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AN ANALYTICAL STUDY OF IMPULSIVE DESTRUCTION OF THE TAIL–ROTOR–DRIVE–SHAFT

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ABSTRACT

The impulsive destruction of a drive shaft caused by the sudden stop of tail rotor has been analyzed. The impulsive destruction sometimes occurs when the tail rotor hits some obstacle like tree during an accident of the rotor craft.

It is assumed that the drive shaft is the uniform bar. The equation of motion for the torsional strain wave is solved. The result indicates that the drive shaft may be destroyed at two places by the reflection of the strain waves, when the tail rotor is suddenly constrained. It is also shown that the drive shaft is destroyed at the only one place near the tail rotor end, when the tail rotor is gradually constrained.

NOMENCLATURE

a_1	constant
a_2	constant
b_2	constant
$\bar{c}$	nondimensional speed of wave ($\bar{t} \leqq \bar{T}_1$) = $GJ/\Omega \ell I$
$\bar{c}_1$	nondimensional speed of wave ($\bar{t} > \bar{T}_1$) = $GJ/\Omega \ell_1 I$
GJ	torsional rigidity of drive shaft
I	inertial moment of drive shaft
ℓ	length of drive shaft ($\bar{t} \leqq \bar{T}_1$)
ℓ_1	length of drive shaft ($\bar{t} > \bar{T}_1$)
t	time
$\bar{T}_0$	nondimensional time when drive shaft is constrained
$\bar{T}_1$	nondimensional time when first destruction occurs
$\bar{T}_2$	nondimensional time when main rotor stops
$\bar{t}$	nondimensional time = Ωt
x	position of drive shaft
$\bar{x}$	nondimensional position of drive shaft = x/ℓ ($\bar{t} \leqq \bar{T}_1$) = x/ℓ_1 ($\bar{t} > \bar{T}_1$)

x_1-y_1-z_1-o_1 rotating frame fixed to drive shaft at E end

x_2-y_2-z_2-o_2 inertial frame at T end

x_3-y_3-z_3-o_3 inertial frame at E end

$\bar{\alpha}$ constant

θ torsional angle

θ_0 steady torsional angle ($\bar{t} \leqq \bar{T}_0$)

θ'_{cr} critical strain

ρ density of drive shaft

Ω rotational speed of drive shaft

$(\dot{\ })$ $\partial(\ )/\partial\bar{t}$

$(\ )'$ $\partial(\ )/\partial\bar{x}$

INTRODUCTION

A few helicopter accidents with the destructions of the tail rotor drive shafts recently occurred in Japan. The drive shaft was destroyed at the two places in the first accident as shown in Fig. 1. The evidence was observed that the tail rotor blade hit the tree, and was constrained. In order to make clear the true cause of this accident, the analytical study was conducted to investigate why the drive shaft was destroyed at the two places. The analytical result indicated that the destruction of the drive shaft is strongly effected by the speed of the constraint as follows: The drive shaft might be destroyed at the two places, when the drive shaft is suddenly constrained, or when the main rotor is also constrained after the first destruction of the drive shaft. When the drive shaft is gradually constrained, it should be destroyed at the only one place near the tail rotor end.

After these conclusions of this analysis, two more helicopter accidents occurred. In these cases, the tail rotor blades were constrained by water, and were destroyed at the only one place near the tail rotor end. Consequently, this coincides well with the present conclusions.

ANALYTICAL MODEL

The drive shaft analyzed in this study is shown in Fig. 2(A). A mathematical model of the shaft is, for simplicity of calculation, considered to be a uniform bar as shown in Fig. 2(B). The geometrical configuration and operating conditions are given in Table 1. Three right-handed, orthogonal Cartesian coordinate systems are used as shown in Fig. 3. The first coordinate system, x_1-y_1-z_1-o_1, is fixed to the drive shaft and is located at the terminal end to the engine shaft, E end. The second coordinate system, x_2-y_2-z_2-o_2, is the inertial frame and is located at the terminal end to the tail rotor, T end. The third coordinate system, x_3-y_3-z_3-o_3, is also the inertial frame and is located at the E end. The origin of the third coordinate system 0_3 is coincident with that of the first coordinate system 0_1. Therefore, the x_1-y_1-z_1-o_1 frame rotates with the angular velocity Ω of the shaft against the x_2-y_2-z_2-o_2 frame and the x_3-y_3-z_3-o_3 frame.

The torsional equation of motion is, then, given by the following simple equation[1];

$$\frac{\partial^2\theta}{\partial\bar{t}^2} = \bar{c}^2 \frac{\partial^2\theta}{\partial\bar{x}^2} \tag{1}$$

The above equation is the one dimensional wave equation. The solution is analytically obtained by using the method of separating variables.[2]

THE SUDDEN CONSTRAINT OF THE TAIL ROTOR

The time sequence of the destruction is considered as follows: The tail rotor blade hits some obstacle like a tree, and suddenly stops at time $\bar{t}=\bar{T}_0$. The T end of the drive shaft is immediately constrained at the same time, because the rigidity of the tail rotor blade and of the gear is high enough in comparison with that of the drive shaft. The first destruction of the drive shaft, then, occurs at time $\bar{t}=\bar{T}_1$.

