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APPROXIMATED METHODS OF HELICOPTER CONTROL OPTIMIZATION

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Abstract

An approximate approach is proposed for investigations of optimal control of the helicopter as a complex object not having a complete analytical description, to use efficient methods and procedures accounting for specific features of applied problems such as degeneration and turnpike nature of their solutions. Approximation of multidimensional numerical data arrays by analytical structures of various complexity and accuracy is envisaged and a search for a rough global solution on their basis with subsequent iterative refinement is made. Investigation of safe non-standard landing maneuvers performed by a real helicopter with determination of a safe zone is presented as an illustrative example.

1. Introduction

Many problems originating in investigation of helicopter flight performance both at helicopter design and operation stages are essentially dynamic problems of optimization connected with selection of optimal maneuvers and corresponding control laws in normal and emergency conditions. Typical criteria of optimality are minimum time values of maneuver completion, take-off and landing distances or typical sizes of dangerous zones, maximum values of flight range, load lifting capability, attainable altitude including all multiple design and operation restrictions [1-3].

In Kamov practice, just as in practice of any other helicopter manufacturer, they use helicopter flight dynamics analysis methods of various complexity levels of applied algorithms for solutions of practical problems. Specific nature of the flight dynamics is investigated using mathematic and semi-full size models. Methods involving nonlinear motion equations with pilot work models have been widely used of late. Possibility of realizing in actual flight the control laws obtained through crude modeling is evaluated by its analysis accounting for control margins and observation of prescribed restrictions.

Application of the above listed dynamic processes investigation methods allows one to solve a lot of problems. But on the other hand, complexity of a rotary wing aircraft as a control object very much complicates the solution process tasks and compels to look for approaches which simplify the helicopter motion equations and methods of their investigation.

One of such approaches is a well known in aircraft aerodynamics energy method based on using a helicopter energy equation that originated in the time when limited computational capabilities did not allow to solve differential motion equations in a wide range of altitude/climatic conditions and aircraft weights even in a simplified form, but it is still used in practice when motion trajectories can be divided in quasi-steady motion segments [4,5]. The corresponding algorithms are implemented in working programs.

However, this method is absolutely inapplicable for investigations of essentially unsteady motions like complex spatial high-response maneuvers or landing OEI or engine failures at low altitudes (rejected takeoff) (fig.1).

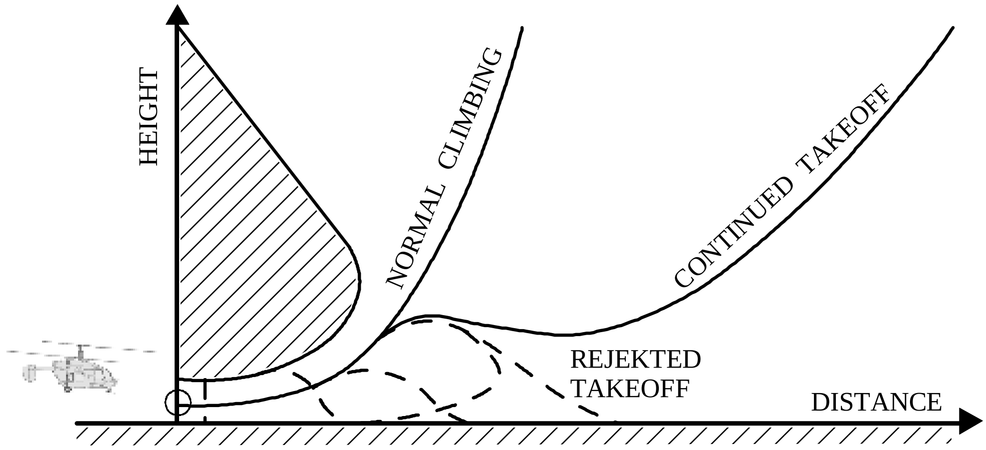


Fig. 1.

A certain positive experience has been accumulated in application of modern optimal control methods for solution of such problems [6-10]. Thus, in [6] a piloting method optimization for Mi-4 type helicopter take-off with increased payload using an iteration method of gradient type resulted in considerable shortening of a take-off distance. In [8-10] similar methods were used to find a series of optimal planar and spatial maneuvers for coaxial helicopters. Wide application of this method is hindered, in our opinion, by a complex and implicit nature of motion models used at present, sometimes in the form of computer programs including a considerable number of empirical dependencies.

Successful solution of the above mentioned practical problems mainly depended upon simplification of motion equation systems that allowed one within an acceptable time period to find approximately optimal control parameters and trajectories even but in any case better than the traditional ones. In [8] a more complicated multistage procedure of successive refinement was applied using ordered series of object models from simpler but crude to more accurate but complicated.

The purpose if this work consisted is to systematize this approximated approach in order to extend the usage of various highly efficient optimal control methods and procedures well proven when applied to solution of aircraft dynamic problems accounting for their specific features like degeneracy and turnpike nature of solutions [11-13], and to demonstrate them on an example of safe landing maneuver investigations.

2. General approach

The proposed approach consists of the following stages:

1) setting of the practical problem;

2) choice an appropriate dynamic model and formulation the mathematical variation problem;

3) analytical approximation of the dynamic model;

4) approximate qualitative analysis using simplifying assumptions to obtain initial approximation of the global solution;

5) iterative refinement of the initial approximation using a suitable method.

The first two stages are preparatory steps usual for investigation of any applied problem by mathematical tools and do not need any special clarifications. Consider the third stage in more detail.

An analytical presentation of motion equations (their rights parts), even if approximate, is required for realization of main stages 4) and 5) of the problem solution while, as a rule, for the discussed class of objects they are in general set up only algorithmically. For this purpose at stage 3) a procedure is proposed that is analogous to statistical patterns for processing empirical data.

