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RECENT TRENDS IN ROTARY-WING AEROELASTICITY

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Abstract

The purpose of this paper is to survey the principal developments which have occurred in the field of rotary-wing aeroelastcity during the past five year period. This period has been one of considerable activity and approximately one hundred papers have been published on this topic. To facilitate this review the field has been divided into a number of areas in which concentrated research activity has taken place. The main areas in which recent research is reviewed are: (1) structural modeling; (2) aerodynamic modeling; (3) aeroelastic problem formulation using automated or computerized methods; (4) aeroelastic analyses in foward flight; (5) coupled rotor/fuselage analyses; (6) active controls and their application to aeroelastic response and stability; (7) application of structural optimization to vibration reduction; and (8) aeroelastic analysis and testing of special configurations. These areas are reviewed with different levels of detail and some useful observations regarding potentially rewarding areas of future research are made.

Nomenclature

$\bar{a}$	= elastic axis aerodynamic center offset, nondimensionalized by semichord
b	= semichord nondimensionalized with respect to blade radius
$C(\bar{s})$	= generalized Theodorsen lift deficiency function
C_1	= inflow parameter
c	= chord
C_T	= thrust coefficient
C_{DU}	= unsteady drag coefficient
C_W	= weight coefficient, approximately equal to C_T
$C_{\theta CT}$	= control system stiffness
c_ζ	= damping coefficient for lag damper
D_ζ	= structural damping in lag
EA	= axial stiffness
EI	= bending stiffness
E	= expected value
e	= blade root offset
GJ	= torsional stiffness
h	= plunge displacement, also hub height above CG
k	= reduced frequency
ℓ_T	= length of swept tip portion of blade
ℓ_i	= length of finite element
L_C	= circulatory lift
M_C	= circulatory moment
M	= Mach number
M_1	= dynamic inflow parameter

n = number of blades
$Q(t)$ = downwash velocity at ¾ chord
R_c = elastic coupling parameter
R = blade radius
R_x, R_y = fuselage translations in x and y directions, respectively
T = transfer or control matrix
$u, u(\mathring{x}, \mathring{y}), u(x, y, \dot{x}, \dot{y})$ = control, reduced state control of hub rate and rate and position
U_{TO} = constant part of velocity of approach in airfoil theory
$U_T(t)$ = velocity of approach in airfoil theory
u, v, w = axial, lag and flap elastic blade displacements
W_Z, W_θ, $W_{\Delta\theta}$ = weighting matrices for vibration levels, control angles and rates control angle, respectively
X_1, X_2 = augmented states
Z_o, Z_i = vector of uncontrolled and controlled vibration levels, respectively
α = angle of attack
α_o = mean angle of attack
$\bar{\alpha}$ = oscillatory part of α
β_R, β_P = regressing and progressing flap modes
β_p = precone angles
β_{BB} = built in blade to beam angle
β_k, ζ_k = flap and lag deflections of the k^{th} blade, respectively
γ = lock number
δ_3 = pitch-flap coupling
ϵ = basis for ordering schemem, magnitude of blade slopes
ζ, η = principal axes of cross section
ζ_R, ζ_P = regressing and progressing lag mode
θ_o = collective pitch
θ_x, θ_y = roll and pitch of fuselage, respectively
θ = pitch mode for coupled rotor/body analysis
θ = pitch angle for airfoil in ONERA model
θ_{HHn} = higher harmonic control pitch angle
$\theta_{HHC}, \theta_{HHS}, \theta_{HHO}$ = various components of higher harmonic control
$\theta_{AC(\psi)}, \theta_{AS(\psi)}$ = components of active control
θ_{Ak} = actively controlled pitch angle of k^{th} blade
λ = inflow mode
Λ_V, Λ_H = vertical and horizontal ply angle orientation
Λ = sweep angle of blade tip
μ = advance ratio
ρ_A = density of air
σ = blade solidity
ϕ = roll mode, coupled rotor/fuselage model
ϕ_c, ϕ_s = phase angles for HHC
$\tilde{\phi}, \phi$ = torsional deformation and torsional quasicoordinate, respectively
ψ = Azimuth angle
$\bar{\omega}_{F1}, \bar{\omega}_{L1}$ = fundamental rotating flap and lag frequencies nondimensionalized with respect to Ω
$\bar{\omega}_\phi, \bar{\omega}_{T1}$ = fundamental torsional, rotating flap and lag frequencies nondimensionalized with respect to Ω
Ω = speed of rotation
$(\dot{\ })$ = derivative with respect to ψ or t

