 94.0381  | -0.47 |
| 7.F                                               | -.000                    | 101.4592 | 0.00 | .000                         | 101.7085 | 0.25  | .000                                    | 101.4144 | -0.04 |
| 8.F                                               | .000                     | 127.7523 | 0.00 | .000                         | 128.1104 | 0.28  | .000                                    | 127.6653 | -0.07 |
| 2.T                                               | -.000                    | 130.7448 | 0.00 | .000                         | 130.5487 | -0.15 | .000                                    | 130.6705 | -0.06 |
| 5.L                                               | -.000                    | 151.6762 | 0.00 | .000                         | 151.7808 | 0.07  | .000                                    | 150.4446 | -0.81 |
| 9.F                                               | -.000                    | 157.3434 | 0.00 | .000                         | 157.8246 | 0.31  | .000                                    | 157.1846 | -0.10 |
| 1.S                                               | .000                     | 158.6406 | 0.00 | .000                         | 158.6210 | -0.01 | .000                                    | 158.6408 | 0.00  |
| 10.F                                              | -.000                    | 190.3084 | 0.00 | .000                         | 190.9214 | 0.32  | .000                                    | 190.0472 | -0.14 |
| 3.T                                               | .000                     | 218.1552 | 0.00 | .000                         | 217.3908 | -0.35 | .000                                    | 218.1114 | -0.02 |
| 6.L                                               | -.000                    | 222.6349 | 0.00 | .000                         | 222.8081 | 0.08  | .000                                    | 219.9212 | -1.22 |
| 11.F                                              | -.000                    | 226.7037 | 0.00 | .000                         | 227.4477 | 0.33  | .000                                    | 226.2980 | -0.18 |

Table 6: The clamped homogeneous *rotating* single blade ( $n = 6.0$  [Hz],  $\alpha_0 = 30.^\circ$ ): *Eigenfrequencies* and *damping* according to the three methods in comparison to the FEM solution

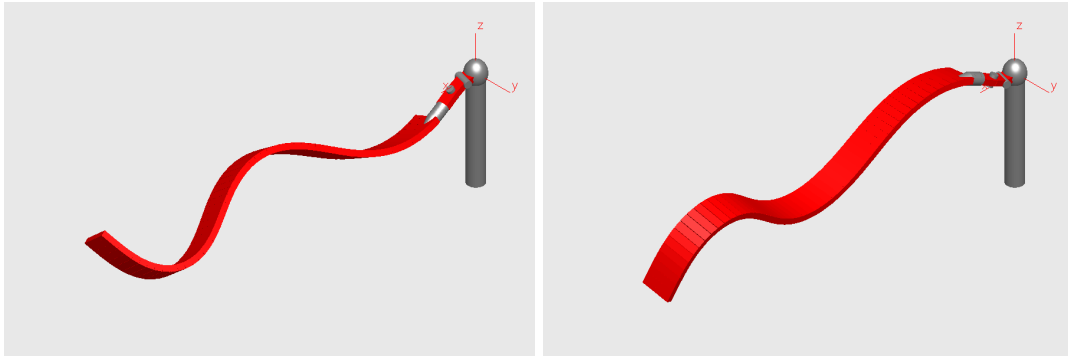


Figure 9: The 6. articulated *blade* eigenmode,  $f = 41.0005$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

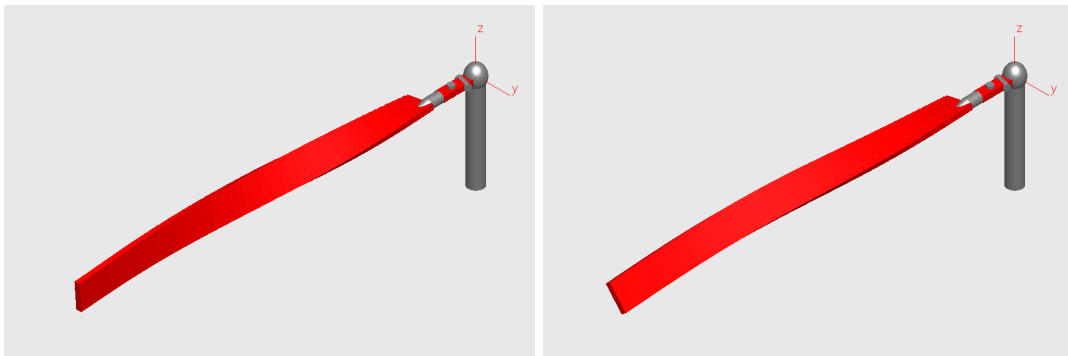


Figure 10: The 7. articulated *blade* eigenmode,  $f = 48.2504$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

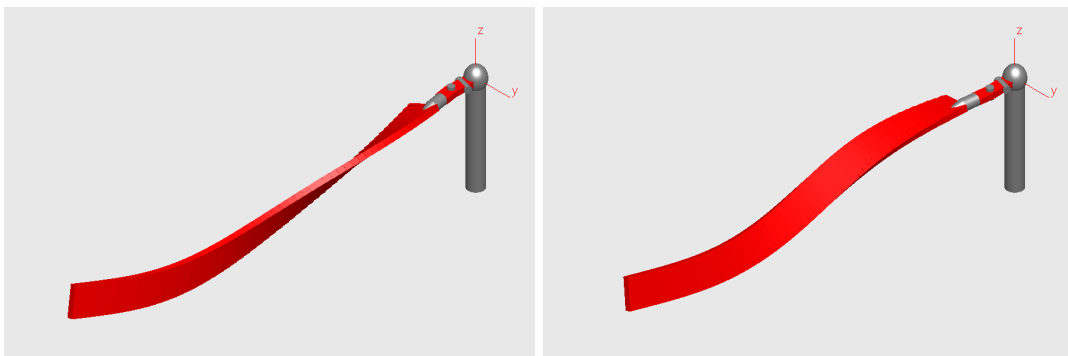


Figure 11: The 8. articulated *blade* eigenmode,  $f = 48.5460$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

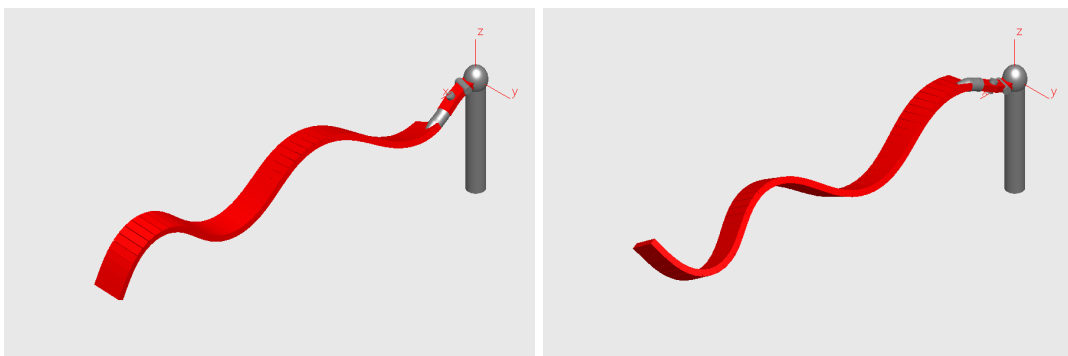


Figure 12: The 9. articulated *blade* eigenmode,  $f = 58.4584$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

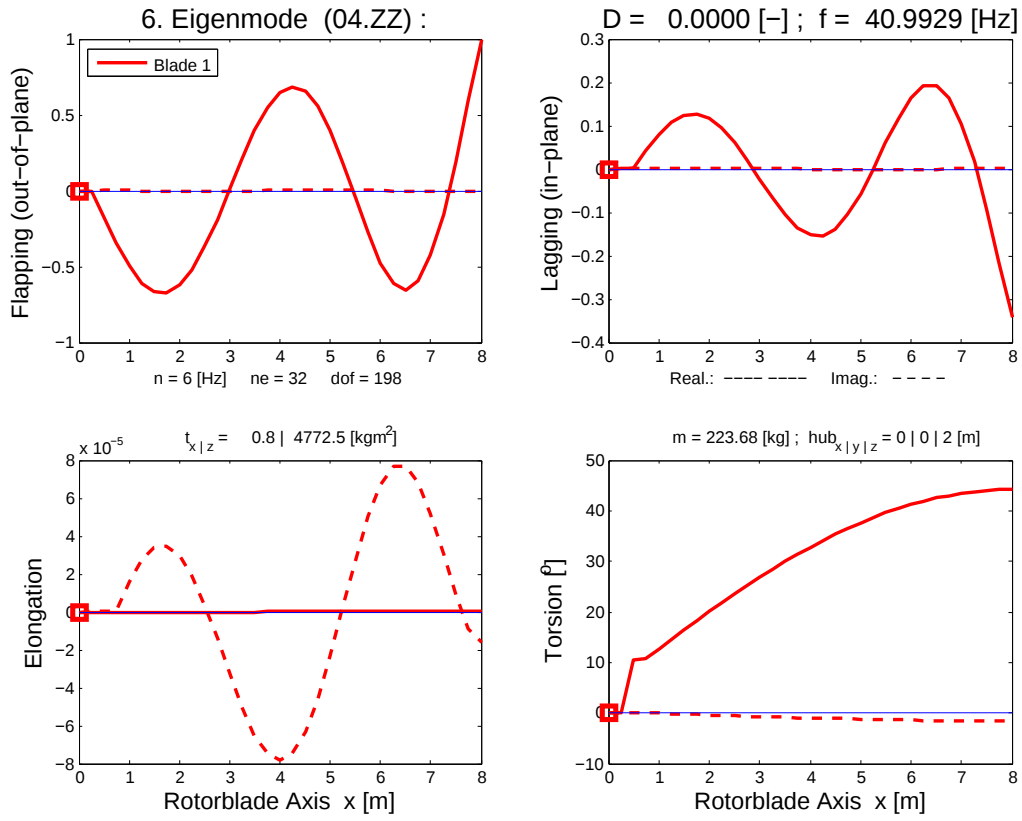


Figure 13: The 6. articulated *blade* eigenmode,  $f = 40.99$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

