The Structural Dynamics of a Free Flying Helicopter in MBS- and FEM-Analysis

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Abstract

Since Multi Body System (MBS) codes have been proved to be potentially powerful simulation tools in the whole range of helicopter rotor dynamics, here the question of modelling the free flying helicopter in a pure MBS as well as in a hybrid FEMBS dynamical simulation model is highlighted. The objective of this research work are modelling techniques for decribing the dynamical behaviour and the struchtural interaction between helicopter rotors — main and tail rotor — and the nacelle of a free flying helicopter. Here the focus lies on the coupling of the rotating structure of the fully elastic main rotor with the non-rotating parts of the body structure via a flexible rotor-nacelle interface. As simulation platform the 9[to] generic model "Helicopter H9" has been developed. Representing the research object for this investigation it serves as a demonstrator model and as dynamic reference configuration for both the MBS and the FEM calculations. The "Helicopter H9" has a five blade main rotor with a diameter of D=16[m], a four blade tail rotor with a diameter of D=2.8[m] and a MTOW of 9118.4[to]. Investigated are modelling techniques for simulating the dynamics of the structural behaviour of the free flying helicopter in the frequency domain. On the MBS side the commecial tool SIMPACK is tested while on the FEM side the scientific rotor code GYRBLAD is used.

For reasons of a better clarification of the rotor-cell coupling effects the center of gravity of the helicopter fuselage exhibits large offsets in all three coordinate directions. As a consequence we get a highly non-symmetrical dynamical system w.r.t the main rotor axis and a rotated principal axes system. By the fact that the main rotor axis does not coincide with any of the three inertial axes all three rigid body rotational modes will be coupled by the main rotor gyroscopic effect. Concerning the specific dynamic coupling effects between rotor and nacelle a survey study with topics like the main rotor suspension (lateral and vertical) or the elasticity of the drive train had been conducted. In systematic variation of the respective stiffness values (over four decades) the results of different parameter studies are presented as numerical results for single constant rotor speeds as well as in frequency fan diagrams for the overall dynamical behaviour under the change of rotor speed. By applying different blade pitch angles the influence of the blade pitch positon on the rotor eigenbehaviour has being tested. By introducing different kinematical and dynamical boundary conditions, cases of stability loss due to ground resonance could be reproduced for the isolated rotor. Even cases of stability loss of the free flying helicopter concerning elastical eigenmodes of the coupeled rotor-nacelle-system — an air resonance type — could be detected in this work.

The validation of the models finally was done by comparing the eigenmodes and the eigenvalue results produced with the two elasto-mechanical methods MBS and FEM. Thus different algorithms and independent tools have been used in the examination. It has been shown that for the non-rotating as well as for the rotating test cases the coupling effects will be reproduced without any restriction in both approaches. Thus the potential of a sophisticated MBS code like SIMPACK as a powerful simulation tool for helicopter dynamics has been demonstrated with respect to the dynamics of the free flying helicopter.

Contents

1 Introduction

In recent time Multi Body System (MBS) codes have found their way into structural analysis within the helicopter industry and the use of commercial MBS tools in the general design and development process seems to become fruitful. These MBS codes combine their inherent property of describing large deflections of the (rigid) structure including full geometric non-linearities with in general high performance time integration algorithms. In combination with special algorithmic features Finite Element Model (FEM) substructures can be incorporated into the MBS model replacing one or several rigid body components. By applying these so called FEMBS techniques consistent elastic properties can be introduced into the structure to any desired amount. Together with these FEMBS structures and additional degrees of freedom added to the — now — hybrid MBS model the total dynamic model can be subjected to any kind of numerical simulation. Thus with MBS and FEM two fundamentally different approaches in structural dynamics can be combined with their respective advantages to potential high power CSD tools.

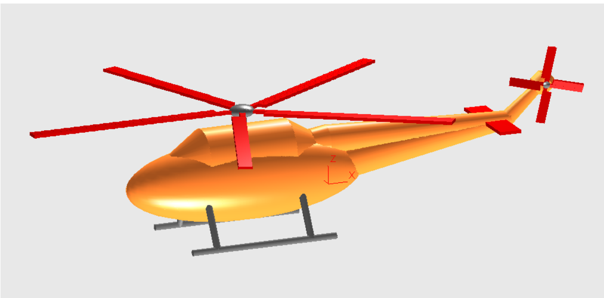


Figure 1: The five-bladed H9 helicopter dynamic demonstrator — *side view*

Since the most MBS codes have not primarily been designed for describing elastic helicopter rotors with their numerous potentially coupling mechanisms, here the question of modelling the free flying helicopter in a MBS dynamical simulation model is highlighted. In order to take into account also the characteristics of the flexible rotor-nacelle interface, the specific modelling features of the MBS code have been used. In this paper these characteristic features have been subjected to a systematic investigation to verify their performance, correctness and reliability. It could be shown that one potential drawback of the MBS approach — the numerical composition of the system matrices in a linearised equation of motion for the consecutive eigenvalue analysis — is in general succsessfully tackled due to numerical high performance differentiating algorithms.

In this paper the structural model of the investigated helicopter demonstrator has been built up parallelly for the three methods MBS, FEM and FEMBS, thus resulting in features independent from each other but with a physical agreement as good as possible. The commercial MBS code SIMPACK has been basically validated by comparisons to the scientific FEM code GYRBLAD, designed for the solution of rotordynamic problems. Additional comparisons have been done to the commercial FEM tool NASTRAN and — in a minor extent — to exact solutions from linearised analytical models.

The effort to demonstrate a low error margin in the results to be compared proved to be successful. Most of the eigenvalue results show a relative error of around 0.1%. To reach values further below this margin would have needed an additional high numerical effort in model resolution. On the other hand error values approaching or passing the 1% margin are to be considered a hint for wrong or physically incomplete modelling on either of the both sides to be compared and has — at least — to be explained thoroughly.

2 Build-up of the "Helicopter H9" dynamic demonstrator

The objectives of this research work are the application and the validation of MBS modelling techniques for decribing on one hand the general dynamical behaviour of a free flying helicopter and on the other hand the struchtural interaction between the helicopter rotors — main and tail rotor — and the nacelle of this free helicopter. In this context the focus lies on the dynamic coupling of the rotating structure of the fully elastic main rotor and the — non-rotating — parts of the body structure. For this purpose a generic model, the 9 [to] "Helicopter H9" (see Fig. 1 until 3), had been developed as simulation platform. Representing the basic research object in this investigation it serves as a strucural demonstrator model and as dynamic reference configuration for both the MBS and the FEM calculations. Since in this investigation the focus was put on the dynamical behaviour of the rotating, elastic structure the influence of the aerodynamic forces had been neglected. Nevertheless one can consider it justified to adress the "free" system also as the "free flying" helicopter since in contrast to a ground fixed system it displays the essential dynamic features like the influence of the fuselage mass or the BC.

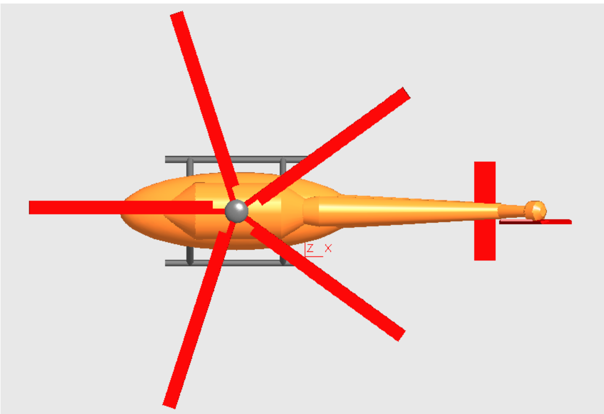


Figure 2: The five-bladed H9 helicopter dynamic demonstrator — *top view*

The "Helicopter H9" has a fully elastic five blade main rotor with a diameter of $D = 16$ [m], a rigid four blade tail rotor with a diameter of $D = 2.8$ [m] and a MTOW of 9118.4 [to]. The rotor transmission ratio had been determined to such a value that the blade tip speed of main and tail rotor will result equal. The main rotor is rigidly mounted on a shaft whose lower end is elastically suspended. As suspension conditions two sets of stiffnesses are available, an isotropic or an orthotropic one. To the basic stiffness values a general factor is to be applied to get a stiffening or a softening effect. In both models (MBS and FEM) discrete springs have been applied in the mounting point with such stiffness values that would cause the respective frequency of a (rigid body) one degree of freedom oscillation. An increase of these basic suspension frequencies by the factor 10 e.g. would thus result in an stiffness increase by the factor 100. These main rotor suspension stiffness sets together with the complete set of the characteristic parameter values for the H9 helicopter are displayed in Tab. 1 and 2.

The helicopter nacelle is modelled as a six degree of freedom rigid body, while the elastic main rotor has up to ~ 1300 degrees of freedom. For reasons of a better clarification and validation of the rotor-cell coupling effects — and to make the task more demanding — the center of gravity of the helicopter fuselage exhibits large offsets. Thus the overall center of gravity of the helicopter does not lie in the sourroundings of the main rotor hub as usual but with distinct offsets in all three coordinate directions. As a consequence we get a highly non-symmetrical dynamical system w.r.t the main rotor axis. It has been experienced that the geometric translation of the system matrices into the coordinate system of the overall center of gravity and the rotation of the coordinate axes about the adequate Euler angles onto the principal axes allow a better control of the rigid body modes. By the fact that now the main rotor axis does not coincide with any of the three inertial axes any longer all three rigid body rotational modes will be coupled by the main rotor gyroscopic effect.

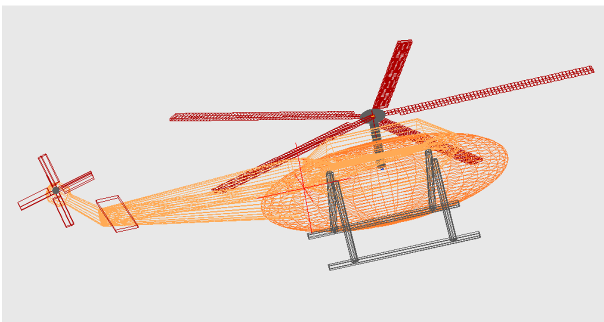


Figure 3: The five-bladed H9 helicopter dynamic demonstrator — rotor shaft, main rotor suspension and the overall CG

total mass	$m_{tot} = 9118.4$	[kg]
position of total CG	$x_{cg} = 2.6320 \quad y_{cg} = -1.7547 \quad z_{cg} = -2.6320$	[m]
total inertia (principal axes)	$\Theta_x = 28724.8 \quad \Theta_y = 64820.2 \quad \Theta_z = 76555.2$	[kgm^2]
Kardan angles (principal axes)	$\alpha = 22.710 \quad \beta = 12.584 \quad \gamma = -11.090$	[°]
Euler angles (principal axes)	$\psi = 16.411 \quad \vartheta = 25.801 \quad \varphi = -30.038$	[°]
fuselage mass	$m_{fus} = 8000.0$	[kg]
position of fuselage CG	$x_{fus} = 3.0 \quad y_{fus} = -2.0 \quad z_{fus} = -3.0$	[m]
fuselage inertia (basic axes)	$\Theta_{xfu} = 7200.0 \quad \Theta_{yfu} = 36000.0 \quad \Theta_{zfu} = 36000.0$	[kgm^2]
5-bladed main rotor mass	$m_{mr} = 1118.4$	[kg]
diameter main rotor	$D_{mr} = 16.0$	[m]
position of main rotor hub	$x_{mr} = 0.0 \quad y_{mr} = 0.0 \quad z_{mr} = 0.0$	[m]
main rotor inertia (basic axes)	$\Theta_{xmr} = 11931.7 \quad \Theta_{ymr} = 11931.7 \quad \Theta_{zmr} = 23862.9$	[kgm^2]
4-bladed tail rotor mass	$m_{tr} = 156.58$	[kg]
diameter tail rotor	$D_{tr} = 2.80$	[m]
position of tail rotor hub	$x_{tr} = 11.540 \quad y_{tr} = -0.400 \quad z_{tr} = -0.300$	[m]
tail rotor inertia (basic axes)	$\Theta_{xtr} = 51.442 \quad \Theta_{ytr} = 102.82 \quad \Theta_{ztr} = 51.442$	[kgm^2]
rotor transmission ratio	$n_{mr}/n_{tr} = \frac{40}{7} = 5.7143$	[–]
position of mr suspension	$x_{smr} = 0.0 \quad y_{smr} = 0.0 \quad z_{smr} = -2.0$	[m]
isotropic main rotor suspension stiffness $/f_s$	$c_u = 2.0 \quad c_v = 2.0 \quad c_w = 3.0$ $c_\alpha = 4.0 \quad c_\beta = 4.0 \quad c_\gamma = 5.0$	[Hz]
orthotropic main rotor suspension stiffness $/f_s$	$c_u = 1.0 \quad c_v = 2.0 \quad c_w = 3.0$ $c_\alpha = 4.0 \quad c_\beta = 5.0 \quad c_\gamma = 6.0$	[Hz]
soft suspension case	$f_s = 1.0$	[–]
medium suspension case	$f_s = 10.0$	[–]
stiff suspension case	$f_s = 100.0$	[–]

Table 1: The basic parameters of the dynamic demonstrator "Helicopter H9"

l	8.0	[m]
b	0.20	[m]
h	0.05	[m]
η	0.843	[–]
E	$71.73 * 10^9$	[N/m^2]
G	$26.90 * 10^9$	[N/m^2]
ν	0.33327	[–]
ρ	2796.0	[kg/m^3]
l/b	40.0	[–]
b/h	4.00	[–]
pitch	$0.0, 15.0, 30.0$	[°]

Table 2: The basic parameters of the homogeneous blade of "Helicopter H9"

Beside the establishing of the principle axes as coordinate axes for the rigid body rotations — the rigid body transversal movement still is aligned along the original basic helicopter coordinate system — additional decoupling measures like small but finite and disparate free body suspensions enable a clear distinction of the rotating and transversal rigid body eigenmodes. As long as the values of these free body suspension stiffnesses remain lower than one tenth of the smallest elastic rotor frequency there will be no significant interaction. (This can easily beeing controlled by a look at the rigid body modes of this "free" system.)

Furthermore the advantage of special joint modelling techniques like the usage of pseudo bodies could be shown. This technique of introducing additional bodies and DOF into the "pure" MBS model could help to reach the aim of placing the required rotor mount positions without any spacial restrictions. (See also [8].)

3 Survey of the single blade and the isolated rotor dynamic eigenbehavior

The validity of the rotor-body coupling in general has been proven in advance by several separate examinations of subsystems like the comparison of the flexible and the rigid isolated rotor. For example the invariance of the *symmetrical* rotor toward *orthogonal* suspension conditions (fuselage mass and hub stiffness effect on boundary conditions) has been verified by comparing the eigenvalues of our refernce system with results of a second rotor system which was built up identical except for the blades beeing mounted to the hub by an offset of $= 25.0^{\circ}$. Both systems rendered (numerically) equal eigenfrequencies. In other precedingly examined control cases like rotor systems with a rigid rotor analytical solutions could be used for verification (See Tab. 3 until 5).

The generic helicopter blade model is originally based on the "Princeton beam" (see also [1]), but has been changed in such a manner that makes it more suitable for the present investigation concerning size, mass allocation and frequency distribution. With choosing MBS and FEM model build-ups two basic approaches for incorporating elastic properties into the rotor blade models are compared with each other. Here the way of mapping the continuously distributed elastic properties of the blade beam structure on dynamic equivalent discrete spring stiffnesses of a "pure" MBS model — in contrast to the strategy of importing separately built upp elastic Finite Element models with modal substructure techniques (FEMBS), thus creating a hybrid MBS model — is demonstrated in order to achieve mechanical equivalent rotor models.

By applying different blade pitch angles the influence of the blade pitch positon on the rotor eigenbehaviour has being tested in this work. The eigenfrequencies of the rotating and the non-rotating single blade are displayed in Tab. 6 until 9 for one single rotor speed and different pich angles. In Fig. 4 the frequency fan diagram of the single blade is shown for these three different pitch angle cases.