1. Introduction and Objectives

During the last twenty years there has been a tremendous proliferation in the literature dealing with rotary-wing aeroelastcity which indicates that understanding of rotor and coupled rotor/fuselage dynamics plays a central role in the design of successful rotorcraft. This vigorous activity has also resulted in a considerable number of survey papers which have dealt with various aspects of rotary-wing aeroelastic stability and response problem. These surveys are listed in chronological orders in Refs. [1-10]. One of the first significant reviews of rotary-wing dynamic and aeroelastic problems was provided by Loewy [1] where a wide range of dynamic problems were reviewed in considerable detail. A more restricted survey emphasizing the role of unsteady aerodynamics and vibration problems in forward flight was presented by Dat [2]. Flight dynamics problems of hingeless rotorcraft including experimental results was treated by Hohenemser [3]. Blade stability was also discussed in Ref. 3, since it is considered to be part of the broader flight dynamics problem. Two comprehensive reviews of rotary-wing aeroelasticity were presented by Friedmann [4,5]. In Ref. 4 a detailed chronological discussion of the flap-lag and coupled flap-lag-torsion problems in hover and forward flight was presented emphasizing the inherently nonlinear nature of the hingeless blade aeroelastic stability problem. The nonlinearities considered were geometrical nonlinearities due to moderate blade deflections. In Ref. 5 the role of unsteady aerodynamics, including dynamic stall, was examined together with the treatment the nonlinear aeroelastic problem in forward flight. Finite element solutions to rotary-wing aeroelastic problems were also considered together with the treatment of coupled rotor/fuselage problems. Another detailed survey by Ormiston [6] discussed the aeroelasticity of hingeless and bearingless rotors, in hover, from an experimental and theoretical point of view.

In addition to these papers which have emphasized primarily the aeroelastic stability problem two other surveys have dealt exclusively with the vibration problem and its control in rotorcraft [7,8]. One could therefore classify these papers as related to the aeroelastic response of the rotor, the vibrations caused by this aeroelastic response and the study of various passive, semi-active and active devices for controlling such vibrations. Finally it should be mentioned that in a very recent comprehensive review paper by Johnson [9,10] both the aeroelastic stability and the rotorcraft vibration problems were reviewed in the context of dynamics of advanced rotor systems.

The purpose of this paper is to survey the principal developments which have occurred in the field of rotary-wing aeroelasticity during the past five year period and thus it represents an extension of the previous two papers written by the author [4,5]. This period has been very productive and over one hundred papers were published on this topic. To facilitate this review the subject matter has been subdivided into a number of areas in which a concentrated research activity had taken place. Each area is reviewed as a separate topic and a list of these topics, including a brief description, is provided below.

(1) Structural Modeling; In this area there was continued interest in geometrically nonlinear structural models for hingeless and bearingless rotor configurations. Finite element models for bearingless rotors have been developed. Structural models for composite blades and curved or swept blades were also introduced.

(2) Aerodynamic Modeling; Previously developed dynamic stall models have been refined and extended. There have been a number of additional studies aimed at the understanding of dynamic inflow. Arbitrary motion aerodynamics for rotary-wing applications were developed and applied to a number of simple problems.

(3) Aeroelastic Problem Formulation; Automated generation of complicated equations of motion using numerical methods or computerized symbolic manipulation was the primary activity.

(4) Aeroelastic Analyses in Forward Flight; A number of studies dealing with aeroelastic stability and response of hingeless and bearingless rotors were performed. There was continued interest in numerical treatment of equations with periodic coefficients.

(5) Coupled Rotor/Fuselage Aeromechanical Analyses; A number of coupled rotor/fuselage analyses have been developed and correlated with experimental data.

(6) Active Controls and their Application to Vibration Alleviation and Blade Stability Augmentation; This area was by far the most vigorous. Many studies, primarily experimental, have been aimed at vibration reduction by higher harmonic control. A few studies have also considered blade stability augmentation.

(7) Application of Structural Optimization to Vibration Reduction; In this area modern structural optimization was used to tailor fundamental blade frequencies such that vibration levels in forward flight were significantly reduced.

(8) Aeroelastic Analysis and Testing of Special Configurations; Such as circulation in controlled rotors, constant lift rotors, hybrid heavy lift helicopters, and bearingless/hingeless configurations which were tested in wind tunnels.

Based on the review of these research areas a number of observations regarding potentially rewarding areas of future research are made. Finally it should be noted that the author apologizes for papers which were inadvertently omitted in this survey.

2. Structural Modeling

Previous research during the last fifteen years [4-6] has established the importance of geometrically nonlinear terms in the analysis of hingeless and bearingless rotors. These geometrically nonlinear terms are associated with the assumption of moderate rotations (or blade slopes) and small strains and require the use of nonlinear beam kinematics in the development of the structural, inertia and aerodynamic operators associated with the rotary-wing aeroelastic problems. This kinematical nonlinearity produces, in many cases, coupling between the bending and torsional motions of the blade. This important coupling effect can not be obtained in an accurate and consistent manner without incorporation of the geometrical nonlinearities. Therefore a considerable number of recent studies have been aimed at providing improved capabilities for dealing with this particular class of problems.