It is assumed that a motion model in general is presented by a controlled system of differential equations in a normal form

(1) $$\frac{dx}{dt} = f(t,x,u), \; (\frac{dx^i}{dt} = f^i(t,x,u))$$

and additional restrictions

$$x \in \mathbf{X}(t) = \{x : x^i(t,x) \le 0\} \subset \mathbf{R}^n, \qquad u \in \mathbf{U}(t,x) = \{u : n^k(t,x) \le 0\} \subset \mathbf{R}^p,$$

with at least a program-algorithmic representation available (computer calculation programs) for the equation right parts (1) and the left parts of inequalities as functions of many variables $f^i(t,x,u),\; x^j(t,x),\; n^k(t,x)$. Using existing programs, tables of their values on nodal grids generated by combinations of each argument values uniformly distributed in its operating range are calculated. The following polynomials are considered as approximating structures: $\sum_a y_a(t) q_a(y),$ for example, algorithmically convenient compositions of one-dimensional polynomials like

2) $$\sum_{j_0=1}^{m_1} (y_1)^{j^1} \left(\sum_{j_2=0}^{m_2} (y_2)^{j^2} \left(\mathbf{L} \sum_{j_r=0}^{m_r} y_{j_1, j_2 \mathbf{L} j_r} (y_r)^{j_r} \right) \right),$$

where $y = (t,x,u),\; r = 1 + n + p.$ Approximation is done using the least-squares method

(3) $$\min_{[y_a]} \sum_b \left([g_a(y_b)][y_a] - f(y_b) \right)^2 \to \min_{\{y_a\}},$$

where $f(y_b)$ - approximated function values in selected nodes.

For the structures under consideration the method is reduced to solution of a linear equation system with respect to coefficients $y_a (y_{j_1, j_2, \ldots, j_r}).$ For the unique of the system each value m_{k} should not the number of nodal values of the corresponding variable y^{k} (in case equality polynomial (2) becomes interpolation one). In order to select a suitable approximation structure from this class a preliminary analysis of table value sensitivity to argument variations in the working range (practically by building of graph families describing dependencies on this or that variable under various typical combinations of other variables) is performed.

It is useful to construct not one but several different approximations. Part of them can be selected in a class allowing for efficient analysis but not providing for high accuracy. For example, it may be a class of linear structures since a much advanced general theory of control linear systems is available, or a class of structures linear with respect to all or part of controls because the corresponding control systems of the type

(4) $$\frac{dx}{dt} = g(t,x,u_1) + k(t,x)u_2$$

allow the transformed to derived systems of a lower order, system decomposition and application of specific effective methods of degenerate problem solution [9, 10].

On the contrary, the other part is aimed to the most accurate object description for the application at the stage 5) in iterative refinement universal methods independent of the model specific nature. Though this stage can be in principal performed using model computer presentation, its analytical presentation has the advantage of independency from the computational system in which the object model is realized, and allows one to implement it in the same system where its refinement algorithm is realized. The final selection of approximating construction can be made based on a natural criterion (3) of the mean square deviation from table values.

Note that as a rule only part of dependencies contained in expressions of the system right parts (1) is described algorithmically, not analytically. Then one can approximating only those very dependencies. Their corresponding tables can apparently be obtained by simple recalculation of the corresponding right parts.

At stage 4), besides using special rough approximations, possible idealizing and simplifying assumptions are adopted based on the content of the dependencies under consideration in order to obtain a simpler system (4). Then the transformation to a derived is made (perhaps multiple one, up to the first order), and thus an approximate decomposition (4) to separate systems and, probably, to separate loosely interconnected equations is performed.

This decomposed system can be investigated in order to evaluate initial system attainability domains, to find approximate turnpike solutions of the original problem [11], with their approximation by the original system solutions serving as initial approximations for the next stage 5). At this stage it is proposed to use iterative methods of control successive improvement similar to [12], applicable directly to both analytical and algorithmic presentations of the object model. The methods are based mainly on approximation of generalized Krotov Lagrangian in the vicinity of the current approximate trajectory. They are described in detail together with the corresponding algorithms in [13].

3. Emergency landing dangerous zone lower boundary problem

Helicopter motion in the vertical plane described by the following equations is examined:

(5)
$$\frac{dx^1}{dt} = f^1(x^1,x^2,x^3,u^1,u^2) = \frac{1}{m}(-X_{BP}\cos q - T\sin u^1),$$
$$\frac{dx^2}{dt} = f^2(x^1,x^2,x^3,u^1,u^2) = \frac{1}{m}(-X_{BP}\sin q + T\cos u^1 - G),$$
$$\frac{dx^3}{dt} = f^3(x^1,x^2,x^3,u^1,u^2,N) = f^3(x^1,x^2,x^3,u^1,u^2,\tilde{N}) + \frac{P}{x^3}(N-\tilde{N}),$$
$$\frac{dx^4}{dt} = f^4(x^2) = x^2,$$

where $x^1, x^2,$ – horizontal and vertical speed vector component,

x^3 – main rotor angular velocity, x^4 – geometrical height, u^1 – thrust vector defection angle from vertical, u^2 – main rotor collective pitch,

$$X_{BP} = Q((x^1)^2 + (x^2)^2), \quad T = F_T(x^3 R)^2, \quad q = arctg(x^2/x^1),$$

N – engine available power (considered to be an external effect in emergency situation),

$\tilde{N}$ - engine require power, m, G, P, Q and R – constants (m and G – correspondingly mass and weight of the helicopter, R - helicopter rotor radius). Dependences

$F_T(x^1, x^2, x^3, u^1, u^2)$, $f^3(x^1, x^2, x^3, u^1, u^2, \tilde{N})$ are calculated beforehand and set up as data arrays for a specific helicopter along with constant parameters.