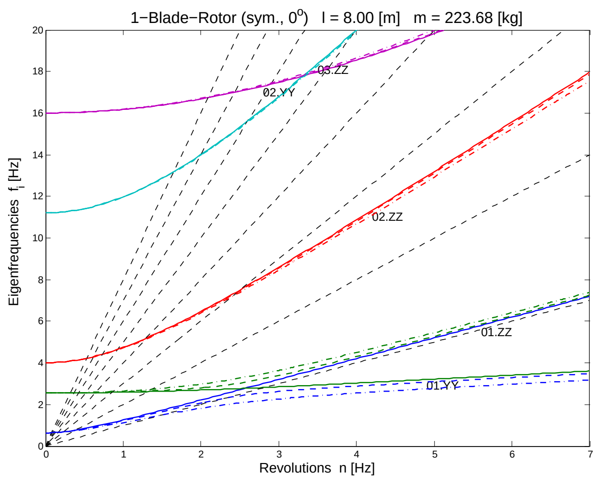


Figure 4: Eigenfrequencies of the single blade (clamped), *pitch angle* $= 0.0^{\circ}$ ——;$= 15.0^{\circ}$ $---$; $= 30.0^{\circ}$ $- \cdot - \cdot -$

The corresponding results are presented for the isolated rotor. The eigenfrequencies of the rotating and the non-rotating isolated rotor with soft isotropic and orthotropic suspension are displayed

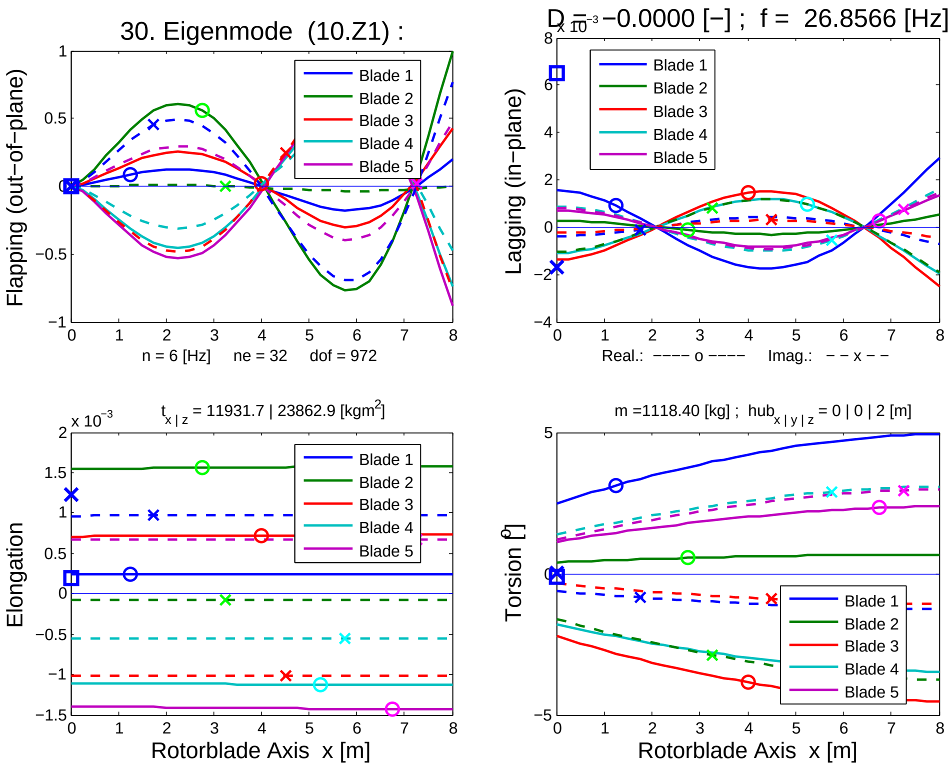


Figure 5: The free H9 helicopter with *rotating* main and tail rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = +34.3$ [Hz], soft orthotropic main rotor suspension): The 30. eigenmode (10. flapping, 26.8566 [Hz]) according to the FEM solution (GYRBLAD) in four component display

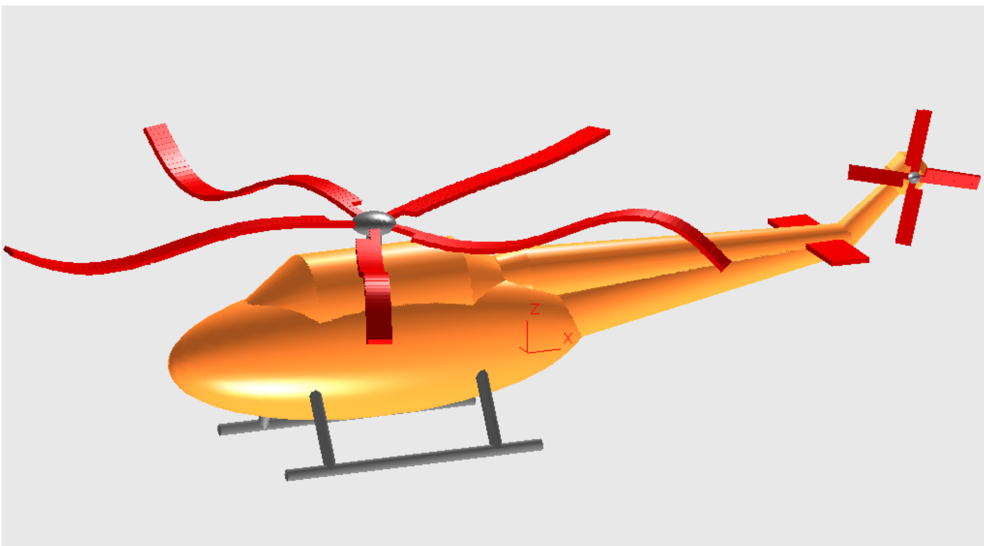


Figure 6: The free H9 helicopter with *rotating* main and tail rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = +34.3$ [Hz], soft orthotropic main rotor suspension): The 30. eigenmode (10. flapping, 26.8667 [Hz]) according to the MBS solution (SIMPACK)

in Tab. 10 until 12 for a single rotor speed and different rotor axis cases. The rotation boundary conditions are different in the sense that the position of the rotor axis is either fixed in space or freely moving withe the rotating body. In Fig. 14 until 17 the fan diagrams of the isolated rotor for these two different rotor axis cases and the two suspension cases (isotropic and orthotropic) are shown. Beside the direct influence of the mounting spring stiffness on the eigenfrequencies compared with the non-rotating case the main difference between the free and the fixed rotor axis cases lies in the subsequent stability behaviour. Large areas with negative damping values occur in the latter case while in the free rotor axis case only small instability bands show up. In the soft orthotropically suspended case with fixed rotor axis (Fig. 17) for rotor speeds between 1 [Hz] and 2 [Hz] even a divergence type of instability occurs by vanishing of the respective eigenfrequency.

4 The free flying system and the dynamic influence of the tail rotor

Investigated are modelling techniques for simulating the dynamics of the structural behaviour of the free flying helicopter in the frequency domain. The results presented are produced by eigenvalue analyses and contain the eigen characteristics like eigenfrequency, natural damping and the eigenmodes. In the case of the MBS simulation code the dynamical model requires a numerical linearisation prior to the eigenvalue analysis. On the MBS side the commecial tool SIMPACK is tested while on the FEM side the scientific tool GYRBLAD is used, which is working on a linear formulation, as well as the imported NASTRAN blade models are linear FEM models.

Results of the eigenvalue analyses of the complete helicopter model are presented in Fig. 7 until 11 as frequency fan diagrams and in Tab. 13 until 19 as eigenvalues (damping and frequency) for a single rotor speed. Eigenmodes for some selected rotor speeds are displayed in Fig. 5, 6, 12 and 13, with the latter ones being instable. To get an impression of the "mere" rotor-nacelle interaction one has to look at the results for the stiff main rotor suspension case, because there the influence of the suspension springs is — at least for the displayed frequency range — almost negligible. In the Tab. 13 until 15 the influence of the rotation of the two rotors on the dynamic behaviour of the free system had been looked at by stepwise "activating" them: First with no rotation at all, then with only the main rotor rotating and finally with main and tail rotor put on.

By lining up the eigenfrequencies in ascending order a clear structure in the results can be perceived: Groups of five very closely neighboured frequencies follow each other where the identical pairs belong to the reactionless modes. These quintuples are based on the eigenfrequencies of the single blade and are eventually disturbed only by rigid body or elastic suspension effects. Obviously the stiffer the rotor suspension is the more pronounced the grouping occurs (and vice versa). Comparing Fig. 4 with the stiff case in Fig. 8 also illustrates the influence of the eigenbehaviour of the underlying rotor blade. Although frequencies of zero belong only to pure rigid body modes a small suspension stiffness does not interfere with the elastisc mode shapes. By hitting the (small) presumable eigenfrequency number in the simulation this value can be used as a control criterion for the quality of the eigensolution (compare e.g. the rigid body values for the non-rotating case, Tab. 13).

Especially in terms of aeroelastic stability the physical completeness of the rotor dynamic model plays a crucial role. By introducing different kinematical and dynamical boundary conditions into it, cases of stability loss due to ground resonance could be produced, as demonstrated for the (elastically mounted) isolated rotor. Even cases of stability loss of the free flying helicopter concerning elastical eigenmodes of the coupled rotor-nacelle-system — an air resonance type — could be detected. Beside the dynamic properties of the elastic rotor blade the design of the main rotor suspension has an essential influence here.

5 Parameter variation of the main rotor suspension and driving shaft stiffness

Concerning the specific dynamic coupling effects between rotor and nacelle a survey study with topics like the main rotor suspension (lateral and vertical), the elasticity of the drive train and the pitch link stiffness has been conducted. Therefore six degrees of freedom in the elastic suspension point of the main rotor can be collocated and adressed at the lower end of the rotorshaft. The pairs of translational and rotational displacement around the $x-$ and the $y-$axis can be used to model the dynamic and kinematic conditions of the rotor mount while the third rotational DOF around the rotor $z-$axis stands for the elastic torsional stiffness of the drive train. Spacial division of these DOF is optional. (The transversal vertical DOF in $z-$direction had been included just for the sake of completeness.) In systematic variation of the respective stiffness values (in three steps over four decades) the results of different parameter studies are presented as numerical results for distinct constant rotor speeds, as well as in fan diagrams (Campbell diagrams) the overall dynamical behaviour with the change of rotor speed is illustrated. The results of the eigenvalue analyses are shown in the figures and tables already mentioned above for the complete free helicopter. When comparing the MBS solution of the first seven eigenvalues of the free helicopter to the FEM or the FEMBS values (Tab. 14 and 15) quite large differences are to be perceived. These deviations in both the damping and the frequency terms have no physical background but are based upon an insufficient numerical differentiation precision in the linearisation process of the MBS model. The results can be improved by increasing the differentiation resolution. (Here the default parameter configuration had been used.) The (small) differences in the blade torsional frequencies are due to the lack of the propeller moment in the MBS blade model.

Like already for the isolated rotor also for the free flying helicopter several instability regions occur over the whole rotor speed range. As a general tendency it can be stated that the softer the main rotor suspension is the more numerous and the more severe instability cases are to be recognised. An influence of the direction of rotation of the tail rotor axis on the stability behaviour could be detected, but was not deeper investigated. Instead the effect of the tail rotor on the eigenfrequencies can be studied in the Tab. 13 until 19. In all proceeding tables the given relative difference of the results represent (roughly) the numerical error in the solutions, but in the last four tables the "difference" values in contrast have a physical meaning: Tab. 16 and 18 give in general (but not "literally") a numerical impression of the influence of the main rotor suspension stiffness (attention: the values are placed in an ascending order without regard to the eventual commutation of the frequency branches). Tab. 17 and 19 show the results for the tail-rotor "off" szenario to be compared with the respective cases with the rotating tail rotor included, one for the positive and one for the negative sense of rotation. The displayed relative difference values give a nice impression of the influence of the rotating tail rotor on the frequency of each (lower) eigenmode.

6 Conclusion

The validation of the models and the proceeding had been done finally by comparing the eigenmodes and the eigenvalue results produced with the two elasto-mechanical methods MBS and FEM. Thus different algorithms and independent tools have been used in the examination. It has been shown that for the non-rotating as well as for the rotating test cases the coupling effects will be reproduced without any restriction in both approaches. The FEMBS modulation of the pure MBS model renders a hybrid formulation which can combine advantages of both sides. Thus the potential of a sophisticated MBS code like SIMPACK as a powerful simulation tool for helicopter dynamics has been demonstrated with respect to the dynamics of the free flying helicopter.

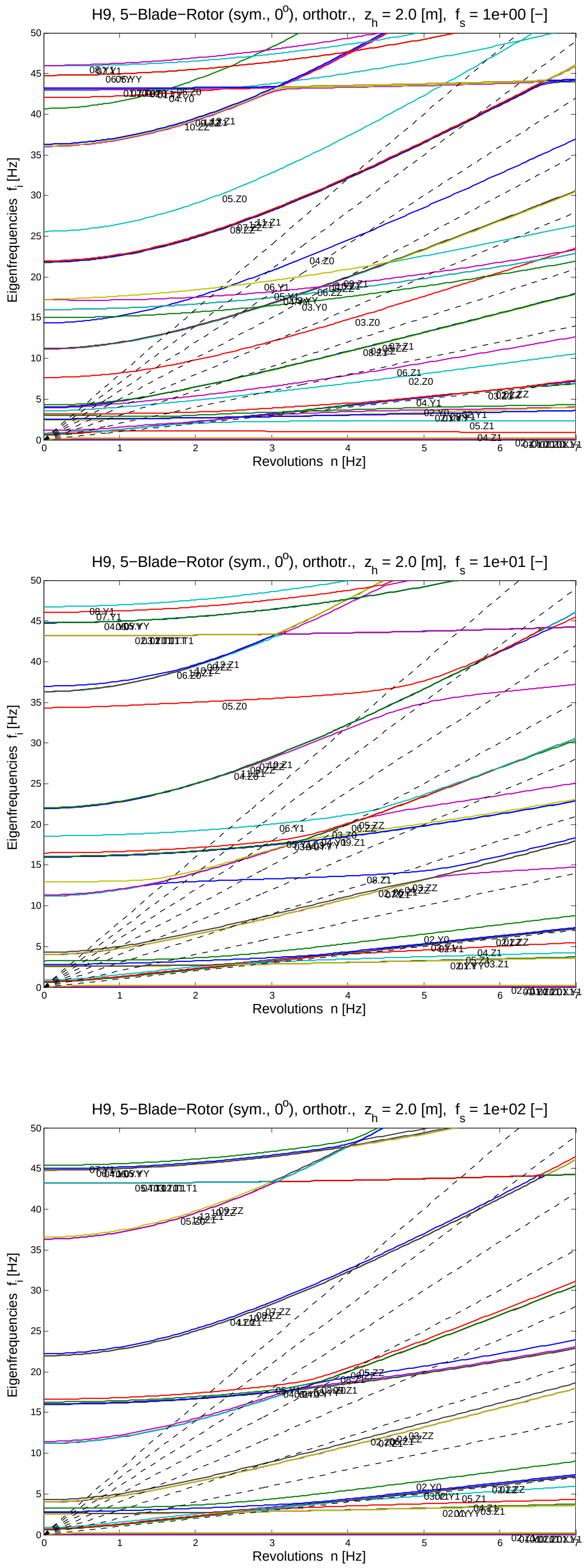


Figure 7: The free flying H9 helicopter: *Eigenfrequencies (upper range)* for the soft ($f_s = 1.$), the medium ($f_s = 10.$) and the stiff ($f_s = 100.$) orthotropic main rotor suspension case (top down)