Alkire [11] extended the analysis presented in Ref. 12 to obtain a better understanding of the role of built-in pretwist and elastic twist in the derivation of transformation matrices which relate the position vectors of the undeformed and deformed states of the blade. Two distinct approaches were considered, one in which the built-in twist is applied initially before the elastic deformations occur and a second approach where the pretwist is combined with the twist due to deformation. It was shown that these two approaches could be related to each other. Furthermore a procedure was developed for evaluating transformation matrices which remain unaffected by rotation sequences or the treatment of pretwist. Hodges [13] in a recent study has developed a nonlinear beam element for the analysis of rotating blades in which the assumption of moderate deflections has been abandoned. His analysis, which is intended to capture large rotations, is based on the systematic simplification of the kinematic relations using a less restrictive assumption whereby extensional strain is neglected compared to unity. Furthermore the transformations used in this study utilized Tait-Bryan orientation angles and Rodrigues parameters instead of Euler angles, which have been used in many previous studies. The final equations are based on the assumptions of isotropic stress-strain relations. This study served as the theoretical basis of the beam element used in the GRASP computer program [14]. This beam element [13] represents an important contribution since it is based on a minimal number of assumptions restricting the magnitude of the deflections experienced by the rotor blade. Associated with this model one finds both mathematical elegance and complexity. Thus the cost effectiveness of this model for rotary-wing aeroelastic analyses remains to be demonstrated. It is quite possible that for most applications the previous models, based on the assumption of moderate deflections, could prove themselves adequate.

Equations of blade equilibrium which were based on moderate deflection beam theories [5] frequently utilize ordering schemes to neglect higher order nonlinear terms. In such ordering schemes the slopes of the blade are assigned an order of magnitude ϵ and terms of order of magnitude ϵ^2 are neglected compared to terms of order one. By assigning orders of magnitude to the various parameters in the problem this approach leads to equations which contain second order nonlinear terms. In a study by Crespo DaSilva and Hodges [15,16] the influence of retaining the next level of terms in the equations of motion was considered, this approach yields more exact equations, which include third order nonlinear terms. In the second part of this study [16] the influence of these third orders on blade response and stability was considered, using a global Galerkin method to solve the equations of motion [5]. The results indicated that at relatively high collective pitch values ($\theta_0 > 0.2$) and for a blade which was very soft in torsion ($\bar{\omega}_\phi = 2.5$) the third order terms can influence both, the equilibrium position and the stability of the blade. This influence which is more pronounced for stiff-in-plane hingeless blades, is mild at practical values of the collective pitch setting for soft-in-plane blades.

Many previous studies of structural models of rotor blades [4-6] were restricted to initially straight blades. To remedy this situation Rosen and Rand [17,18] developed a structural nonlinear model for the behavior of curved helicopter blades. The model is very general and it allows for complicated geometries, boundary conditions and structural property distributions. In this model large deformations of the blade are accounted for. A somewhat restrictive assumption is that the undeformed rod lies initially in a plane. This assumption, combined with the pretwist of the blade and the retention of the curvature terms causes these equations to be cumbersome. Large deflections are treated using Euler angles and it is assumed that strains are small and negligible compared to unity. This model appears to be an improvement on a previous model which was used by the same authors [19].

Finite element modeling of hingeless and bearingless blade configuration was another area where a fair amount of activity occurred. Sivaneri and Chopra [20] developed a finite element model for hingeless blades which was very similar to an earlier model developed by Friedmann and Straub [21]. Subsequently Sivaneri and Chopra [22] have extended this finite element model to bearingless rotors. The flexbeam type of bearingless rotor is modeled by using regular beam finite elements for the outboard portion, a rigid clevis, and multiple beams to represent the flexbeam and the torque tube, as shown in Fig. 1. Special displacement compatibility conditions are enforced at the clevis. The fifteen degree of freedom finite element model used for modeling the outboard portion of the beam, shown in Fig. 2, is based on a cubic interpolation for the bending degrees of freedom, v, w and the axial degree of freedom u, and a quadratic interpolation for torsion ϕ. This method consists essentially of developing a special redundant root element for the flexbeam.

Finite elements for bearingless rotor modeling have been also developed by Hodges et al. [14] as part of a general rotorcraft aeromechanical program called GRASP. The structural modeling capability in GRASP is quite general enabling one to model any bearingless rotor configuration. Reference 14 utilizes higher order finite elements as opposed to the conventional finite elements used in Refs. 20-22. Another, sixteen degree of freedom, finite element model has been used in Ref. 23 to study the influence of a compressible lifting surface theory on the coupled flap-lag-torsional aeroelastic stability of a hingeless rotor blade in hover.

All the structural models discussed above were restricted to isotropic blades. One of the more important recent developments was the emergence of structural models suitable for the analysis of composite rotor blades, which are widely used on modern helicopters. Mansfield and Sobey [24] made a pioneering attempt to develop the stiffness properties of graphite fiber composite rotor blades and they also tried to explore the potential of this model for aeroelastic tailoring. Despite its innovative nature this study fell short of it stated objectives.

A comprehensive and important study by Hong and Chopra [25] presented, for the first time, an aeroelastic model for a composite rotor blade in hover. In this study a moderate deflection, coupled flap-lag-torsional analysis of a laminated box beam was developed in which terms up to the second order in blade slopes were retained. The nonlinear strain displacement relations were taken from Hodges and Dowell [26]. Each laminate wall of the box beam, representing the blade spar shown in Fig. 3, was assumed to consist of a number of composite plies at arbitrary orientation of the ply angles. Constitutive relations were obtained assuming that each lamina of the laminates is orthotropic and there is no shear stiffness through the thickness distribution. The equations of blade motion were obtained using Hamilton's principle. In these equations the axial stiffness EA, the bending stiffness EI and the torsional stiffness GJ are effective section stiffnesses which depend on ply lay-up and orientation.