Initial values of state variables, control limits and state variable values at the end of the maneuver are assigned as:

$$u^{i-} \le u^i \le u^{i+}, i = 1,2, x^1(t_F) \le x^{1+}, x^2(t_F) \ge x^{2-}, x^3(t_F) \ge x^{3-}.$$

It is required to minimize the final height $h(t_F) = x^4(t_F)$, that is equivalent to maximization of the emergency landing dangerous zone lower border.

The described simplified helicopter dynamic model is often used in helicopter flight performance preliminary evaluation problems [5, 6]. On the one hand, it is comparatively simple and on the other hand, it allows to considerably increase the accuracy of take-off/landing characteristic calculation in transient conditions (in particular, when determining dangerous zone borders in coordinates $h - v$) in comparison with the energy method due to including the main rotor dynamic characteristics.

For the conditions under consideration rather hard restrictions for transients are typical at which more or less narrow working ranges of these parameter variations are obtained. It permits to accept a linear structure as a comparatively crude approximation of motion model at the stage of qualitative analysis: $\dot{x} = Ax + Bu + C$,where A, B и C – (4×4), (4×2) and , (4×1) matrices, and at the itineration refinement stage --- approximation for types F_T and f^3

$$\sum_{k=0}^{2} n_k (x^3)^k + \sum_{j=1}^{3} y^j \sum_{k=0}^{2} y_{jk} (x^3)^k, \quad y = (x^1, x^2, x^3).$$

At the qualitative analysis stage the following simplifying assumptions are accepted:

$T \sin u^1 = G u^1$, $X_{BP} = 0$, (remembering that in the conditions under investigation $T \approx G$, and values $|u^{1\pm}|$, x^1, x^2 are comparatively small). Under these assumptions, the model linear approximation looks like

$$\frac{dx^1}{dt} = -9.8u^1, \quad \frac{dx^2}{dt} = a_{21}x^1 + a_{22}x^2 + a_{23}x^3 + b_{22}u^2 + c_2,$$

(6)
$$\frac{dx^3}{dt} = a_{31}x^1 + a_{32}x^2 + a_{33}x^3 + b_{32}u^2 + c_3, \quad \frac{dx^4}{dt} = x^2.$$

In accordance with theory [11] the following transformations are made. Equation for x^1 is excluded since it is absolutely independent.

An integral of a back-up (limit) system is found

$$\frac{dx^2}{dx^3} = q = b_{22} / b_{32}, \quad y = x^2 - qx^3,$$

and a derived system is written as

$$\frac{dy}{dt} = \frac{dx^2}{dt} - q\frac{dx^3}{dt} = a_{yy}y + a_{y1}x^1 + a_{y3}x^3 + c_2 - qc_3, \quad \frac{dx^4}{dt} = y + qx^3.$$

In this system x^1, x_3 play the role of controls, and y, x^4- the role of state variables.

Taking into account that the right parts do not depend on time t, it can be eliminated, passing to argument $x^4 = h$ assuming that $x^2 < 0$ (that is typical for the kind of maneuver under examination):

(7) $$\frac{dy}{dh} = h(y, x^1, x^3) = \frac{a_{yy} y + a_{y1} x^1 + a_{y3} x^3 + c_y}{y + q x^3}, \quad y = x^2 - q x^3.$$

Thus we get a first order system with the boundaries of its possible solutions being determined by boundaries of the right part (accounting for the given set restrictions):

(8) $$\frac{dy_{u,l}}{dh} = \max, \min h(y, x^1, x^3).$$

To determine the boundaries where operations max, min are performed, equation (8) is supplemented by equations relative to x^1, x^3:

(9) $$\frac{dx^1}{dh} = x^1(y, x^3, u^1) = \frac{-9.8 u^1}{y + q x^3},$$

(10) $$\frac{dx^3}{dh} = x^3(y, x^1, x^3, u^2) = \frac{g_{3y} y + g_{31} x^1 + g_{33} x^3 + k_{32} u^2 + l_3}{y + q x^3}.$$

As a result, we get an initial system (with argument h) written in new variables (x^1, y^2, x^3).

Boundaries $x^1_{u,l}(h),\ x^3_{u,l}(h)$ are determined as solutions of equations

(11) $$\frac{dx^1_{u,l}(h)}{dh} = \max, \min x^1(y, x^3, u^1), \quad \frac{dx^3_{u,l}(h)}{dh} = \max, \min x^3(y, x^1, x^3, u^2),$$

$x^i_l \le x^i \le x^i_u,\ \ u^{j-} \le u^j \le u^{j+}$ under assigned boundary conditions and state bounds. Here $y_F \ge y^- = x_F^{2-} + 3x^{3-}$.

The turnpike solution of the variation problem of minimum $x^4 = h$ is obtained as one of the boundaries to the extreme (in projection on axis x^4) point of crossing with the set multitude. Then this solution is approximated by an allowable one under controls $u^1(h),\ u^2(h)$, assigned in the process of building of the indicated boundary that is taken as an initial approximation $m^I = (x(h), u^1(h), u^2(h))^I$ for subsequent iterative refinement.

Further on, system (8), (11) is integrated from right to left and from left to right at various values of h_F; that allows one to find external boundaries of a feasible domain. It may be seen that the turnpike solution corresponds with the upper boundary y and value h at which it crosses the a priori lower bound y^-. In other projections there are no limitations for the h lower value, so for determination of target minimum value h it is enough to examine the situation in plane (h, y). In general everything is reduced to building of a one-parameter family of indicated boundaries $x^1_{u,l}(h),\ x^3_{u,l}(h)$ as system (11) solutions and corresponding boundaries $y_u(h)$. From this family the solution is selected where $y_u(h_F) = y^-$.

Thus we obtain control laws $u^1(h),\ u^2(h)$, accepted with the found h_F as an initial approximation. Its further iterative refinement by an algorithm using more accurate analytic description of the model and then the initial algorithmic presentation leads to the final solution.

