A mathematical model for landing a single-rotor helicopter with a wheeled chassis in standard conditions and special cases

Mikhail Kruchinin, JSC Helicopters Mil & Kamov,
e-mail: mmkruchinin@mi-helicopter.ru
Boris Artamonov, Moscow Aviation Institute, e-mail: k102@mai.ru

At the present stage of development of the design of modern aviation equipment, in conditions of fierce rivalry, there are obvious trends to reduce the design time, to lower the final cost, as well as to increase the safety of the product being designed. One of the ways to solve this problem, along with increasing the functionality of the calculation software, is to reduce the volume of certification work by replacing full – scale stand and flight tests with mathematical modeling.

This paper continues a number of scientific works of the authors related to the problems of constructing reliable mathematical models in the study of helicopter landings. In the article [1], the simulation of copra tests of the main and front landing gear of the Mi-38 helicopter using software packages *LMS.Amesim.Imagine.Lab* и *LMS.Virtual.Lab* was considered. The stages of constructing a model and conducting a numerical experiment are shown. The simulation results were compared with the experimental data of field tests. In the paper [2], the features of using the *LMS.Imagine.Lab.AMESim* platform are considered in the design of helicopter shock absorbers consisting of mechanical and hydraulic and pneumatic elements. A variant of including a simulation model of fluid flow through the hole obtained from experimental data in the calculation algorithm was proposed.

Drop tests provide information about the energy consumption of the chassis supports and allow you to confirm the design loads and the stroke of the wheel axis. However, they do not allow us to obtain data on the behavior of the entire helicopter as a whole when landing and moving on the surface. Since the method used for modeling shock absorbers and landing gear supports gives a good match with the data of a full-scale experiment, it was decided to conduct a numerical simulation of the landing of the entire helicopter with a three-point wheeled landing gear.

The most general approach that opens up wide opportunities for engineering analysis of complex aircraft systems is provided by the *Siemens Simcenter* software package.

The Siemens Simcenter 3D Motion software product allows you to create complex kinematic calculation models using submechanisms technology. This approach allows you to use previously created models of landing gear supports in a single helicopter model. Since submechanisms can contain elements for independent calculations, after importing them into the general kinematic model, all unused elements of submechanisms (applied forces, links, etc.) can be deactivated, and they will not be taken into account when analyzing motion.

When modeling the motion of a system of solids relative to an absolute coordinate system in the *Simcenter Motion* module, the position of the bodies in space is determined by the vector $\boldsymbol{S}$:

$$\boldsymbol{S} = \begin{pmatrix} S_1 \\ \vdots \\ S_n \end{pmatrix}, \tag{1}$$

Where n is the number of bodies in the system.

Two-way connections between bodies can be represented as a vector $\boldsymbol{C}$, of dimension d, equal to the number of connections in the system, and [3]:

$$C_k(S_i, S_j) = 0; \quad 1 \le k \le d. \tag{2}$$

The Jacobi matrix $\boldsymbol{J}$ for the vector of communication functions $\boldsymbol{C}$ looks like this:

$$\boldsymbol{J} = \begin{pmatrix} \frac{\partial C_1}{\partial S_1} & \cdots & \frac{\partial C_1}{\partial S_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial C_d}{\partial S_1} & \cdots & \frac{\partial C_d}{\partial S_n} \end{pmatrix}. \tag{3}$$

The vector of reactions in bonds $\boldsymbol{R}$:

$$\boldsymbol{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_n \end{pmatrix}. \tag{4}$$

Vector of indeterminate Lagrange multipliers $\boldsymbol{\lambda}$:

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_d \end{pmatrix}. \quad (5)$$

The reactions in the bonds are determined from the equilibrium equation:

$$\boldsymbol{R} - \boldsymbol{J}^{\boldsymbol{T}}\boldsymbol{\lambda} = 0. \quad (6)$$

The differential equations of the dynamics of a system of solids used for modeling in the *Simcenter Motion* module can generally be written as:

$$\boldsymbol{M}\frac{d^2\boldsymbol{S}}{dt^2} - \boldsymbol{F} - \boldsymbol{R} = 0, \quad (7)$$

where $\boldsymbol{F}$ is a vector of external forces, of dimension *n*,

$\boldsymbol{M}$ – diagonal mass matrix:

$$\boldsymbol{M} = \begin{pmatrix} M_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_n \end{pmatrix}. \quad (8)$$

Since this approach was implemented in the helicopter industry for the first time, this problem was solved within the framework of the Mi-38 project, for which there are data from full-scale bench and flight tests used to confirm the reliability of the model.

To build a mathematical model of helicopter landing, at the first stage of the work, a solid-state model of the helicopter fuselage was taken, to which models of landing gear supports were added as submechanisms (See Figure 1), which were previously used for modeling copra tests. The submechanisms are positioned relative to each other, and when the position of one of the mechanisms changes, for example, the center of gravity of the helicopter fuselage changes relative to the surface, or the pitch angle, the other submechanisms automatically change their position accordingly.