Identification of coupling effects due to the composite structure of the blade was facilitated by the introduction of six constants $\kappa_{p1},\ldots\kappa_{p6}$ which are unique to the composite blade and depend on laminate orientation and layup. The

κ_{p1} resembles a pitch-lag coupling term, κ_{p2} resembles a pitch-flap coupling terms and κ_{p3} is due to nonlinear extension-torsion coupling terms. The other three constants were less significant. Using quasisteady aerodynamics, and a finite element model, illustrated by Fig. 2, the dynamic equations of motion were solved in a conventional manner [5] to obtain stability boundaries, which were presented as root locus plots. Results were calculated for a hingeless rotor with the following properties: $\gamma = 5.0$; $\sigma = 0.10$; $c/R = 0.08$; $\beta_p = 0$; $C_T/\sigma = 0.10$. Some typical results are shown in Figs. 4 and 5. The root locus plot for the lag mode eigenvalue for a symmetric case is shown in Fig. 4. For this case the side flanges of the box in Fig. 3 have non zero-ply angles. The full lines show the results for positive ply angles and the heavy broken lines correspond to the negative ply angels. The influence of the ply angle (Λ_V) variation is considerable. A positive ply angle destabilizies the lag mode and a negative angle stabilizes the lag mode. It turns out that this effect is primarily due to the κ_{p1} coupling term which represents pitch-lag coupling. The other coupling terms are either zero or play a negligible role. The light broken line in Fig. 4 represents the results with the κ_{p1} coupling term neglected and thus is shows only the influence of play angle variation on the stiffness terms corresponding to an equivalent isotropic blade analysis. Figure 5 shows similar results for the lag degree of freedom for antisymmetric ply angles on the side flanges and zero ply angles on the top and bottom flanges. In this case the major coupling term is κ_{p3} which is due to the extension-torsion coupling. This is a nonlinear coupling term and it indicates the importance of a nonlinear analysis for this class of structures.

Another important practical and complicated theoretical problem is the structural modeling of the aeroelastic behavior of rotor blades with swept tips. An analytical study which illustrates the effect of blade sweep on rotor vibratory hub, blade and control system loads was conducted by Tarzanin and Vlaminck [27]. The portion of the blade tip which was swept was located at 0.87R and two sweep angles, 10° and 20°, were considered. Sweep introduces powerful inertia, aerodynamic and structural coupling effects. The analytical model used in Ref. 27 could not represent in a consistent manner all the effects due to sweep therefore a technique called simulated sweep was used in which local inertia, elastic and aerodynamic axes were adjusted in an approximate manner. The authors concluded that tip sweep influences both blade vibrations and stability and recommended the development of improved analytical methods needed for a better fundamental understanding of dynamics of blades with swept tips.

A hingeless rotor with a swept tip is shown in Fig. 6. An important study capable of simulating such a hingeless rotor blade configuration was recently completed by Celi [28]. The model developed in this study is based on the dynamic equations of equilibrium presented in Ref. [29]. The blade is modeled using the Galerkin finite element technique [21] and a special element for the structural, inertia and aerodynamic properties of the swept tip was developed. Typical results showing blade equilibrium and stability for hover are presented in Fig. 7 and 8, for a stiff-in-plane rotor blade at a thrust coefficient of $C_T = 0.005$ (corresponding to a collective pitch setting of $\theta_0 = 0.1432$). Figure 7 shows the static blade equilibrium in flap, lag and torsion for zero precone and $\beta_p = 3$ degrees. It is evident that presence of precone significantly changes the equlibrium position in torsion as the 0.10R portion of the blade is gradually swept back. The curve of the torsional equilibrium (ϕ) has a characteristic concave shape. Precone interacts with sweep to change the nose-down torsional moment due to lift and the nose up moment due to centrifugal force. The influence of sweep and precone on the root-locus of first torsion and first lag mode are presented in Fig. 8. For zero precone frequency coalesce occurs between the first lag and first torsion mode. As frequencies coalesce, the torsional damping increases considerably, while the lag mode becomes unstable. This lag instability is not eliminated by small amounts of structural damping [4] indicating that this is a strong instability. For β_p = 3 degrees, increasing sweep increases the imaginary part of the first torsion eigenvalue, instead of decreasing it, as was the case for zero precone. Thus frequency coalesce does not occur, and the lag mode remains stable. Many additional results are presented in Ref. 28, which represents an important contribution to the literature since it contains the first detailed and systematic study of the effect of sweep on blade stability in hover and in forward flight.