4. Specific example

Calculations were performed based on a conventional helicopter similar in its characteristics to Ка-226 helicopter [19, 20, 22] as regards the following numerical values of parameters, limitations and initial conditions (in flight at pressure altitude of 2000 m, ISA +20°C):

1. Available controls U(x):

Thrust vector defection angle from vertical 1|< u^1_max}; u^1_max=0.349 rad;
main rotor collective pitch < u^2 < u^2_max; u^2_min=0.08 rad;
u^2_max=0.349 rad;

2. Admission conditions X:
horizontal speed: x^1(t)>0 m/s;
vertical speed : x^2(t)>a; a=-3.2 (при x^1<15,5) m/s;
main rotor angular velocity : 24.6< x^3(t)< 30.8 rad/s;
3. Admission final conditions Г: x^1(t_F)<7.5; (t_F - конечный момент);
x^2(t_F)>-3.2;
4. Initial conditions:
x^1(0)=1 km/h;
x^2(0)=0 m/s;
x^3(0)=29.635 rad/s;
5. Minimum duration of control movement - $\Delta t_{\text{перекладки}} \geq 0{,}5$ sek.
6. Minimum pilot reaction - $\Delta t_{\text{зад}}=1$ сек.

For this object a proven Fortran program is available for calculation of the right parts equations. Using this program, a table of their values in the nodal point grid, generated by combinations of each argument values uniformly distributed in it working range, was calculated. The following coefficients of linear approximation (6) were obtained:

$$a_{21} = 0.1,\ a_{22} = -0.24,\ a_{23} = +0.35,\ b_{22} = +39,\ c_2 = -20.5,$$
$$a_{31} = 0.019,\ a_{32} = -0.13,\ a_{33} = -0.19,\ b_{32} = -13,$$

which correspond to concrete system (7),(9),(10):

(12) $$\frac{dy}{dh} = h(y, x^1, x^3) = \frac{-0.18y + 0.16x^1 + 0.32x^3 - 20.5 + 3c_3}{y - 3(x^3)}, \quad y = x^2 + 3(x^3),$$

(13) $$\frac{dx^1}{dh} = x^1(y, x^3, u^1) = \frac{-9.8u^1}{y - 3(x^3)},$$

(14) $$\frac{dx^3}{dh} = x^3(y, x^1, x^3, u^2) = \frac{-0.13y + 0.019x^1 + 0.20x^3 - 13u^2 + c_3}{y - 3(x^3)},$$

and concrete system (8), (11) for the bounds, where c_3 is left as a parameter defined by the available power which value is varied depending upon the emergent situation scenario.

Let us examine the expression in the right part (12). It is evident that within the examined domain it is a decreasing function x^1 and x^3 (fig. 2).

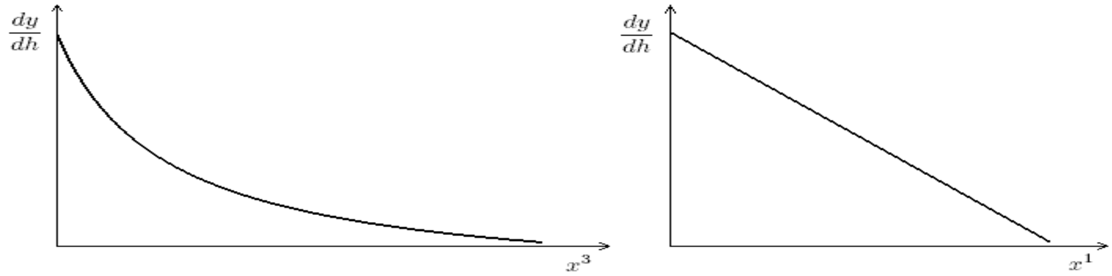


Fig. 2.

Hence it may be seen that the maximum is attained at the lower boundaries of these variables and the minimum - at their upper boundaries. The bounds for x^3 were calculated accounting for the constraint $y = x^2 + 3(x^3)$ and x^2 bounds. For example, one of upper bounds is determined on the base of prescribed restrictions as $x_u^3 = \min(\frac{1}{3} y - x^{2-}, x^{3+})$. Other, more accurate ones , are determined in the same way accounting for equations (12)–(14), containing control variables u^1, u^2.

According the above described rule a family of bounds as solutions of this system was built, and the bound corresponding to the least value of h_F was selected (fig.3).

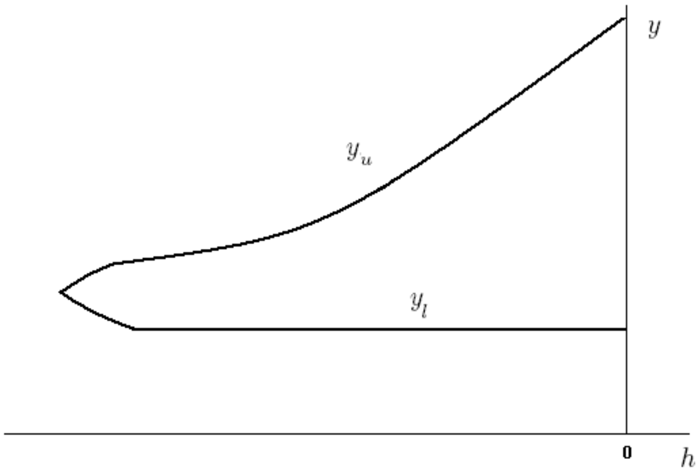


Fig. 3.

The corresponding set $(t_F, u^1(t), u^2(t))_0$ (after recalculating to argument t) can be considered as a qualitative analysis result – an initial approximation for the further improvement.