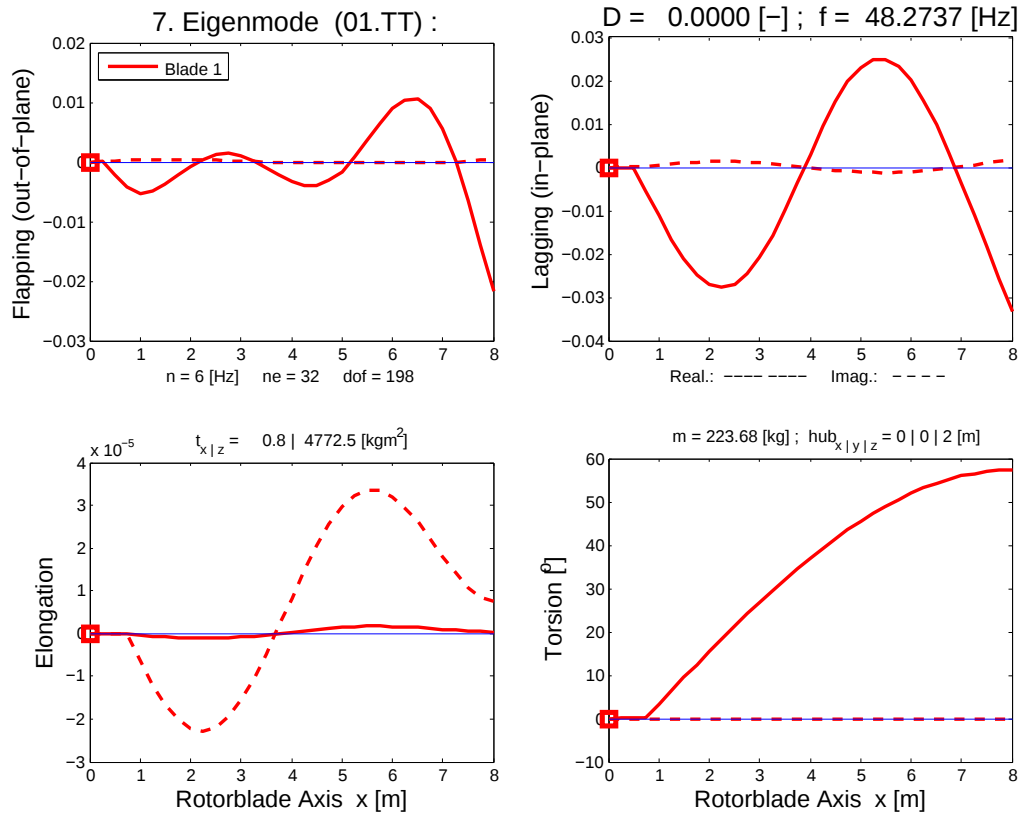
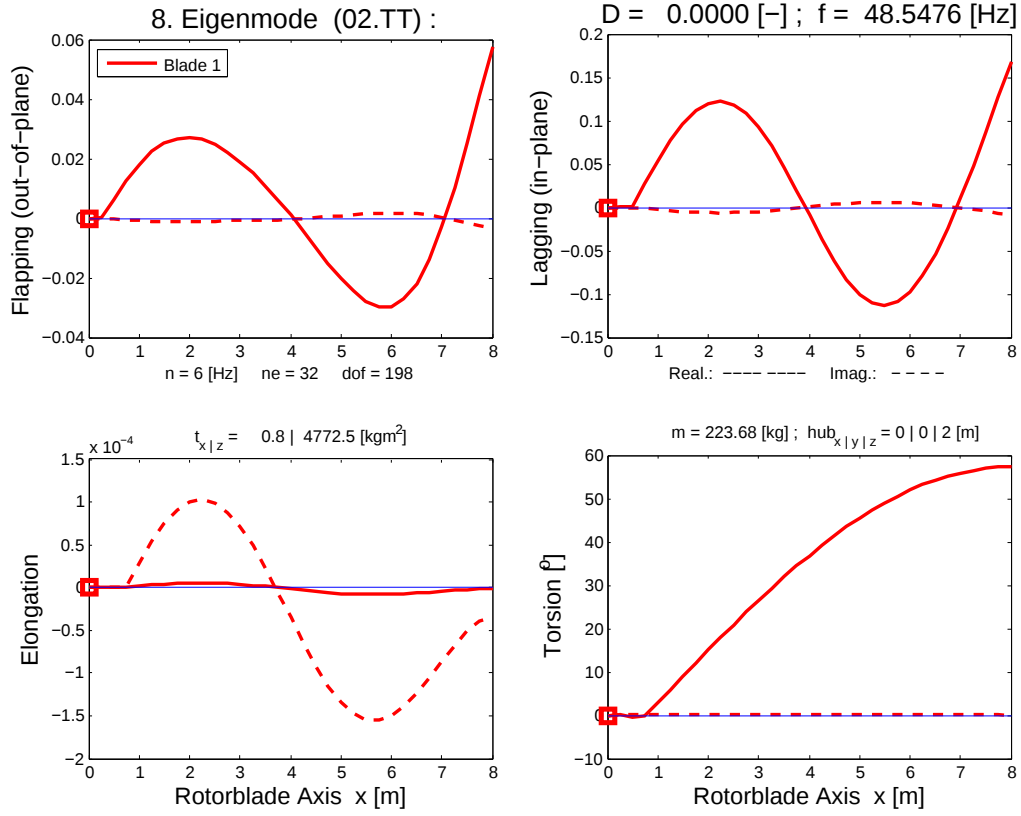
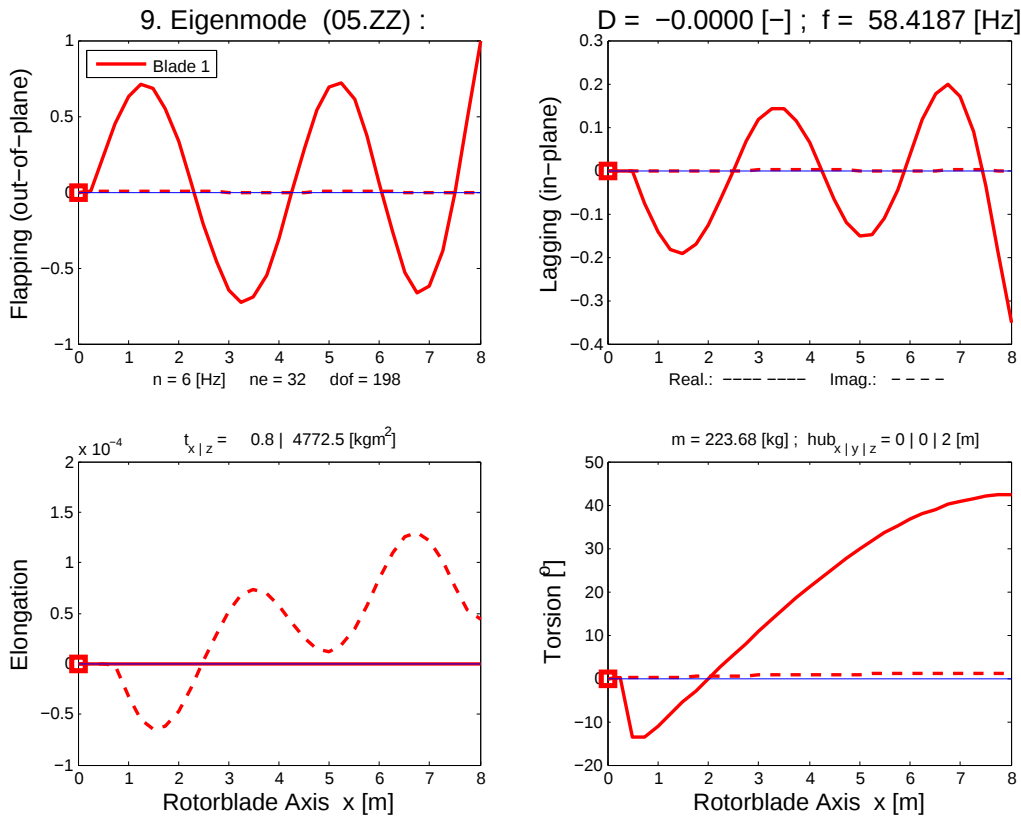


