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A SIMPLIFIED APPROACH TO THE FREE WAKE ANALYSIS OF A HOVERING ROTOR

R. H. Miller

Massachusetts Institute of Technology
Cambridge, Massachusetts

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Garmisch-Partenkirchen
Federal Republic of Germany

Deutsche Gesellschaft fur Luft- und Raumfahrt e.V.
Goethestr. 10, D-5000 Koln 51, F.R.G.

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R.H. MILLER
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

ABSTRACT

In order to predict rotor performance, blade loading and acoustic signatures, it is necessary to use methods of analysis which account for the true wake geometry and the finite number of blades. This paper discusses simplified approaches to the free wake aerodynamic analysis of hovering rotors which permit rapid evaluation of rotor aerodynamic characteristics. Analytical results are compared with existing measurements of blade bound circulation and wake geometry.

1. Introduction

It has been known for some time that the aerodynamic characteristics of a helicopter rotor blade are critically dependent on the vortex structure in the wake and particularly on the position of the tip vortex generated by a blade relative to the immediately following blade (Fig. 1). Experimental evidence [1,2] indicates that this first blade/vortex encounter is closer than would be predicted by classical vortex or momentum theories and that the vortices start descending at the expected rate only after this first encounter. Furthermore, the wake contracts rapidly prior to first encounter (and more slowly thereafter) which places the encounter well inboard of the tip.

Recognizing the necessity of establishing the wake structure in some detail, free wake analysis techniques are being developed (for example, references 3 to 7) in which the wake is allowed to assume a position determined by the time history of the velocities in the wake. Such analyses are important for optimizing rotor performance, determining blade loads and estimating acoustic signatures.

This paper is concerned with a simplified approach to the problem of determining free wake geometry. This approach is being developed in order to help obtain a better understanding of the physics of the problem of wake structure and as a guide for more elaborate aerodynamic modelling.

2. Theoretical Development

It has been shown [8] that, in the forward flight case, blade loads can be determined by replacing the curved vortex as it approaches the blade by a straight vortex line at the point of closest encounter. A similar approximation suggests itself for the hovering case in which the spiral wake is replaced by straight infinite vortex lines located below the blade (Fig. 2). This leads to a two dimensional solution that may be readily shown to give the same results as conventional momentum theory or the classical vortex theory of, for example, Ref. 9.

The two dimensional model postulated above can be readily extended to a three dimensional solution, using the relationships for hovering flight of reference 10, in which the infinite vortex lines are replaced by vortex rings and the far wake by semi-infinite vortex cylinders. Since most of the elliptical integrals resulting from this modelling can be readily expanded by Cayley's logarithmic series truncated after the first two or three terms, the computational time involved in the three dimensional solution is not appreciably longer than for the two dimensional case, for which the contributions of the vortex lines have simple closed form solutions. Results from both models will be presented here.

The complete theoretical development and computational algorithms are described in detail in reference 11, and may be briefly summarized as follows:

1) The wake is divided into 3 sections: the near wake attached to the blade, an intermediate wake and a far wake.

2) The near wake is composed of a series of straight semi-infinite vortex filaments extending from the blade nodes, with the blade treated as a lifting line, as in reference 10.

3) After leaving the blade, it is assumed that this distributed wake rolls up almost immediately according to Betz's theory of conservation of momentum [12, 13]. Since the wake initially contracts rapidly it may be expected that a distinct maximum in circulation will occur inboard of the blade tip. It is logical to roll up the wake from the tip to this maximum. A second roll up will most likely occur between this point of maximum circulation and the point where $d\Gamma/dr$ again starts changing abruptly. The remaining circulation to the root must then roll up into a third vortex. These three vortices form the intermediate wake.

4) The far wake is represented by semi-infinite vortex sheets in the case of the two dimensional model and by semi-infinite vortex cylinders for the three dimensional case, both starting at a distance from the rotor one vortex spacing below the last intermediate wake vortex. The solutions are not sensitive to the number of spirals in the intermediate wake beyond a minimum of four.