At the next stage (initial approximation refinement) a more accurate model was used obtained through crude approximation of F_T and f^3 for variant B:

$$\frac{dx^1}{dt} = -0.00044x^1 + (-9.8u^1);$$

$$\frac{dx^2}{dt} = -0.00044x^2 + 0.12(x^3)^2(-0.0046 - 0.00041x^3 + x^1(0.0013 - 0.58 \cdot 10^{-5} x^3) + x^2(-0.055 + $$
$$+ 0.00010x^3)) - 9.8 + 0.12(x^3)^2(0.087 - 0.0015x^3)u^1 + 0.12(x^3)^2(0.35 + 0.0013x^3)u^2;$$

$$\frac{dx^3}{dt} = -0.16x^3 + 6.80 + x^1(-0.021 + 0.0014x^3) + x^2(0.035 - 0.063x^3) + (-0.36 + 0.093x^3)u^1 + $$
$$+ (5.5 - 0.65x^3)u^2 + 0.19(N - 357)/x^3;$$

$$\frac{dx^4}{dt} = x^2,$$

Substitution of the obtained set $(t_F, u^1(t), u^2(t))_0$ into this model resulted in the trajectory not satisfying exactly to given restrictions (due to approximation errors). To eliminate this drawback the u^2-dynamics was corrected to observe the restrictions on x^2, x^3. Thus more rigorous initial approximation was obtained. It is presented on fig. 4 for different values of N.

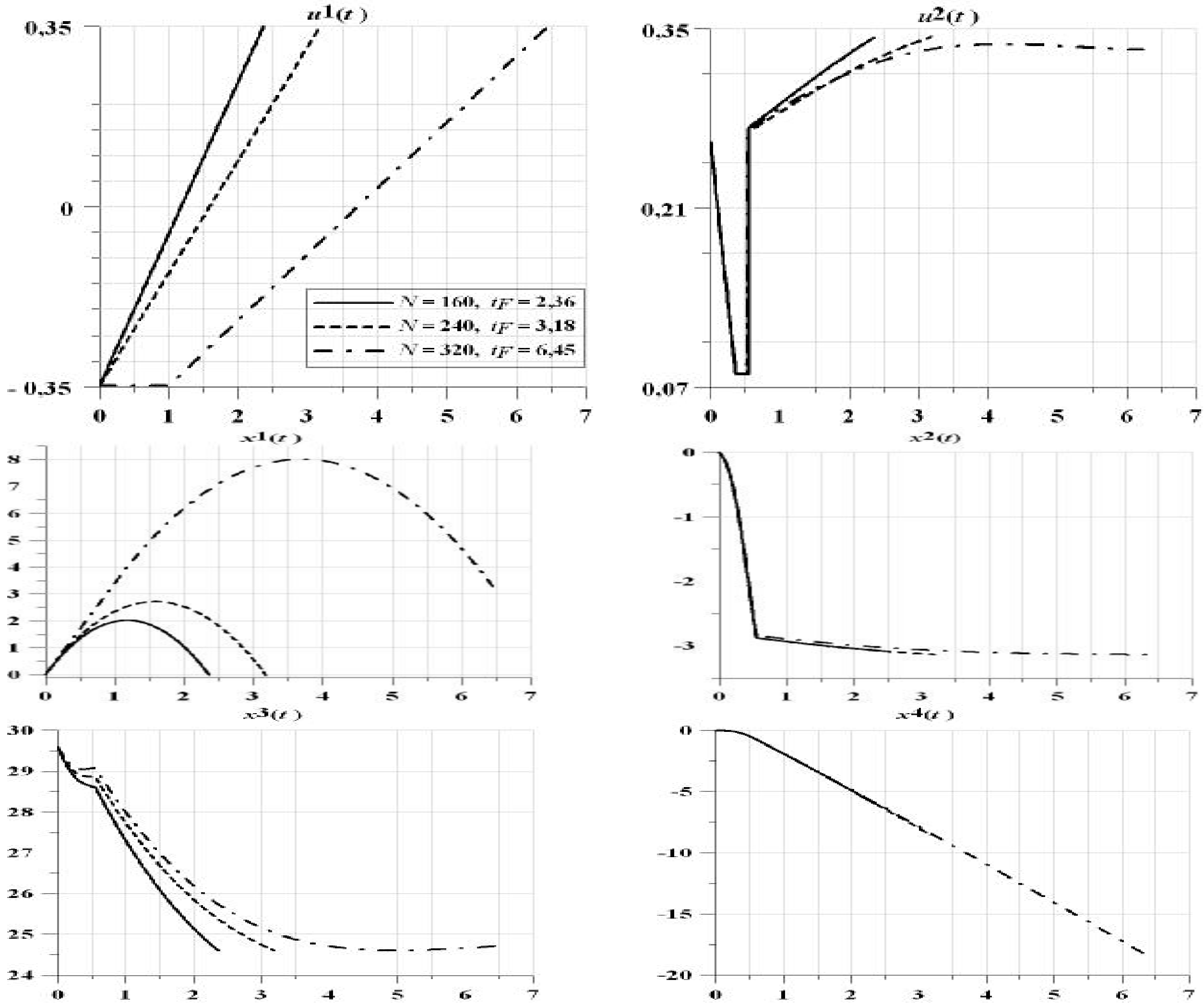
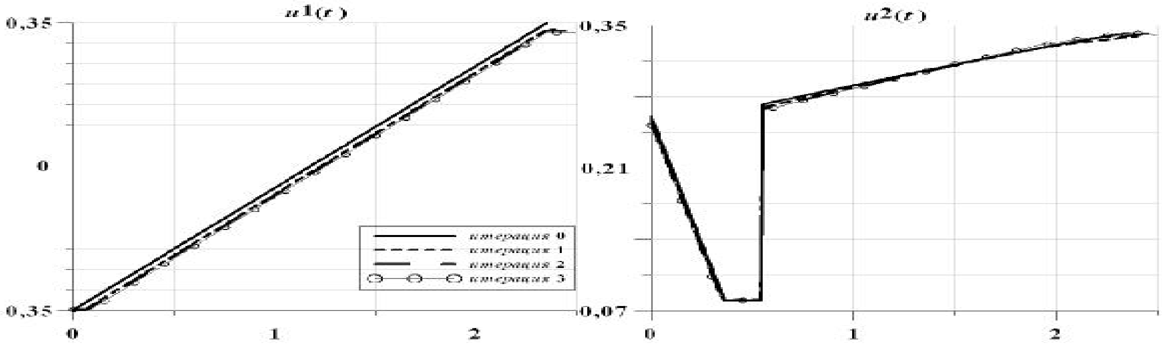


Fig. 4

It is seen that for sufficiently large N (in this example for $N > 320$) h_F has no lower bound; this means that there is no dangerous zone whereas for smaller N its lower bound is determined (in first approximation).