Another new study by Kosmatka [30] combines a capability of modeling highly curved and swept blades undergoing moderate deflections, with the ability to deal with blades having a general, anisotropic, composite construction. This model was developed for the structural dynamic modeling of advanced composite propellers (prop-fans) however it is equally applicable to the analysis of composite, pretwisted, rotor blades. A curved pretwisted blade, is modeled by straight beam elements which are aligned with the curved line of shear centers of the blade. Each straight pretwisted beam finite element is derived, using Hamilton's principle, assuming that the beam undergoes moderate deflection, is composed of anisotropic materials, has an arbitrary shaped cross section, and rotates about a vector in space. Combined with this beam model a companion isoparametric eight node quadrilateral finite element model has been developed which is capable of calculating the shear center and the structural constants of an arbitrary shaped cross section, built up from anisotropic materials. The finite element model can also predict shear stress distribution over the composite cross section and it provides insight on the effects of ply orientation and material selection on the stress distribution within the cross section. A representative example used in this study was a composite blade cross section which consists of uni-directional Kevlar, laminated Kevlar and aluminum strip shown in Fig. 9. The location of the shear, area and mass center for different ply orientations are shown in Fig. 10. The shear center location can be easily moved within the cross section by varying the ply orientation. The structural properties of the blade can be also greatly modified by varying the ply orientation as indicated in Fig. 11. The axial stiffness and bending stiffness decrease as ply orientation is increased. On the other hand the torsional stiffness of the blade increases significantly with ply orientation.

Another interesting study associated with the structural dynamics of rotating pretwisted beams was the recognition that the use of twisted principal coordinates can lead to increased effectiveness in frequency and mode shape calculations [31].

From the studies reviewed in this section it is evident that very substantial advances in structural modeling capabilities have taken place during the time period considered.

3. Aerodynamic Modeling

Accurate modeling of the unsteady aerodynamic loads required for aeroelastic analyses continues to be one of the major challenges facing the analyst. An excellent review of some of major issues in blade unsteady aerodynamics have been presented in a paper written by Dat [32]. The accuracy with which the unsteady aerodynamic loading phenomena environment needs to be determined depends to a large extent on the dynamic problems which are investigated. Thus for example the coupled flap-lag-torsional aeroelastic instability in hover is frequently investigated using quasi-steady aerodynamics [5] whereas the periodic loads in forward flight must be evaluated using more precise aerodynamic models. The combination of the blade advancing and rotational speeds is a formidable source of complexity for the flow. At large values of advance ratio, the aerodynamic field around the blades undergoes such variations that there are problems of transonic flow, with shock waves, at the advancing blade tip, problems of speed reversal and low speed unsteady stall on the retreating blade, and problems due to the high blade sweep angle in the fore and aft positions. Furthermore the geometry of the wake, which is an important source of vibration and noise, is much more complicated than the fixed wing wake geometry.

The empirical and semi-empirical treatment of the unsteady, two dimensional, dynamic stall problem has played an important role in rotary wing aeroelastcity during the last twenty years. The review of three relatively recent dynamic stall models can be found in Ref. [5]. Continued research on these dynamic stall models has led to improved predictive capabilities which are described below.

Beddoes [33] has continued his work on indicial formulation of unsteady lift which was a basic ingredient in his dynamical stall model during the attached flow regime. Subsequently this work was incorporated by Leishman and Beddoes [34] in an improved generalized model for airfoil unsteady aerodynamic behavior and dynamic stall using the indicial method. This improved model provides the methodology for the computation of two dimensional unsteady airfoil lift, pitching moment and drag for an airfoil undergoing arbitrary forcing in the time domain, using an indicial response formulation. The linearized unsteady aerodynamic response on the attached flow regime is separated into two components, namely circulatory and impulsive loading, which are computed independently using indicial aerodynamic transfer functions. In the separated flow regime the nonlinear lift characteristics of the trailing edge separation was evaluated using the concept of Kirckhoff flow. The Kirchoff model was also used to evaluate the nonlinear effects on chordwise force and pressure drag response. The onset of of vortex shedding during dynamic stall was captured using a generalized criterion for the onset of leading edge or shock induced separation. Furthermore this leading edge separation was also coupled with the trailing edge separation calculation. These feature were incorporated in a general numerical algorithm for predicting airfoil unsteady aerodynamic behavior and dynamic stall, for arbitrary forcing or motion in the time domain. Extensive validation of the model was conducted by comparing it with other available analytical results and two dimensional unsteady test data. This model represents a major improvement on the previous model developed by Beddoes. One of the important attributes of the new new model is the capability for simulating, in a fairly accurate manner, the unsteady drag hysteresis loop of an airfoil undergoing either light or deep stall.

Gangwani [36] continued developing his original dynamic stall model which was initially reviewed in Ref. 5. The most important contribution of this new study was synthesized unsteady drag data which provides a basis for the computation of unsteady pressure drag of airfoils and rotor blades, in the time domain. A typical unsteady drag coefficient loop data for the SC1095 airfoil, at $M = 0.30$, a mean angle of attack of $\alpha_0 = 12°$ and oscillation amplitude of $\bar{\alpha} = 8.0°$ at a reduced frequency of $k = 0.10$ is shown in Fig. 12. The method for generating the unsteady aerodynamic coefficients for such an airfoil depends on the predictions of three major events associated with dynamic stall, namely: (1) stall onset; (2) vortex at trailing edge; and (3) reattachment. These events are usually closely related with the unsteady pitching moment coefficient, and they are also shown in Fig. 12. The unsteady drag coefficient C_{DU} is represented by an algebraic relation which depends on eight coefficients which are computed by using linear least square curve fitting with experimentally determined unsteady drag data. Numerous results presented in Ref. 35 indicate good agreement with experimental data. This method is considerably less sophisticated than that described in Ref. 34, however one of its attractive features is its relative simplicity compared to other dynamic stall models.