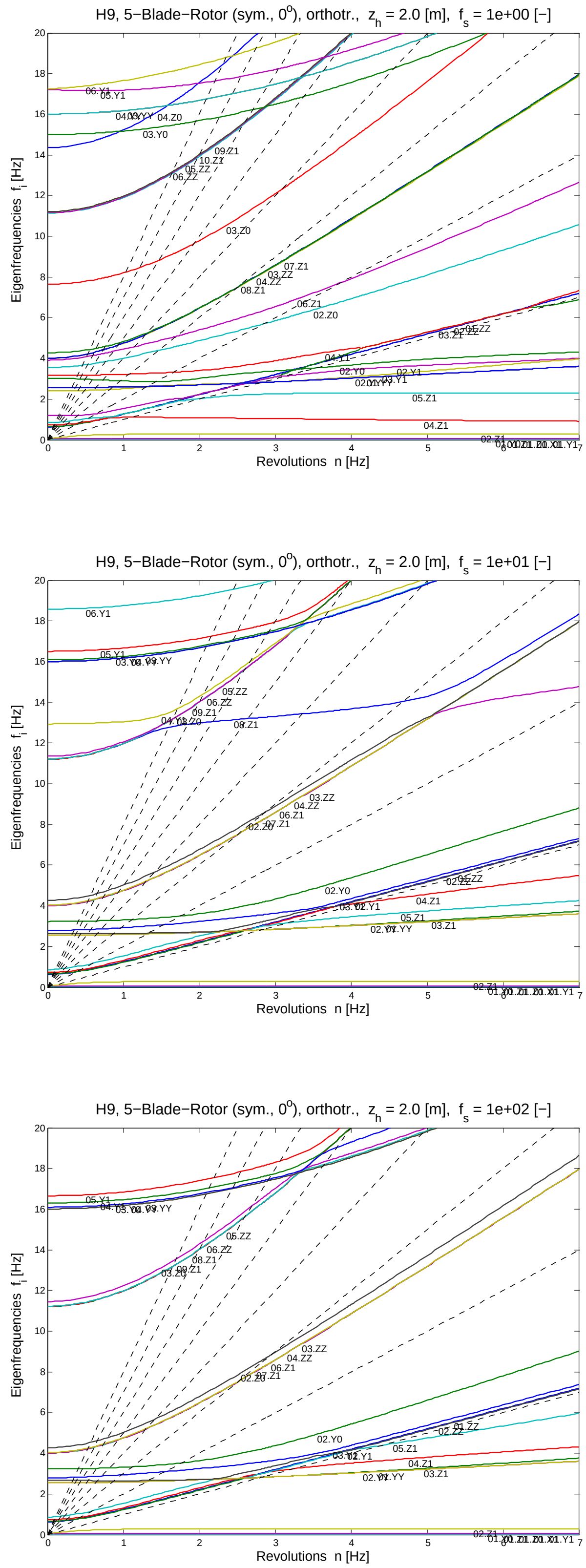
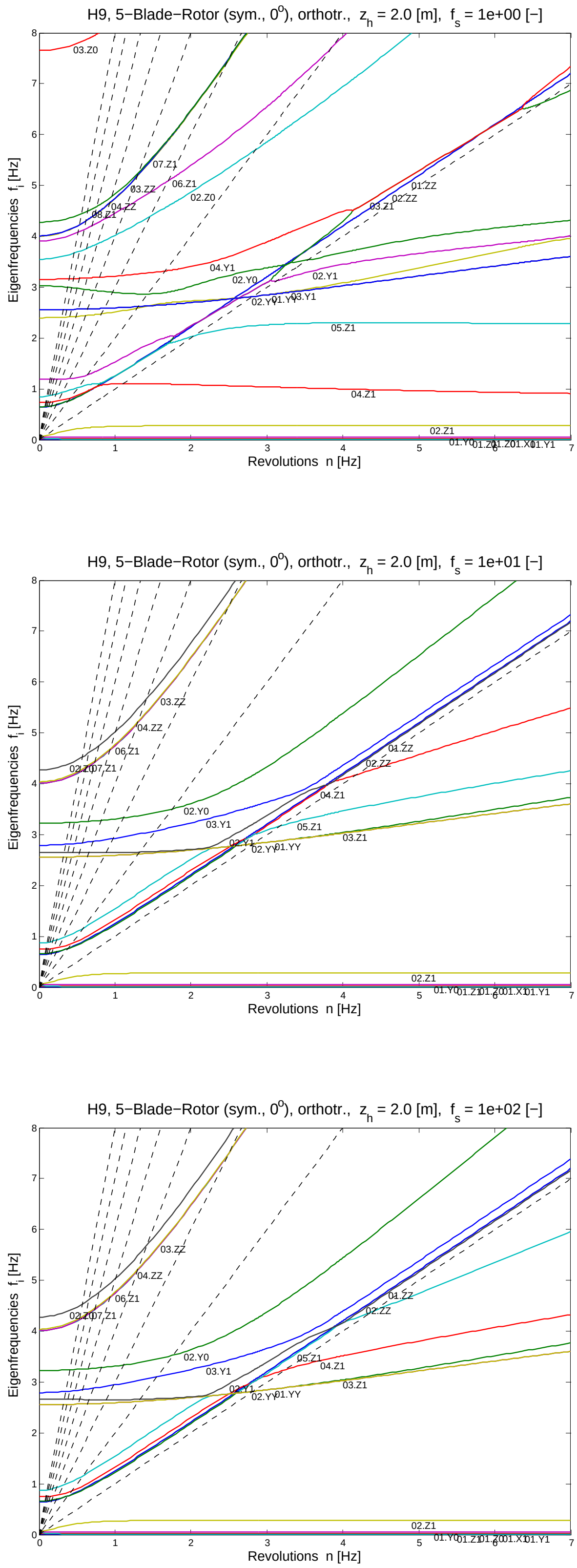


Figure 8: The free flying H9 helicopter: *Eigenfrequencies (medium range)* for the soft ($f_s = 1.$), the medium ($f_s = 10.$) and the stiff ($f_s = 100.$) orthotropic main rotor suspension case (top down)



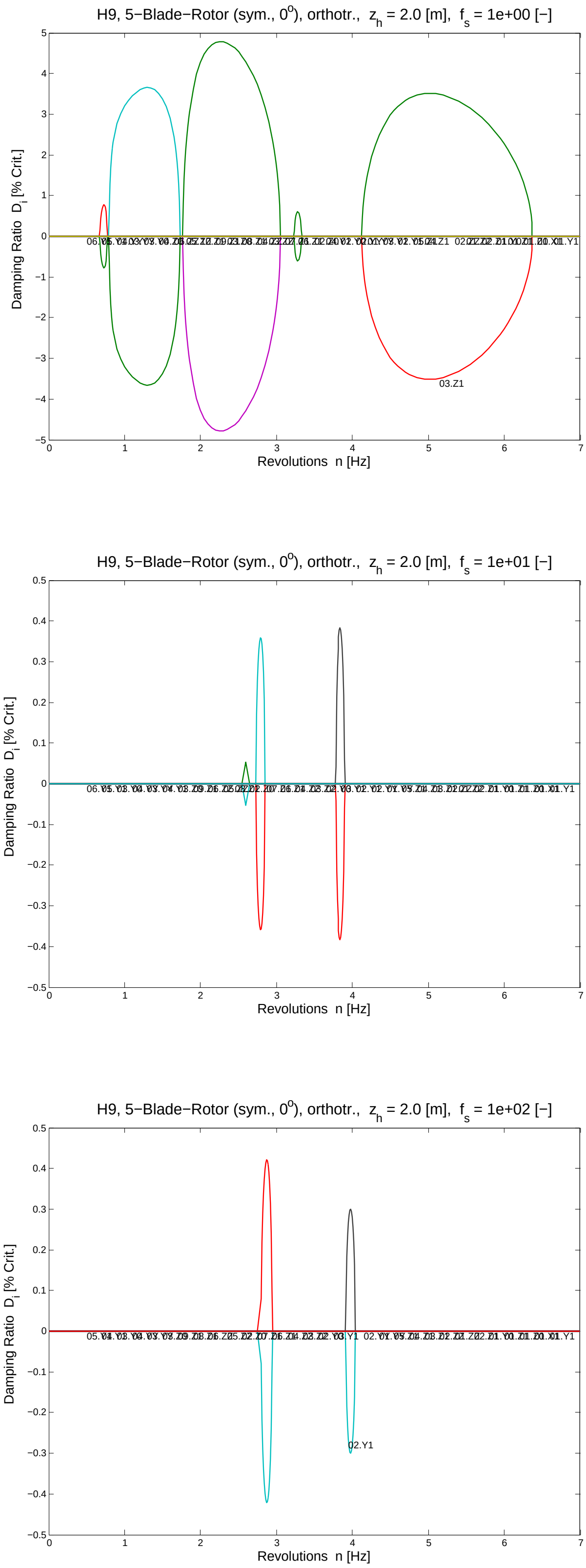
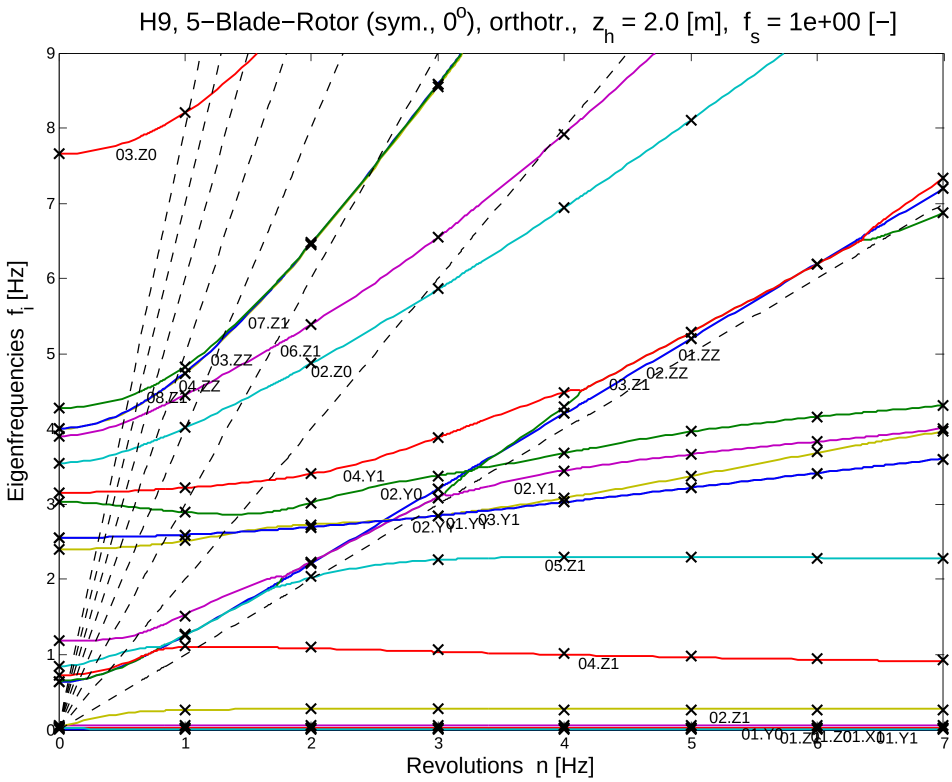


Figure 9: The free flying H9 helicopter: *Eigenfrequencies (lower range)* for the soft ($f_s = 1.$), the medium ($f_s = 10.$) and the stiff ($f_s = 100.$) orthotropic main rotor suspension case (top down)

Figure 10: The free flying H9 helicopter: *Natural Damping* for the soft ($f_s = 1.$), the medium ($f_s = 10.$) and the stiff ($f_s = 100.$) orthotropic main rotor suspension case (top down)



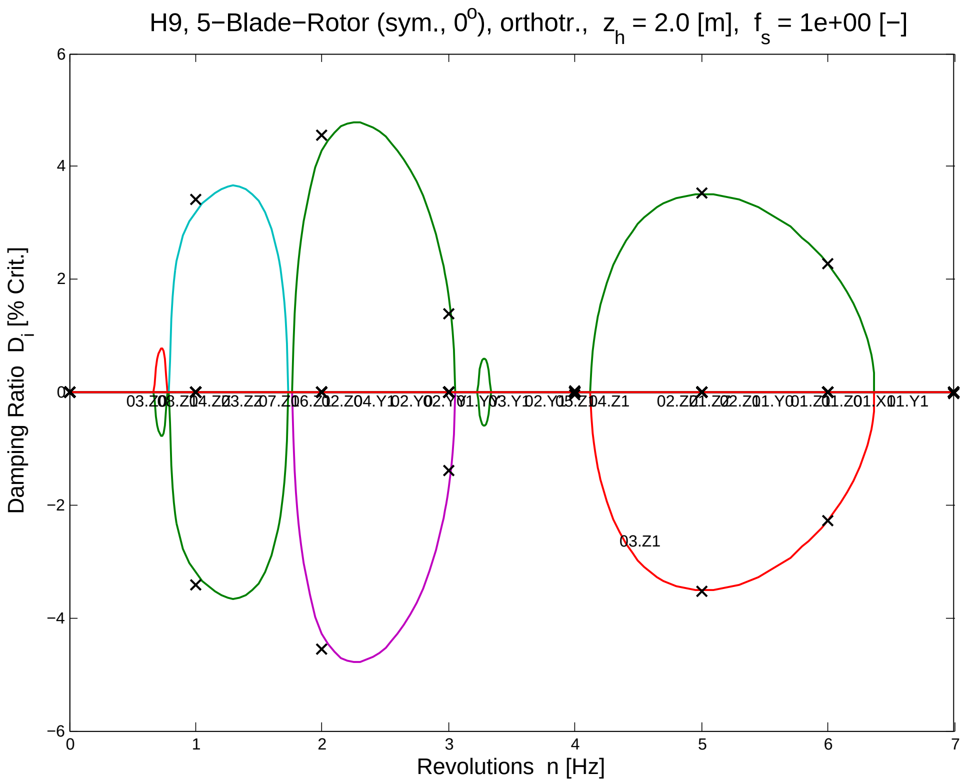


Figure 11: The free flying H9 helicopter: Comparison of the *eigenfrequencies* and the *damping* for the soft main rotor suspension case (x = FEMBS SIMPACK/NASTRAN, — = FEM GYRBLAD)

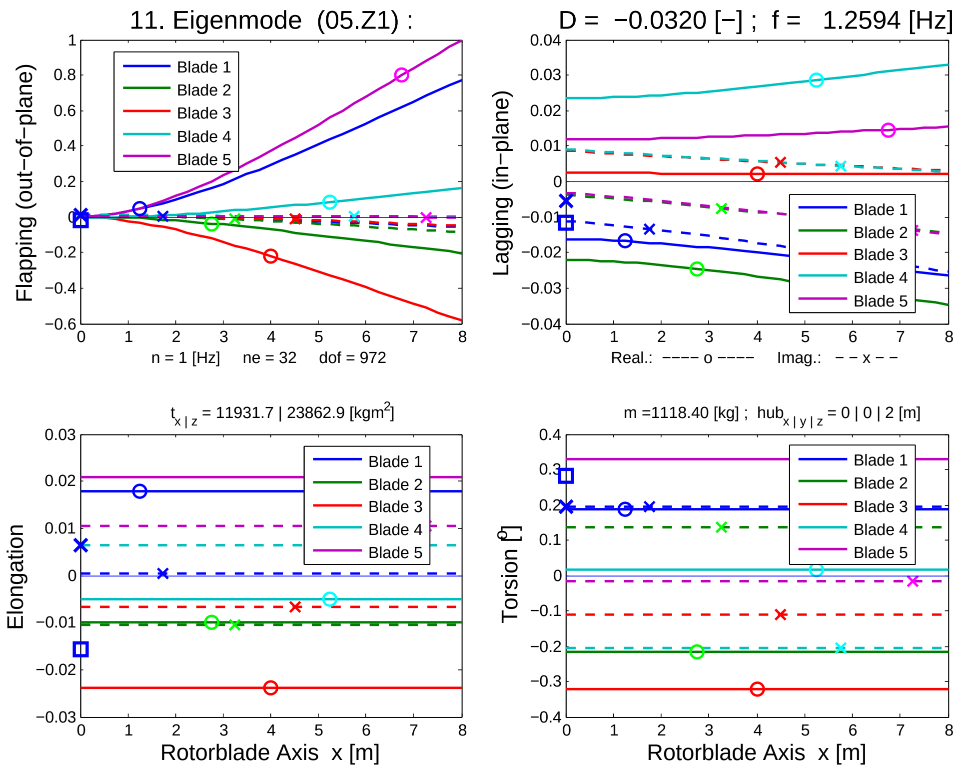


Figure 12: The free H9 helicopter with soft orthotropic main rotor suspension: The 1.26 [Hz] *instable* eigenmode at $n_{mr} = 1.$ [Hz] in four component display