Equation (1) must be solved for corresponding boundary and initial conditions for the various periods as follows:

1) The Period Before the Constraint ($\bar{t}<\bar{T}_0$)

During this period, the drive shaft rotates at the constant angular velocity Ω, and transmits the necessary torque from the engine to the tail rotor. The drive shaft is steadily twisted by the corresponding torsional angle θ_0. By using x_1-y_1-z_1-o_1 frame, the boundary conditions is expressed as

$$\left.\begin{aligned} \theta(0,\ \bar{t}) &= 0 \\ \theta(1,\ \bar{t}) &= \theta_0 \end{aligned}\right| \tag{2}$$

and the initial condition is given by

$$\dot{\theta}\ (\bar{x},\ \bar{t}) = 0 \tag{3}$$

The solution of equation (1) is then given by

$$\theta\ (\bar{x},\ \bar{t}) = \theta_0\bar{x} \tag{4}$$

This result is shown in Fig. 4.

2) The Period After the Constraint and Before the First Destruction ($\bar{T}_0<\bar{t}<\bar{T}_1$)

During this period, it is assumed that the drive shaft at the E end is continuously rotating with the constant angular velocity Ω, because the angular momentum of the engine is large enough in comparison with that of the drive shaft.

When the T end is suddenly constrained at time $\bar{t} = \bar{T}_0$, it is observed in the x_1-y_1-z_1-o_1 frame that the T end suddenly starts to rotate with the angular velocity $-\Omega$. Therefore, the boundary conditions are given by

$$\left.\begin{aligned} &\theta\ (0,\ \bar{t}) = 0 \\ &\theta\ (1,\ \bar{t}) = (\bar{t} - \bar{T}_0) + \theta_0 \end{aligned}\right| \qquad (5)$$

The initial conditions are given from the previous period 1) as

$$\left.\begin{aligned} &\dot{\theta}\ (\bar{x},\ \bar{T}_0) = 0 \\ &\theta\ (\bar{x},\ \bar{T}_0) = \theta_0 \bar{x} \end{aligned}\right| \qquad (6)$$

The results of solutions for θ, $\theta' = \partial\theta/\partial\bar{x}$ and $\dot{\theta} = \partial\theta/\partial\bar{t}$ obtained from equations (1), (5) and (6) are shown in Fig. 5.

It is assumed here that the drive shaft may be destroyed when and where the torsional strain θ' $(\bar{x},\ \bar{t})$ becomes larger than the critical strain $\theta'cr$. It is observed from Fig. 5 that the step input of the torsional strain does not change the wave form and that the amplitude of the strain wave discretely increases at every reflection at the opposite ends. In the present example, the value of $\theta'cr$ shown in Table 1 includes some uncertainty, and the analytical model includes the assumptions. Consequently, it is impossible to determine exactly when the first destruction occurs. It is, however, definite that the drive shaft is destroyed before the third reflection of the strain wave (twice at the E end and once at T end) in comparison θ' with $\theta'cr$. Therefore, the only three cases are considered as follows:

2-1) The destruction occurs before the first reflection. In this case, the wave moves from the T end to the E end as shown in Fig. 5- ② and ③ . The destruction should occur just at the front of the wave (P-P section) or at some small distance behind the front of the wave (P'-P' section). Consequently, in this case, the drive shaft is destroyed always near the T end. The reason of the destruction which might occur at the P'-P' section is that the destruction starts when the front of the wave reaches at a certain point and it is completed after the part of the wave moves through this point.

2-2) The destruction occurs before the second reflection. In this case, the wave moves from the E end to the T end as shown in Fig. 5- 4 and ⑤ . Therefore, the drive shaft is destroyed near the E end.

2-3) The destruction occurs after the second reflection. In this case, the wave moves from the T end to the E end as shown in Fig. 5- 6 . The drive shaft is destroyed near the T end.

3) The Period After the First Destruction ($\bar{T}_1<\bar{t}$)

The x_1-y_1-z_1-o_1 frame is used after the first destruction occurred near the T end (Cases 2-1) and 2-3)). The x_2-y_2-z_2-o_2 frame is used after the first destruction occurred near the E end (Case 2-2)). The length of the remaining drive shaft is defined by ℓ_1, and x is nondimensionalized by ℓ_1 as $\bar{x} = x/\ell_1$. The equation of motion is again given by equation (1). The terminal end of the drive shaft is fixed at $\bar{x}$=0, and it is free at $\bar{x}$=1. Therefore, the boundary conditions are given by

$$\left.\begin{aligned} \theta\ (0,\ \bar{t}) &= 0 \\ \theta'\ (1,\ \bar{t}) &= 0 \end{aligned}\right| \qquad (7)$$

The initial conditions are different between the places where the first destruction occurred as follows:

3-1) When the first destruction occurred at the P-P section, the following initial conditions are given in either three cases, 2-1), 2-2) or 2-3):

$$\left.\begin{aligned} \theta'\ (\bar{x},\ \bar{T}_1) &= a_1 \\ \dot{\theta}\ (\bar{x},\ \bar{T}_1) &= 0 \end{aligned}\right| \qquad (8)$$

The solution of equations (1), (7) and (8) is the free torsional vibration as shown in Fig. 6. It is seen that the torsional strain θ' is always equal to its initial value at $\bar{t} = \bar{T}_1$. Therefore, the second destruction does not occur in this case.