Figure 14: The 7. articulated *blade* eigenmode,  $f = 48.27$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$


 Figure 15: The 8. articulated *blade* eigenmode,  $f = 48.55 \text{ [Hz]}$ ,  $n = 6 \text{ [Hz]}$ ,  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$ 

 Figure 16: The 9. articulated *blade* eigenmode,  $f = 58.42 \text{ [Hz]}$ ,  $n = 6 \text{ [Hz]}$ ,  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

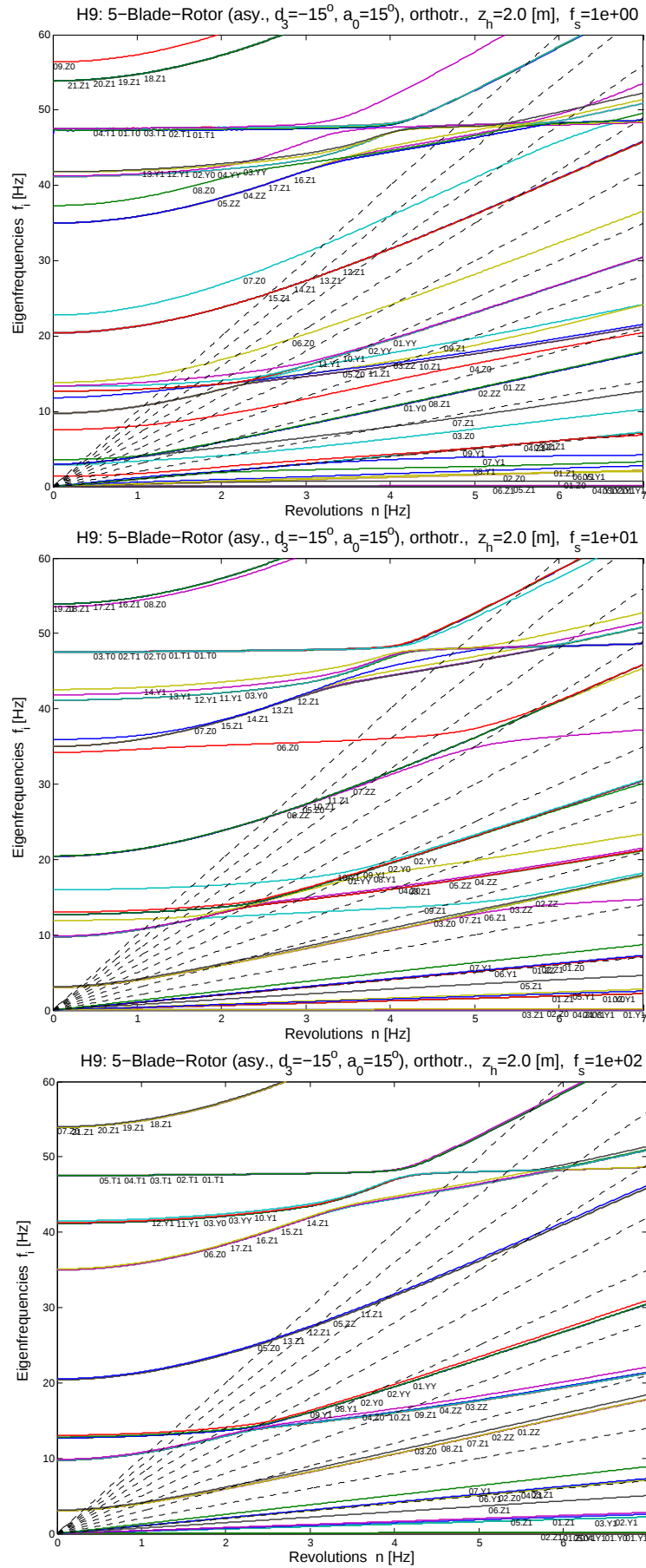


Figure 17: The H9 with articulated rotor ( $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$ ): Eigenfrequencies (60 [Hz] range) for the main rotor *soft*, *medium* and *stiff* ( $f_s = 1.$ , 10. and 100.) orthotropic suspension case (top down)

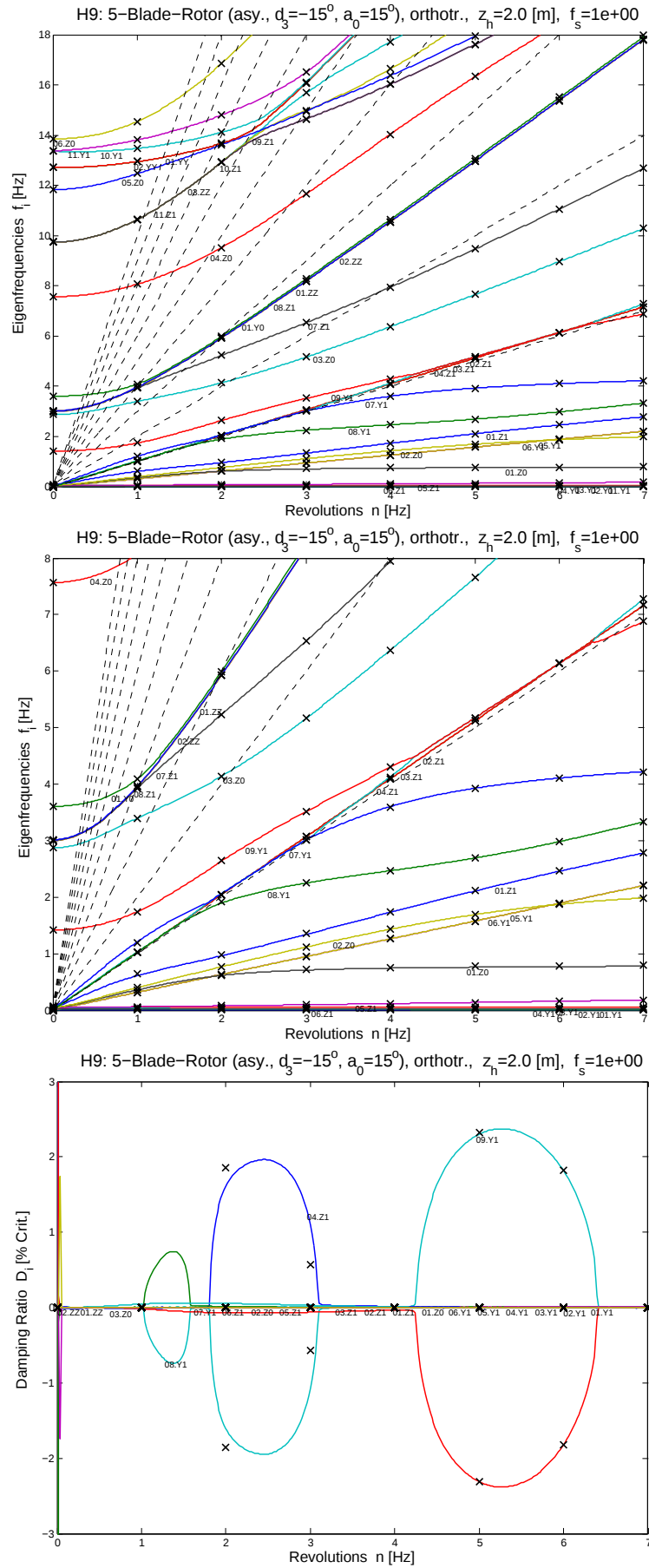


Figure 18: The H9 with articulated rotor ( $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$ ): Comparison of the *eigenfrequencies* and the *damping* for the main rotor *soft* suspension case (x = MBS SIMPACK, — = FEM GYRBLAD)