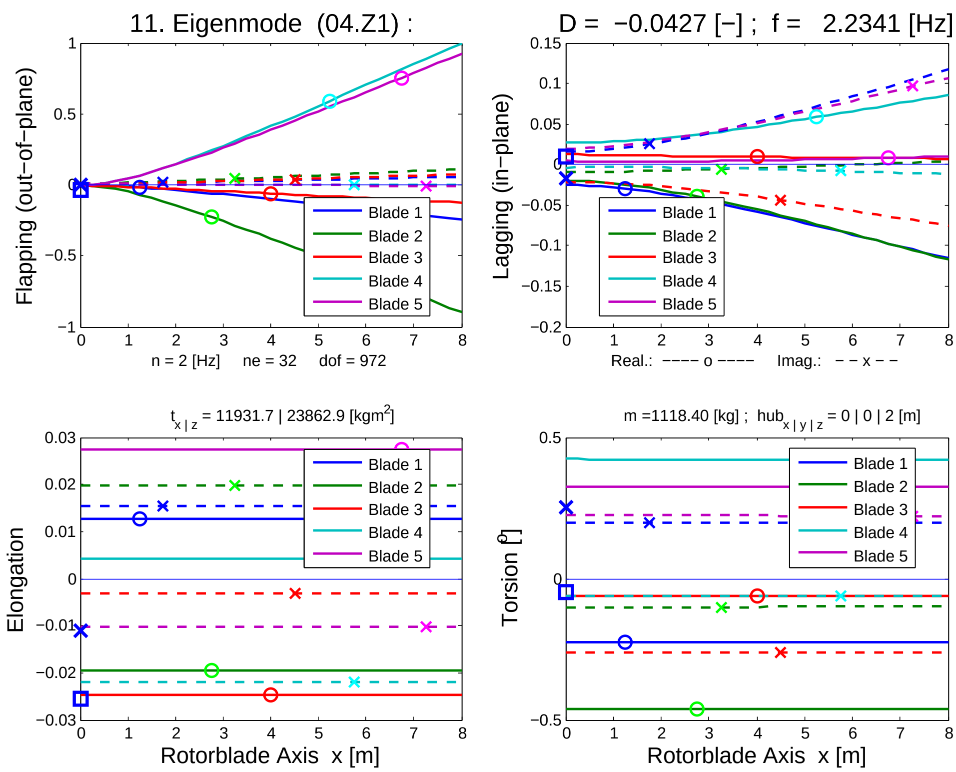


Figure 13: The free H9 helicopter with soft orthotropic main rotor suspension: The 2.23 [Hz] *instable* eigenmode at $n_{mr} = 2.$ [Hz] in four component display

7 Appendix: Additional Figures and Tables

Tab. 3 until 5:	Eigenvalues of the rigid rotor systems
Fig. 14 until 17:	Frequency fan diagrams of the isolated rotor
Tab. 6 until 9:	Eigenvalues of the single blade
Tab. 10 until 12:	Eigenvalues of the isolated rotor
Tab. 13 until 15:	Eigenvalues of the free helicopter
Tab. 16 until 19:	Eigenvalues of parameter variations

8 Copyright Statement

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c_a = .004 [Hz] om = 0 [Hz]		c_a = .004 [Hz] om = 6 [Hz]		c_a = 4.0 [Hz] om = 0 [Hz]		c_a = 4.0 [Hz] om = 6 [Hz]	
Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]
I S O T R O P I C S U P P O R T at z_c = 0.0 [m]							
0.0000	0.0020	0.0000	0.0000	0.0000	2.0000	0.0000	1.2111
0.0000	0.0020	0.0000	0.0020	0.0000	2.0000	0.0000	2.0000
0.0000	0.0030	0.0000	0.0020	0.0000	3.0000	0.0000	2.0000
0.0000	0.0040	0.0000	0.0030	0.0000	4.0000	0.0000	3.0000
0.0000	0.0040	0.0000	0.0050	0.0000	4.0000	0.0000	5.0000
0.0000	0.0050	0.0000	11.9998	0.0000	5.0000	0.0000	13.2109
O R T H O T R O P I C S U P P O R T at z_c = 0.0 [m]							
0.0000	0.0010	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000
0.0000	0.0020	0.0000	0.0010	0.0000	2.0000	0.0000	1.4792
0.0000	0.0030	0.0000	0.0020	0.0000	3.0000	0.0000	2.0000
0.0000	0.0040	0.0000	0.0030	0.0000	4.0000	0.0000	3.0000
0.0000	0.0050	0.0000	0.0060	0.0000	5.0000	0.0000	6.0000
0.0000	0.0060	0.0000	11.9998	0.0000	6.0000	0.0000	13.5206

Table 3: The isolated main rotor as rigid body: Six DOF suspension in the *hub point* ($z_c = 0.0$ [m])

c_a = .004 [Hz] om = 0 [Hz]		c_a = .004 [Hz] om = 6 [Hz]		c_a = 4.0 [Hz] om = 0 [Hz]		c_a = 4.0 [Hz] om = 6 [Hz]	
Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]
I S O T R O P I C S U P P O R T at z_c = -2. [m]							
0.0000	0.0019	0.0000	0.0000	0.0000	1.8893	0.0000	1.1584
0.0000	0.0019	0.0000	0.0020	0.0000	1.8893	0.0000	1.9592
0.0000	0.0030	0.0000	0.0020	0.0000	3.0000	0.0000	2.1175
0.0000	0.0042	0.0000	0.0030	0.0000	4.2344	0.0000	3.0000
0.0000	0.0042	0.0000	0.0050	0.0000	4.2344	0.0000	5.0000
0.0000	0.0050	0.0000	11.9998	0.0000	5.0000	0.0000	13.3165
O R T H O T R O P I C S U P P O R T at z_c = -2. [m]							
0.0000	0.0010	0.0000	0.0000	0.0000	0.9923	0.0000	0.9872
0.0000	0.0019	0.0000	0.0010	0.0000	1.8893	0.0000	1.4284
0.0000	0.0030	0.0000	0.0020	0.0000	3.0000	0.0000	2.0884
0.0000	0.0042	0.0000	0.0030	0.0000	4.2344	0.0000	3.0000
0.0000	0.0050	0.0000	0.0060	0.0000	5.0389	0.0000	6.0000
0.0000	0.0060	0.0000	11.9998	0.0000	6.0000	0.0000	13.5828

Table 4: The isolated main rotor as rigid body: Six DOF suspension in the *shaft base point* ($z_c = -2.0$ [m])

	c_a = 4 [Hz]		c_a = 40 [Hz]		c_a = 400 [Hz]	
	Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]	Nat.Damp. [-]	Frequency [Hz]
	om = 0 [Hz]					
1	0.0000	0.0100	0.0000	0.0100	0.0000	0.0100
2	0.0000	0.0200	0.0000	0.0200	0.0000	0.0200
3	0.0000	0.0300	0.0000	0.0300	0.0000	0.0300
4	0.0000	0.0400	0.0000	0.0400	0.0000	0.0400
5	0.0000	0.0500	0.0000	0.0500	0.0000	0.0500
6	0.0000	0.0600	0.0000	0.0600	0.0000	0.0600
7	0.0000	5.9848	0.0000	59.8469	0.0000	598.4688
8	0.0000	6.9402	0.0000	69.3995	0.0000	693.9949
9	0.0000	7.4772	0.0000	74.7707	0.0000	747.7068
	om = 6 [Hz]					
1	0.0000	0.0006	0.0000	0.0006	0.0000	0.0006
2	0.0000	0.0100	0.0000	0.0100	0.0000	0.0100
3	0.0000	0.0200	0.0000	0.0200	0.0000	0.0200
4	0.0000	0.0300	0.0000	0.0300	0.0000	0.0300
5	0.0000	0.0585	0.0000	0.0585	0.0000	0.0585
6	0.0000	2.1853	0.0000	3.2140	0.0000	3.2322
7	0.0000	5.2292	0.0000	59.4026	0.0000	598.4203
8	0.0000	7.3617	0.0000	70.1411	0.0000	694.0752
9	0.0000	11.9363	0.0000	74.9581	0.0000	747.7235
	om = 6 [Hz] + TR (40:7)					
1	0.0000	0.0006	0.0000	0.0006	0.0000	0.0006
2	0.0000	0.0100	0.0000	0.0100	0.0000	0.0100
3	0.0000	0.0200	0.0000	0.0200	0.0000	0.0200
4	0.0000	0.0300	0.0000	0.0300	0.0000	0.0300
5	0.0000	0.0586	0.0000	0.0587	0.0000	0.0587
6	0.0000	2.1902	0.0000	3.2213	0.0000	3.2395
7	0.0000	5.2304	0.0000	59.4027	0.0000	598.4203
8	0.0000	7.3656	0.0000	70.1457	0.0000	694.0756
9	0.0000	11.9315	0.0000	74.9533	0.0000	747.7231

Table 5: The free helicopter with rigid main rotor: Orthotropic soft, medium and rigid three DOF (c_α, c_β, c_γ) suspension in the *shaft base point* ($z_c = -2.0$ [m])

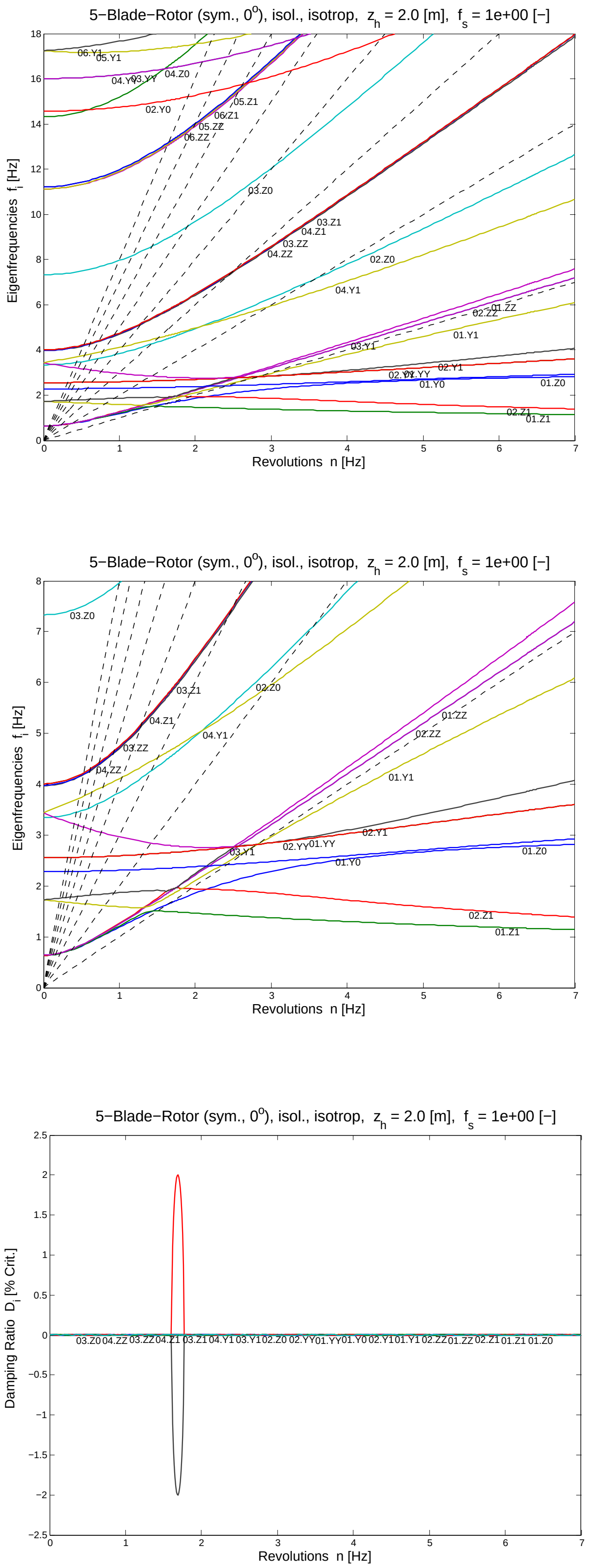

Figure 14: The rotating 5-bladed isolated rotor ($z_h = 2$ [m]) with soft *isotropic* suspension (*free* rotor axis): Eigenfrequencies and damping

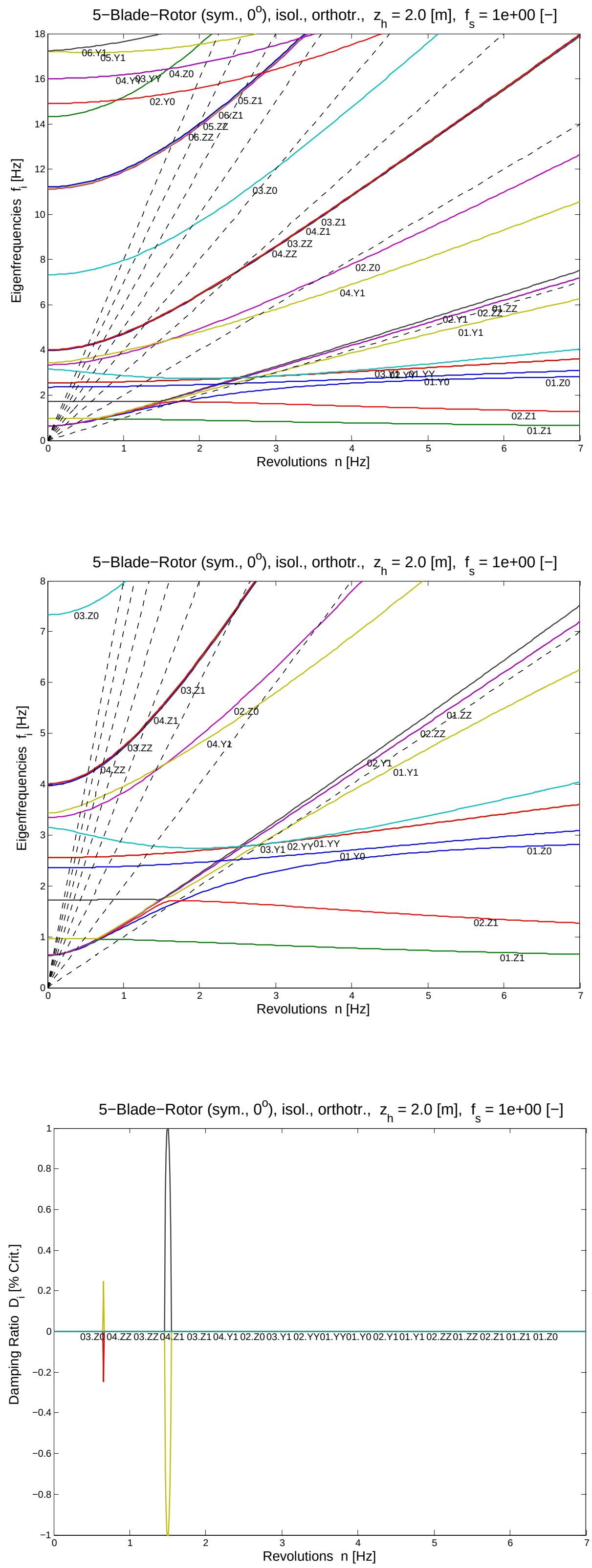

Figure 15: The rotating 5-bladed isolated rotor ($z_h = 2$ [m]) with soft *orthotropic* suspension (*free* rotor axis): Eigenfrequencies and damping