3-2) When the first destruction occurred at the P'-P' section, the following initial conditions are obtained in either three cases, 2-1), 2-2) or 2-3):

$$\left.\begin{aligned} \theta'\ (\bar{x},\ \bar{T}_1) &\left|\begin{aligned} &= a_1\ \ (0 < \bar{x} < \bar{x}_2) \\ &= a_2\ \ (\bar{x}_2 < \bar{x} < 1) \end{aligned}\right. \\ \dot{\theta}\ (\bar{x},\ \bar{T}_1) &\left|\begin{aligned} &= 0\ \ \ \ (0 < \bar{x} < \bar{x}_2) \\ &= b_2\ \ (\bar{x}_2 < \bar{x} < 1) \end{aligned}\right. \end{aligned}\right| \qquad (9)$$

The solution of equations (1), (7) and (9) is shown in Fig. 7. For simplicity, it is assumed to be $a_1 = 0$ in this figure. When $a_1 \fallingdotseq 0$, the wave form can be given by superposing the wave form of Fig. 6 and that of Fig. 7. Then, the superposed wave form is very similar to that of Fig. 7, because a_2 is very small in the present example. Therefore, the following discussion about Fig. 7 may be extended to the entire cases of equation (9): The free end in Fig. 7 corresponds physically to the end of the first destruction. The fixed end corresponds to the E end in cases 2-1) and 2-3), and it corresponds to the T end in case 2-2). It is clearly shown in Fig. 7 that the reflection of the wave makes the amplitude of the torsional strain increase greater than the initial value near the fixed end. This is because the angular momentum of $\dot{\theta}$ changes to the strain of θ' at the fixed end. This maximum strain is observed at 5 in Fig. 7. Therefore, there is the possibility that the second destruction may occur at the fixed end.

SOFT CONSTRAINT OF THE TAIL ROTOR

When the tail rotor is gradually constrained, or when the rigidity of the tail rotor blade is low, the drive shaft is gradually constrained. In this case, the wave form of $\dot{\theta}$ is assumed to have the first order delay at the T end. Before the constraint $\bar{t} < \bar{T}_0$, the solution is the same as shown in Fig. 4. After the constraint and before the first destruction $\bar{T}_0 \leq \bar{t} < \bar{T}_1$, by using the x_1-y_1-z_1-o_1 frame, the equation of the motion and the initial conditions are again given by equations (1) and (6), respectively. The boundary conditions are given by

$$\left.\begin{aligned} \theta\ (0,\ \bar{t}) &= 0 \\ \dot{\theta}\ (1,\ \bar{t}) &= 1-e^{-\bar{\alpha}\bar{t}} \end{aligned}\right| \qquad (10)$$

The solutions of equations (1), (6) and (10) are shown in Fig. 8 through 10 for $\bar{\alpha}$ = 10, 3 and 1/5, respectively. When $\bar{\alpha}$ increases, the wave form becomes similar to the ramp input, and the angular momentum of $\dot{\theta}$, which is transmitted to the remaining part of the drive shaft through the destroyed section, becomes small. In addition, the maximum strain is observed only at the T end as shown in Fig. 11(b). This leads to the only one destruction, which is limited near the T end. When $\bar{\alpha}$ decreases, the wave form becomes similar to the step input. Consequently, the results given in the previous section concerning "the sudden constraint", is again obtained.

THE SECOND DESTRUCTION CAUSED BY THE SUDDEN STOP OF THE MAIN ROTOR

There is another possibility that the second destruction of the drive shaft may occur. After the first destruction occurred near the T end, the considerable time has passed. The remaining drive shaft, the length of which is ℓ_1, is assumed to rotate with constant angular velocity, and to have no strain θ' along the entire drive shaft. The main rotor, then, hits some obstacle and suddenly stops at $\bar{t} = \bar{T}_2$. This causes the sudden constraint of the drive shaft at the E end, because the drive shaft of the tail rotor is connected with the main rotor. The equation of motion is again given by equation (1). By using the x_3-y_3-z_3-o_3 frame, the terminal end of the drive shaft is fixed at $\bar{x} = 0$, and it is free at $\bar{x} = 1$. Therefore, the boundary conditions are given by equation (7). The initial conditions are given by

$$\left.\begin{aligned} \theta'(\bar{x},\ \bar{T}_2) &= 0 \quad (0 < \bar{x} < 1) \\ \dot{\theta}\ (\bar{x},\ \bar{T}_2) &= b_2 \quad (0 < \bar{x} < 1) \end{aligned}\right| \qquad (10)$$

The solutions of equations (1), (7) and (10) is shown in Fig. 11. It can be seen that the maximum strain appears at the E end, because the angular momentum $\dot{\theta}$ changes to the strain θ'. Therefore, there is the possibility of the second destruction near the E end.

CONCLUSIONS

The impulsive destruction of the drive shaft caused by the sudden stop of the tail rotor was analyzed. The drive shaft was assumed to be uniform, and the equation of the torsional motion was solved. The following conclusions were then obtained:

1) When the tail rotor is suddenly constrained (like a step input), there is a possibility that the drive shaft is destroyed at two places by the reflection of the strain waves. These destructions should occur near the opposite ends of the drive shaft.
2) When the tail rotor is gradually constrained (like a ramp input), the drive shaft is destroyed at the only one place near the T end.
3) When the first destruction of the drive shaft occurs near the T end, and when the main rotor hits some obstacle and thus suddenly stops, there is a possibility of the second destruction of the drive shaft near the E end.

REFERENCES

1) W. Johnson, Impact Strength of Materials, Edward Arnold, London, 1972

2) E. Kreyszig, Advanced Engineering Mathematics, Fifth Edition, John Wiley and Sons, New York, 1983.