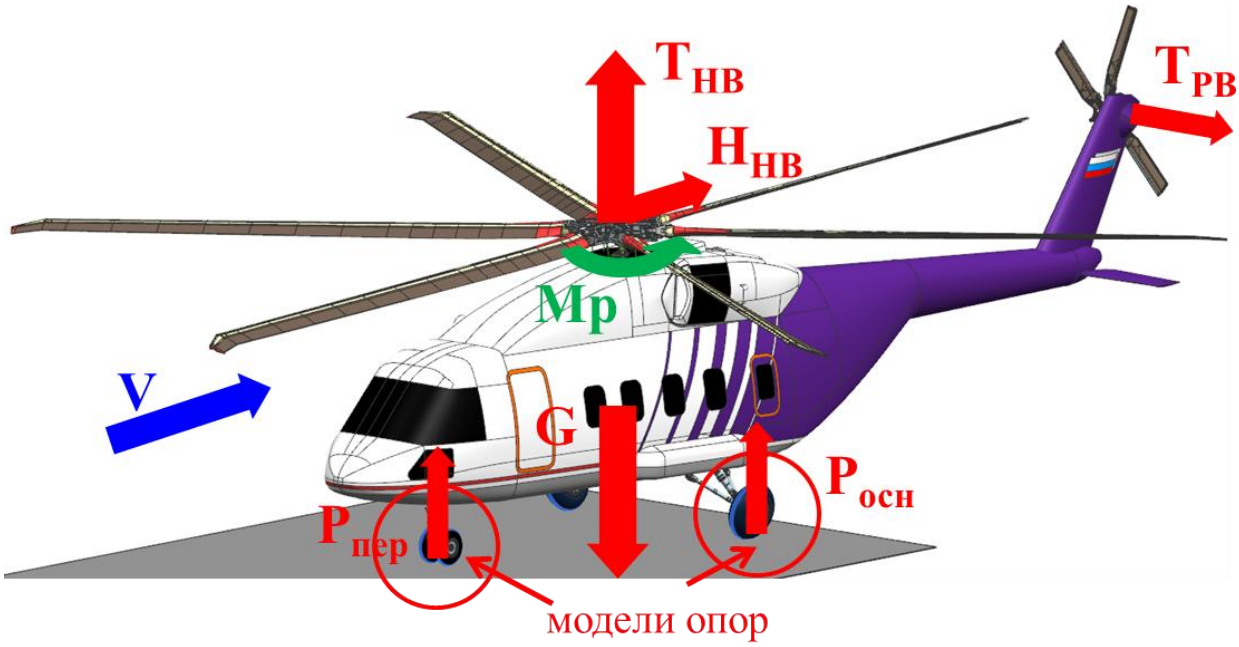

Figure 1 - Landing model with a solid-state fuselage

By setting the initial conditions and the laws of control of the main rotor (MR), the movement of the system of solids under the influence of external forces is calculated – this is the weight of the helicopter, the reaction forces when the wheels interact with the surface and the force factors from the MR.

At the same time, the following assumptions are made:

1) the friction in the joints does not affect the calculation results;
2) the reactive moment from MR is compensated by the tail rotor (TR);
3) the calculation is carried out from the moment the wheel touches the surface.

The synthesis of mathematical models of shock absorbers, as a combination of pneumatic, hydraulic and mechanical systems, and the modeling of copra tests of wheel chassis supports by means of joint calculation of mathematical models of shock absorbers, models of kinematics of chassis supports and models of pneumatics are considered in detail in previous papers of the authors [1, 2].

When constructing a mathematical model of landing a helicopter on a 3-point wheeled chassis with both an absolutely rigid and elastic fuselage of a helicopter, it is necessary to calculate the thrust force of the MR taking into account the influence of the earth.

To correctly account for the influence of an MR operating near the ground on the forces and moments transmitted to the fuselage, it is necessary to have an algorithm for calculating the force factors acting on the helicopter from the MR side under various landing modes.

The basis for calculating the aerodynamic characteristics of MR is the method for determining inductive velocities. In this study, the linear disk vortex theory of MR [4-6] is used, which allows us to calculate the field of inductive velocities in a wide range of angles of attack and flight speeds.

The influence of the earth is taken into account in the calculation by a reflected vortex cylinder with a circulation of γ (see Figure 2). There are two possible options for the inclination of the vortex cylinder δ: positive, as well as negative, which is typical for landing modes on the MR self-rotation mode with longitudinal speed and with the fuselage tilt for cabring.