Then the most simple algorithm from [12] (of first order, second type), related to known fast descend algorithm. To account for restrictions cut-off penalty functions were used.

Results for the most hard scenarios under consideration, $N = 160$ are presented on fig5.



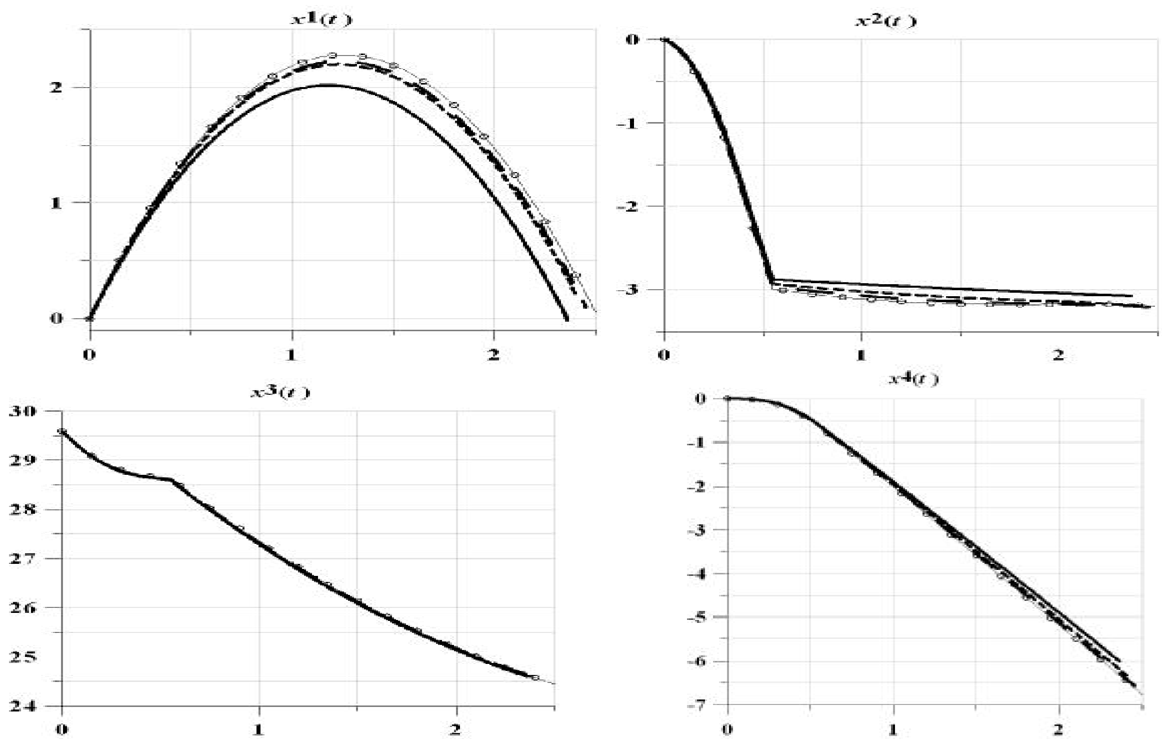


Fig. 5.

It is seen that after 3-d iteration h_F reduced by 1 m, which corresponds to increasing of the dangerous zone lower bound by 15% from initial approximation whereas .the qualitative character of state and control dynamics was preserved. This tells about comparatively high accuracy of the initial approximation as a result of qualitative analysis.

On the base of above investigations and calculations for a conventional example several versions of simple control laws were elaborated for a real helicopter, Ka-226, approved with the use of the original Fortran-program. Two of them are presented on fig. 6.