| +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+         |               |          |               |          |       |                     |          |       |  |
|-----------------------------------------------------------------------|---------------|----------|---------------|----------|-------|---------------------|----------|-------|--|
| H9 w/hinges (15/15 [o]) : c_u = 1 [Hz] n_MR = 0 [Hz] n_TR = 00.0 [Hz] |               |          |               |          |       |                     |          |       |  |
|                                                                       | GYRBLAD       |          | SIMPACT 895   |          |       | NASTRAN-SIMPACT 895 |          |       |  |
|                                                                       | FEM (32 Elm.) |          | MBS (64 Bod.) |          |       | FEMBS (120 Mod.)    |          |       |  |
|                                                                       | Nat.Dmp.      | Frequen. | Nat.Dmp.      | Frequen. | Dif.  | Nat.Dmp.            | Frequen. | Dif.  |  |
|                                                                       | [-]           | [Hz]     | [-]           | [Hz]     | [%]   | [-]                 | [Hz]     | [%]   |  |
| +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+         |               |          |               |          |       |                     |          |       |  |
| 1                                                                     | .0000         | 0.0000   | .0000         | 0.0000   | 0.00  | .0000               | 0.0000   | 0.00  |  |
| 2                                                                     | .0000         | 0.0000   | .0000         | 0.0000   | 0.00  | .0000               | 0.0000   | 0.00  |  |
| 3                                                                     | .0000         | 0.0000   | .0000         | 0.0000   | 0.00  | .0000               | 0.0000   | 0.00  |  |
| 4                                                                     | .0000         | 0.0000   | .0000         | 0.0000   | 0.00  | .0000               | 0.0000   | 0.00  |  |
| 5                                                                     | .0000         | 0.0000   | .0000         | 0.0000   | 0.00  | -.0000              | 0.0000   | 0.00  |  |
| 6                                                                     | .0000         | 0.0000   | .0000         | 0.0000   | 0.00  | .0000               | 0.0000   | 0.00  |  |
| 7                                                                     | -.0002        | 0.0001   | .0000         | 0.0000   | 0.00  | -.0000              | 0.0000   | 0.00  |  |
| 8                                                                     | .0002         | 0.0001   | .0000         | 0.0000   | 0.00  | .0000               | 0.0003   | 0.00  |  |
| 9                                                                     | .0003         | 0.0001   | .0000         | 0.0000   | 0.00  | .0000               | 0.0003   | 0.00  |  |
| 10                                                                    | -.0000        | 0.0000   | .0000         | 0.0003   | 0.00  | .0000               | 0.0003   | 0.00  |  |
| 11                                                                    | .0000         | 0.0102   | .0000         | 0.0102   | 0.00  | .0000               | 0.0102   | 0.00  |  |
| 12                                                                    | .0000         | 0.0204   | .0000         | 0.0204   | 0.00  | .0000               | 0.0204   | 0.00  |  |
| 13                                                                    | -.0000        | 0.0313   | .0000         | 0.0313   | 0.00  | .0000               | 0.0313   | 0.00  |  |
| 14                                                                    | .0000         | 0.0561   | .0000         | 0.0561   | 0.00  | .0000               | 0.0561   | 0.00  |  |
| 15                                                                    | .0000         | 0.0656   | .0000         | 0.0656   | 0.00  | .0000               | 0.0656   | 0.00  |  |
| 16                                                                    | .0000         | 0.0757   | .0000         | 0.0757   | 0.00  | .0000               | 0.0757   | 0.00  |  |
| 17                                                                    | .0000         | 1.4200   | .0000         | 1.4200   | 0.00  | .0000               | 1.4200   | 0.00  |  |
| 18                                                                    | -.0000        | 2.8692   | .0000         | 2.8691   | -0.00 | .0000               | 2.8692   | 0.00  |  |
| 19                                                                    | -.0000        | 3.0032   | .0000         | 3.0031   | -0.00 | .0000               | 3.0032   | 0.00  |  |
| 20                                                                    | .0000         | 3.0032   | .0000         | 3.0031   | -0.00 | .0000               | 3.0032   | 0.00  |  |
| 21                                                                    | .0000         | 3.0090   | .0000         | 3.0089   | -0.00 | .0000               | 3.0091   | 0.00  |  |
| 22                                                                    | -.0000        | 3.0150   | .0000         | 3.0149   | -0.00 | .0000               | 3.0151   | 0.00  |  |
| 23                                                                    | .0000         | 3.6071   | .0000         | 3.6071   | 0.00  | .0000               | 3.6072   | 0.00  |  |
| 24                                                                    | .0000         | 7.5664   | .0000         | 7.5662   | -0.00 | .0000               | 7.5664   | 0.00  |  |
| 25                                                                    | -.0000        | 9.7403   | .0000         | 9.7397   | -0.01 | .0000               | 9.7397   | -0.01 |  |
| 26                                                                    | -.0000        | 9.7403   | .0000         | 9.7398   | -0.01 | .0000               | 9.7398   | -0.01 |  |
| 27                                                                    | -.0000        | 9.7408   | .0000         | 9.7402   | -0.01 | .0000               | 9.7403   | -0.01 |  |
| 28                                                                    | .0000         | 9.7409   | .0000         | 9.7402   | -0.01 | .0000               | 9.7403   | -0.01 |  |
| 29                                                                    | -.0000        | 11.8290  | .0000         | 11.8289  | -0.00 | .0000               | 11.8285  | -0.00 |  |
| 30                                                                    | -.0000        | 12.7079  | .0000         | 12.7078  | -0.00 | .0000               | 12.7076  | -0.00 |  |
| 31                                                                    | .0000         | 12.7079  | .0000         | 12.7078  | -0.00 | .0000               | 12.7076  | -0.00 |  |
| 32                                                                    | .0000         | 13.3581  | .0000         | 13.3580  | -0.00 | .0000               | 13.3582  | 0.00  |  |
| 33                                                                    | .0000         | 13.3764  | .0000         | 13.3763  | -0.00 | .0000               | 13.3765  | 0.00  |  |
| 34                                                                    | .0000         | 13.8478  | .0000         | 13.8478  | 0.00  | .0000               | 13.8478  | 0.00  |  |
| 35                                                                    | .0000         | 20.4290  | .0000         | 20.4267  | -0.01 | .0000               | 20.4254  | -0.02 |  |
| 36                                                                    | -.0000        | 20.4290  | .0000         | 20.4267  | -0.01 | .0000               | 20.4254  | -0.02 |  |
| 37                                                                    | .0000         | 20.4313  | .0000         | 20.4290  | -0.01 | .0000               | 20.4278  | -0.02 |  |
| 38                                                                    | -.0000        | 20.4347  | .0000         | 20.4314  | -0.02 | .0000               | 20.4302  | -0.02 |  |
| 39                                                                    | .0000         | 22.7861  | .0000         | 22.7846  | -0.01 | .0000               | 22.7820  | -0.02 |  |
| 40                                                                    | -.0000        | 34.9832  | .0000         | 34.9746  | -0.02 | .0000               | 34.9683  | -0.04 |  |
| 41                                                                    | .0000         | 34.9832  | .0000         | 34.9827  | -0.00 | .0000               | 34.9765  | -0.02 |  |
| 42                                                                    | -.0000        | 35.0063  | .0000         | 35.0003  | -0.02 | .0000               | 34.9940  | -0.04 |  |
| 43                                                                    | .0000         | 35.0066  | .0000         | 35.0003  | -0.02 | .0000               | 34.9940  | -0.04 |  |
| 44                                                                    | .0000         | 37.3191  | .0000         | 37.3137  | -0.01 | .0000               | 37.3048  | -0.04 |  |
| 45                                                                    | -.0000        | 41.1755  | .0000         | 41.1741  | -0.00 | .0000               | 41.1309  | -0.11 |  |
| 46                                                                    | .0000         | 41.1755  | .0000         | 41.1741  | -0.00 | .0000               | 41.1309  | -0.11 |  |
| 47                                                                    | .0000         | 41.2728  | .0000         | 41.2712  | -0.00 | .0000               | 41.2294  | -0.11 |  |
| 48                                                                    | .0001         | 41.7895  | .0000         | 41.7918  | 0.01  | .0000               | 41.7486  | -0.10 |  |
| 49                                                                    | -.0001        | 41.7895  | .0000         | 41.7955  | 0.01  | .0000               | 41.7523  | -0.09 |  |
| 50                                                                    | -.0002        | 47.3122  | .0000         | 47.2402  | -0.15 | .0000               | 47.2463  | -0.14 |  |
| 51                                                                    | .0002         | 47.3123  | .0000         | 47.3367  | 0.05  | .0000               | 47.3429  | 0.06  |  |
| 52                                                                    | -.0000        | 47.5482  | .0000         | 47.5417  | -0.01 | .0000               | 47.5480  | -0.00 |  |
| 53                                                                    | -.0002        | 47.5666  | .0000         | 47.5417  | -0.05 | .0000               | 47.5480  | -0.04 |  |
| 54                                                                    | .0002         | 47.5671  | .0000         | 47.5652  | -0.00 | .0000               | 47.5710  | 0.01  |  |
| +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+         |               |          |               |          |       |                     |          |       |  |