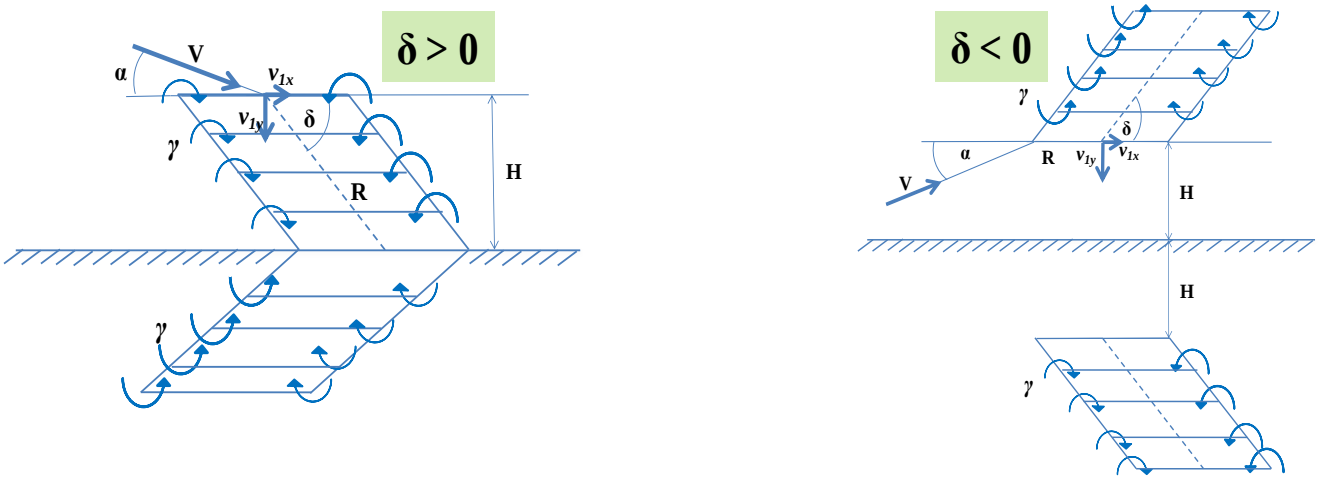

Figure 2 - Vortex model of the main rotor near the ground

To determine the inductive velocities, a system of equations is solved jointly with respect to the linear circulation of vortex rings γ and the angle of inclination of the vortex cylinder δ [7]

$$\begin{cases} \frac{\tilde{\gamma}}{2}\left|\tilde{V}\cos(\alpha+\delta)+\frac{\tilde{\gamma}}{2}F_{10}(\bar{H},\delta)\right| = 1 \\ \tilde{V}\sin(\alpha+\delta)+\frac{\tilde{\gamma}}{2}\left(F_{1x}(\bar{H},\delta)\sin\delta - F_{1y}(\bar{H},\delta)\cos\delta\right) = 0 \end{cases}, \quad (9)$$

where $\bar{H}$ – is the relative distance from the plane of the disk MR to the earth,

$$\bar{H} = \frac{H}{R},$$

where H – the distance from the plane of the disk to the earth,

R – radius MR,

$F_{10}(\bar{H},\delta)$, $F_{1x}(\bar{H},\delta)$, $F_{1y}(\bar{H},\delta)$ – dimensionless functions,

$$F_{10}(\bar{H},\delta) = F_{1x}(\bar{H},\delta)\cos\delta + F_{1y}(\bar{H},\delta)\sin\delta. \qquad (10)$$

Function graphs$F_{1x}(\bar{H},\delta)$ и $F_{1y}(\bar{H},\delta)$ they are shown in Figures 3 and 4.