5) Since the blade generates a spiral wake the vortex lines or vortex circles should be inclinded at an angle λ/r where λ is the local velocity ratio and r is the radial position of the vortex element. The effect of this inclination is to decrease slightly the vertical displacement of the vortex due to slip and to reduce the inplane thrust due to the reduction in tangential velocities, effects which may readily be shown to be second order in λ. When this additional refinement was introduced into the calculations it was found to result in a negligible change in wake geometry and rotor thrust for typical helicopter rotor loadings.

6) Since the problem is nonlinear, an iterative approach has to be used for its solution. To start this iteration the wake spacing determined from three dimensional momentum theory for non-uniform inflow and the corresponding blade circulations are used. The axial and radial velocities induced at each vortex from all the sections of the wake are then computed. The rolled up near wake is used in computing its contributions to velocities everywhere in the wake. A new wake geometry is then established using the average velocity between vortices to determine their spacing.

7) The new induced velocities at the blade are then computed and a new distribution of circulation along the blade determined. The iteration is continued until the induced velocities everywhere on the blade have converged. Usually a convergence test of 5% on geometry will result in thrust convergence of a fraction of a percent.

8) The slope of the lift curve has been taken from reference 14 as .98 x 2π. The self-induced velocities on the curved vortex have been computed from reference 15 using a core size from reference 16.

3. Comparison of Experimental Results

In order to test the validity of the theoretical approach, the results were compared to those of reference 17, which presents experimentally determined blade circulation distributions and wake geometries for a two bladed rotor and for two radically different sets of rotor blades. One set has a constant chord, the other has a rapidly varying "ogee" chord distribution over the outer 10% of the blade. (Table 1).

The experimental results for the straight blade are compared in figure 3 with blade circulation and wake geometry predicted by the two dimensional model. It is interesting to observe the rapid inboard movement of the tip vortex and its proximity to the blade at first encounter.

Examining the analytical solutions indicates that this geometry is primarily influenced by two factors. As the bound circulation leaves the blade at the tip (Fig. 1) and forms the strong tip vortex, its position is determined primarily by the induced velocity field from the vortices in the wake, generated by preceding blade passages, which lie below it. This induced velocity has strong inward radial components and relatively smaller vertical components. Consequently, it can be expected that the vortex will remain close to the tip path plane, but move rapidly inward. After passage of the following blade, the position of this first vortex is influenced by the new vortex formed immediately above and out-board of it, (Fig. 1). This new vortex then initiates a more rapid downward migration of the first vortex, as well as slowing its rate of inward displacement since the horizontal component of velocity induced by the new vortex at the first vortex is opposite to that induced by the wake below.

As may be expected from this geometry, the bound circulation distribution shows rapid variations with span. Neither the inflow, nor the circulation distributions are close to the uniform values assumed in classical vortex theories for the "ideally twisted" rotor blade. Preliminary indications are that the twist distribution and taper required for optimum performance of a hovering rotor are appreciably different from this ideal twist, as might be expected. The theory represented here should be sufficiently simple computationally to permit heuristic and possibly formal techniques to be used for such rotor optimization.

As predicted by classical vortex theory, the center vortex descends at a more rapid rate than the tip vortex. Of particular interest is the root vortex, whose position is strongly influenced by the upflow at the center of the rotor frequently observed on hovering helicopters. The migration of this root vortex (with its attendent far wake) through the blade, causes computational difficulties due to the singularities involved. And yet its influence upon either the blade loading or rotor performance is negligible, as shown in Figure 3 and Table 2. Furthermore, it is probable that this vortex does not exist in practice in the form postulated, since it trails into a region where the induced velocities are of the same order of magnitude as the local blade velocities and the flow is highly disturbed by the hub and root fittings. The resultant mixing and diffusion of the root vortices from all blades makes their true contributions to the wake velocities uncertain. Since the effects of the root vortex on the solutions are in any case negligible, their strength has been set to zero for the rest of the solutions.

Figure 4 shows a comparison between the two dimensional and three dimensional solutions. There is little to choose between the two methods as to accuracy of results for the cases considered here.

The results for the "ogee" blade are shown in Figure 5.

Table 2 contains a comparison of the experimental rotor performance with that obtained from the two mathematical models and from three dimensional momentum theory.

4. Extensions of Theory

Figure 6 lists some of the extensions to existing theory and additional experimental data required as an aid to further development of rotor aerodynamic design techniques.