Among the three dynamic stall models reviewed in Ref. 5 the model developed at ONERA by Dat, Tran and Petot had a number of features which caused it to be suitable for inclusion in rotary wing aeroelastic response and stability calculations, because it is a time domain theory for an airfoil performing completely arbitrary motions. Furthermore the model utilizes the properties of differential equations to simulate the different effects which can be identified on an oscillating airfoil such as pseudo elastic, viscous and inertial effects, and the effect of the flow time history [32]. The theory also recognizes that in the linear range of airfoil motions the Theodorsen lift deficiency function represents the aerodynamic transfer function for the airfoil relating the three quarter chord downwash velocity to circulatory lift. Furthermore the theory is based on approximating the aerodynamic transfer function by rational fractions. For convenience, the input variables for the system of equations which are plunge (h) and pitch (θ), are combined in a single almost equivalent variable defined as $\alpha = \bar{h}+\theta$, where $\bar{h}$ is nondimensionalized in a suitable manner which is, range the model consists of a system of differential equations containing the angle of attack or downwash at the forward quarter chord. In the nonlinear range the model consists of a system of differential euqaitons containing unsteady linear terms whose coefficients are functions of the angle of attack and steady flow nonlinear terms. A lucid description of this model together with an outline for imbedding it in unsteady aerodynamic calculation including the effects of three dimensional flow can be found in a paper by Dat [32].

Further work aimed at an improved physical understanding of this model was carried out by Rogers [36] and Peters [37]. Rogers considered primarily the equation for unsteady lift and verified the validity of the model by reproducing previously published lift hysteresis data [39]. He also considered simplifications to the model and concluded that a term representing apparent mass in the model could be neglected without influencing the results. To determine some of the practical aspects of the model he incorporated it into a very simple dynamic model representing single section flapping dynamics of a rotor blade in forward flight. He concluded that Floquet theory, based upon linearized perturbation equations, could provide useful information on stability behavior when the ONERA model is used to represent the unsteady aerodynamics.

More recently Peters [37] continued to study the unsteady lift equation associated with the ONERA model. He noted that the original model lumped the pitch and plunge into one variable which can cause difficulty when attempting to compare the model to classical Theodorsen or Greenberg theory [5]. He introduced certain modifications in the theory so that it reduces to Greenberg's theory for small angles of attack, and it reduces to Theodorsen's theory for steady free stream. He also introduced modifications which remove certain ambiguities in the model at large angles of attack.

The ONERA stall model was also used in a recent test [38] to compare the measured and calculated stall flutter behavior of a one bladed model rotor. The agreement between the calculated and measured results showed good agreement, further illustrating the utility of the model for aeroelastic calculations.

It appears that the ONERA dynamic stall model is gaining acceptance in rotary wing aeroelasticity as other researchers introduce refinements in the model. Recent research by Leiss [40,41] has emphasized the role of unsteady sweep in the semi-empirical simulation of rotor blade aerodynamic loading. Thus the introduction of sweep effects into the ONERA model could produce another potential improvement.

Another significant portion of recent research in unsteady aerodynamics has been aimed at developing two dimensional unsteady airfoil theories in the time domain. Two dimensional aerodynamic theories, which provide analytic expressions for unsteady loads on a moving airfoil are usually based on the assumption of simple harmonic motion. Representative theories of this category are Theodorsen's and Greenberg's theories for fixed wings and Loewy and Shipman and Wood's theory for rotary wings [5]. These theories which deal with the linear, attached flow regime, have a significant deficiency when applying them to aeroelastic stability calculations, since the assumption of simple harmonic motion, upon which they are based, implies that they are strictly valid only at the stability boundary, and thus they provide no information on system damping before or after the flutter condition is reached. Thus standard stability analyses, such as the root locus method cannot be used in conjunction with these theories. Another important limitation of these theories is evident when one tries to apply them to the rotary-wing aeroelastic problem in forward flight, which is governed by equations with periodic coefficients. In this case the complex lift deficiency function associated with frequency domain unsteady aerodynamics is not consistent with the numerical methods employed in the treatment of periodic systems [5]. Thus many rotary-wing analyses in forward flight are based upon quasi-steady aerodynamics [5]. To remedy this situation a number of recent studies [42-48] were aimed at developing arbitrary motion unsteady aerodynamic theories in the time domain. In these studies the term arbitrary motion was used to denote motion with growing or decaying oscillation with a certain frequency.