Table 7: The H9 with *non-rotating* 5-bladed articulated rotor ( $n = 0$ . [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$ ; *soft orthotropic* rotor suspension) and tail rotor: *Eigenfrequencies* and *damping* according to the three methods in comparison to the FEM solution

| +-----+<br>  H9 w/hinges (15/15 [o]) : c_u = 1 [Hz] n_MR = 6 [Hz] n_TR = 34.3 [Hz]  <br>+-----+ |               |          |               |          |       |                     |          |       |
|-------------------------------------------------------------------------------------------------|---------------|----------|---------------|----------|-------|---------------------|----------|-------|
|                                                                                                 | GYRBLAD       |          | SIMPACK 895   |          |       | NASTRAN-SIMPACK 895 |          |       |
|                                                                                                 | FEM (32 Elm.) |          | MBS (64 Bod.) |          |       | FEMBS (120 Mod.)    |          |       |
|                                                                                                 | Nat.Dmp.      | Frequen. | Nat.Dmp.      | Frequen. | Dif.  | Nat.Dmp.            | Frequen. | Dif.  |
|                                                                                                 | [-]           | [Hz]     | [-]           | [Hz]     | [%]   | [-]                 | [Hz]     | [%]   |
| 1                                                                                               | .0000         | 0.0083   | -.0003        | 0.0083   | 0.00  | -.0021              | 0.0083   | 0.00  |
| 2                                                                                               | .0001         | 0.0122   | -.0036        | 0.0122   | 0.00  | -.0258              | 0.0123   | 0.82  |
| 3                                                                                               | -.0000        | 0.0169   | .0010         | 0.0169   | 0.00  | .0070               | 0.0169   | 0.00  |
| 4                                                                                               | -.0000        | 0.0300   | .0000         | 0.0300   | 0.00  | .0000               | 0.0300   | 0.00  |
| 5                                                                                               | .0000         | 0.0498   | .0001         | 0.0498   | 0.00  | .0010               | 0.0498   | 0.00  |
| 6                                                                                               | .0001         | 0.1589   | -.0018        | 0.1556   | -2.08 | -.0131              | 0.1551   | -2.39 |
| 7                                                                                               | -.0000        | 0.7700   | .0001         | 0.7855   | 2.01  | .0009               | 0.7854   | 2.00  |
| 8                                                                                               | .0000         | 1.8735   | .0001         | 1.8729   | -0.03 | .0005               | 1.8729   | -0.03 |
| 9                                                                                               | .0000         | 1.8919   | .0000         | 1.8906   | -0.07 | .0000               | 1.8907   | -0.06 |
| 10                                                                                              | .0000         | 1.8919   | .0000         | 1.8906   | -0.07 | .0000               | 1.8907   | -0.06 |
| 11                                                                                              | -.0000        | 2.4590   | .0000         | 2.4596   | 0.02  | .0001               | 2.4597   | 0.03  |
| 12                                                                                              | .0000         | 2.9834   | .0000         | 2.9855   | 0.07  | .0000               | 2.9857   | 0.08  |
| 13                                                                                              | .0000         | 4.1079   | .0000         | 4.1054   | -0.06 | .0001               | 4.1059   | -0.05 |
| 14                                                                                              | -.0182        | 6.1365   | -.0182        | 6.1356   | -0.01 | -.0182              | 6.1353   | -0.02 |
| 15                                                                                              | .0182         | 6.1368   | .0182         | 6.1358   | -0.02 | .0182               | 6.1356   | -0.02 |
| 16                                                                                              | .0000         | 6.1432   | .0000         | 6.1429   | -0.00 | .0000               | 6.1425   | -0.01 |
| 17                                                                                              | -.0000        | 6.1432   | .0000         | 6.1429   | -0.00 | .0000               | 6.1425   | -0.01 |
| 18                                                                                              | -.0001        | 8.9680   | .0000         | 8.9659   | -0.02 | .0000               | 8.9678   | -0.00 |
| 19                                                                                              | .0000         | 11.0595  | .0000         | 11.0559  | -0.03 | .0000               | 11.0577  | -0.02 |
| 20                                                                                              | .0000         | 15.3880  | .0000         | 15.3609  | -0.18 | .0001               | 15.3728  | -0.10 |
| 21                                                                                              | -.0000        | 15.3880  | .0000         | 15.3797  | -0.05 | .0001               | 15.3924  | 0.03  |
| 22                                                                                              | .0000         | 15.3972  | .0000         | 15.3797  | -0.11 | .0001               | 15.3924  | -0.03 |
| 23                                                                                              | .0002         | 15.4921  | .0000         | 15.5130  | 0.13  | .0001               | 15.5262  | 0.22  |
| 24                                                                                              | -.0000        | 18.4544  | .0000         | 18.4489  | -0.03 | .0000               | 18.4617  | 0.04  |
| 25                                                                                              | -.0000        | 19.3823  | .0000         | 19.3791  | -0.02 | .0000               | 19.3933  | 0.06  |
| 26                                                                                              | -.0000        | 19.3823  | .0000         | 19.3791  | -0.02 | .0000               | 19.3933  | 0.06  |
| 27                                                                                              | -.0000        | 19.7113  | .0000         | 19.7033  | -0.04 | .0000               | 19.7182  | 0.04  |
| 28                                                                                              | -.0000        | 21.3381  | .0000         | 21.3286  | -0.04 | .0000               | 21.3416  | 0.02  |
| 29                                                                                              | -.0001        | 21.9255  | .0000         | 21.9271  | 0.01  | .0000               | 21.9428  | 0.08  |
| 30                                                                                              | .0000         | 26.7588  | .0000         | 26.7111  | -0.18 | .0001               | 26.7038  | -0.21 |
| 31                                                                                              | .0000         | 26.7588  | .0000         | 26.7492  | -0.04 | .0001               | 26.7421  | -0.06 |
| 32                                                                                              | .0000         | 26.7589  | .0000         | 26.7492  | -0.04 | .0001               | 26.7421  | -0.06 |
| 33                                                                                              | .0001         | 26.8183  | .0000         | 26.8467  | 0.11  | .0001               | 26.8399  | 0.08  |
| 34                                                                                              | -.0000        | 32.3714  | .0000         | 32.3659  | -0.02 | .0000               | 32.3613  | -0.03 |
| 35                                                                                              | -.0000        | 40.9115  | .0000         | 40.9038  | -0.02 | .0009               | 40.9268  | 0.04  |
| 36                                                                                              | -.0000        | 40.9419  | .0000         | 40.9634  | 0.05  | .0010               | 40.9905  | 0.12  |
| 37                                                                                              | .0000         | 40.9929  | .0000         | 41.0005  | 0.02  | .0010               | 41.0260  | 0.08  |
| 38                                                                                              | -.0000        | 40.9929  | .0000         | 41.0005  | 0.02  | .0010               | 41.0260  | 0.08  |
| 39                                                                                              | .0000         | 45.8160  | .0000         | 45.8234  | 0.02  | .0015               | 45.8365  | 0.04  |
| 40                                                                                              | -.0001        | 48.0090  | .0000         | 47.9710  | -0.08 | .0001               | 47.6767  | -0.69 |
| 41                                                                                              | -.0000        | 48.1307  | .0000         | 48.1144  | -0.03 | .0001               | 47.8168  | -0.65 |
| 42                                                                                              | -.0000        | 48.2322  | .0000         | 48.2090  | -0.05 | .0009               | 47.9413  | -0.60 |
| 43                                                                                              | -.0000        | 48.2731  | .0000         | 48.2504  | -0.05 | .0003               | 47.9829  | -0.60 |
| 44                                                                                              | .0000         | 48.2737  | .0000         | 48.2504  | -0.05 | .0003               | 47.9829  | -0.60 |
| 45                                                                                              | -.0000        | 48.5474  | .0000         | 48.5460  | -0.00 | -.0002              | 48.5116  | -0.07 |
| 46                                                                                              | .0000         | 48.5474  | .0000         | 48.5460  | -0.00 | -.0002              | 48.5116  | -0.07 |
| 47                                                                                              | -.0000        | 49.0174  | .0000         | 49.0089  | -0.02 | -.0001              | 49.0004  | -0.03 |
| 48                                                                                              | .0000         | 49.4532  | .0000         | 49.4580  | 0.01  | -.0009              | 49.4353  | -0.04 |
| 49                                                                                              | -.0000        | 49.7221  | .0000         | 49.7312  | 0.02  | .0000               | 49.7208  | -0.00 |
| 50                                                                                              | -.0000        | 58.2981  | .0000         | 58.3301  | 0.05  | -.0003              | 58.2650  | -0.06 |
| 51                                                                                              | -.0000        | 58.3651  | .0000         | 58.4123  | 0.08  | -.0003              | 58.3494  | -0.03 |
| 52                                                                                              | .0000         | 58.4187  | .0000         | 58.4584  | 0.07  | -.0004              | 58.3936  | -0.04 |
| 53                                                                                              | -.0000        | 58.4187  | .0000         | 58.4584  | 0.07  | -.0004              | 58.3936  | -0.04 |
| 54                                                                                              | -.0000        | 64.1693  | .0000         | 64.2140  | 0.07  | -.0002              | 64.1455  | -0.04 |
| +-----+                                                                                         |               |          |               |          |       |                     |          |       |