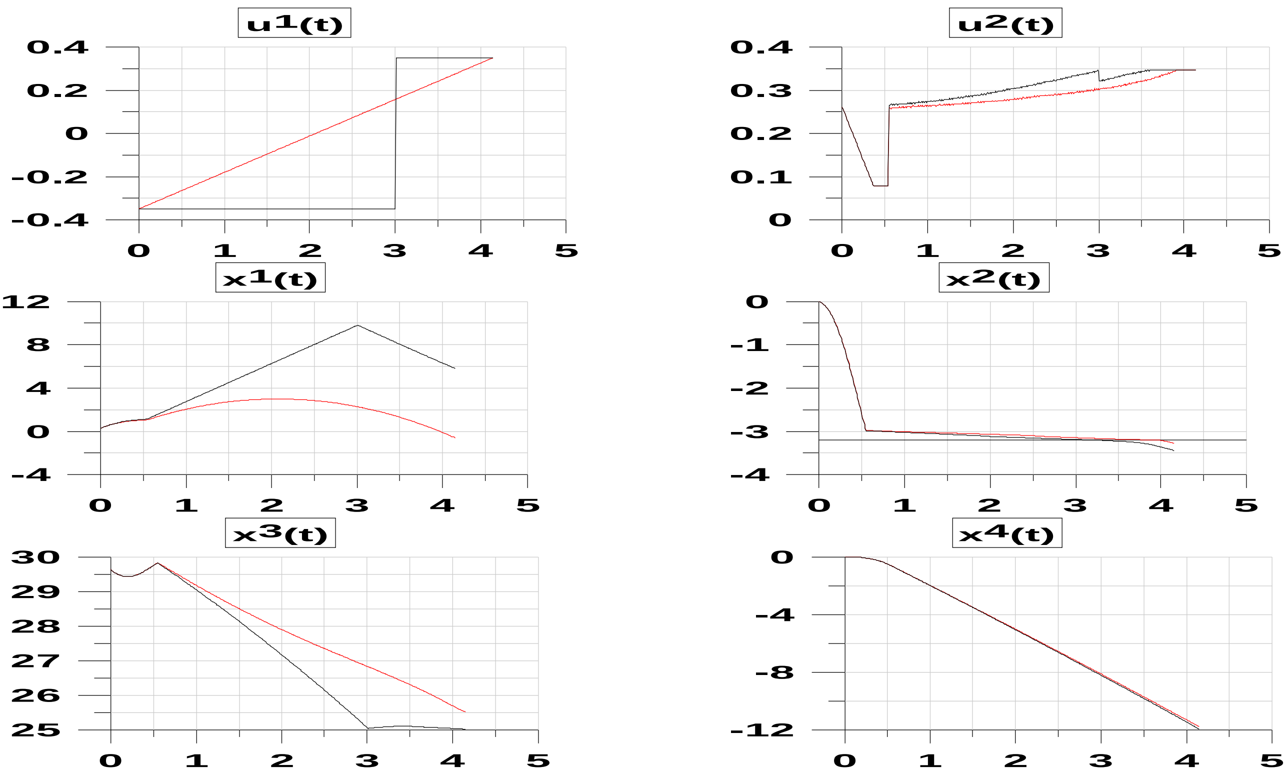


Рис. 6.

Fig. 7 presents a variant of control and flight trajectory accounting for a pilot reaction to an engine failure (1 sec) and physical limitations of the control change rate. The height of the lower boundary of the dangerous zone is approximately 7,5..8 m that correspond to the Ka-226 helicopter in examined conditions.

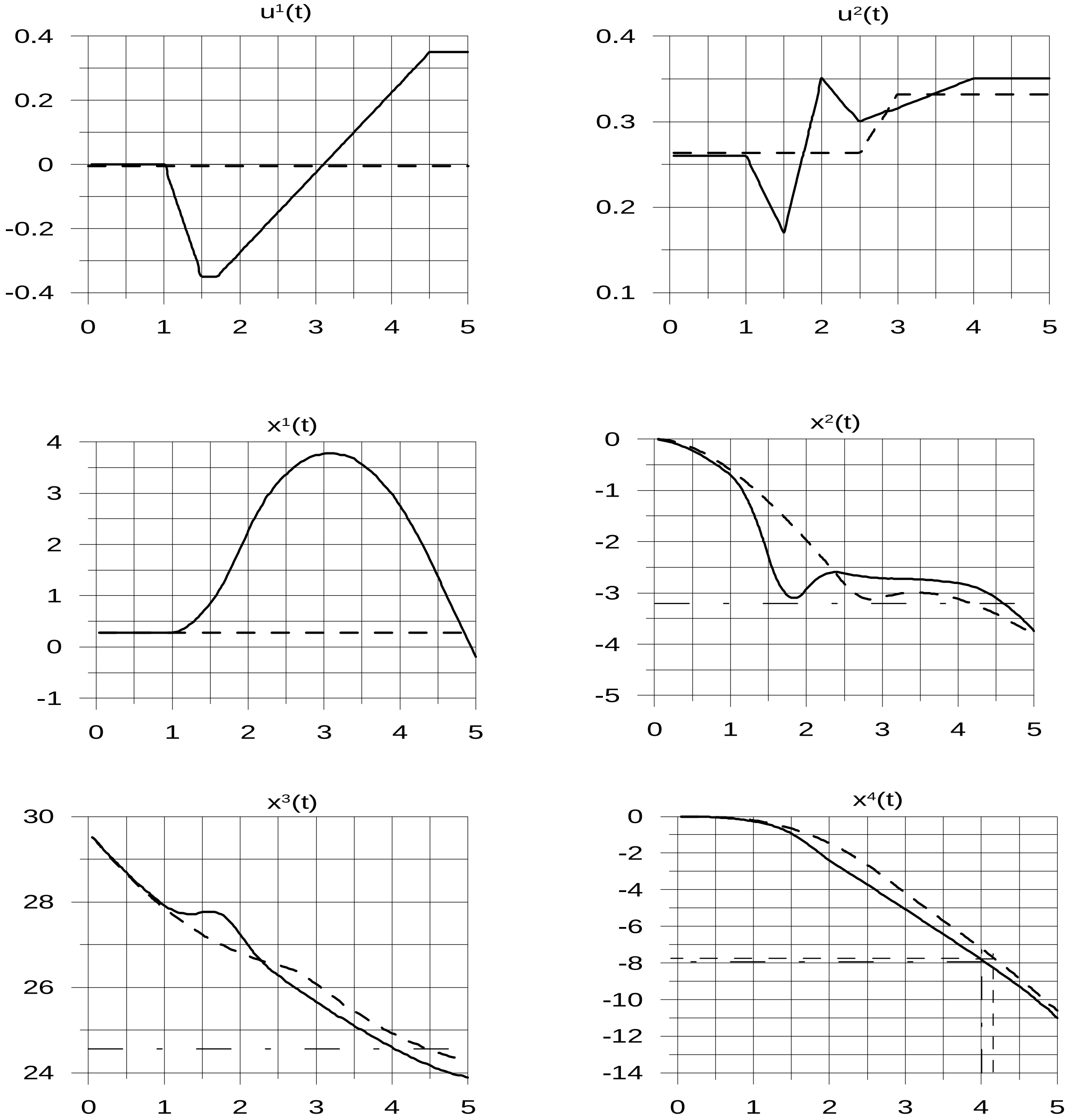


Рис. 7.