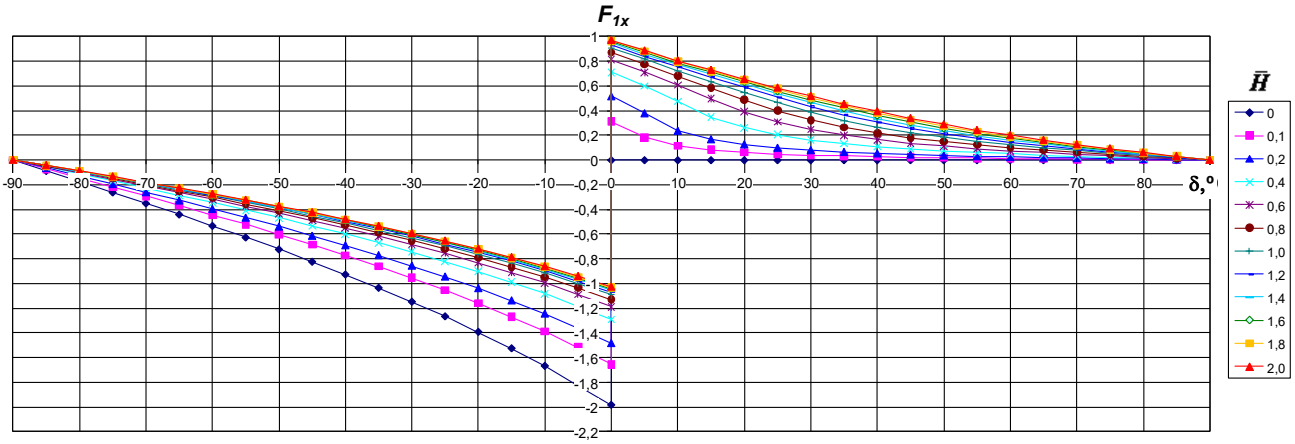

Figure 3 - Graphs of a dimensionless function F_{1x}

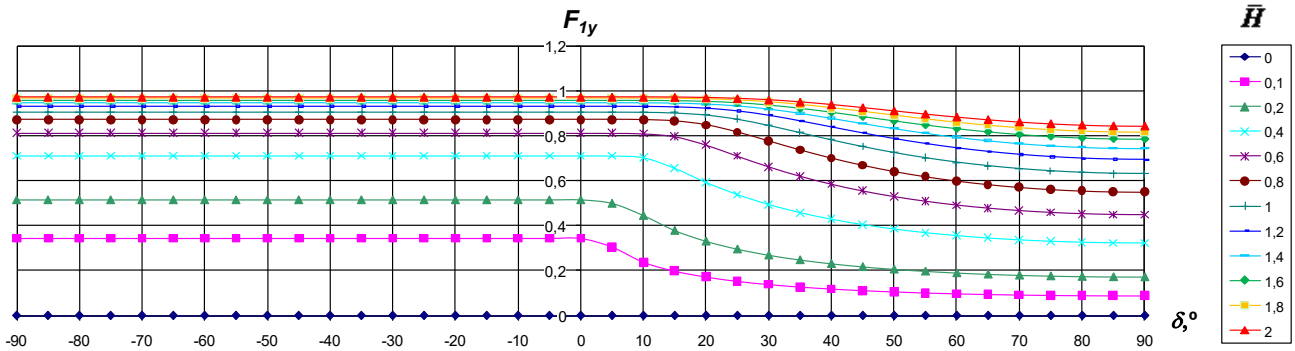

Figure 4 - Graphs of a dimensionless function F_{1y}

The solution of system (9) with respect to $\tilde{\gamma}$ is as follows

$$\tilde{\gamma} = \frac{1}{F_{10}}\left[-\tilde{V}\,cos(\alpha+\delta) + \sqrt{\tilde{V}^2 cos^2(\alpha+\delta) + 4F_{10}}\right] \qquad (11)$$

There are such areas of angles of attack α in which the solution of system (9) does not exist. The approximation of the dependence $\delta(\bar{H},\alpha,\tilde{V})$ in these regions is performed by Artamonov B. L. (See Figure 5).

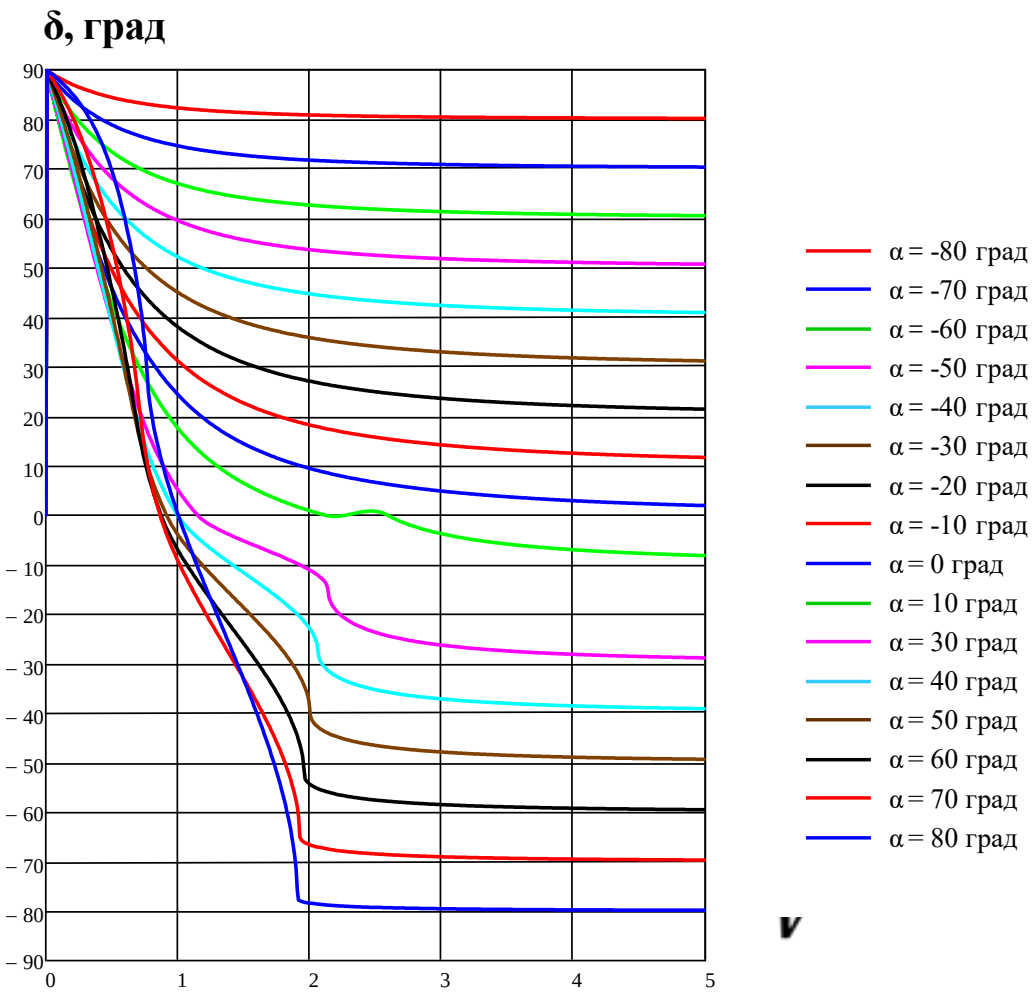

Figure 5 - The dependence of the angle of inclination of the vortex cylinder δ on the flight speed and the angle of attack α a $\bar{H} = 2$

The formulas for finding the inductive velocities in the Earth's zone of influence are as follows

$$\tilde{v}_{1y} = \frac{\bar{v}_{1y}}{\bar{v}_{1в}} = \frac{\tilde{\gamma}}{2} F_{1y}(\bar{H}, \delta),$$
$$\tilde{v}_{1x} = \frac{\bar{v}_{1x}}{\bar{v}_{1в}} = \frac{\tilde{\gamma}}{2} F_{1x}(\bar{H}, \delta), \quad (12)$$

where $\bar{v}_{1в}$ is the average inductive velocity on the disk MR in the hovering mode outside the influence of the earth

$$\bar{v}_{1в} = \frac{1}{2}\sqrt{\frac{c_T}{\varkappa}},$$

where $\varkappa$ – is the coefficient of end losses MR.