Table 1. Drive shaft geometrical configulation and operating conditions

Items	Dimensions
rotational speed rad./sec, Ω	200π
density $\frac{Kg \cdot S^2}{m^4}$, ρ	286
length of drive shaft m ($\bar{t} \leq \bar{T}_1$), ℓ	3.45
($\bar{t} > \bar{T}_1$), ℓ_1	3.1
speed of wave ($\bar{t} \leq \bar{T}_1$), $\bar{C}$	1.42
($\bar{t} > \bar{T}_1$), $\bar{C}_1$	1.55
torsional angle rad., θ_0	0.395
critical strain rad. ($\bar{t} \leq \bar{T}_1$), θ'_{cr}	1.43
($\bar{t} > \bar{T}_1$)	1.29
torsional rigidity $kg \cdot m^2$, GJ	41.5

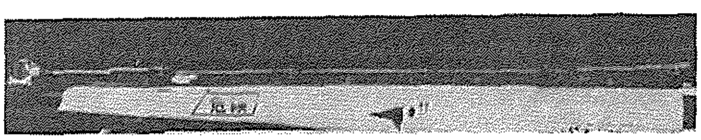


Fig. 1 Destroyed drive shaft

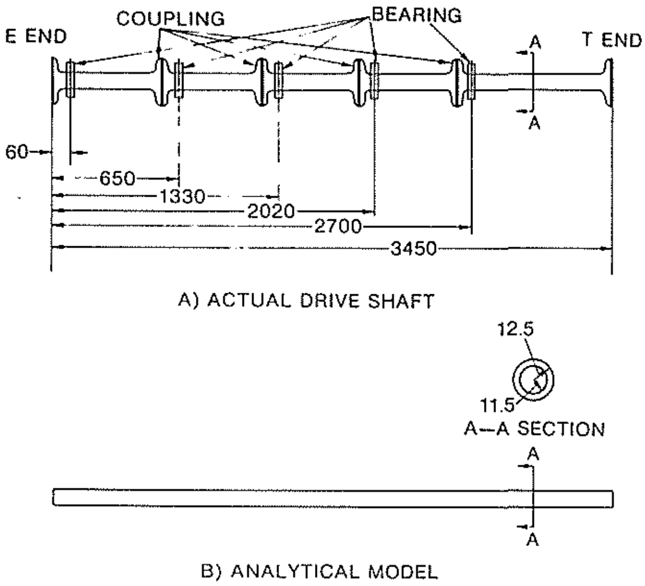


Fig. 2 Geometries of drive shaft

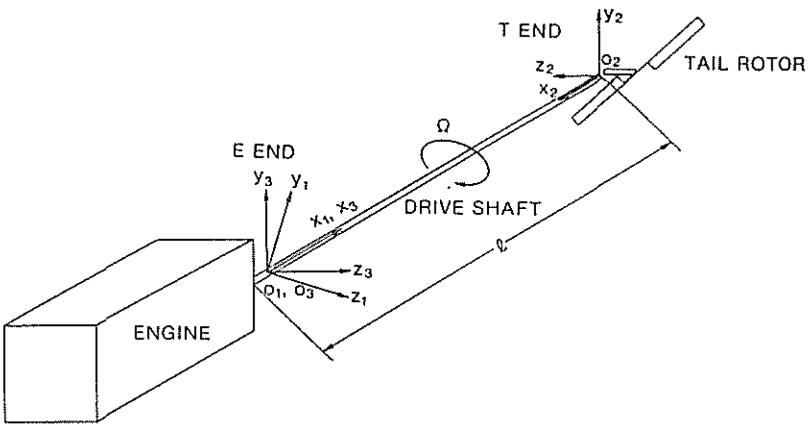


Fig. 3 Coordinate systems

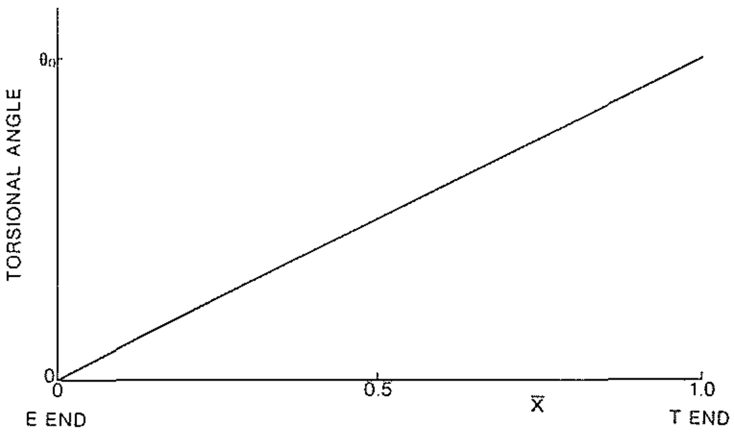
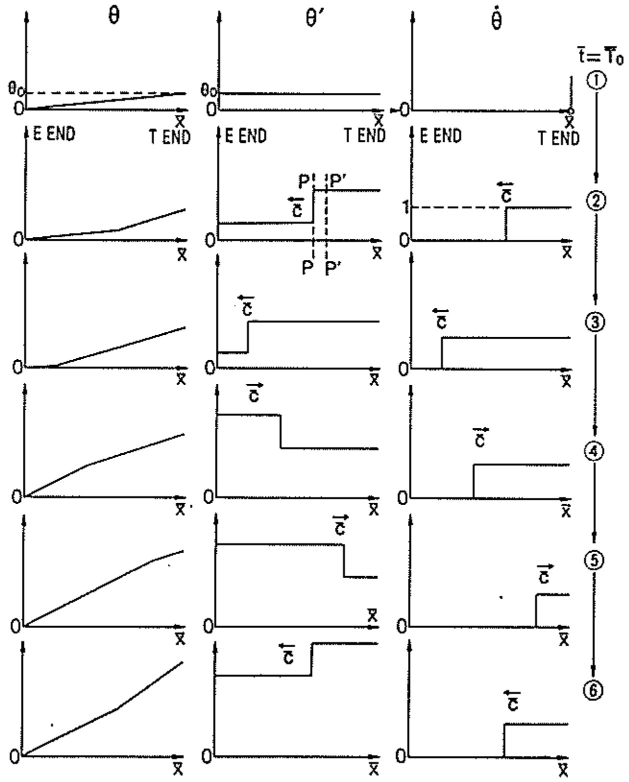
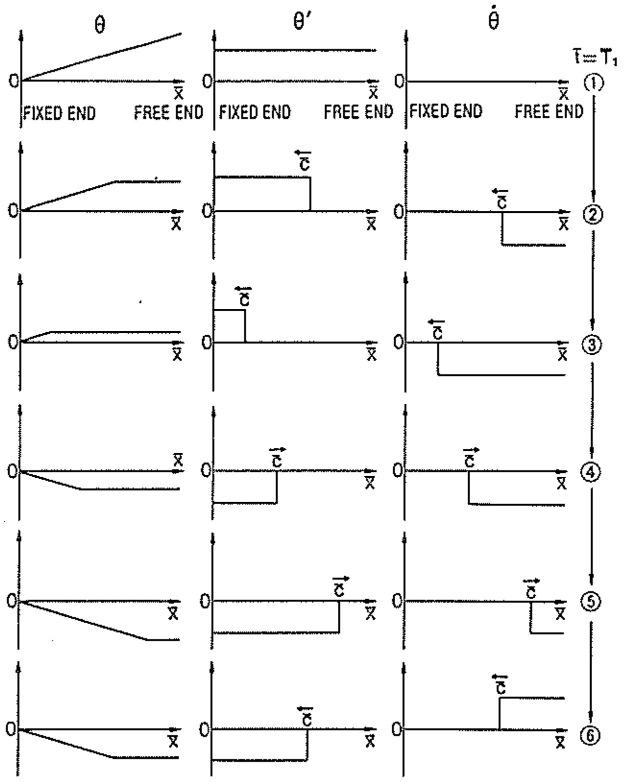