Probably one of the most serious limitations to a more extensive use of free wake theory is a lack of information on the vortex structure, in particular, the distribution of vorticity outside the vortex core and the size of this core. In reference 5 it was shown that reasonable agreement between free wake analyses of the rotor in forward flight and experimental results could only be obtained if it were postulated that the vortex burst before the first blade encounter. This assumption seems to have been borne out by the test results of reference 18, where enlargement of the vortex core prior to first blade encounter is clearly indicated. It may be expected that a similar phenomenon could occur with rotors in hovering flight under conditions where very close proximity exists between vortex and blade at first encounter.

For the two bladed rotor considered here, first blade/vortex encounter occurs at approximately one chord length below the rotor, well outside the vortex core and also in the irrotational flow field outside the distributed vorticity predicted by the Betz roll up criteria. However, for multi-bladed lightly-loaded rotors, the first encounter could occur much closer to the blade. Under such conditions, it may be necessary to treat the distribution of vorticity outside the vortex core more carefully [11] and to have some knowledge of the time history of roll up.

It may also be necessary to extend the search techniques used in the present program to locate the maximum blade circulation at the tip to include more stationary points along the blade since it may be expected that vortex roll up will occur wherever $d\Gamma/dr$ changes sign. Such changes will occur more frequently with closer blade/vortex encounters.

The lifting line theory used in the present analyses is an adequate representation of the blade for the cases considered. Closer first encounters may, however, require the use of lifting surface theory. The Weissinger approximation (single panel with control point at the 3/4 chord location) is easily applied and gives highly accurate results for the hovering case where the vortex at first encounter is essentially perpendicular to the blade. Only a slight additional correction is needed for the effects of wake curvature, providing a sufficiently fine trailing wake distribution has been used. However, no theory is as yet available for predicting the initial roll up as the tip vortex leaves the blade and the resultant "tip loss" effects. For the purposes of this paper, the same tip losses have been assumed as were estimated in reference 16 from the experimental data, that is, .985 equivalent span for the straight rotor and .94 for the ogee.

Finally, real fluids effects occur due to flow separation resulting from the rapid span-wise flows induced by a close vortex/blade encounter which could result in an appreciable over-estimation of the peak loads along the blade, as discussed in reference 19. The development of theories capable of predicting real fluids effects together with more experimental data to substantiate and guide such theoretical analyses would be highly desirable.

5. Summary and Conclusions

This paper has shown that it is possible to model the free wake of a hovering rotor to a reasonable degree of accuracy using fairly simple mathematical models of the wake structure. The algorithms proposed permit ready identification of the physical factors affecting rotor aerodynamic characteristics and can serve as a useful guide in the development of more elaborate theories, particularly those including real fluid effects on the structure of the vortices in the rotor wake and on the blade airloads.

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Table 1

Rotor Characteristics

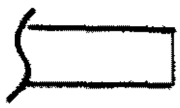
	Straight	Ogee
		
Twist	- 11°	- 11°
Collective pitch at 75% Radius	9.8°	9.8°
Solidity	.0464	.0396

Table 2

C_T Comparison

Experimental[17]	.00459	.00408
2 D Model (with root vortex)	.00456	---
2 D Model (no root vortex)	.00460	.00411
3 D Model	.00454	.00404
3 D Momentum Theory	.00442	.00414