The calculation of the coefficients of the thrust force c_T and the torque m_k on the MR was carried out in the *Mathcad* programming environment.

The result of calculating the dimensionless inductive velocities $\tilde{v}_{1y}(\tilde{V},\alpha)$, $\tilde{v}_{1x}(\tilde{V},\alpha)$ is shown in Figure 6.

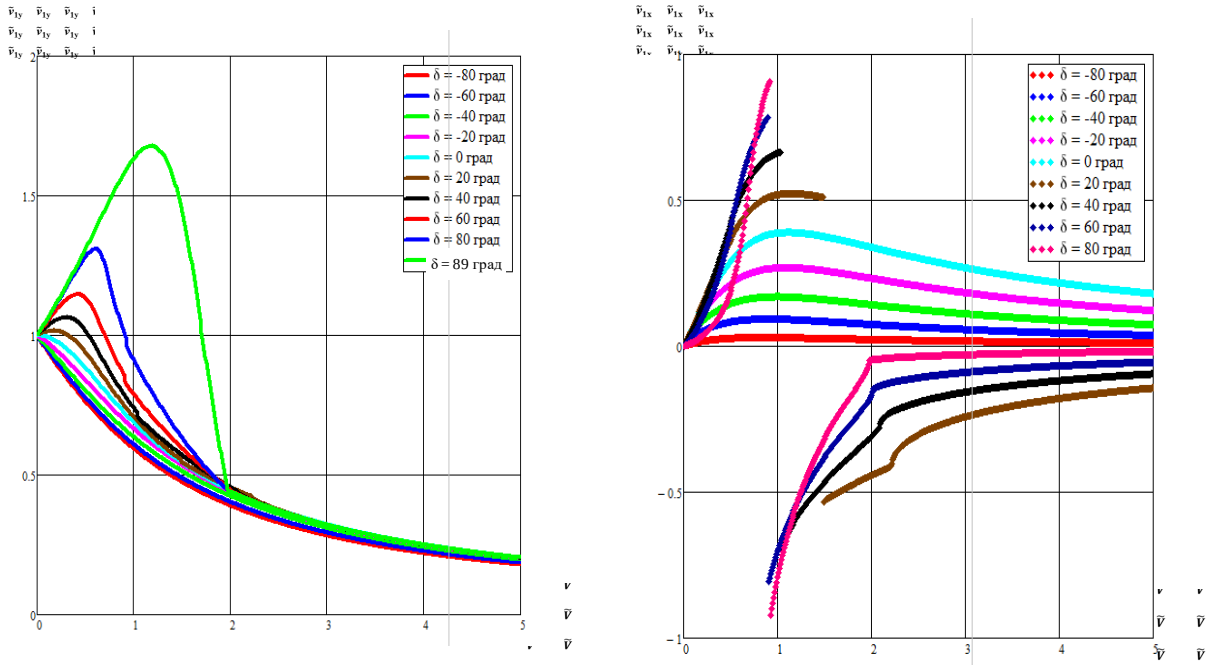

Figure 6-Inductive velocities taking into account the influence of the earth at $\overline{H}$ = 2

In this work, the joint work of CAD modeling programs (*Simcenter Engineering Desktop*), *CAE* analysis (*Simcenter Motion Modeling*, *Simcenter Structures*), *1D* modelingя (*Simcenter AMESim*), *Mathcad*, *Excel* is implemented within a single information space (See Figure 7).

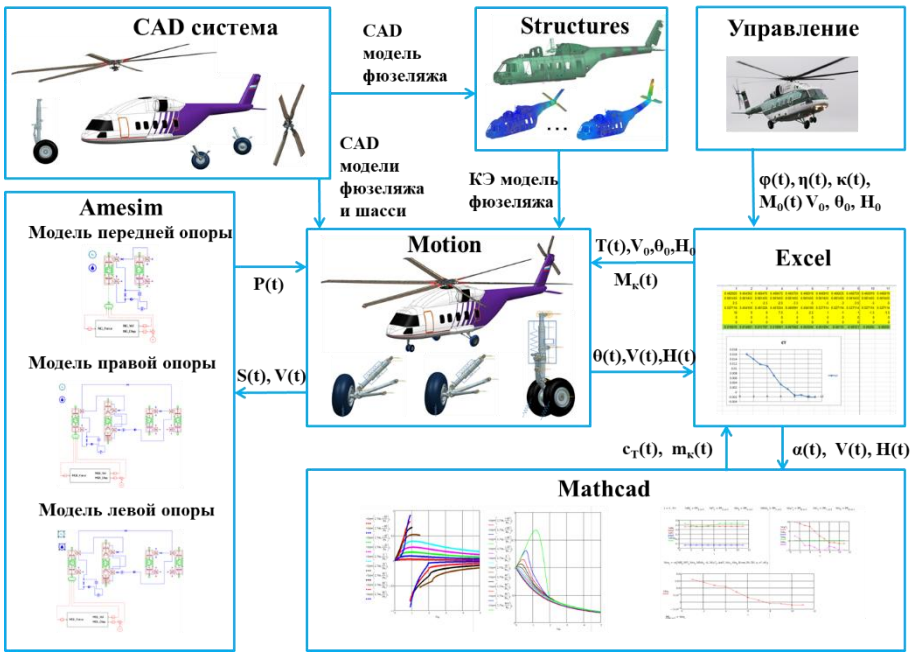

Figure 7 - Joint work of software modules

Simcenter Engineering Desktop modeling module serves as a source of geometric data for creating models of mechanisms in the *Simcenter Motion Modeling* and creating CAD models in the *Simcenter Structures*. Formal mathematical models of shock absorbers have been created in the *Simcenter AMESim* module. The *Mathcad* program has created a mathematical model of the main rotor, taking into account the influence of the screen.