Table 8: The H9 with *rotating* 5-bladed articulated rotor ( $n = 6.0$  [Hz],  $\delta_3 = -15.^\circ$ ,  $\alpha_0 = 15.^\circ$ ; *soft orthotropic* rotor suspension) and tail rotor: *Eigenfrequencies* and *damping* according to the three methods in comparison to the FEM solution

| +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+                 |                |          |       |               |          |       |              |          |       |
|-------------------------------------------------------------------------------|----------------|----------|-------|---------------|----------|-------|--------------|----------|-------|
| H9 w/hinges (15/15 [o]) :    z_H = 2 [m]    n_MR = 6 [Hz]    n_TR = 34.3 [Hz] |                |          |       |               |          |       |              |          |       |
| GYR-<br>BLAD<br>FEM                                                           | c_u = 100 [Hz] |          |       | c_u = 10 [Hz] |          |       | c_u = 1 [Hz] |          |       |
|                                                                               | Nat.Dmp.       | Frequen. | Dif.  | Nat.Dmp.      | Frequen. | Dif.  | Nat.Dmp.     | Frequen. | Dif.  |
|                                                                               | [-]            | [Hz]     | [%]   | [-]           | [Hz]     | [%]   | [-]          | [Hz]     | [%]   |
| +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+                 |                |          |       |               |          |       |              |          |       |
| 1                                                                             | -.0000         | 0.0083   | 0.00  | .0000         | 0.0083   | 0.00  | .0000        | 0.0083   | 0.00  |
| 2                                                                             | .0001          | 0.0122   | 0.00  | .0001         | 0.0122   | 0.00  | .0001        | 0.0122   | 0.00  |
| 3                                                                             | .0000          | 0.0169   | 0.00  | .0000         | 0.0169   | 0.00  | -.0000       | 0.0169   | 0.00  |
| 4                                                                             | .0000          | 0.0300   | 0.00  | .0000         | 0.0300   | 0.00  | -.0000       | 0.0300   | 0.00  |
| 5                                                                             | .0000          | 0.0498   | 0.60  | .0000         | 0.0498   | 0.60  | .0000        | 0.0498   | 0.60  |
| 6                                                                             | .0001          | 0.1620   | -0.49 | .0001         | 0.1620   | -0.49 | .0001        | 0.1589   | -0.06 |
| 7                                                                             | -.0000         | 1.8919   | 0.00  | -.0000        | 1.8919   | 0.00  | -.0000       | 0.7700   | -0.04 |
| 8                                                                             | -.0000         | 1.8919   | 0.00  | -.0000        | 1.8919   | 0.00  | .0000        | 1.8735   | 0.03  |
| 9                                                                             | .0000          | 2.2027   | 0.19  | .0000         | 2.1854   | 0.17  | .0000        | 1.8919   | 0.00  |
| 10                                                                            | -.0000         | 2.4101   | -0.48 | -.0000        | 2.3976   | -0.48 | .0000        | 1.8919   | 0.00  |
| 11                                                                            | .0000          | 4.3301   | 0.37  | .0000         | 4.0475   | 0.42  | -.0000       | 2.4590   | -0.10 |
| 12                                                                            | -.0000         | 6.1188   | -0.00 | .0000         | 6.1264   | 0.00  | .0000        | 2.9834   | -0.02 |
| 13                                                                            | -.0000         | 6.1432   | 0.00  | -.0000        | 6.1432   | 0.00  | .0000        | 4.1079   | -0.08 |
| 14                                                                            | .0000          | 6.1432   | 0.00  | .0000         | 6.1432   | 0.00  | -.0182       | 6.1365   | -0.01 |
| 15                                                                            | -.0001         | 6.2760   | 0.00  | -.0001        | 6.2474   | 0.00  | .0182        | 6.1368   | -0.01 |
| 16                                                                            | -.0002         | 7.6319   | 0.08  | -.0002        | 7.5106   | 0.09  | .0000        | 6.1432   | 0.00  |
| 17                                                                            | -.0000         | 15.3880  | 0.00  | .0002         | 14.0597  | -0.02 | -.0000       | 6.1432   | 0.00  |
| 18                                                                            | .0000          | 15.3880  | 0.00  | -.0000        | 15.3880  | 0.00  | -.0001       | 8.9680   | 0.00  |
| 19                                                                            | .0000          | 15.3919  | 0.00  | .0000         | 15.3880  | 0.00  | .0000        | 11.0595  | -0.00 |
| 20                                                                            | .0005          | 15.4212  | 0.00  | .0003         | 15.3950  | 0.00  | .0000        | 15.3880  | 0.00  |
| 21                                                                            | .0005          | 15.9341  | 0.00  | .0004         | 15.5656  | 0.00  | -.0000       | 15.3880  | 0.00  |
| 22                                                                            | -.0000         | 19.3823  | 0.00  | .0001         | 15.9498  | -0.01 | .0000        | 15.3972  | 0.00  |
| 23                                                                            | .0000          | 19.3823  | 0.00  | -.0001        | 19.3253  | 0.00  | .0002        | 15.4921  | 0.00  |
| 24                                                                            | .0000          | 19.4082  | -0.00 | -.0000        | 19.3823  | 0.00  | -.0000       | 18.4544  | 0.00  |
| 25                                                                            | -.0002         | 19.5247  | -0.00 | .0000         | 19.3823  | 0.00  | -.0000       | 19.3823  | 0.00  |
| 26                                                                            | -.0007         | 20.1351  | 0.01  | -.0004        | 19.6373  | 0.00  | -.0000       | 19.3823  | 0.00  |
| 27                                                                            | -.0000         | 26.7588  | 0.00  | -.0003        | 21.5843  | -0.01 | -.0000       | 19.7113  | 0.00  |
| 28                                                                            | .0000          | 26.7588  | 0.00  | -.0001        | 26.6142  | 0.00  | -.0000       | 21.3381  | 0.00  |
| 29                                                                            | .0000          | 26.7620  | 0.00  | -.0000        | 26.7588  | 0.00  | -.0001       | 21.9255  | 0.00  |
| 30                                                                            | .0001          | 26.7791  | 0.00  | .0000         | 26.7588  | 0.00  | .0000        | 26.7588  | 0.00  |
| 31                                                                            | .0001          | 27.1972  | 0.00  | .0001         | 26.7835  | 0.00  | .0000        | 26.7588  | 0.00  |
| 32                                                                            | -.0000         | 40.9929  | 0.00  | .0004         | 26.9083  | -0.00 | .0000        | 26.7589  | -0.00 |
| 33                                                                            | .0000          | 40.9929  | 0.00  | -.0001        | 36.3373  | 0.00  | .0001        | 26.8183  | 0.00  |
| 34                                                                            | .0000          | 40.9936  | 0.00  | .0002         | 40.9087  | 0.00  | -.0000       | 32.3714  | 0.00  |
| 35                                                                            | .0000          | 40.9971  | -0.00 | -.0000        | 40.9929  | 0.00  | -.0000       | 40.9115  | 0.00  |
| 36                                                                            | .0000          | 41.3147  | 0.00  | -.0000        | 40.9929  | 0.00  | -.0000       | 40.9419  | 0.00  |
| 37                                                                            | -.0000         | 48.2736  | 0.00  | .0000         | 41.0028  | 0.00  | .0000        | 40.9929  | 0.00  |
| 38                                                                            | -.0000         | 48.2738  | -0.00 | -.0002        | 41.1119  | -0.00 | -.0000       | 40.9929  | 0.00  |
| 39                                                                            | -.0000         | 48.2751  | -0.00 | .0000         | 48.2704  | -0.00 | .0000        | 45.8160  | 0.00  |
| 40                                                                            | -.0000         | 48.2938  | -0.00 | -.0000        | 48.2737  | 0.00  | -.0001       | 48.0090  | -0.00 |
| 41                                                                            | .0000          | 48.3154  | -0.00 | .0000         | 48.2737  | 0.00  | -.0000       | 48.1307  | -0.00 |
| 42                                                                            | -.0000         | 48.5474  | 0.00  | -.0000        | 48.3083  | 0.00  | -.0000       | 48.2322  | 0.00  |
| 43                                                                            | .0000          | 48.5474  | 0.00  | .0000         | 48.3168  | 0.00  | -.0000       | 48.2731  | 0.00  |
| 44                                                                            | -.0000         | 48.5535  | 0.00  | -.0000        | 48.4798  | 0.00  | .0000        | 48.2737  | 0.00  |
| 45                                                                            | -.0000         | 48.6437  | 0.00  | .0000         | 48.5474  | 0.00  | -.0000       | 48.5474  | 0.00  |
| 46                                                                            | -.0001         | 48.9561  | 0.00  | .0000         | 48.5475  | -0.00 | .0000        | 48.5474  | 0.00  |
| 47                                                                            | .0000          | 58.4171  | 0.00  | .0000         | 49.1330  | 0.00  | -.0000       | 49.0174  | 0.00  |
| 48                                                                            | -.0000         | 58.4187  | 0.00  | .0000         | 50.2301  | 0.00  | .0000        | 49.4532  | 0.00  |
| 49                                                                            | .0000          | 58.4187  | 0.00  | .0000         | 57.4656  | 0.00  | -.0000       | 49.7221  | 0.00  |
| 50                                                                            | .0000          | 58.4308  | 0.00  | -.0000        | 58.4187  | 0.00  | -.0000       | 58.2981  | 0.00  |
| 51                                                                            | .0000          | 58.6686  | 0.00  | .0000         | 58.4187  | 0.00  | -.0000       | 58.3651  | -0.00 |
| 52                                                                            | -.0000         | 78.8713  | -0.00 | -.0000        | 58.4419  | 0.00  | .0000        | 58.4187  | 0.00  |
| 53                                                                            | -.0000         | 78.8723  | 0.00  | -.0000        | 58.4754  | 0.00  | -.0000       | 58.4187  | 0.00  |
| 54                                                                            | .0000          | 78.8723  | 0.00  | -.0000        | 76.6821  | 0.00  | -.0000       | 64.1693  | 0.00  |
| +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+                 |                |          |       |               |          |       |              |          |       |