In Ref. 42 the basic procedures for generalizing Greenberg's and Loewy's theories to the time domain, for airfoils undergoing arbitrary motion were presented. In Ref. 43 the generalized Greenberg theory was incorporated in the simple flap-lag analysis of a hingeless rotor blade in forward flight and the influence of unsteady aerodynamics on blade response and stability was obtained. When using a second order rational approximant for the generalized Theodorsen lift deficiency function, the finite state time domain representation of the circulatory aerodynamic lift and moment can be written in the following form [43]

$$L_C(t) = 2\pi\rho_A bRU_T(t)\left[0.00685(U_{TO}/bR)^2 X_1(t) + 0.10805(U_{TO}/bR)X_2(t)\right] + \pi\rho_A bRU_T(t)Q(t) \tag{1}$$

$$M_C = bR(0.5 + \bar{a})bRL_C(t) \tag{2}$$

These expressions are written in terms of the downwash velocity Q(t) at the ¾ chord, which depends on the airfoil degrees of freedom and their time derivatives, and two additional augmented state variables X_1 and X_2. The augmented state variables are governed by a system of first order differential equations which depend on Q(t), as shown below

$$\begin{Bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{Bmatrix} = \left[\begin{array}{c|c} 0 & 1 \\ \hline -0.01365(U_{TO}/bR)^2 & -0.3455(U_{TO}/bR) \end{array}\right] \begin{Bmatrix} X_1(t) \\ X_2(t) \end{Bmatrix} + \begin{Bmatrix} Q \\ Q(t) \end{Bmatrix} \tag{3}$$

The additional augmented state variables X_1 and X_2 convey information regarding the unsteady wake history. Such an aerodynamic theory provides a good approximation to both low frequency and high frequency regimes of blade motion. Furthermore it should be noted that this time domain unsteady aerodynamic model bears a close resemblance to the unsteady aerodynamic loads, used for the attached flow case, in the dynamic stall model for rotor blades developed by Dat, Tran and Petot [5,32,37].

To assess the influence of arbitrary motion unsteady aerodynamics on blade aeroelastic stability and response in forward flight a simple problem consisting of an offset hinged, spring restrained model of an isolated hingeless blade was selected [43]. Typical results for blade response and stability are shown in Figs. 13 and 14. Figure 13 illustrates the steady state flap response of the blade over one revolution using both time domain unsteady aerodynamics and quasisteady aerodynamics. There is a pronounced unsteady aerodynamic effect on the flap response. The effect of phase lag and amplitude modulation associated with unsteady aerodynamics are both evident in Fig. 13. In Ref. 43 the same effect, on the lag degree of freedom, was also examined and found to be small. The influence of unsteady aerodynamics on blade stability is shown in Fig. 14, where the real part of the characteristic exponents (which is a measure of damping in a periodic system) is plotted as a function of the advance ratio μ. The interesting result in this plot is the instability in the flap degree of freedom which occurs at an advance ratio of $\mu = 0.45$. When quasisteady aerodynamics are used this instability does not occur. It was shown [43,45] that this instability can be associated with an unsteady lift deficiency function which represents the ratio between unsteady lift and quasisteady lift. The important conclusion from these plots is that unsteady aerodynamics influences primarily the flap response of the blade [48].

It was also shown in Refs. 42 and 44 that generalizing a rotary-wing theory such as Loewy's to the time domain is more complicated that the extension of Greenberg's theory. To overcome this problem a novel technique for formulating high quality finite state unsteady aerodynamic models, based upon the Bode plot, was developed [46,47]. This technique is based on recognizing that the circulatory portion of the lift, per unit span, of the airfoil in the Laplace domain can be written as

$$L_C(\bar{s}) = \alpha\rho_A bRUC(\bar{s})Q(\bar{s}) \tag{4}$$

where $Q(\bar{s})$ represents the Laplace transform of the ¾ chord downwash velocity. The Bode plot method used in control systems engineering is a useful tool for constructing approximations to complicated transfer functions. It can also be used to construct approximations to lift deficiency function, which has the role of an aerodynamic transfer function, according to Eq. (4). Using this technique good approximations to Loewy's lift deficiency function were obtained [46,47] and used to obtain a rotary-wing indicial response function. These indicial response functions are oscillatory and thus are different from fixed wing indicial response functions which are non oscillatory [46,47].

Another contribution made in Refs. 44 and 45 was the development of an arbitrary motion unsteady cascade airfoil theory for helicopters rotors in hover.

It was pointed out in Ref. 48 that dynamic inflow also represents an arbitrary motion type of approximate unsteady aerodynamic theory, which captures low frequency aerodynamic effects associated with the wake. A comprehensive review of the dynamic inflow models available in hover and forward flight together with their correlation with experimental data was presented by Gaonkar and Peters [49].

In addition to the theories considered in this section, more complicated theories such as unsteady prescribed wake models, lifting surface models and more sophisticated models based on computational fluid mechanics are also needed for more accurate aeroelastic stability and response calculations. A newly developed unsteady prescribed wake model for helicopter rotor blades in hover and forward flight was presented by Rand and Rosen [50]. Unsteady lifting surface theories were also considered in Ref. [32]. A detailed survey on the role of computational fluid mechanics for rotorcraft was given by Davis [51].

4. Aeroelastic Problem Formulation

The derivation of equations of motion for aeroelastic stability and response calculations, for an isolated rotor blade in forward flight including geometrical nonlinearities, is a relatively complicated task from an algebraic point of view. When the fuselage degrees of freedom are added to the problem this task tends to become very arduous, even when an ordering scheme is used to simplify the equations. Good representative examples showing the complexity of the equations which model a coupled rotor/fuselage system in forward flight can be found in Refs. 52-54. The solution process of such equations leads to additional complications since use of a global Rayleigh-Ritz or Galerkin method, combined with the multiblade coordinate transformation, frequently used in coupled rotor/fuselage analyses, requires considerable algebraic effort [5]. Substantial increases in raw computing power, as represented by high computational speeds and availability of large core memory at low cost, which have taken place during the last five years imply that the time has come to delegate these algebraic tasks to the computer. Only a few papers were published on the automatic generation of helicopter equations of motion using computers, however, the number of such papers is increasing. From these papers it is evident that two different approaches are being used to achieve the same goal.