In the control module, the dependences on the time of the total, cyclic step and angular velocity of rotation MR are set – $\varphi(t)$, $\eta(t)$, $\kappa(t)$, $M_0(t)$. Here, the initial conditions for modeling the landing are set – the speed, the angle of inclination of the helicopter trajectory and the height of the center of mass MR above the surface – V_0, θ_0, H_0.

In the control module, using the *Excel* program, as well as in the *Motion* module, arrays of initial data are formed for calculating the coefficient of traction and torque on the MR in the *Mathcad* program. The values of thrust and torque on the MR from *Excel* tables are transmitted to the *Motion* module to calculate the movement of the helicopter under the influence of external forces. The forces on the landing gear shock absorbers that occur when the landing shock is extinguished are calculated in the *AMESim* module and transmitted to the *Motion* module.

To confirm the reliability of the developed mathematical model of landing, a simulation was performed with the initial landing conditions taken from the flight experiment.

In 2014 and 2016, landing tests were conducted for the Mi-38 helicopter in the autorotation mode.

During the tests, at the moment of touching the ground, the helicopter has a vertical speed of $V_{y0} \approx 1.2$ m/s and a horizontal speed $V_x \approx 56$ km/h = 15.5 m/s. The angle of inclination of the trajectory of the Mi-38 helicopter is $\theta_0 = 7.5°$. The data of flight tests of landing on autorotation are shown in Figure 8.

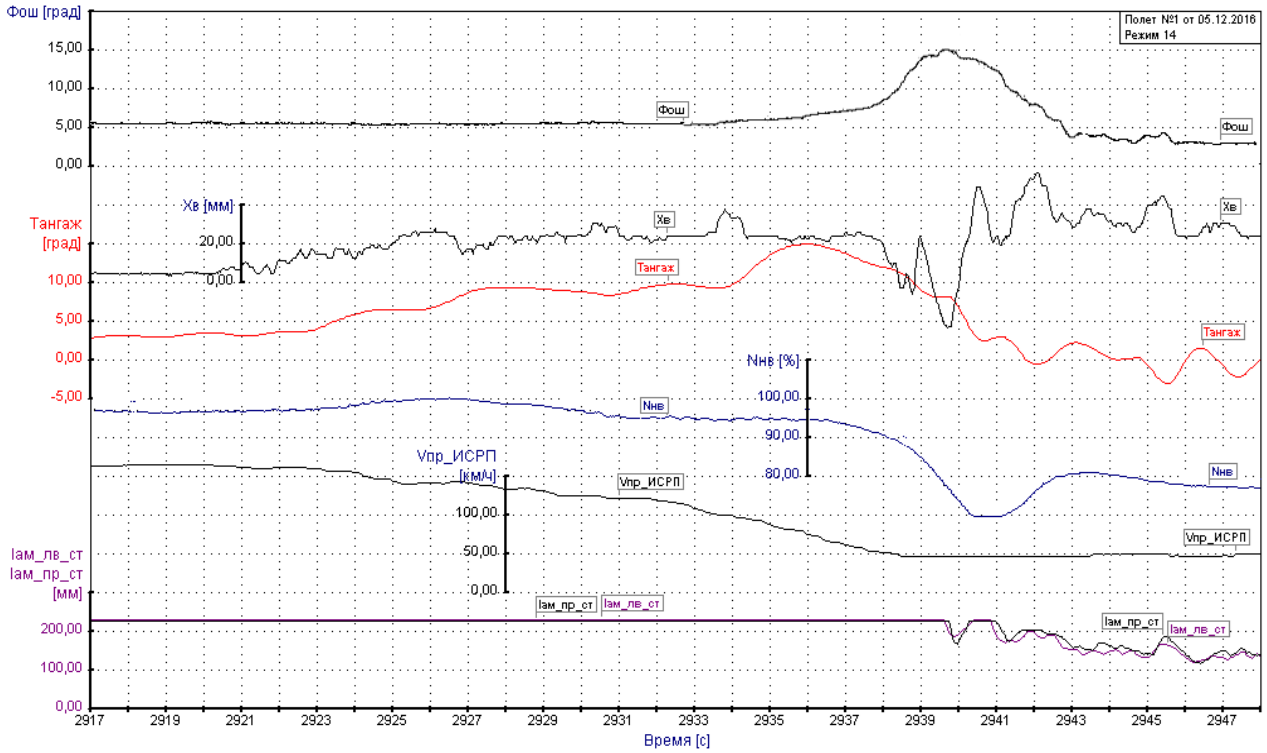

Figure 8 - Data of flight tests of the Mi-38 helicopter

A comparison of the calculation results and flight test data is shown on the graphs of compression of the shock absorber rod over time (See Figure 9). The calculation results are shown as solid lines, the data of the flight experiment are shown as dots.