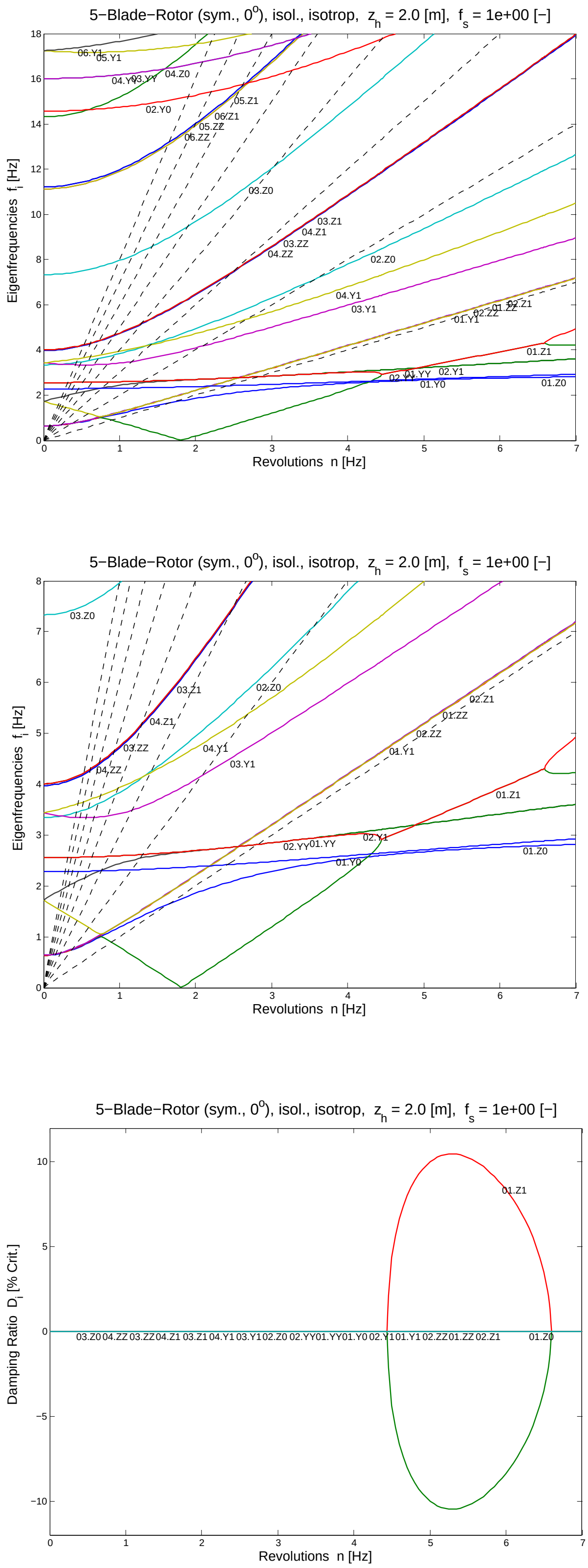

Figure 16: The rotating 5-bladed isolated rotor ($z_h = 2$ [m]) with soft *isotropic* suspension (*fixed* rotor axis): Eigenfrequencies and damping

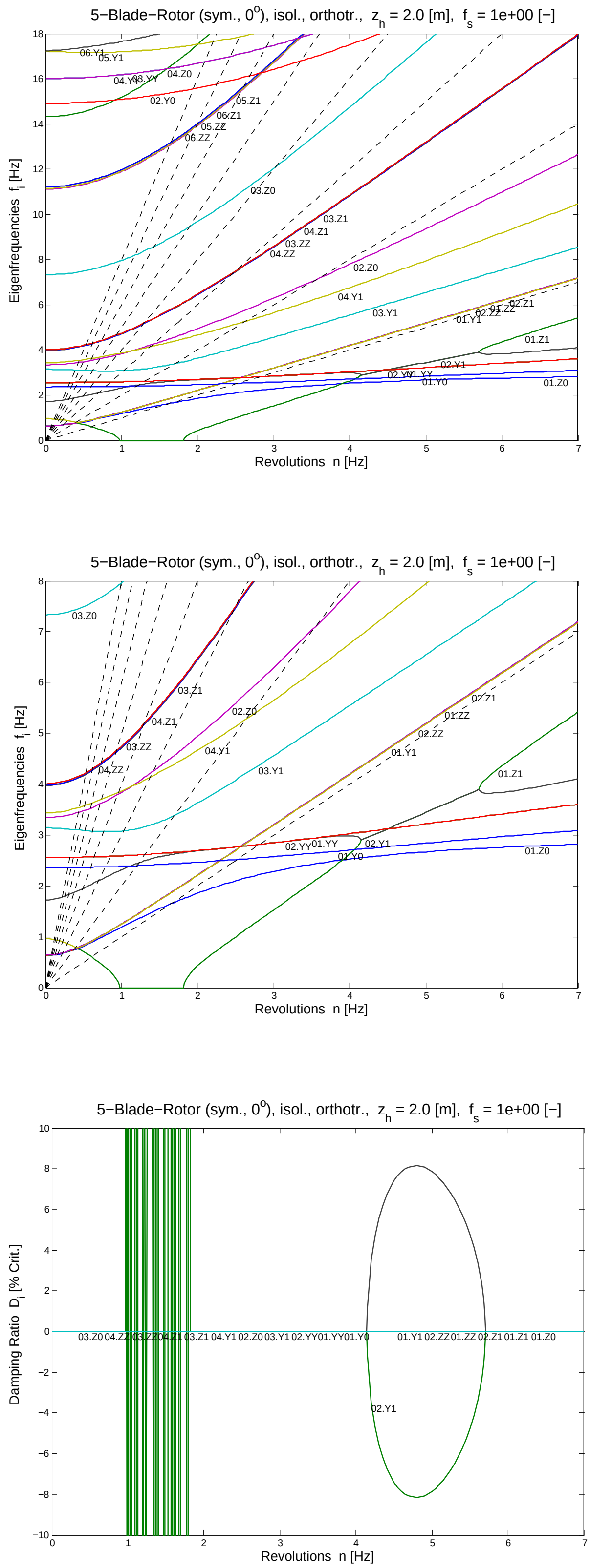

Figure 17: The rotating 5-bladed isolated rotor ($z_h = 2$ [m]) with soft *orthotropic* suspension (*fixed* rotor axis): Eigenfrequencies and damping

SINGLE BLADE : clamped n = 0 [Hz] pitch = 00. [o]									
	FEM (32 Elm.)			MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]
1.F	.000	0.6392	0.00	.000	0.6393	0.02	.000	0.6389	-0.05
1.L	-.000	2.5566	0.00	.000	2.5567	0.00	.000	2.5558	-0.03
2.F	-.000	4.0057	0.00	.000	4.0065	0.02	.000	4.0053	-0.01
3.F	-.000	11.2154	0.00	.000	11.2189	0.03	.000	11.2130	-0.02
2.L	.000	16.0103	0.00	.000	16.0134	0.02	.000	15.9878	-0.14
4.F	-.000	21.9754	0.00	.000	21.9850	0.04	.000	21.9670	-0.04
5.F	-.000	36.3226	0.00	.000	36.3424	0.05	.000	36.3004	-0.06
1.T	.000	43.1738	0.00	.000	43.1684	-0.01	.000	43.1738	0.00
3.L	.000	44.7771	0.00	.000	44.7911	0.03	.000	44.6287	-0.33
6.F	-.000	54.2521	0.00	.000	54.2867	0.06	.000	54.2042	-0.09
7.F	.000	75.7624	0.00	.000	75.8157	0.07	.000	75.6713	-0.12
4.L	.000	87.5968	0.00	.000	87.6342	0.04	.000	87.0667	-0.61
8.F	-.000	100.8521	0.00	.000	100.9262	0.07	.000	100.6944	-0.16
9.F	.000	129.5210	0.00	.000	129.4792	-0.03	.000	129.2664	-0.20
2.T	.000	129.6255	0.00	.000	129.6141	-0.01	.000	129.6255	0.00
5.L	.000	144.4883	0.00	.000	144.5635	0.05	.000	143.1133	-0.95
1.S	.000	158.2980	0.00	.000	158.2784	-0.01	.000	158.2980	0.00
10.F	-.000	161.7701	0.00	.000	161.8742	0.06	.000	161.3813	-0.24
11.F	-.000	197.6026	0.00	.000	197.7003	0.05	.000	197.0349	-0.29
6.L	.000	215.2665	0.00	.000	215.3922	0.06	.000	212.3310	-1.36
3.T	-.000	216.3902	0.00	.000	215.7120	-0.31	.000	216.3896	-0.00

Table 6: The clamped *non-rotating* single blade ($n = 0.0$ [Hz]): The eigenvalues according to the three methods compared to the FEM solution

SINGLE BLADE : clamped n = 6 [Hz] pitch = 00. [o]									
	FEM (32 Elm.)			MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]
1.L	-.000	3.4146	0.00	.000	3.4153	0.02	.000	3.4087	-0.17
1.F	-.000	6.1988	0.00	.000	6.2017	0.05	.000	6.1976	-0.02
2.F	.000	15.5739	0.00	.000	15.5734	-0.00	.000	15.5695	-0.03
2.L	.000	21.2790	0.00	.000	21.2811	0.01	.000	21.2654	-0.06
3.F	.000	26.9831	0.00	.000	26.9904	0.03	.000	26.9779	-0.02
4.F	.000	41.2738	0.00	.000	41.3115	0.09	.000	41.2705	-0.01
1.T	-.000	43.9811	0.00	.000	43.5348	-1.01	.000	43.5800	-0.91
3.L	-.000	51.0314	0.00	.000	51.0508	0.04	.000	50.9205	-0.22
5.F	.000	58.3910	0.00	.000	58.4789	0.15	.000	58.3787	-0.02
6.F	-.000	78.4173	0.00	.000	78.5755	0.20	.000	78.3930	-0.03
4.L	-.000	94.4371	0.00	.000	94.4902	0.06	.000	93.9896	-0.47
7.F	.000	101.5047	0.00	.000	101.7531	0.24	.000	101.4554	-0.05
8.F	-.000	127.7888	0.00	.000	128.1458	0.28	.000	127.6975	-0.07
2.T	-.000	130.8052	0.00	.000	129.6019	-0.92	.000	130.6701	-0.10
5.L	.000	151.6450	0.00	.000	151.7493	0.07	.000	150.4141	-0.81
9.F	-.000	157.3732	0.00	.000	157.8535	0.31	.000	157.2099	-0.10
1.S	-.000	158.6410	0.00	.000	158.6220	-0.01	.000	158.6410	0.00
10.F	-.000	190.3332	0.00	.000	190.9452	0.32	.000	190.0669	-0.14
3.T	.000	218.1916	0.00	.000	215.7856	-1.10	.000	218.1136	-0.04
6.L	.000	222.6136	0.00	.000	222.7861	0.08	.000	219.8991	-1.22
11.F	-.000	226.7246	0.00	.000	227.4678	0.33	.000	226.3136	-0.18

Table 7: The clamped *rotating* single blade ($n = 6.0$ [Hz], pitch angle $= 0.°$): The eigenvalues according to the three methods compared to the FEM solution

SINGLE BLADE : clamped n = 6 [Hz] pitch = 15. [o]									
	FEM (32 Elm.)			MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]
1.L	-.000	3.2964	0.00	.000	3.2976	0.04	.000	3.2910	-0.16
1.F	.000	6.2624	0.00	.000	6.2650	0.04	.000	6.2611	-0.02
2.F	.000	15.4838	0.00	.000	15.4835	-0.00	.000	15.4798	-0.03
2.L	.000	21.3449	0.00	.000	21.3468	0.01	.000	21.3313	-0.06
3.F	.000	26.9371	0.00	.000	26.9447	0.03	.000	26.9327	-0.02
4.F	.000	41.2440	0.00	.000	41.2820	0.09	.000	41.2414	-0.01
1.T	.000	43.9328	0.00	.000	43.4859	-1.02	.000	43.5803	-0.80
3.L	.000	51.0558	0.00	.000	51.0752	0.04	.000	50.9447	-0.22
5.F	-.000	58.3699	0.00	.000	58.4581	0.15	.000	58.3587	-0.02
6.F	-.000	78.4016	0.00	.000	78.5600	0.20	.000	78.3784	-0.03
4.L	-.000	94.4503	0.00	.000	94.5033	0.06	.000	94.0026	-0.47
7.F	.000	101.4924	0.00	.000	101.7411	0.25	.000	101.4444	-0.05
8.F	-.000	127.7790	0.00	.000	128.1364	0.28	.000	127.6889	-0.07
2.T	.000	130.7891	0.00	.000	129.5854	-0.92	.000	130.6702	-0.09
5.L	.000	151.6534	0.00	.000	151.7583	0.07	.000	150.4223	-0.81
9.F	.000	157.3652	0.00	.000	157.8458	0.31	.000	157.2031	-0.10
1.S	.000	158.6409	0.00	.000	158.6207	-0.01	.000	158.6409	0.00
10.F	.000	190.3265	0.00	.000	190.9389	0.32	.000	190.0616	-0.14
3.T	-.000	218.1819	0.00	.000	215.7755	-1.10	.000	218.1130	-0.03
6.L	.000	222.6193	0.00	.000	222.7919	0.08	.000	219.9050	-1.22
11.F	-.000	226.7189	0.00	.000	227.4624	0.33	.000	226.3094	-0.18

Table 8: The clamped *rotating* single blade ($n = 6.0$ [Hz], pitch angle $= 15.^{\circ}$): The eigenvalues according to the three methods compared to the FEM solution

SINGLE BLADE : clamped n = 6 [Hz] pitch = 30. [o]									
	FEM (32 Elm.)			MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]
1.L	-.000	2.9811	0.00	.000	2.9837	0.09	.000	2.9757	-0.18
1.F	-.000	6.4185	0.00	.000	6.4205	0.03	.000	6.4170	-0.02
2.F	-.000	15.2462	0.00	.000	15.2462	0.00	.000	15.2432	-0.02
2.L	-.000	21.5158	0.00	.000	21.5173	0.01	.000	21.5019	-0.06
3.F	-.000	26.8120	0.00	.000	26.8200	0.03	.000	26.8094	-0.01
4.F	.000	41.1625	0.00	.000	41.2014	0.09	.000	41.1619	-0.00
1.T	-.000	43.8007	0.00	.000	43.3521	-1.02	.000	43.5813	-0.50
3.L	-.000	51.1221	0.00	.000	51.1415	0.04	.000	51.0105	-0.22
5.F	.000	58.3126	0.00	.000	58.4013	0.15	.000	58.3041	-0.01
6.F	.000	78.3587	0.00	.000	78.5178	0.20	.000	78.3386	-0.03
4.L	.000	94.4861	0.00	.000	94.5389	0.06	.000	94.0381	-0.47
7.F	-.000	101.4592	0.00	.000	101.7085	0.25	.000	101.4144	-0.04
8.F	.000	127.7523	0.00	.000	128.1104	0.28	.000	127.6653	-0.07
2.T	-.000	130.7448	0.00	.000	129.5406	-0.92	.000	130.6705	-0.06
5.L	-.000	151.6762	0.00	.000	151.7807	0.07	.000	150.4446	-0.81
9.F	-.000	157.3434	0.00	.000	157.8246	0.31	.000	157.1846	-0.10
1.S	.000	158.6406	0.00	.000	158.6210	-0.01	.000	158.6408	0.00
10.F	-.000	190.3084	0.00	.000	190.9214	0.32	.000	190.0472	-0.14
3.T	.000	218.1552	0.00	.000	215.7480	-1.10	.000	218.1114	-0.02
6.L	-.000	222.6349	0.00	.000	222.8078	0.08	.000	219.9212	-1.22
11.F	-.000	226.7037	0.00	.000	227.4477	0.33	.000	226.2980	-0.18

Table 9: The clamped *rotating* single blade ($n = 6.0$ [Hz], pitch angle $= 30.^{\circ}$): The eigenvalues according to the three methods compared to the FEM solution

ISOL.ROT.: c_u = 1 [Hz] z_h = 2 [m] n_mr = 0 [Hz] Rot. Axis = ./.