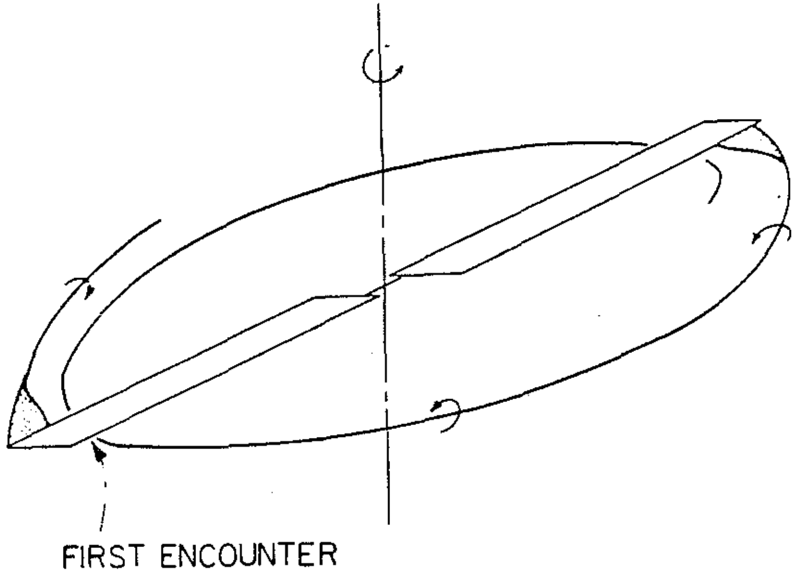


FIG. 1 FIRST BLADE VORTEX ENCOUNTER

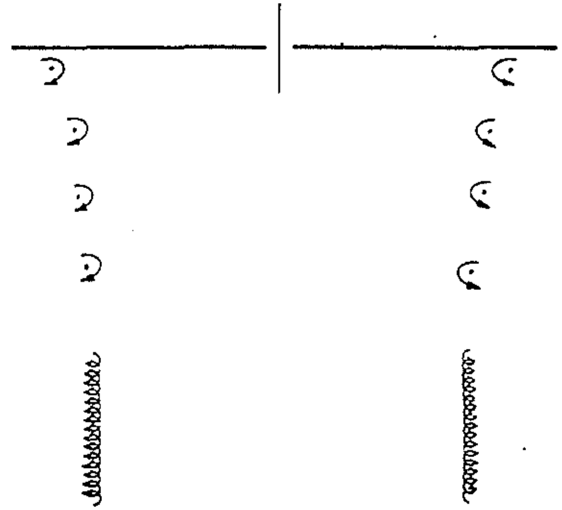


FIG. 2 TWO DIMENSIONAL MODEL OF HOVERING ROTOR

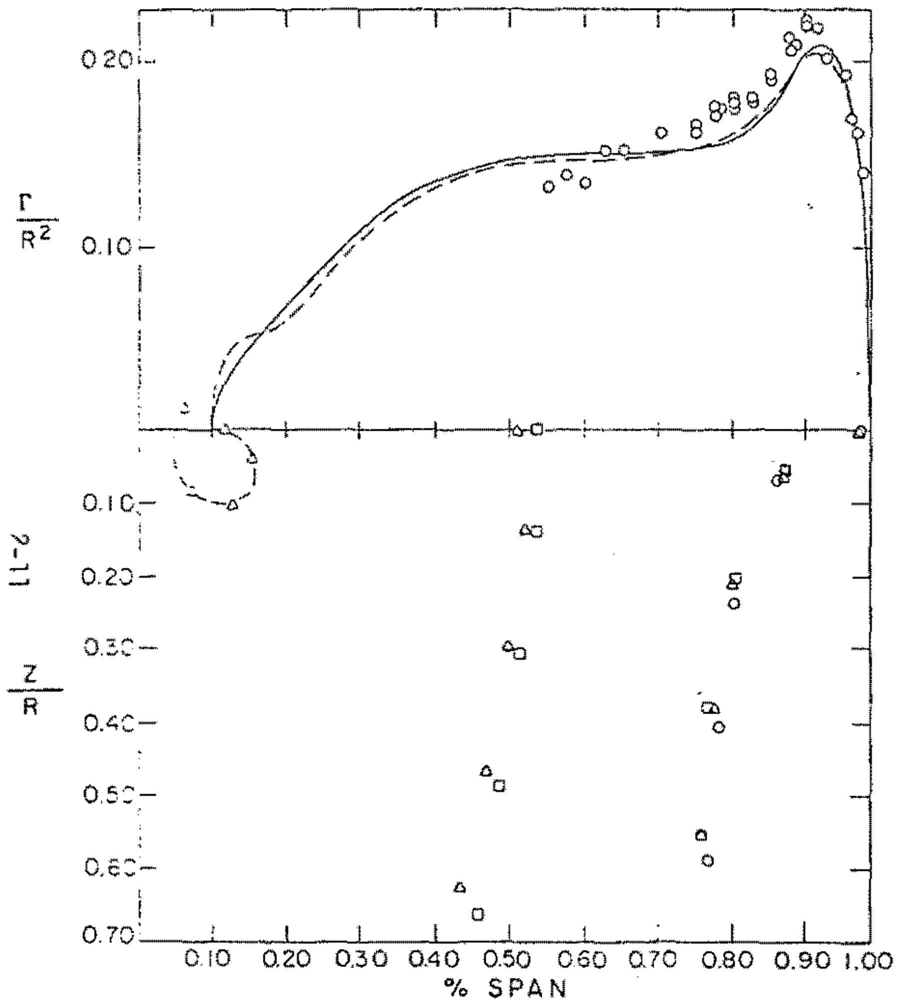


FIG. 3 BLADE BOUND CIRCULATION DISTRIBUTION AND LOCATION OF WAKE VORTICES FOR TWO DIMENSIONAL MODEL - STRAIGHT BLADE

- — — — △ WITH ROOT VORTICES
- ——— □ ROOT VORTICES NEGLECTED
- ○ TEST DATA