Table 9: The H9 with *rotating* 5-bladed articulated rotor ( $n = 6.0$  [Hz],  $\delta_3 = -15.^\circ$ ,  $\alpha_0 = 15.^\circ$ ) and tail rotor: *Eigenfrequencies* and *damping* w.r.t. the rotor suspension stiffness (*stiff*, *medium* and *soft orthotropic*) and the respective difference to the zero tail rotor rotation solution

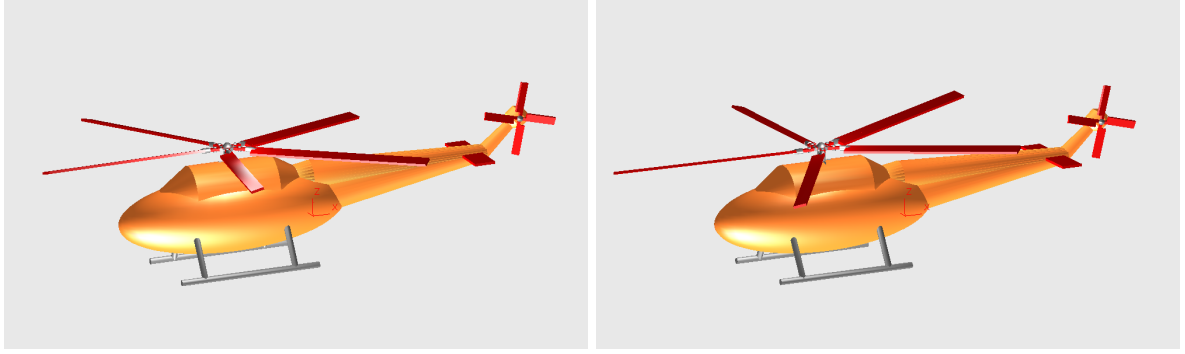


Figure 19: The 10. articulated *rotor* eigenmode,  $f = 2.3928$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

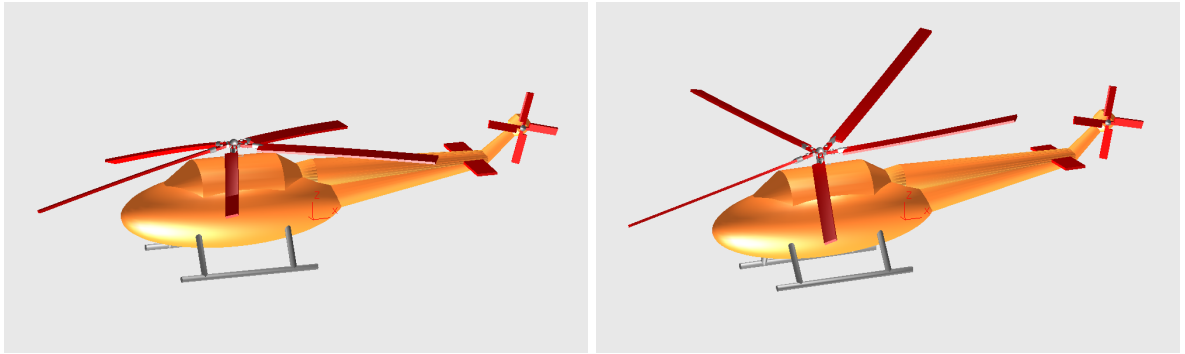


Figure 20: The 16. articulated *rotor* eigenmode,  $f = 7.5785$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

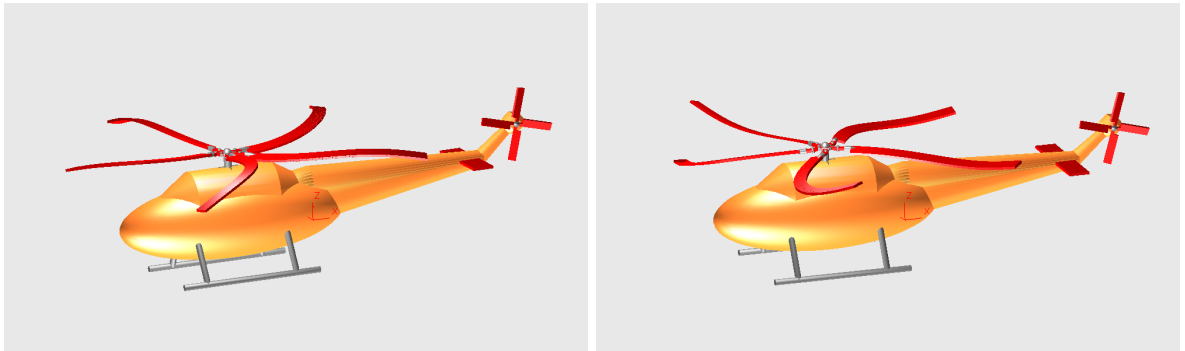


Figure 21: The 26. articulated *rotor* eigenmode,  $f = 20.1356$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

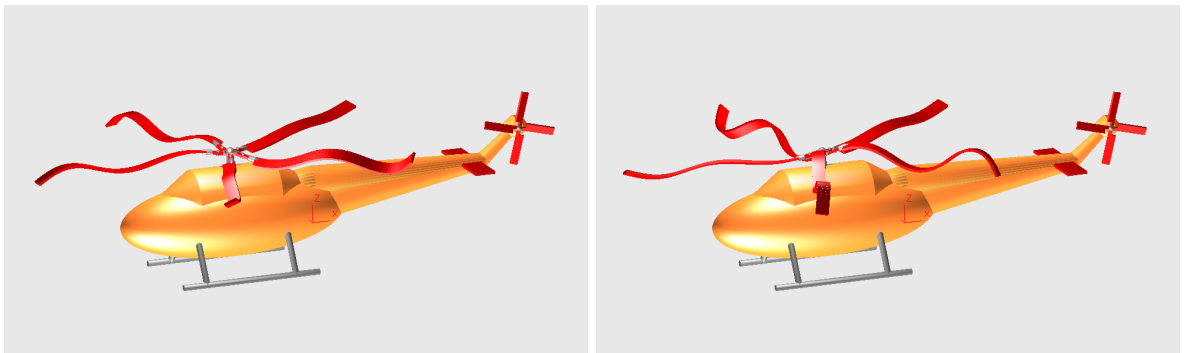
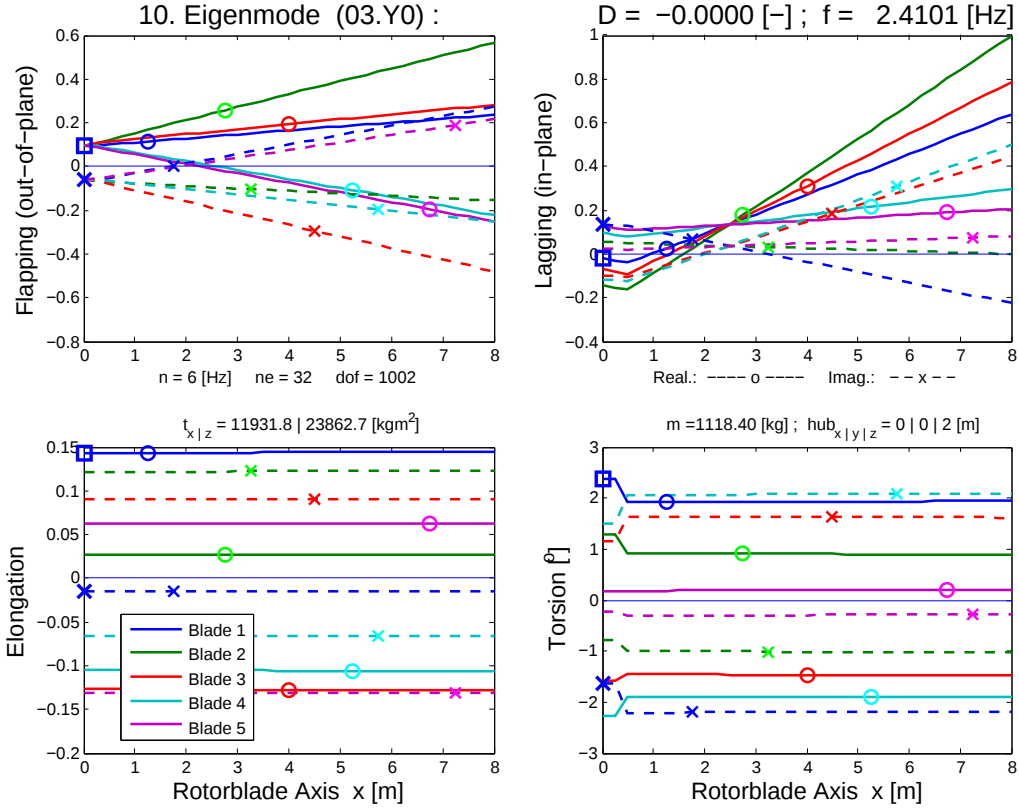
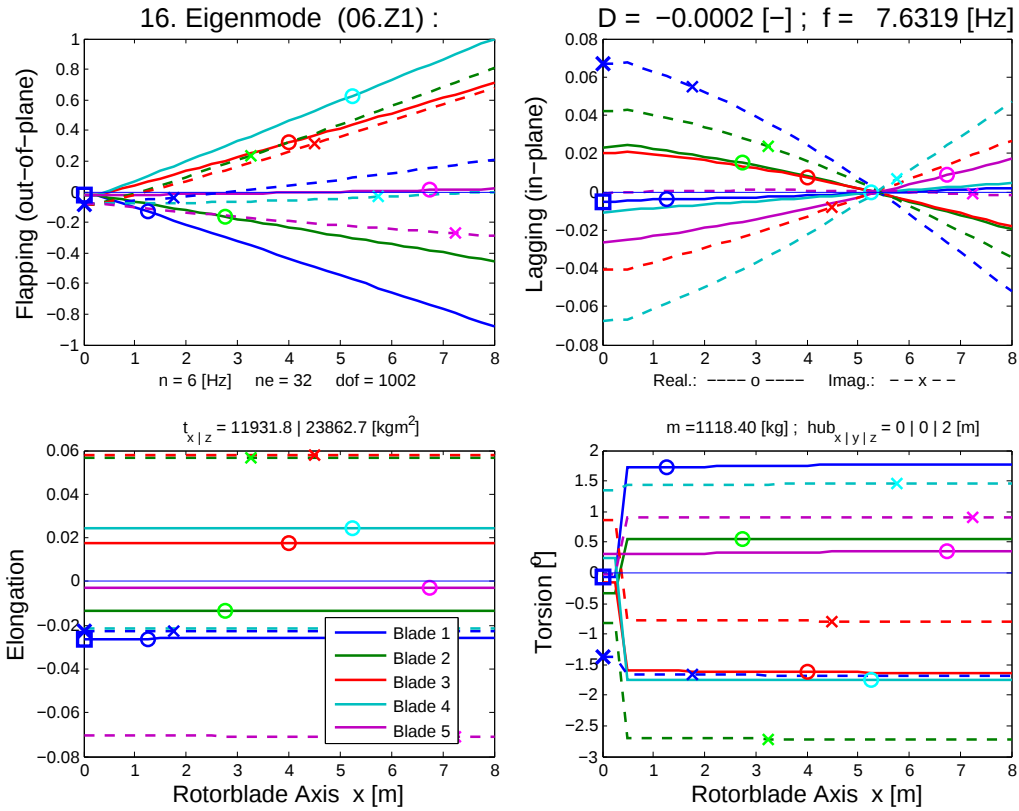