	FEM (32 Elm.)		MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequen. [Hz]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]
1	.0000	0.6304	.0000	0.6304	0.00	.0000	0.6301	-0.05
2	-.0000	0.6335	.0000	0.6335	0.00	.0000	0.6332	-0.05
3	-.0000	0.6355	.0000	0.6356	0.02	.0000	0.6353	-0.03
4	-.0000	0.6392	.0000	0.6393	0.02	.0000	0.6389	-0.05
5	.0000	0.6392	.0000	0.6393	0.02	.0000	0.6389	-0.05
6	-.0000	0.9698	.0000	0.9699	0.01	.0000	0.9698	0.00
7	-.0000	1.7262	.0000	1.7262	0.00	.0000	1.7260	-0.01
8	.0000	2.3565	.0000	2.3566	0.00	.0000	2.3559	-0.03
9	.0000	2.5566	.0000	2.5567	0.00	.0000	2.5558	-0.03
10	-.0000	2.5566	.0000	2.5567	0.00	.0000	2.5558	-0.03
11	-.0000	3.1445	.0000	3.1446	0.00	.0000	3.1439	-0.02
12	-.0000	3.3446	.0000	3.3452	0.02	.0000	3.3444	-0.01
13	.0000	3.4332	.0000	3.4332	0.00	.0000	3.4329	-0.01
14	.0000	3.9749	.0000	3.9757	0.02	.0000	3.9745	-0.01
15	-.0000	3.9833	.0000	3.9841	0.02	.0000	3.9829	-0.01
16	-.0000	4.0057	.0000	4.0065	0.02	.0000	4.0053	-0.01
17	.0000	4.0057	.0000	4.0065	0.02	.0000	4.0053	-0.01
18	.0000	7.3307	.0000	7.3308	0.00	.0000	7.3304	-0.00
19	-.0000	11.1245	.0000	11.1279	0.03	.0000	11.1222	-0.02
20	-.0000	11.1530	.0000	11.1564	0.03	.0000	11.1507	-0.02
21	-.0000	11.2154	.0000	11.2189	0.03	.0000	11.2130	-0.02
22	.0000	11.2154	.0000	11.2189	0.03	.0000	11.2130	-0.02
23	-.0000	14.3223	.0000	14.3214	-0.01	.0000	14.3209	-0.01
24	-.0000	14.9097	.0000	14.9119	0.01	.0000	14.8936	-0.11
25	.0000	16.0103	.0000	16.0134	0.02	.0000	15.9878	-0.14
26	-.0000	16.0103	.0000	16.0134	0.02	.0000	15.9878	-0.14
27	-.0000	17.2190	.0000	17.2217	0.02	.0000	17.1979	-0.12
28	.0000	17.2372	.0000	17.2398	0.02	.0000	17.2162	-0.12
29	-.0000	21.7989	.0000	21.8079	0.04	.0000	21.7908	-0.04
30	.0000	21.8541	.0000	21.8633	0.04	.0000	21.8459	-0.04
31	.0000	21.9754	.0000	21.9850	0.04	.0000	21.9670	-0.04
32	-.0000	21.9754	.0000	21.9850	0.04	.0000	21.9670	-0.04
33	-.0000	25.6059	.0000	25.6046	-0.01	.0000	25.5996	-0.02
34	-.0000	36.0282	.0000	36.0465	0.05	.0000	36.0070	-0.06
35	.0000	36.1208	.0000	36.1396	0.05	.0000	36.0993	-0.06
36	-.0000	36.3226	.0000	36.3424	0.05	.0000	36.3004	-0.06
37	.0000	36.3226	.0000	36.3424	0.05	.0000	36.3004	-0.06
38	.0000	40.6981	.0000	40.6956	-0.01	.0000	40.6799	-0.04
39	-.0000	42.0198	.0000	42.0285	0.02	.0000	41.9125	-0.26
40	.0000	42.9630	.0000	42.9577	-0.01	.0000	42.9629	-0.00
41	.0000	43.0271	.0000	43.0218	-0.01	.0000	43.0271	0.00
42	.0000	43.1737	.0000	43.1684	-0.01	.0000	43.1738	0.00
43	-.0000	43.1737	.0000	43.1684	-0.01	.0000	43.1738	0.00
44	.0000	43.1737	.0000	43.1684	-0.01	.0000	43.1738	0.00
45	.0000	44.7771	.0000	44.7911	0.03	.0000	44.6287	-0.33
46	-.0000	44.7771	.0000	44.7911	0.03	.0000	44.6287	-0.33
47	-.0000	45.9700	.0000	45.9828	0.03	.0000	45.8259	-0.31
48	-.0000	45.9726	.0000	45.9852	0.03	.0000	45.8285	-0.31
49	.0000	53.8385	.0000	53.8699	0.06	.0000	53.7924	-0.09
50	-.0000	53.9632	.0000	53.9957	0.06	.0000	53.9166	-0.09

Table 10: The *non-rotating* 5-bladed isolated rotor ($z_h = 2$ [m], $n = 0.$ [Hz]) with soft orthotropic rotor suspension: The eigenvalues according to the three methods compared to the FEM solution

ISOL.ROT.: c_u = 1 [Hz] z_h = 2 [m] n_mr = 6 [Hz] Rot. Axis = free								
	FEM (32 Elm.)		MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequen. [Hz]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]
1	-.0000	0.6895	.0000	0.6897	0.03	.0000	0.6888	-0.10
2	-.0000	1.3390	.0000	1.3387	-0.02	-.0001	1.3367	-0.17
3	-.0000	2.7596	.0000	2.7600	0.01	.0000	2.7595	-0.00
4	.0000	2.9698	.0000	2.9696	-0.01	.0000	2.9655	-0.14
5	-.0000	3.4146	.0001	3.4149	0.01	.0000	3.4087	-0.17
6	.0000	3.4146	.0000	3.4152	0.02	.0000	3.4087	-0.17
7	.0000	3.7044	.0000	3.7039	-0.01	.0002	3.7053	0.02
8	.0000	5.4928	.0000	5.4872	-0.10	.0000	5.4985	0.10
9	-.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
10	.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
11	-.0000	6.4354	.0000	6.4428	0.11	.0000	6.4357	0.00
12	.0000	9.3102	.0000	9.3084	-0.02	.0002	9.2976	-0.14
13	.0000	10.9865	.0000	10.9830	-0.03	.0000	10.9846	-0.02
14	.0000	15.5014	.0000	15.4963	-0.03	.0000	15.4952	-0.04
15	-.0000	15.5617	.0000	15.5625	0.01	.0000	15.5593	-0.02
16	.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
17	-.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
18	-.0000	20.2822	.0000	20.2817	-0.00	.0000	20.2772	-0.02
19	.0000	20.5548	.0000	20.5437	-0.05	.0000	20.5513	-0.02
20	.0000	21.2790	.0000	21.2807	0.01	.0000	21.2654	-0.06
21	-.0000	21.2790	.0000	21.2809	0.01	.0000	21.2654	-0.06
22	-.0000	21.8014	.0000	21.8059	0.02	.0000	21.7842	-0.08
23	.0000	24.4020	.0000	24.3990	-0.01	.0000	24.4036	0.01
24	.0000	26.8526	.0000	26.8626	0.04	.0000	26.8532	0.00
25	-.0000	26.9416	.0000	26.9415	-0.00	.0000	26.9297	-0.04
26	-.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
27	.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
28	-.0000	32.7036	.0000	32.6992	-0.01	.0000	32.7047	0.00
29	.0000	41.0751	.0000	41.0868	0.03	.0000	41.0588	-0.04
30	-.0000	41.1565	.0000	41.2016	0.11	.0000	41.1671	0.03
31	.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
32	-.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
33	-.0000	43.7637	.0000	43.3264	-1.00	.0000	43.3472	-0.95
34	.0000	43.8570	.0000	43.4173	-1.00	.0000	43.4617	-0.90
35	.0000	43.9809	.0000	43.5348	-1.01	.0000	43.5800	-0.91
36	.0000	43.9810	.0000	43.5348	-1.01	.0000	43.5800	-0.91
37	.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5803	-0.91
38	.0000	47.5936	.0000	47.6120	0.04	.0000	47.5935	-0.00
39	-.0000	48.4695	.0000	48.4796	0.02	.0000	48.4040	-0.14
40	-.0000	51.0314	.0000	51.0504	0.04	.0000	50.9205	-0.22
41	.0000	51.0314	.0000	51.0507	0.04	.0000	50.9205	-0.22
42	.0000	51.8577	.0000	51.8804	0.04	.0000	51.7496	-0.21
43	-.0000	52.9616	.0000	52.9727	0.02	.0000	52.8604	-0.19
44	.0000	58.0826	.0000	58.1800	0.17	.0000	58.0910	0.01
45	-.0000	58.2387	.0000	58.3006	0.11	.0000	58.2057	-0.06
46	-.0000	58.3910	.0000	58.4789	0.15	.0000	58.3787	-0.02
47	.0000	58.3910	.0000	58.4789	0.15	.0000	58.3787	-0.02
48	-.0000	65.2609	.0000	65.3166	0.09	.0000	65.2570	-0.01
49	-.0000	78.0215	.0000	78.1463	0.16	.0000	77.9839	-0.05
50	.0000	78.1395	.0000	78.3018	0.21	.0000	78.1356	-0.00

Table 11: The *rotating* 5-bladed isolated rotor ($z_h = 2$ [m], $n = 6.$ [Hz]) with soft orthotropic rotor suspension (*free* rotor axis): The eigenvalues according to the three methods compared to the FEM solution

ISOL.ROT.: c_u = 1 [Hz] z_h = 2 [m] n_mr = 6 [Hz] Rot. Axis = fixed

	FEM (32 Elm.)		MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequen. [Hz]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]
1	-.0000	2.7596	.0000	2.7600	0.01	.0000	2.7595	-0.00
2	.0000	2.9698	.0000	2.9696	-0.01	.0000	2.9655	-0.14
3	-.0000	3.4146	.0001	3.4149	0.01	.0000	3.4087	-0.17
4	.0000	3.4146	.0000	3.4152	0.02	.0000	3.4087	-0.17
5	-.0000	3.8393	.0000	3.8388	-0.01	.0001	3.8513	0.31
6	.0000	4.3593	.0000	4.3598	0.01	.0000	4.3470	-0.28
7	-.0000	6.1797	.0000	6.1819	0.04	.0000	6.1785	-0.02
8	.0000	6.1861	.0000	6.1885	0.04	.0000	6.1849	-0.02
9	.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
10	-.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
11	.0000	7.5444	.0000	7.5443	-0.00	-.0001	7.5479	0.05
12	-.0000	9.1861	.0000	9.1856	-0.01	.0001	9.1686	-0.19
13	-.0000	10.9865	.0000	10.9830	-0.03	.0000	10.9846	-0.02
14	.0000	15.5285	.0000	15.5266	-0.01	.0000	15.5249	-0.02
15	-.0000	15.5422	.0000	15.5407	-0.01	.0000	15.5384	-0.02
16	.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
17	-.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
18	-.0000	20.2823	.0000	20.2817	-0.00	.0000	20.2772	-0.03
19	-.0000	20.5548	.0000	20.5437	-0.05	.0000	20.5513	-0.02
20	-.0000	21.2790	.0000	21.2807	0.01	.0000	21.2654	-0.06
21	.0000	21.2790	.0000	21.2809	0.01	.0000	21.2654	-0.06
22	-.0000	21.8045	.0000	21.8089	0.02	.0000	21.7855	-0.09
23	.0000	24.3925	.0000	24.3895	-0.01	.0000	24.3960	0.01
24	.0000	26.8856	.0000	26.8897	0.02	.0000	26.8792	-0.02
25	-.0000	26.9151	.0000	26.9201	0.02	.0000	26.9091	-0.02
26	-.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
27	.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
28	.0000	32.7036	.0000	32.6992	-0.01	.0000	32.7047	0.00
29	.0000	41.0881	.0000	41.1168	0.07	.0000	41.0876	-0.00
30	.0000	41.1467	.0000	41.1786	0.08	.0000	41.1458	-0.00
31	.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
32	-.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
33	-.0000	43.7828	.0000	43.3463	-1.00	.0000	43.3817	-0.92
34	.0000	43.8396	.0000	43.3995	-1.00	.0000	43.4410	-0.91
35	.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5800	-0.91
36	-.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5800	-0.91
37	.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5803	-0.91
38	.0000	47.5936	.0000	47.6120	0.04	.0000	47.5935	-0.00
39	.0000	48.4697	.0000	48.4796	0.02	.0000	48.4040	-0.14
40	.0000	51.0314	.0000	51.0504	0.04	.0000	50.9205	-0.22
41	-.0000	51.0314	.0000	51.0507	0.04	.0000	50.9205	-0.22
42	-.0000	51.8580	.0000	51.8807	0.04	.0000	51.7511	-0.21
43	.0000	52.9610	.0000	52.9721	0.02	.0000	52.8580	-0.19
44	.0000	58.1230	.0000	58.2005	0.13	.0000	58.1097	-0.02
45	.0000	58.2023	.0000	58.2829	0.14	.0000	58.1894	-0.02
46	-.0000	58.3910	.0000	58.4789	0.15	.0000	58.3787	-0.02
47	.0000	58.3910	.0000	58.4789	0.15	.0000	58.3787	-0.02
48	-.0000	65.2609	.0000	65.3166	0.09	.0000	65.2570	-0.01
49	-.0000	78.0219	.0000	78.1641	0.18	.0000	78.0033	-0.02
50	.0000	78.1395	.0000	78.2863	0.19	.0000	78.1193	-0.03

Table 12: The *rotating* 5-bladed isolated rotor ($z_h = 2$ [m], $n = 6.$ [Hz]) with soft orthotropic rotor suspension (sl fixed rotor axis): The eigenvalues according to the three methods compared to the FEM solution

H9 : c_u = 1 [Hz] z_h = 2 [m] n_mr = 0 [Hz] n_tr = 00.0 [Hz]								
	FEM (32 Elm.)		MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequen. [Hz]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]
1	-.0000	0.0100	.0000	0.0100	0.00	.0000	0.0100	0.00
2	-.0000	0.0200	.0000	0.0200	0.00	.0000	0.0200	0.00
3	-.0000	0.0300	.0000	0.0300	0.00	.0000	0.0300	0.00
4	.0000	0.0400	.0000	0.0400	0.00	.0000	0.0400	0.00
5	.0000	0.0500	.0000	0.0500	0.00	.0000	0.0500	0.00
6	-.0000	0.0600	.0000	0.0600	0.00	.0000	0.0600	0.00
7	.0000	0.6392	.0000	0.6393	0.02	.0000	0.6389	-0.05
8	.0000	0.6392	.0000	0.6393	0.02	.0000	0.6389	-0.05
9	-.0000	0.6469	.0000	0.6469	0.00	.0000	0.6466	-0.05
10	-.0000	0.7296	.0000	0.7297	0.01	.0000	0.7293	-0.04
11	.0000	0.8473	.0000	0.8474	0.01	.0000	0.8470	-0.04
12	-.0000	1.1850	.0000	1.1850	0.00	.0000	1.1850	0.00
13	-.0000	2.3977	.0000	2.3978	0.00	.0000	2.3973	-0.02
14	-.0000	2.5566	.0000	2.5567	0.00	.0000	2.5558	-0.03
15	.0000	2.5566	.0000	2.5567	0.00	.0000	2.5558	-0.03
16	-.0000	3.0300	.0000	3.0301	0.00	.0000	3.0292	-0.03
17	.0000	3.1525	.0000	3.1525	0.00	.0000	3.1519	-0.02
18	-.0000	3.5481	.0000	3.5484	0.01	.0000	3.5478	-0.01
19	.0000	3.9087	.0000	3.9093	0.02	.0000	3.9084	-0.01
20	.0000	4.0018	.0000	4.0025	0.02	.0000	4.0014	-0.01
21	.0000	4.0057	.0000	4.0065	0.02	.0000	4.0053	-0.01
22	-.0000	4.0057	.0000	4.0065	0.02	.0000	4.0053	-0.01
23	.0000	4.2776	.0000	4.2780	0.01	.0000	4.2774	-0.00
24	.0000	7.6534	.0000	7.6537	0.00	.0000	7.6530	-0.01
25	-.0000	11.1493	.0000	11.1527	0.03	.0000	11.1470	-0.02
26	-.0000	11.1586	.0000	11.1621	0.03	.0000	11.1563	-0.02
27	.0000	11.2154	.0000	11.2189	0.03	.0000	11.2130	-0.02
28	-.0000	11.2154	.0000	11.2189	0.03	.0000	11.2130	-0.02
29	-.0000	14.3631	.0000	14.3622	-0.01	.0000	14.3617	-0.01
30	.0000	15.0033	.0000	15.0055	0.01	.0000	14.9868	-0.11
31	.0000	16.0103	.0000	16.0134	0.02	.0000	15.9878	-0.14
32	-.0000	16.0103	.0000	16.0134	0.02	.0000	15.9878	-0.14
33	.0000	17.2190	.0000	17.2217	0.02	.0000	17.1979	-0.12
34	.0000	17.2378	.0000	17.2404	0.02	.0000	17.2168	-0.12
35	.0000	21.8105	.0000	21.8195	0.04	.0000	21.8023	-0.04
36	.0000	21.8568	.0000	21.8660	0.04	.0000	21.8486	-0.04
37	.0000	21.9754	.0000	21.9850	0.04	.0000	21.9670	-0.04
38	-.0000	21.9754	.0000	21.9850	0.04	.0000	21.9670	-0.04
39	.0000	25.6118	.0000	25.6105	-0.01	.0000	25.6055	-0.02
40	.0000	36.0352	.0000	36.0535	0.05	.0000	36.0139	-0.06
41	.0000	36.1225	.0000	36.1413	0.05	.0000	36.1009	-0.06
42	.0000	36.3226	.0000	36.3424	0.05	.0000	36.3004	-0.06
43	-.0000	36.3226	.0000	36.3424	0.05	.0000	36.3004	-0.06
44	.0000	40.6994	.0000	40.6969	-0.01	.0000	40.6813	-0.04
45	.0000	42.0456	.0000	42.0544	0.02	.0000	41.9381	-0.26
46	-.0000	42.9663	.0000	42.9611	-0.01	.0000	42.9663	0.00
47	.0000	43.0280	.0000	43.0226	-0.01	.0000	43.0279	-0.00
48	.0000	43.1738	.0000	43.1684	-0.01	.0000	43.1738	0.00
49	.0000	43.1738	.0000	43.1684	-0.01	.0000	43.1738	0.00
50	-.0000	43.1738	.0000	43.1684	-0.01	.0000	43.1738	0.00