Fig. 4 Distribution of torsional angle before constraint.



① $\bar{t}=\bar{T}_0$ → ② → ③ → REFLECTION AT E END → ④ → ⑤ → REFLECTION AT T END → ⑥

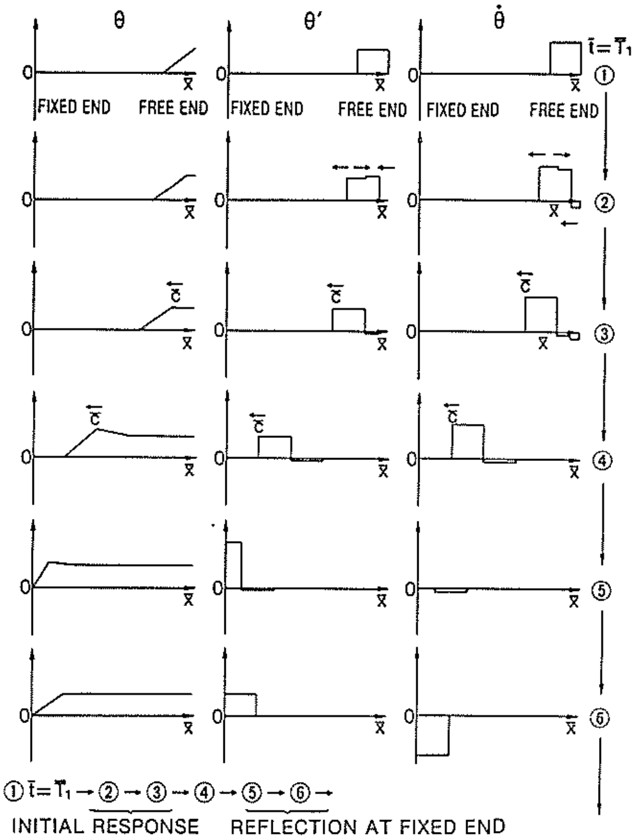
Fig. 5 Time-histories of θ, θ' and $\dot{\theta}$ for sudden constraint



① $\bar{t}=T_1$ → ② → ③ → REFLECTION AT FIXED END → ④ → ⑤ → REFLECTION AT FREE END → ⑥

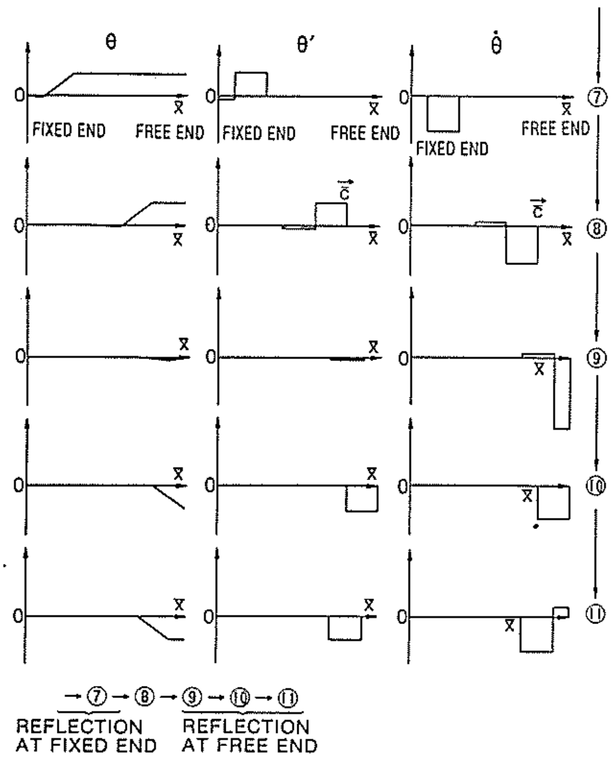
Free end: First destruction
Fixed end: E end or T end