Figure 22: The 34. articulated *rotor* eigenmode,  $f = 41.0012$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

Figure 23: The 10. articulated rotor eigenmode,  $f = 2.4101$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$ Figure 24: The 16. articulated rotor eigenmode,  $f = 7.6319$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

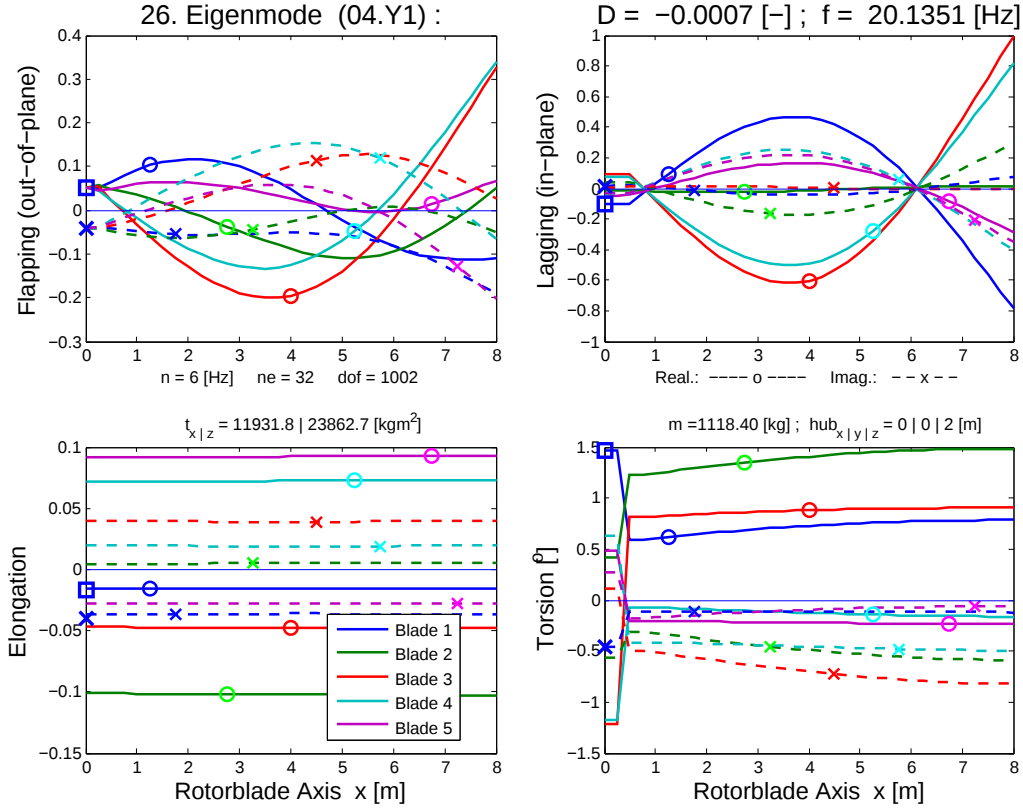


Figure 25: The 26. articulated rotor eigenmode,  $f = 20.1351$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$

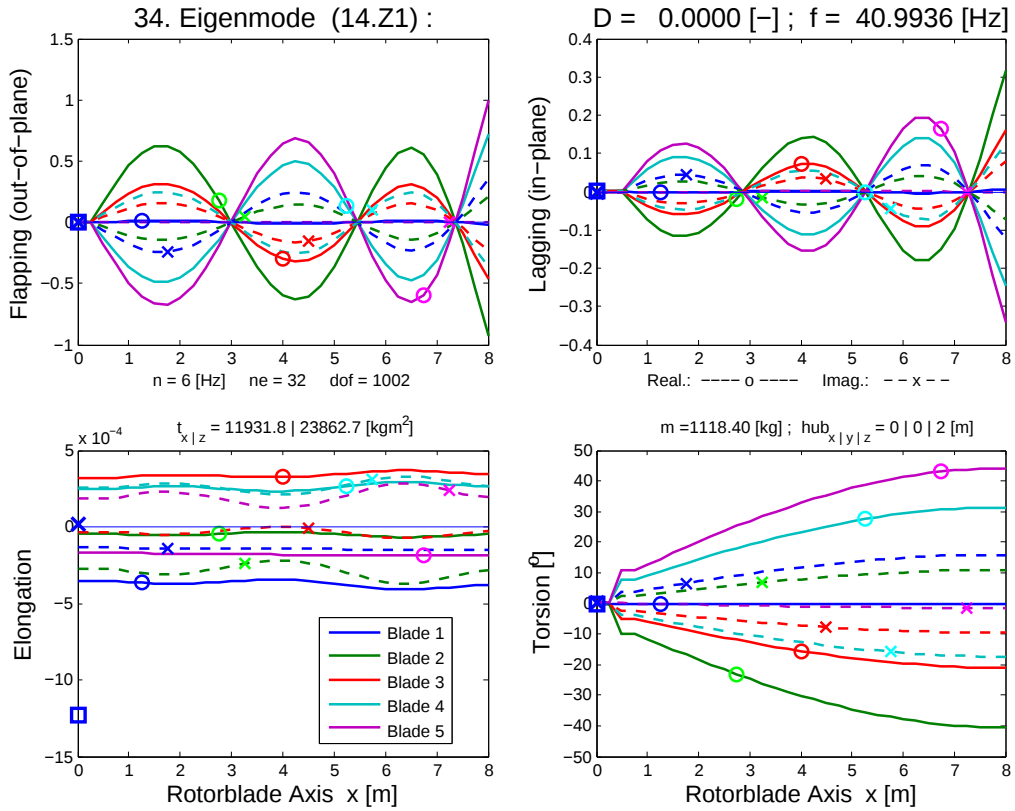


Figure 26: The 34. articulated rotor eigenmode,  $f = 40.9936$  [Hz],  $n = 6$  [Hz],  $\delta_3 = -15^\circ$ ,  $\alpha_0 = 15^\circ$