Table 13: The free H9 helicopter with *non-rotating* rotors ($n_{mr} = 0.$ [Hz], $n_{tr} = 0.$ [Hz], soft orthotropic main rotor suspension): The eigenvalues according to the three methods compared to the FEM solution

H9 :	c_u = 1 [Hz]	z_h = 2 [m]	n_mr = 6 [Hz]	n_tr = 00.0 [Hz]				
	FEM (32 Elm.)		MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequen. [Hz]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]
1	-.0000	0.0094	-.0055	0.0096	2.13	.0000	0.0094	0.00
2	.0000	0.0095	-.1462	0.0158	66.32	.0000	0.0096	1.05
3	.0000	0.0189	.0564	0.0182	-3.70	.0000	0.0189	0.00
4	-.0000	0.0300	.0000	0.0300	0.00	.0000	0.0300	0.00
5	-.0000	0.0548	.0205	0.0577	5.29	.0001	0.0548	0.00
6	.0000	0.2517	.0007	0.2566	1.95	.0000	0.2459	-2.30
7	.0000	0.9350	.0000	0.9551	2.15	.0001	0.9544	2.07
8	.0000	2.2885	.0001	2.2858	-0.12	-.0001	2.2851	-0.15
9	-.0000	3.4146	.0001	3.4149	0.01	.0000	3.4087	-0.17
10	-.0000	3.4146	.0000	3.4152	0.02	.0000	3.4087	-0.17
11	.0000	3.6850	.0000	3.6860	0.03	-.0001	3.6861	0.03
12	.0000	3.8355	.0000	3.8361	0.02	.0000	3.8327	-0.07
13	.0000	4.1536	.0000	4.1510	-0.06	.0001	4.1538	0.00
14	.0226	6.1926	-.0228	6.1950	0.04	-.0226	6.1912	-0.02
15	-.0226	6.1926	.0228	6.1950	0.04	.0226	6.1912	-0.02
16	-.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
17	-.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
18	-.0000	9.3235	.0000	9.3226	-0.01	-.0001	9.3116	-0.13
19	-.0000	11.0309	.0000	11.0274	-0.03	.0000	11.0290	-0.02
20	-.0000	15.5079	.0000	15.5032	-0.03	.0000	15.5019	-0.04
21	.0000	15.5627	.0000	15.5634	0.00	.0000	15.5602	-0.02
22	.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
23	-.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
24	.0000	20.3245	.0000	20.3240	-0.00	.0000	20.3192	-0.03
25	-.0000	20.5626	.0000	20.5515	-0.05	.0000	20.5590	-0.02
26	-.0000	21.2790	.0000	21.2807	0.01	.0000	21.2654	-0.06
27	.0000	21.2790	.0000	21.2809	0.01	.0000	21.2654	-0.06
28	.0000	21.8015	.0000	21.8059	0.02	.0000	21.7842	-0.08
29	.0000	24.4023	.0000	24.3992	-0.01	.0000	24.4038	0.01
30	.0000	26.8566	.0000	26.8667	0.04	.0000	26.8573	0.00
31	-.0000	26.9426	.0000	26.9426	0.00	.0000	26.9308	-0.04
32	.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
33	-.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
34	-.0000	32.7058	.0000	32.7014	-0.01	.0000	32.7069	0.00
35	-.0000	41.0784	.0000	41.0903	0.03	.0000	41.0621	-0.04
36	.0000	41.1574	.0000	41.2026	0.11	.0000	41.1680	0.03
37	.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
38	-.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
39	-.0000	43.7664	.0000	43.3289	-1.00	.0000	43.3499	-0.95
40	.0000	43.8579	.0000	43.4181	-1.00	.0000	43.4626	-0.90
41	-.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5800	-0.91
42	.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5800	-0.91
43	-.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5803	-0.91
44	-.0000	47.5944	.0000	47.6127	0.04	.0000	47.5943	-0.00
45	.0000	48.4867	.0000	48.4969	0.02	.0000	48.4210	-0.14
46	-.0000	51.0314	.0000	51.0504	0.04	.0000	50.9205	-0.22
47	.0000	51.0314	.0000	51.0507	0.04	.0000	50.9205	-0.22
48	.0000	51.8577	.0000	51.8804	0.04	.0000	51.7496	-0.21
49	-.0000	52.9616	.0000	52.9727	0.02	.0000	52.8605	-0.19
50	-.0000	58.0847	.0000	58.1822	0.17	.0000	58.0932	0.01

Table 14: The free H9 helicopter with *rotating* main rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = 0.$ [Hz], soft orthotropic main rotor suspension): The eigenvalues according to the three methods compared to the FEM solution

H9 : c_u = 1 [Hz] z_h = 2 [m] n_mr = 6 [Hz] n_tr =+34.3 [Hz]								
	FEM (32 Elm.)		MBS (64 Bod.)			FEMBS (120 Mod.)		
	Nat.Dmp. [-]	Frequen. [Hz]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequen. [Hz]	Dif. [%]
1	-.0000	0.0094	-.0064	0.0096	2.13	.0000	0.0094	0.00
2	.0000	0.0097	.1095	0.0174	79.38	.0000	0.0097	0.00
3	-.0000	0.0189	-.0941	0.0177	-6.35	.0000	0.0189	0.00
4	.0000	0.0300	.0000	0.0300	0.00	.0000	0.0300	0.00
5	.0000	0.0533	.0215	0.0533	0.00	.0000	0.0533	0.00
6	.0000	0.2528	-.0051	0.2574	1.82	.0000	0.2463	-2.57
7	.0000	0.9357	-.0001	0.9590	2.49	.0001	0.9582	2.40
8	-.0000	2.2899	.0001	2.2868	-0.14	-.0001	2.2861	-0.17
9	-.0000	3.4146	.0001	3.4149	0.01	.0000	3.4087	-0.17
10	.0000	3.4146	.0000	3.4152	0.02	.0000	3.4087	-0.17
11	.0000	3.6854	.0000	3.6848	-0.02	-.0001	3.6848	-0.02
12	.0000	3.8311	.0000	3.8361	0.13	-.0001	3.8329	0.05
13	.0000	4.1601	.0000	4.1544	-0.14	.0001	4.1571	-0.07
14	-.0227	6.1932	-.0229	6.1956	0.04	.0227	6.1917	-0.02
15	.0227	6.1932	.0229	6.1956	0.04	-.0227	6.1918	-0.02
16	.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
17	-.0000	6.1988	.0000	6.2017	0.05	.0000	6.1976	-0.02
18	.0000	9.3235	.0000	9.3227	-0.01	-.0001	9.3117	-0.13
19	-.0000	11.0310	.0000	11.0275	-0.03	.0000	11.0290	-0.02
20	-.0000	15.5079	.0000	15.5032	-0.03	.0000	15.5019	-0.04
21	.0000	15.5627	.0000	15.5634	0.00	.0000	15.5602	-0.02
22	.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
23	-.0000	15.5739	.0000	15.5734	-0.00	.0000	15.5695	-0.03
24	.0000	20.3245	.0000	20.3240	-0.00	.0000	20.3192	-0.03
25	.0000	20.5626	.0000	20.5515	-0.05	.0000	20.5590	-0.02
26	-.0000	21.2790	.0000	21.2807	0.01	.0000	21.2654	-0.06
27	.0000	21.2790	.0000	21.2809	0.01	.0000	21.2654	-0.06
28	.0000	21.8015	.0000	21.8059	0.02	.0000	21.7842	-0.08
29	.0000	24.4023	.0000	24.3992	-0.01	.0000	24.4038	0.01
30	-.0000	26.8566	.0000	26.8667	0.04	.0000	26.8573	0.00
31	.0000	26.9426	.0000	26.9426	0.00	.0000	26.9308	-0.04
32	.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
33	-.0000	26.9831	.0000	26.9904	0.03	.0000	26.9779	-0.02
34	-.0000	32.7058	.0000	32.7014	-0.01	.0000	32.7069	0.00
35	-.0000	41.0784	.0000	41.0903	0.03	.0000	41.0621	-0.04
36	.0000	41.1574	.0000	41.2026	0.11	.0000	41.1680	0.03
37	.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
38	-.0000	41.2738	.0000	41.3115	0.09	.0000	41.2705	-0.01
39	-.0000	43.7664	.0000	43.3289	-1.00	.0000	43.3499	-0.95
40	.0000	43.8578	.0000	43.4181	-1.00	.0000	43.4626	-0.90
41	.0000	43.9810	.0000	43.5348	-1.01	.0000	43.5800	-0.91
42	-.0000	43.9810	.0000	43.5348	-1.01	.0000	43.5800	-0.91
43	-.0000	43.9811	.0000	43.5348	-1.01	.0000	43.5803	-0.91
44	-.0000	47.5944	.0000	47.6127	0.04	.0000	47.5943	-0.00
45	-.0000	48.4867	.0000	48.4969	0.02	.0000	48.4210	-0.14
46	-.0000	51.0314	.0000	51.0504	0.04	.0000	50.9205	-0.22
47	-.0000	51.0314	.0000	51.0507	0.04	.0000	50.9205	-0.22
48	.0000	51.8577	.0000	51.8804	0.04	.0000	51.7496	-0.21
49	-.0000	52.9616	.0000	52.9727	0.02	.0000	52.8605	-0.19
50	.0000	58.0848	.0000	58.1822	0.17	.0000	58.0932	0.01

Table 15: The free H9 helicopter with *rotating* main and tail rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = +34.3$ [Hz], soft orthotropic main rotor suspension): The eigenvalues according to the three methods compared to the FEM solution

H9 : FEM (32 Elm.) z_h = 2 [m] n_mr = 6 [Hz] n_tr =+34.3 [Hz]									
	c_u = 100 [Hz]			c_u = 10 [Hz]			c_u = 1 [Hz]		
	Nat.Dmp.	Frequn.	Dif.	Nat.Dmp.	Frequn.	Dif.	Nat.Dmp.	Frequn.	Dif.
	[-]	[Hz]	[%]	[-]	[Hz]	[%]	[-]	[Hz]	[%]
1	-.0000	0.0094	0.00	.0000	0.0094	0.0	-.0000	0.0094	0.0
2	.0000	0.0097	0.00	-.0000	0.0097	0.0	.0000	0.0097	0.0
3	-.0000	0.0189	0.00	.0000	0.0189	0.0	-.0000	0.0189	0.0
4	.0000	0.0300	0.00	-.0000	0.0300	0.0	.0000	0.0300	0.0
5	-.0000	0.0533	0.00	-.0000	0.0533	0.0	.0000	0.0533	0.0
6	-.0000	0.2589	0.00	.0000	0.2589	0.0	.0000	0.2528	-2.4
7	-.0000	3.4146	0.00	-.0000	3.4146	0.0	.0000	0.9357	-72.6
8	.0000	3.4146	0.00	-.0000	3.4146	0.0	-.0000	2.2899	-32.9
9	.0000	3.5164	0.00	.0000	3.4961	-0.6	-.0000	3.4146	-2.9
10	.0000	4.0827	0.00	-.0000	4.0211	-1.5	.0000	3.4146	-16.4
11	-.0000	5.3272	0.00	-.0000	5.0151	-5.9	.0000	3.6854	-30.8
12	-.0000	6.1649	0.00	-.0000	6.1756	0.2	.0000	3.8311	-37.9
13	-.0000	6.1988	0.00	.0000	6.1988	0.0	.0000	4.1601	-32.9
14	.0000	6.1988	0.00	-.0000	6.1988	0.0	-.0227	6.1932	-0.1
15	.0000	6.3823	0.00	.0000	6.3360	-0.7	.0227	6.1932	-3.0
16	-.0000	7.7891	0.00	.0000	7.6551	-1.7	.0000	6.1988	-20.4
17	.0000	15.5739	0.00	-.0000	14.2316	-8.6	-.0000	6.1988	-60.2
18	.0000	15.5739	0.00	.0000	15.5733	-0.0	.0000	9.3235	-40.1
19	-.0000	15.5741	0.00	-.0000	15.5739	-0.0	-.0000	11.0310	-29.2
20	-.0000	15.5777	0.00	.0000	15.5739	-0.0	-.0000	15.5079	-0.4
21	.0000	16.1704	0.00	-.0000	15.5768	-3.7	.0000	15.5627	-3.8
22	-.0000	21.2790	0.00	-.0000	16.1015	-24.3	.0000	15.5739	-26.8
23	.0000	21.2790	0.00	-.0000	21.2790	0.0	-.0000	15.5739	-26.8
24	.0000	21.3242	0.00	-.0000	21.2790	-0.2	.0000	20.3245	-4.7
25	.0000	21.4707	0.00	.0000	21.2998	-0.8	.0000	20.5626	-4.2
26	.0000	22.2373	0.00	-.0000	21.5319	-3.2	-.0000	21.2790	-4.3
27	-.0000	26.9831	0.00	.0000	23.5844	-12.6	.0000	21.2790	-21.1
28	.0000	26.9831	0.00	-.0000	26.9513	-0.1	.0000	21.8015	-19.2
29	.0000	26.9833	0.00	.0000	26.9831	-0.0	.0000	24.4023	-9.6
30	-.0000	26.9856	0.00	-.0000	26.9831	-0.0	-.0000	26.8566	-0.5
31	.0000	27.4814	0.00	.0000	26.9838	-1.8	.0000	26.9426	-2.0
32	.0000	41.2738	0.00	-.0000	27.0189	-34.5	.0000	26.9831	-34.6
33	-.0000	41.2738	0.00	.0000	36.2746	-12.1	-.0000	26.9831	-34.6
34	.0000	41.2740	0.00	-.0000	41.1835	-0.2	-.0000	32.7058	-20.8
35	-.0000	41.2755	0.00	.0000	41.2735	-0.0	-.0000	41.0784	-0.5
36	.0000	41.6494	0.00	.0000	41.2738	-0.9	.0000	41.1574	-1.2
37	-.0000	43.9811	0.00	.0000	41.2738	-6.2	.0000	41.2738	-6.2
38	.0000	43.9811	0.00	.0000	41.3278	-6.0	-.0000	41.2738	-6.2
39	.0000	43.9811	0.00	.0000	43.9804	-0.0	-.0000	43.7664	-0.5
40	.0000	43.9818	0.00	-.0000	43.9810	-0.0	.0000	43.8578	-0.3
41	-.0000	43.9840	0.00	.0000	43.9811	-0.0	.0000	43.9810	-0.0
42	-.0000	51.0314	0.00	-.0000	43.9811	-13.8	-.0000	43.9810	-13.8
43	.0000	51.0314	0.00	-.0000	43.9857	-13.8	-.0000	43.9811	-13.8
44	-.0000	51.0475	0.00	.0000	51.0303	-0.0	-.0000	47.5944	-6.8
45	.0000	51.1934	0.00	.0000	51.0314	-0.3	-.0000	48.4867	-5.3
46	-.0000	51.7153	0.00	.0000	51.0314	-1.3	-.0000	51.0314	-1.3
47	.0000	58.3910	0.00	.0000	51.9929	-11.0	-.0000	51.0314	-12.6
48	.0000	58.3910	0.00	-.0000	53.4984	-8.4	.0000	51.8577	-11.2
49	.0000	58.3910	0.00	-.0000	57.1150	-2.2	-.0000	52.9616	-9.3
50	-.0000	58.3923	0.00	-.0000	58.3897	-0.0	.0000	58.0848	-0.5