5. Conclusion

Thus two types of the object model approximations were considered: linear and nonlinear. The first one, more rough, can be used not only for the complex simulation models but also for the models described completely analytically to simplify them and to carry out an effective qualitative analysis. The last one is strictly important for practice leading to comparatively simple near-optimal control laws applicable in emergent situation.

In the whole the approach proposed is seen as effective one for the wide class of problems of helicopter dynamics and control. It allows one to take into account such circumstances and physical features as:

- delayed pilot reaction to failure situation;
- physical limitations on motion speed of control structural elements.

On the base of analysis and generalization of the obtained qualitative results and numerical data it is possible:

- to reduce the scope of recommendations for the pilot (it is not expedient to include into RFM too complicated recommendations on piloting techniques for one-engine failure in various high-hot conditions and helicopter weights;
- to use more accurate, though more complicated math helicopters models for validation of the data on next stages of the work;
- to establish requirements for the main flight parameters (for example those associated with emergency engine modes).

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REFERENCES

1. Браверман А.С., Вайнтруб А.П. Динамика вертолета (Helicopter dynamics). Предельные режимы полета. М., Машиностроение, 1988 .

2. Акимов А.И. Аэродинамика и летные характеристики вертолетов (Aerodynamics and performance of the helicopter). М., Машиностроение, 1988 .

3. Петросян Э.А. Аэродинамика соосного вертолета (Aerodynamics of coaxial helicopter). Балансировка, устойчивость, управляемость, маневренность, автоматическая стабилизация и автоматическое управление. Полигон-пресс, Москва, 2004 .

4. Jepson W.D. Some considerations on the landing and take-off characteristics of twin engine helicopters. Journal of the American Helicopter Society. Vol. 7, N 4,1962.

5. Факторович И.О. Исследование переходных режимов вертолета методом баланса энергии (Energy method of research helicopter transitional process). Труды ЦАГИ, вып. 2026, 1979 г.

6. Чуклов Б.Т. Применение вариационного метода последовательных улучшений по управлению для оптимизации взлетной траектории вертолета (Application of variation methods for takeoff helicopter optimization). Труды ЛИИ. 1972. \No 221. С. 1--26.

7. Gurman V., Nikiforova L. Extension principle in modeling and optimal control: case study for helicopter maneuvering. // A postprint volume from the 10th IFAC workshop, Haifa, Israel, 19-21. December, 1995. Elsevier, London, 1996.

8. Никифорова Л.Н., Ухин М.Ю., Феофилов Е.Б. Оптимизация пространственных траекторий полёта вертолёта (Space flight helicopter optimization). В сборнике <Системный анализ, информатика и оптимизация>. Сборник научных трудов Международной академии информатизации. Москва, РосЗИТЛП, 1999. С. 58-68.

9. Л.Н. Никифорова, М.Ю. Ухин. Синтез оптимального управления вертолетом на маневре в вертикальной плоскости (Optimal helicopter control synthesis on vertical maneuvering). - Научное обозрение, No 2, 2006. С. 63-72.

10. Л.Н. Никифорова, М.Ю. Ухин. Синтез оптимального управления вертолетом на посадке с режима авторотации (Optimal helicopter control synthesis in autorotation landing). - Научное обозрение, No 6, 2005. С. 65-72.

11. Гурман В.И. Вырожденные задачи оптимального управления (Degenerate optimal control problems) . М. Наука, 1977.

12. Гурман В.И. Принцип расширения в задачах управления (Extension principle in optimal control). М. Наука. Физматлит, 1985.

13. Гурман В.И. Магистральные решения в процедурах поиска оптимальных управлений (Turnpike decisions in procedure of optimal control search) . Автоматика и телемеханика, 2003, No 3. С. 61-71.

14. Гурман В.И., Трушкова Е.А., Ухин М.Ю. Улучшение управления, реализующего скользящий режим (Control improving for realization of sliding mode). АИТ. 2007 (в печати)

15. Ухин М.Ю. Приближенный синтез оптимального управления (Approximated synthesis of optimal control). М. Физматлит, 2006.

16. Летников В.Б. Влияние конструктивных параметров на нижнюю границу опасной зоны вертолета при отказе одного двигателя (Influence of structural parameters on. safe zone low boundary by engine failures). Тематический сборник научных трудов института «Проблемы проектирования современных вертолетов». Изд. МАИ, Москва, 1981 г. С. 53-60.

17. Летников В.Б., Рогов Ю.Н. Исследование траекторий прерванного и продолженного полета вертолета при отказе одного двигателя (Rejected and continued helicopter motion trajectories research by engine failures). Труды научных чтений, посвященных памяти академика Б.Н. Юрьева (Москва, 13-14 ноября 1989 г.). Изд-во ИИЕТ АН СССР, Москва, 1990 г. С. 146-156.

18. Кротов В.Ф., Гурман В.И. Методы и задачи оптимального управления (Methods and problems of optimal control). М. Наука, 1973.

19. Лалетин К.Н. Практическая аэродинамика вертолета Ка-26 (Practical aerodynamics of helicopter Ка-26). Москва, «Транспорт», 1974 г.

20. Н.Ф.Суриков, Г.И.Иоффе, Ф.Ф.Дмитриев, Е.Г.Пак. Вертолет Ка-26 (Helicopter Ка-26), М., Транспорт, 1982 г.

21. Eric B. Carlson Sidney Xue, Sr. Engineer Specialist Principal Engineer, Joshua Keane Kevin Burns, H-1 Upgrades height-velocity diagram development through flight test and trajectory optimization, AHS International 62nd Annual Forum, May 9-11, 2006.

22. Вертолетный мир России (Russia’s Helicopter World), 2003 г., С. 72-73.