Fig. 6 Time-histories of θ, θ' and $\dot{\theta}$ for sudden constraint



Free end: First destruction
Fixed end: E end or T end

Fig. 7 Time-histores of θ, θ' and $\dot{\theta}$ for sudden constraint (Continued)



Free end: First destruction
Fixed end: E end or T end

Fig. 7 Time-histories of θ, θ' and $\dot{\theta}$ for sudden constraint (Concluded)

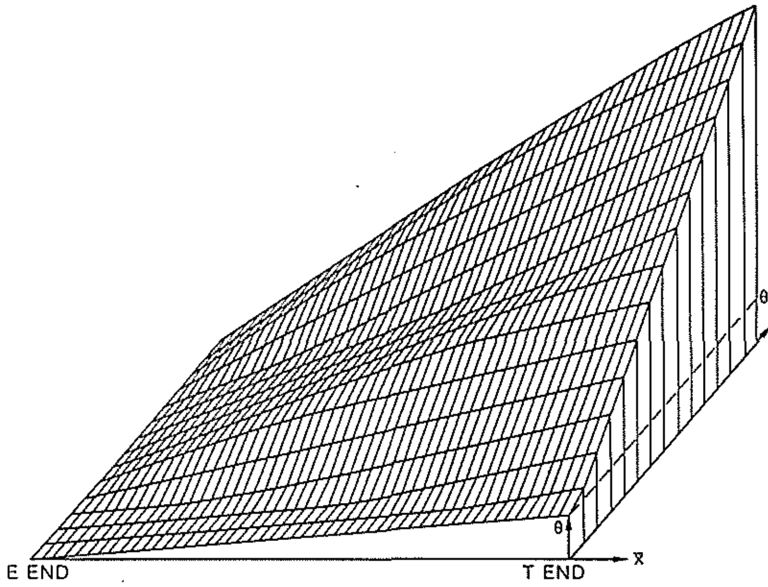


Fig. 8(A) Time-history of θ for gradual constraint: $\bar{\alpha}=10$

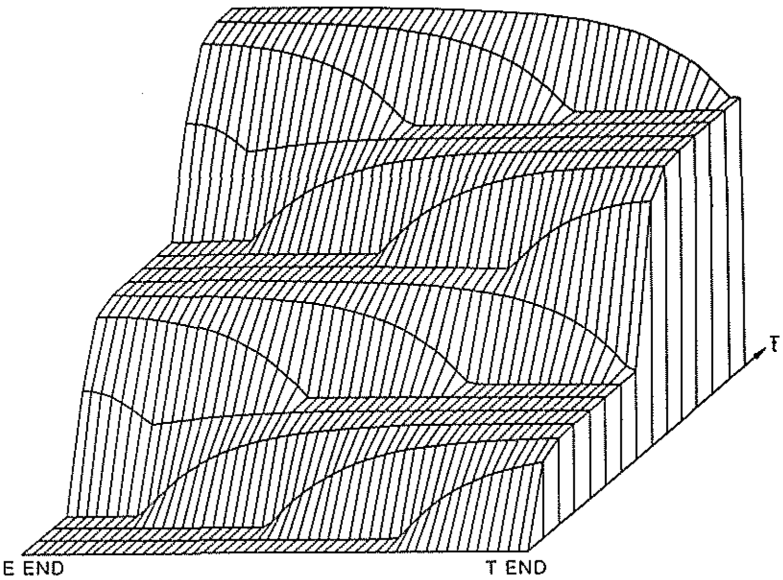


Fig. 8(B) Time-history of θ′ for gradual constraint: $\bar{\alpha}=10$

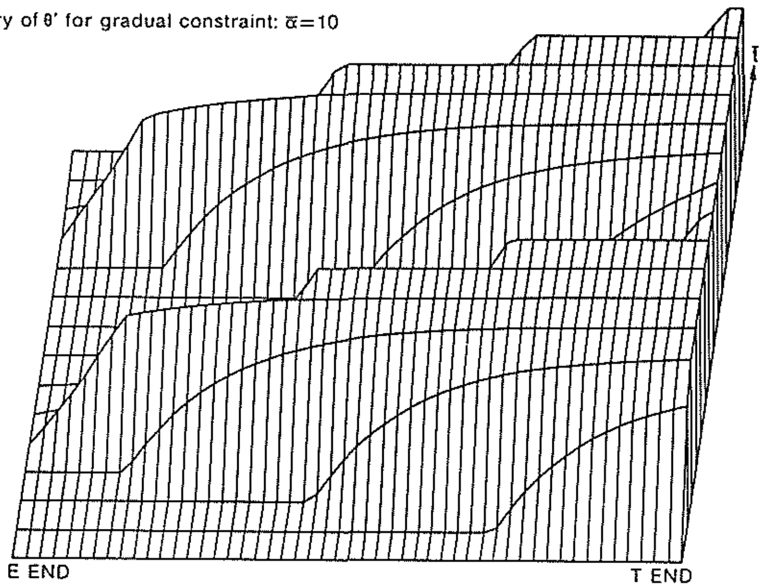


Fig. 8(C) Time-history of $\dot{\theta}$ for gradual constraint: $\bar{\alpha}=10$