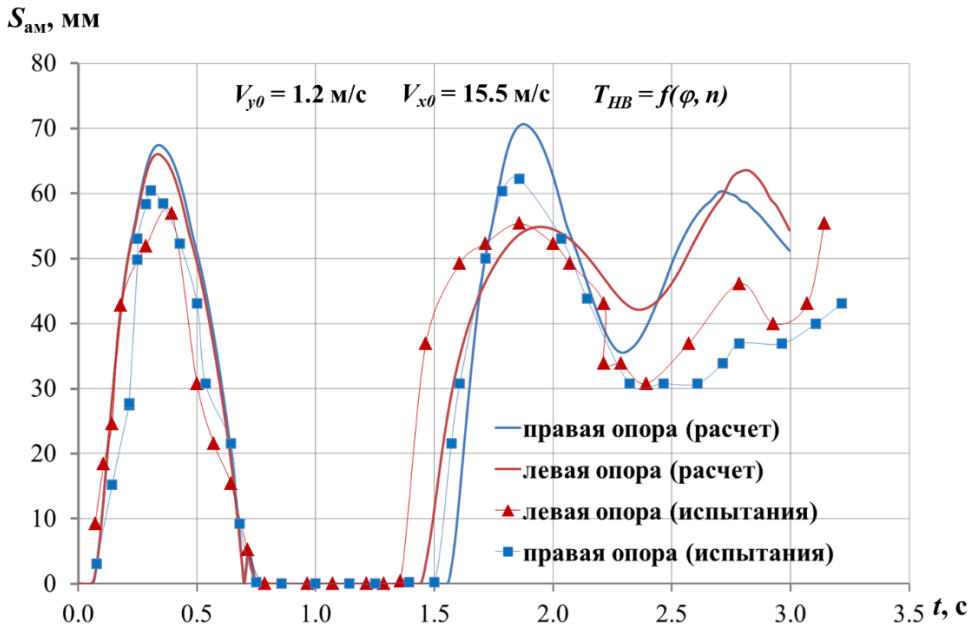

Figure 9 - Compression of the shock absorber rod of the main landing gear supports of the Mi-38 helicopter during landing in autorotation mode

The simulation results coincide well with full-scale tests when the helicopter landing gear hits the surface and the landing gear briefly detaches from the surface after the first impact, therefore, the helicopter landing model can be considered reliable. The discrepancy between the calculation results and the experiment during the run is due to the fact that the balancing of the helicopter in the track channel is considered completed, the influence of control in the track channel is not taken into account in accordance with the accepted assumptions.

A solid-state model of a helicopter airframe is not enough to conduct studies of the strength of the fuselage and tail beam during landing. It is necessary to evaluate the stress-strain state (SSS) of the structure under the influence of emerging loads. For these purposes, it is advisable to use an elastic model of the helicopter airframe.

When mathematically modeling the movement of a mechanism in which solids move with specified degrees of freedom in accordance with restrictions, it is impossible to take into account the dynamic effects that arise as a result of conditions such as sharp shocks, sudden changes in the movement of bodies or when a component of the structure is sufficiently malleable.

During the landing of the helicopter, just such dynamic processes occur – the impact of the wheel on the surface, the change in the direction of movement of the entire system after touching the surface. At the same time, the pliability of the tail beam of the helicopter, which is significantly higher relative to the central part of the airframe, can significantly affect the simulation result.

To account for these effects, the functional of calculating the malleability of the links of the mechanism is used in the work in order to combine both elastic deformation and the movement of a rigid body. The pliability of the links is calculated using dynamic reduction by the Craig-Bampton method for a finite element (FE) model of the link and its integration into the analysis of the movement of the mechanism. In the process of this solution, the dynamic behavior of a pliable body in the form of a certain set of oscillation forms is reduced to modal mass and modal stiffness.

As an elastic body for taking into account the deformations of the airframe in the process of analyzing the movement of the system, a detailed FE model of the helicopter airframe was used, connected to the submechanisms of the landing gear supports. In the strength department of JSC "NCV Miles and Kamov" for a number of years, global FE models of the helicopter airframe have been developed and used in calculations. Figure 10 shows a new model of the Mi-38 helicopter, verified by the results of bench tests conducted earlier. Table 1 shows the main characteristics of the FE model of the Mi-38 helicopter.

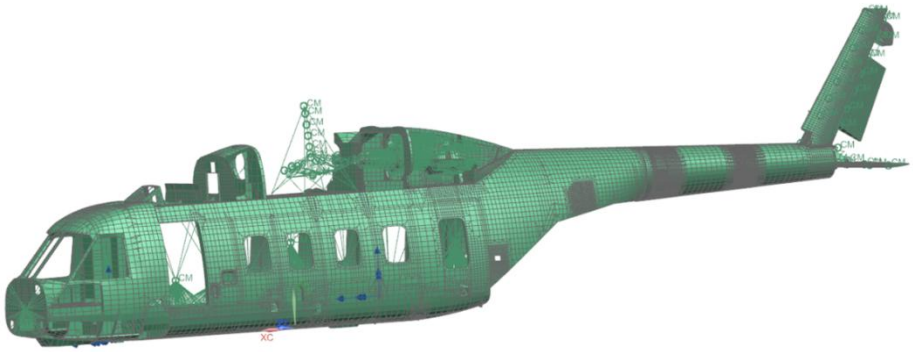

Figure 10 - FE model of the Mi-38 helicopter airframe

Table 1 - Main characteristics of the FE model

Total number of elements	103373
Total number of nodes	68988
The number of elements of the concentrated mass	805
The number of elements of the BAR type	18899
Number of beam elements	12918
Number of shaft elements	363
The number of elements of the hard link	77
Number of elements Quad4	58965
Number of elements Tri3	6083
Number of elements Tetra10	4440
Number of elements Hex8	416
Number of elements Wedge6	360
Number of materials	26