Table 16: The free H9 helicopter with *rotating* main and (*positive*) tail rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = +34.3$ [Hz]): Parameter variation of the orthotropic main rotor suspension stiffness and comparison of the eigenvalues to the stiff case results (according to the FEM method)

H9 :	FEM (32 Elm.)		z_h = 2 [m]		n_mr = 6 [Hz]		n_tr =+00.0 [Hz]		
	c_u = 100 [Hz]			c_u = 10 [Hz]			c_u = 1 [Hz]		
	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]
1	-.0000	0.0094	0.00	.0000	0.0094	0.00	-.0000	0.0094	0.00
2	-.0000	0.0095	-2.06	-.0000	0.0095	-2.06	.0000	0.0095	-2.06
3	-.0000	0.0189	0.00	.0000	0.0189	0.00	.0000	0.0189	0.00
4	.0000	0.0300	0.00	-.0000	0.0300	0.00	-.0000	0.0300	0.00
5	-.0000	0.0548	2.81	.0000	0.0548	2.81	-.0000	0.0548	2.81
6	.0000	0.2570	-0.73	.0000	0.2569	-0.77	.0000	0.2517	-0.44
7	-.0000	3.4146	0.00	.0000	3.4146	0.00	.0000	0.9350	-0.07
8	.0000	3.4146	0.00	-.0000	3.4146	0.00	.0000	2.2885	-0.06
9	.0000	3.5165	0.00	.0000	3.4962	0.00	-.0000	3.4146	0.00
10	-.0000	4.0767	-0.15	-.0000	4.0146	-0.16	-.0000	3.4146	0.00
11	.0000	5.3365	0.17	-.0000	5.0268	0.23	.0000	3.6850	-0.01
12	.0000	6.1648	-0.00	-.0000	6.1755	-0.00	.0000	3.8355	0.11
13	-.0000	6.1988	0.00	-.0000	6.1988	0.00	.0000	4.1536	-0.16
14	.0000	6.1988	0.00	.0000	6.1988	0.00	.0226	6.1926	-0.01
15	.0000	6.3823	0.00	.0000	6.3360	0.00	-.0226	6.1926	-0.01
16	-.0000	7.7965	0.10	.0000	7.6634	0.11	-.0000	6.1988	0.00
17	.0000	15.5739	0.00	-.0000	14.2273	-0.03	-.0000	6.1988	0.00
18	-.0000	15.5739	0.00	-.0000	15.5733	0.00	-.0000	9.3235	0.00
19	.0000	15.5741	0.00	-.0000	15.5739	0.00	-.0000	11.0309	-0.00
20	-.0000	15.5777	0.00	.0000	15.5739	0.00	-.0000	15.5079	0.00
21	-.0000	16.1706	0.00	.0000	15.5768	0.00	.0000	15.5627	0.00
22	-.0000	21.2790	0.00	.0000	16.1003	-0.01	.0000	15.5739	0.00
23	.0000	21.2790	0.00	.0000	21.2790	0.00	-.0000	15.5739	0.00
24	.0000	21.3243	0.00	-.0000	21.2790	0.00	.0000	20.3245	0.00
25	-.0000	21.4702	-0.00	.0000	21.2998	0.00	-.0000	20.5626	0.00
26	.0000	22.2397	0.01	.0000	21.5321	0.00	-.0000	21.2790	0.00
27	-.0000	26.9831	0.00	-.0000	23.5822	-0.01	.0000	21.2790	0.00
28	.0000	26.9831	0.00	-.0000	26.9511	-0.00	.0000	21.8015	0.00
29	-.0000	26.9833	0.00	-.0000	26.9831	0.00	.0000	24.4023	0.00
30	.0000	26.9856	0.00	.0000	26.9831	0.00	.0000	26.8566	0.00
31	.0000	27.4816	0.00	.0000	26.9838	0.00	-.0000	26.9426	0.00
32	.0000	41.2738	0.00	.0000	27.0187	0.00	.0000	26.9831	0.00

Table 17: The free H9 helicopter with *rotating* main rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = +0.$ [Hz]): Parameter variation of the orthotropic main rotor suspension stiffness and comparison of the eigenvalues to the respective cases with the *positive* tail rotor rotation (according to the FEM method)

H9 : FEM (32 Elm.) z_h = 2 [m] n_mr = 6 [Hz] n_tr =-34.3 [Hz]

	c_u = 100 [Hz]			c_u = 10 [Hz]			c_u = 1 [Hz]		
	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]
1	-.0000	0.0085	0.0	-.0000	0.0085	0.0	.0000	0.0085	0.0
2	-.0000	0.0094	0.0	-.0000	0.0094	0.0	.0000	0.0094	0.0
3	-.0000	0.0189	0.0	.0000	0.0189	0.0	-.0000	0.0189	0.0
4	.0000	0.0300	0.0	-.0000	0.0300	0.0	.0000	0.0300	0.0
5	-.0000	0.0558	0.0	-.0000	0.0558	0.0	-.0000	0.0557	-0.2
6	.0000	0.2797	0.0	.0000	0.2797	0.0	-.0000	0.2745	-1.9
7	.0000	3.4146	0.0	-.0000	3.4146	0.0	-.0000	0.9346	-72.6
8	-.0000	3.4146	0.0	-.0000	3.4146	0.0	-.0000	2.2889	-33.0
9	-.0000	3.5166	0.0	-.0000	3.4963	-0.6	-.0000	3.4146	-2.9
10	-.0000	4.0711	0.0	.0000	4.0082	-1.5	.0000	3.4146	-16.1
11	.0000	5.3466	0.0	.0000	5.0396	-5.7	.0000	3.6840	-31.1
12	.0000	6.1647	0.0	-.0000	6.1755	0.2	.0000	3.8331	-37.8
13	-.0000	6.1988	0.0	-.0000	6.1988	0.0	-.0000	4.1598	-32.9
14	.0000	6.1988	0.0	-.0000	6.1988	0.0	-.0227	6.1929	-0.1
15	-.0000	6.3825	0.0	.0000	6.3360	-0.7	.0227	6.1929	-3.0
16	-.0000	7.8049	0.0	-.0000	7.6723	-1.7	-.0000	6.1988	-20.6
17	-.0000	15.5739	0.0	.0000	14.2231	-8.7	.0000	6.1988	-60.2
18	-.0000	15.5739	0.0	.0000	15.5733	-0.0	-.0000	9.3235	-40.1
19	.0000	15.5741	0.0	.0000	15.5739	-0.0	-.0000	11.0310	-29.2
20	.0000	15.5777	0.0	-.0000	15.5739	-0.0	-.0000	15.5079	-0.4
21	-.0000	16.1709	0.0	-.0000	15.5768	-3.7	.0000	15.5627	-3.8
22	-.0000	21.2790	0.0	.0000	16.0991	-24.3	.0000	15.5739	-26.8
23	-.0000	21.2790	0.0	.0000	21.2790	0.0	.0000	15.5739	-26.8
24	.0000	21.3243	0.0	-.0000	21.2790	-0.2	.0000	20.3245	-4.7
25	.0000	21.4697	0.0	-.0000	21.2998	-0.8	.0000	20.5626	-4.2
26	-.0000	22.2422	0.0	.0000	21.5324	-3.2	-.0000	21.2790	-4.3
27	-.0000	26.9831	0.0	-.0000	23.5801	-12.6	.0000	21.2790	-21.1
28	.0000	26.9831	0.0	-.0000	26.9510	-0.1	.0000	21.8015	-19.2
29	.0000	26.9833	0.0	.0000	26.9831	-0.0	-.0000	24.4023	-9.6
30	-.0000	26.9856	0.0	-.0000	26.9831	-0.0	-.0000	26.8566	-0.5
31	.0000	27.4819	0.0	.0000	26.9838	-1.8	.0000	26.9426	-2.0
32	.0000	41.2738	0.0	.0000	27.0186	-34.5	.0000	26.9831	-34.6
33	.0000	41.2738	0.0	.0000	36.2759	-12.1	-.0000	26.9831	-34.6
34	-.0000	41.2740	0.0	.0000	41.1836	-0.2	-.0000	32.7058	-20.8
35	-.0000	41.2754	0.0	-.0000	41.2735	-0.0	-.0000	41.0784	-0.5
36	-.0000	41.6495	0.0	-.0000	41.2738	-0.9	.0000	41.1574	-1.2
37	-.0000	43.9810	0.0	.0000	41.2738	-6.2	-.0000	41.2738	-6.2
38	.0000	43.9811	0.0	-.0000	41.3278	-6.0	.0000	41.2738	-6.2
39	-.0000	43.9811	0.0	.0000	43.9805	-0.0	.0000	43.7664	-0.5
40	.0000	43.9820	0.0	.0000	43.9811	-0.0	-.0000	43.8579	-0.3
41	.0000	43.9840	0.0	.0000	43.9811	-0.0	-.0000	43.9811	-0.0
42	-.0000	51.0314	0.0	-.0000	43.9811	-13.8	.0000	43.9811	-13.8
43	.0000	51.0314	0.0	-.0000	43.9857	-13.8	.0000	43.9811	-13.8
44	.0000	51.0475	0.0	-.0000	51.0303	-0.0	.0000	47.5944	-6.8
45	.0000	51.1932	0.0	-.0000	51.0314	-0.3	-.0000	48.4867	-5.3
46	-.0000	51.7163	0.0	.0000	51.0314	-1.3	.0000	51.0314	-1.3
47	-.0000	58.3910	0.0	.0000	51.9928	-11.0	-.0000	51.0314	-12.6
48	.0000	58.3910	0.0	-.0000	53.4985	-8.4	.0000	51.8577	-11.2
49	-.0000	58.3911	0.0	.0000	57.1150	-2.2	-.0000	52.9616	-9.3
50	.0000	58.3923	0.0	.0000	58.3897	-0.0	-.0000	58.0847	-0.5

Table 18: The free H9 helicopter with *rotating* main and (*negative*) tail rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = -34.3$ [Hz]): Parameter variation of the orthotropic main rotor suspension stiffness and comparison of the eigenvalues to the stiff case results (according to the FEM method)

H9 : FEM (32 Elm.) z_h = 2 [m] n_mr = 6 [Hz] n_tr =-00.0 [Hz]									
	c_u = 100 [Hz]			c_u = 10 [Hz]			c_u = 1 [Hz]		
	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]	Nat.Dmp. [-]	Frequn. [Hz]	Dif. [%]
1	-.0000	0.0094	10.59	.0000	0.0094	10.59	-.0000	0.0094	10.59
2	-.0000	0.0095	1.06	-.0000	0.0095	1.06	.0000	0.0095	1.06
3	-.0000	0.0189	0.00	.0000	0.0189	0.00	.0000	0.0189	0.00
4	.0000	0.0300	0.00	-.0000	0.0300	0.00	-.0000	0.0300	0.00
5	-.0000	0.0548	-1.79	.0000	0.0548	-1.79	-.0000	0.0548	-1.62
6	.0000	0.2570	-8.12	.0000	0.2569	-8.15	.0000	0.2517	-8.31
7	-.0000	3.4146	0.00	.0000	3.4146	0.00	.0000	0.9350	0.04
8	.0000	3.4146	0.00	-.0000	3.4146	0.00	.0000	2.2885	-0.02
9	.0000	3.5165	-0.00	.0000	3.4962	-0.00	-.0000	3.4146	0.00
10	-.0000	4.0767	0.14	-.0000	4.0146	0.16	-.0000	3.4146	0.00
11	.0000	5.3365	-0.19	-.0000	5.0268	-0.25	.0000	3.6850	0.03
12	.0000	6.1648	0.00	-.0000	6.1755	0.00	.0000	3.8355	0.06
13	-.0000	6.1988	0.00	-.0000	6.1988	0.00	.0000	4.1536	-0.15
14	.0000	6.1988	0.00	.0000	6.1988	0.00	.0226	6.1926	-0.00
15	.0000	6.3823	-0.00	.0000	6.3360	0.00	-.0226	6.1926	-0.00
16	-.0000	7.7965	-0.11	.0000	7.6634	-0.12	-.0000	6.1988	0.00
17	.0000	15.5739	0.00	-.0000	14.2273	0.03	-.0000	6.1988	0.00
18	-.0000	15.5739	0.00	-.0000	15.5733	0.00	-.0000	9.3235	0.00
19	.0000	15.5741	0.00	-.0000	15.5739	0.00	-.0000	11.0309	-0.00
20	-.0000	15.5777	0.00	.0000	15.5739	0.00	-.0000	15.5079	0.00
21	-.0000	16.1706	-0.00	.0000	15.5768	0.00	.0000	15.5627	0.00
22	-.0000	21.2790	0.00	.0000	16.1003	0.01	.0000	15.5739	0.00
23	.0000	21.2790	0.00	.0000	21.2790	0.00	-.0000	15.5739	0.00
24	.0000	21.3243	0.00	-.0000	21.2790	0.00	.0000	20.3245	0.00
25	-.0000	21.4702	0.00	.0000	21.2998	0.00	-.0000	20.5626	0.00
26	.0000	22.2397	-0.01	.0000	21.5321	-0.00	-.0000	21.2790	0.00
27	-.0000	26.9831	0.00	-.0000	23.5822	0.01	.0000	21.2790	0.00
28	.0000	26.9831	0.00	-.0000	26.9511	0.00	.0000	21.8015	0.00
29	-.0000	26.9833	0.00	-.0000	26.9831	0.00	.0000	24.4023	0.00
30	.0000	26.9856	0.00	.0000	26.9831	0.00	.0000	26.8566	0.00
31	.0000	27.4816	-0.00	.0000	26.9838	0.00	-.0000	26.9426	0.00
32	-.0000	41.2738	0.00	-.0000	27.0187	0.00	.0000	26.9831	0.00
33	-.0000	41.2738	0.00	-.0000	36.2751	-0.00	-.0000	26.9831	0.00
34	-.0000	41.2740	0.00	-.0000	41.1835	-0.00	-.0000	32.7058	0.00
35	.0000	41.2754	0.00	.0000	41.2735	0.00	-.0000	41.0784	0.00
36	-.0000	41.6494	-0.00	-.0000	41.2738	0.00	.0000	41.1574	0.00
37	-.0000	43.9811	0.00	.0000	41.2738	0.00	.0000	41.2738	0.00
38	.0000	43.9811	0.00	.0000	41.3278	0.00	-.0000	41.2738	0.00
39	-.0000	43.9811	0.00	-.0000	43.9805	0.00	-.0000	43.7664	0.00
40	-.0000	43.9818	-0.00	.0000	43.9810	-0.00	.0000	43.8579	0.00
41	.0000	43.9840	0.00	-.0000	43.9811	0.00	-.0000	43.9811	0.00
42	.0000	51.0314	0.00	.0000	43.9811	0.00	.0000	43.9811	0.00
43	-.0000	51.0314	0.00	.0000	43.9858	0.00	-.0000	43.9811	0.00
44	.0000	51.0475	0.00	-.0000	51.0303	0.00	-.0000	47.5944	0.00
45	-.0000	51.1933	0.00	.0000	51.0314	0.00	.0000	48.4867	0.00
46	.0000	51.7158	-0.00	.0000	51.0314	0.00	-.0000	51.0314	0.00
47	.0000	58.3910	0.00	.0000	51.9928	0.00	.0000	51.0314	0.00
48	-.0000	58.3910	0.00	-.0000	53.4984	-0.00	.0000	51.8577	0.00
49	-.0000	58.3911	0.00	.0000	57.1150	0.00	-.0000	52.9616	0.00
50	.0000	58.3926	0.00	.0000	58.3897	0.00	-.0000	58.0847	0.00

Table 19: The free H9 helicopter with *rotating* main rotor ($n_{mr} = 6.$ [Hz], $n_{tr} = -0.$ [Hz]): Parameter variation of the orthotropic main rotor suspension stiffness and comparison of the eigenvalues to the respective cases with the *negative* tail rotor rotation (according to the FEM method)