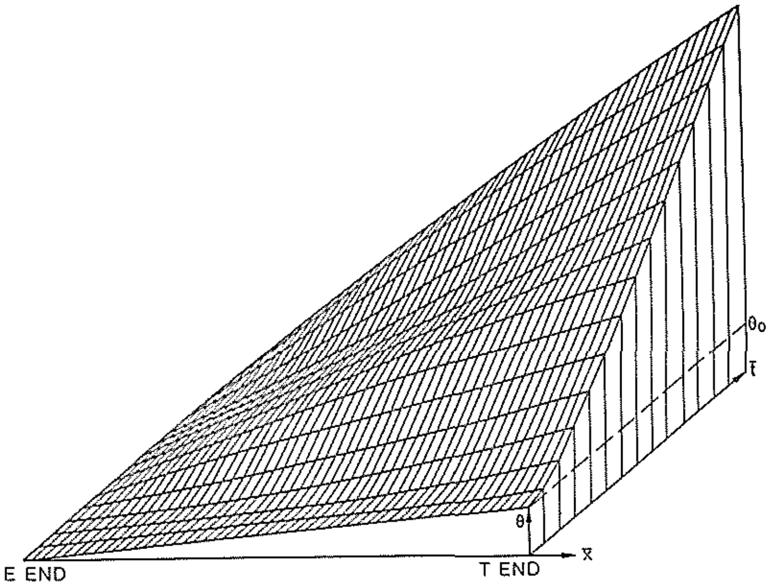


Fig. 9(A) Time-history of θ for gradual constraint: $\bar{\alpha}=3$

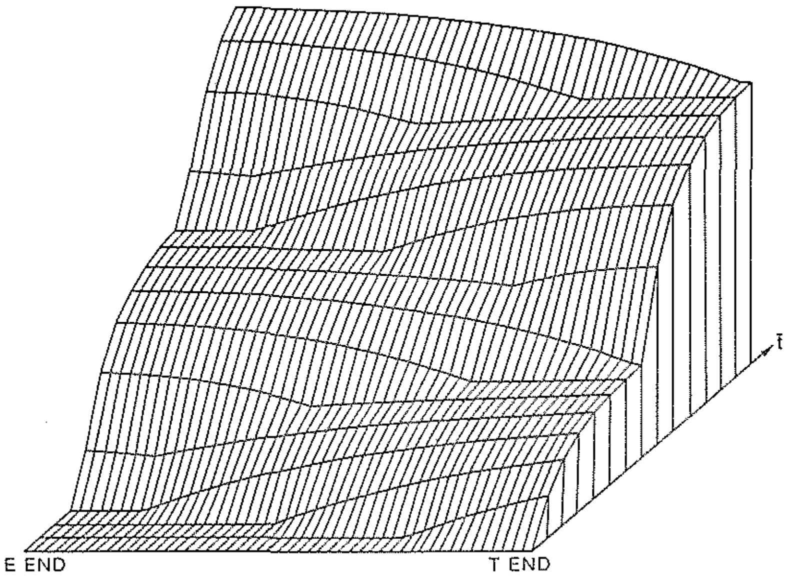


Fig. 9(B) Time-history of θ' for gradual constraint: $\bar{\alpha}=3$

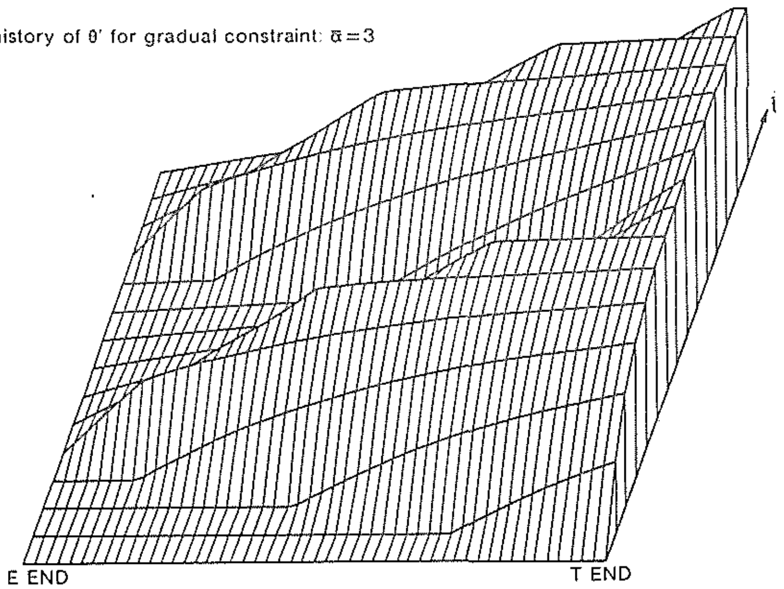


Fig. 9(C) Time-history of $\dot{\theta}$ for gradual constraint: $\bar{\alpha}=3$