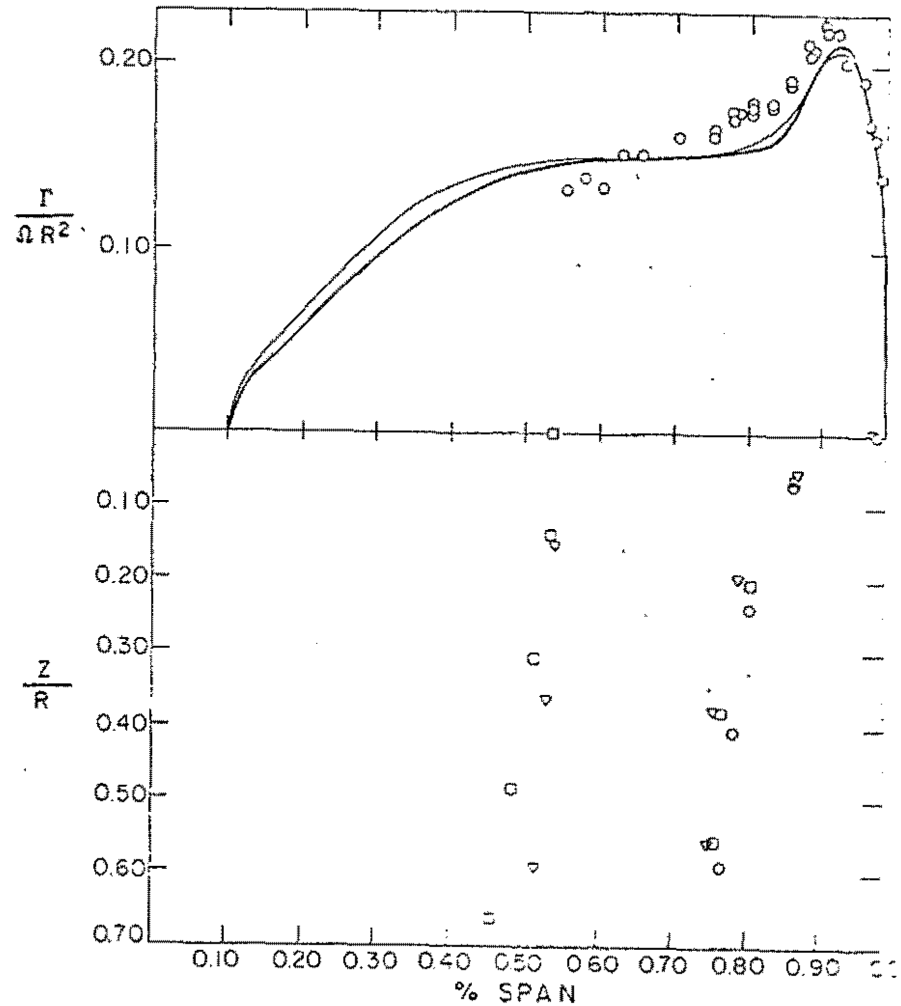


FIG. 4 BLADE BOUND CIRCULATION DISTRIBUTION AND LOCATION OF WAKE VORTICES - STRAIGHT BLADE

- ——— □ TWO DIMENSIONAL SOLUTION
- ▬▬▬ ▽ THREE DIMENSIONAL SOLUTION
- ○ TEST DATA

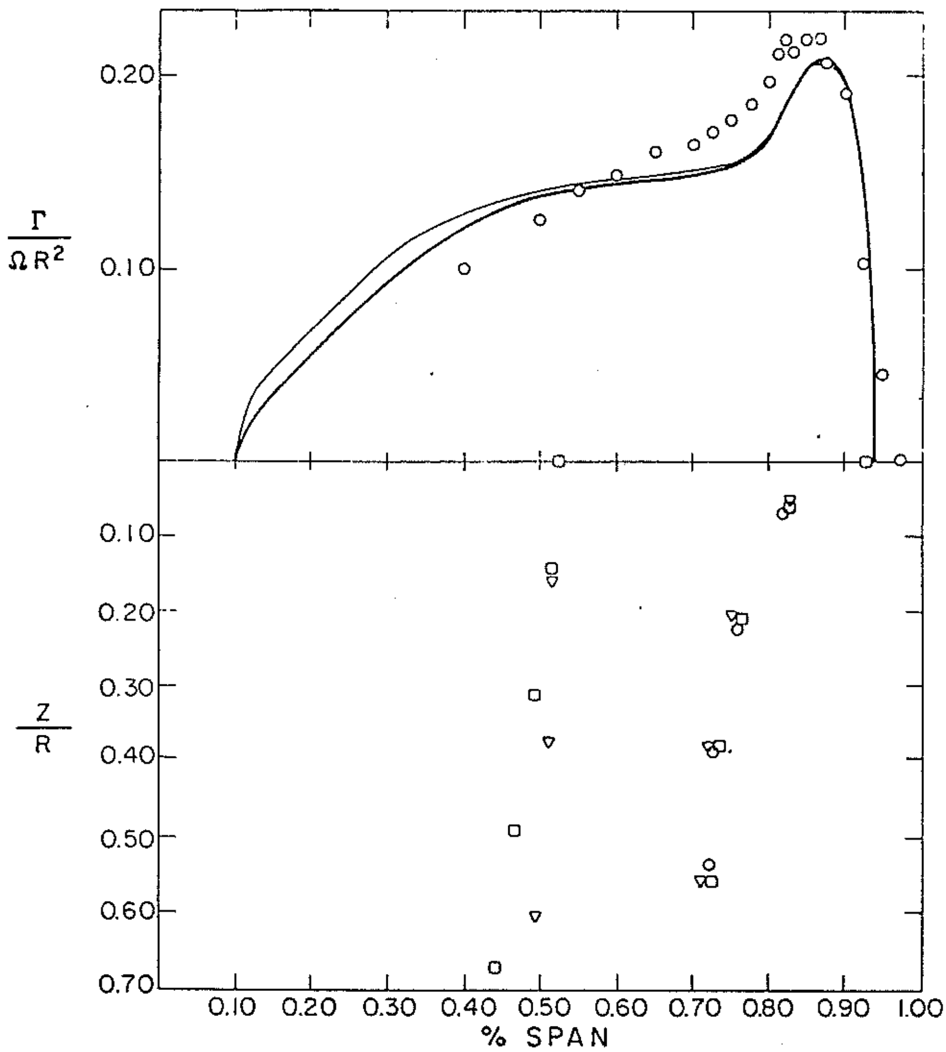


FIG. 5 BLADE BOUND CIRCULATION DISTRIBUTION AND LOCATION OF WAKE VORTICES - OGEE BLADE

□ TWO DIMENSIONAL SOLUTION

▽ THREE DIMENSIONAL SOLUTION

○ TEST DATA

FIGURE 6:

REQUIRED EXTENSIONS TO DATA BASE

1. Definition of vortex core size at first encounter
2. Time history of vortex roll-up
3. Vorticity distribution outside vortex core
4. Number of vortex formations in intermediate wake
5. Formation of tip vortex at blade - tip losses
6. Viscous flow effects on bound circulation due to vortex encounters
7. Experimental data on blade circulation for four and more bladed rotors