During the calculation of the helicopter landing in the *Motion* module, the solid-state mass-dimensional model of the helicopter airframe is replaced with an

elastic one, while the internal elastic forces of the airframe are added to equation (7), and it takes the following form:

$$\boldsymbol{M}\frac{\boldsymbol{d^2S}}{\boldsymbol{dt^2}} + \boldsymbol{C}\frac{\boldsymbol{dS}}{dt} + \boldsymbol{KS} - \boldsymbol{F} - \boldsymbol{R} = 0,$$

where $\boldsymbol{M}$ – mass matrix;

$\boldsymbol{C}$ – матрица демпфирования;

$\boldsymbol{K}$ – damping matrix;

$\boldsymbol{F}$ – vector of external forces;

$\boldsymbol{R}$ – reaction vector in hinges.

The elastic model of the helicopter airframe allows you to estimate the SSS during emergency landings. Figure 11 shows the dependence on the maneuver time of stresses in the area of the tail boom junction with the fuselage on the maneuver time for different vertical descent speeds V_y. The critical stress $\sigma_{крит}$ = 220 МПа, at which the destruction of the stringer panels begins, is obtained from the results of static tests of the helicopter airframe.

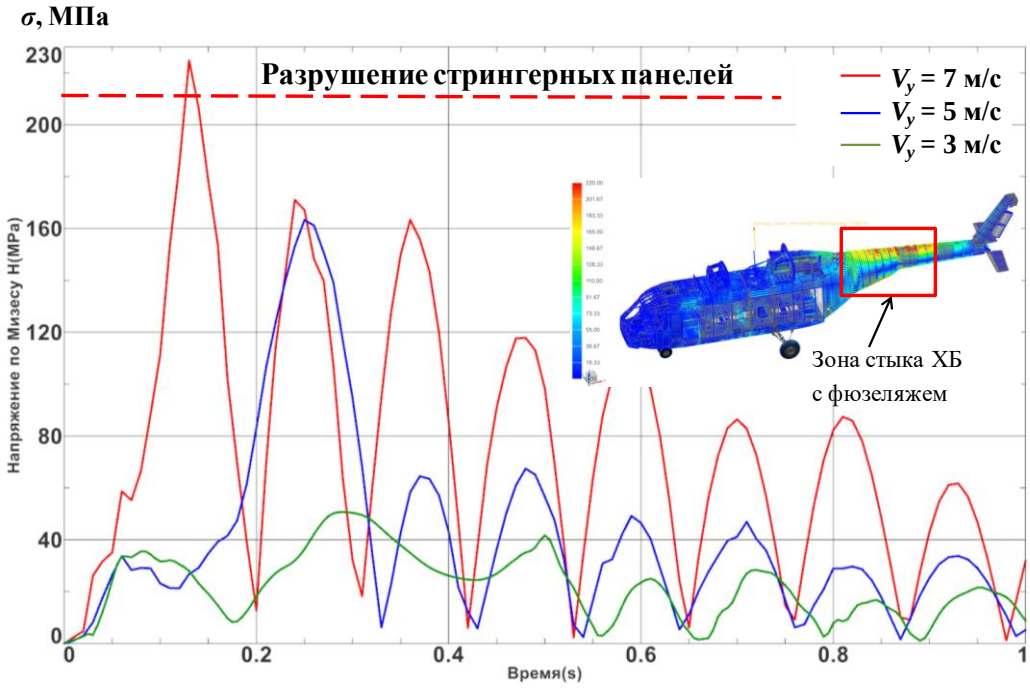

Figure 11 - Stresses in the joint zone of the tail boom with the fuselage for different descent speeds

The analysis of the dependencies shows that at vertical landing speeds $V_y \geq 7$ m/s, the destruction of the helicopter airframe in the area of the tail beam is possible.

Thus, a joint engineering analysis of the behavior of several complex aircraft systems, for example, such as the landing gear and airframe of a helicopter, allows you to get a reliable result at the early stages of design, which reduces the time for product development and certification. The developed methodology can be adopted as a standard in the development of helicopter systems. At the same time, it becomes possible to simulate and investigate critical or emergency situations that cannot be reproduced during flight tests.

In conclusion, it should be noted the following.

The developed mathematical model is used to refine the calculation of loads during landing and running of the helicopter on the ground in various conditions. In the calculations of aerodynamic forces and moments on the MR, a mathematical model developed on the basis of the linear disk vortex theory of the MR, taking into account the influence of the earth, is used, working both in the climb and descent modes.

In the tasks where the glider SSS assessment is carried out, it is necessary to use an elastic model of the helicopter fuselage, which allows you to analyze the SSS in the most loaded places of the structure.

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