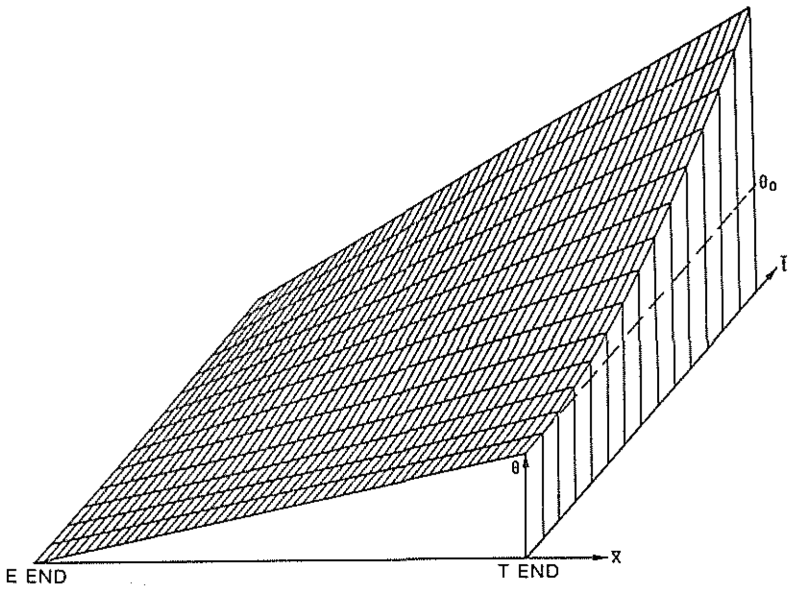


Fig. 10(A) Time-history of θ for gradual constraint: $\bar{\alpha}=1/5$

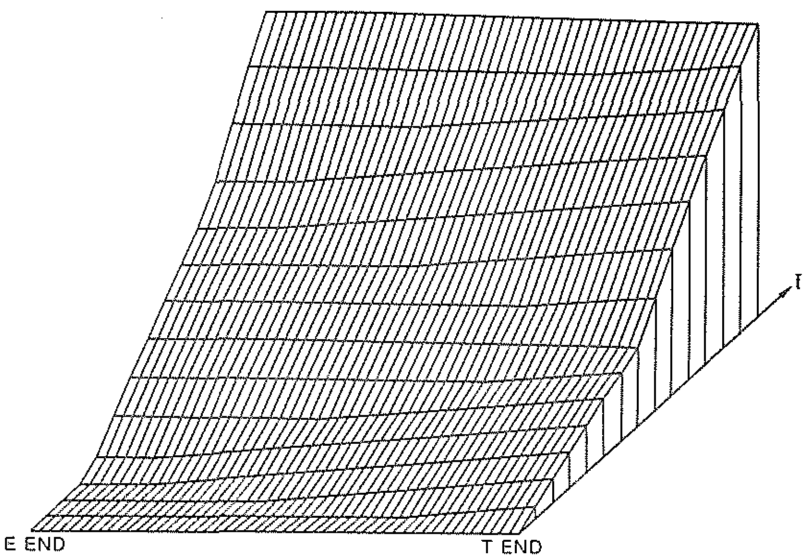


Fig. 10(B) Time-history of θ' for gradual constraint: $\bar{\alpha}=1/5$

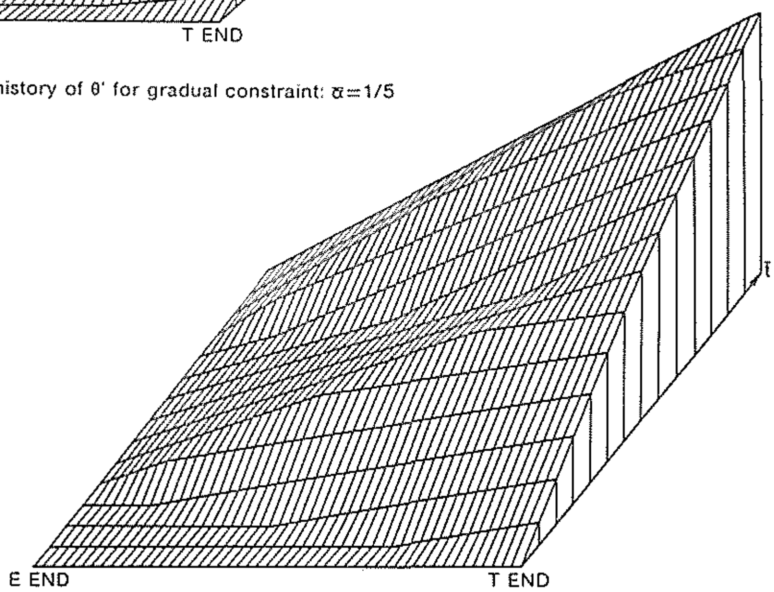
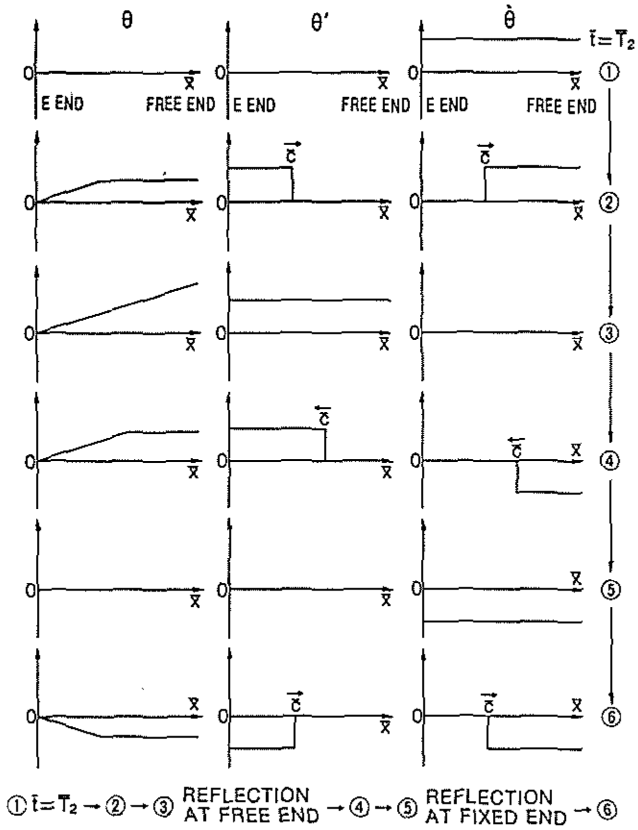


Fig. 10(C) Time-history of $\dot{\theta}$ for gradual constraint: $\bar{\alpha}=1/5$



Free end: First destruction

Fig. 11 Time-histories of θ, θ' and θ̇ for constraint of main rotor