One approach is based on generating equations of motion in explicit form. This can be accomplished by developing special purpose symbolic manipulators written in FORTRAN to automatically generate equations of motion for rotary-wing aeroelastic applications. One of the first studies based on this approach was done by Nagabhushanam, Gaonkar and Reddy [55]. Using this approach the complete equations of motion are obtained in fully explicit nonlinear form, directly from the computer.

The approach developed originally in Ref. 55 has been extended to the problem of coupled flap-lag-torsional dynamics of hingeless rotor blades in forward flight [56,57]. A detailed description of the symbolic processor program principles was presented by Reddy [57]. The program generates the steady state and linearized perturbation equations in symbolic form and then codes them into FORTRAN subroutines. These equations are obtained in explicit form. The coefficients for each equation and for each mode are identified through a numerical program. A Lagrangian formulation is used to obtain equations in generalized coordinates. The coupled flap-lag-torsion equations with dynamic inflow are converted to equations in a multiblade coordinate system by deriving explicit multiblade equations in symbolic form. The whole process, from derivation to numerical calculation, is automated with minimum user interface. The equations have been carefully validated in Ref. 57 by comparing results obtained for hover with other results available in the literature. Many useful results for forward flight were generated with the program [56,57]. These results will be discussed in the next section of this paper.

Another explicit approach is discussed by Crespo Da Silva and Hodges [58]. This approach is based on utilizing a commercially available symbolic manipulation program called MACSYMA, running on a dedicated LISP workstation. The general methodology of deriving flexible blade equations using MACSYMA are discussed and the process is illustrated by a simple example associated with the flap motion of a rotor blade in forward flight.

The second approach to generating rotary-wing aeroelastic equations of motion is based on the implicit approach. In this approach one generates automatically the coefficient matrices for equations of motion linearized in perturbation coordinates about an equilibrium approach. This approach, which was used by Done and his associates in two recent papers [59,60], does not require that the equations be explicitly written out at any stage of the analysis. The first paper by Gibbons and Done [59] presented the theoretical background for the method. The procedure consists of writing down the appropriate transformations governing the dynamics of a mass point and combining it with with Lagrange's equations to obtain the mass, aerodynamic and stiffness terms needed to calculate the equilibrium position. Subsequently perturbation equations about this equilibrium position are generated. The differentiations and integrations required in this process are performed numerically. The equations generated are in numerical form, and their solution is obtained by iterative algorithms. Only a few simple results in hover were used to validate the program. In a second paper [60] three practical examples were treated by the computer program which was developed and results were compared to results generated by Westland Helicopters. Among these the most complicated example was a Lynx ground resonance calculation and comparison between the two sets of results was satisfactory.

Finally it is important to mention that implicit formulations have been used in recent finite element analyses of rotary-wing aeroelastic problems [14,28]. The implicit nature of Ref. 28 is principally associated with two features:

1. The algebraic expressions for the aerodynamic loads are not expanded explicitly . They are coded separately in the computer program and combined numerically with the inertia and structural terms during the solution of the response problem.
2. *The approximate set of generalized coordinates obtained during one iteration* of the solution procedure is used to generate the aerodynamic loads for the next iteration.

The implicit nature of the GRASP program [14] is primarily due to its hybrid finite element/multibody nature which allows the theatment of complicated configuration, without explicitly writing out the governing equations.

5. Aeroelastic Analyses in Forward Flight

The general methodology for the aeroelastic analysis of rotor blades in forward flight was reviewed in detail in Ref. 5. This aeroelastic problem is governed by nonlinear equations with periodic coefficients and furthermore the aeroelastic problem is coupled to the trim state of the helicopter. A considerable number of recent studies were aimed at an improved understanding of the coupled flap-lap-torsional problem of an isolated blade in forward flight [56,57,61-63]. A common element among these studies is a solution procedure which is similar to that first presented in Ref. 64. The solution consists of the following steps: (a) calculation of trim, (b) oalculation of the nonlinear time dependent equilbirium posisiton, (c) linearization of the perturbation equations about this time dependent equilibrium position, and (d) calculation of blade stability using Floquet theory.

A comprehensive study of the coupled flap-lag-torsional aeroelastic behavior of hingless rotor blades in forward flight was done by Reddy and Warmbrodt [56,57]. They used symbolically generated equations and studied the influence of: dynamic inflow, trim, as well as various approximations to the complete coupled-flap-lag-torsional equations. Figure 15 shows the effect of number of degrees of freedom used in trim analysis on lead-lag damping plotted as a function of the advance ratio. It can be seen that a flap-lag-torsion stability analysis based upon a flap trim [64] tends to underpredict the lead-lap damping. Another feature of this plot is the instability, in the lag degree of freedom observed in a stiff-in-plane blade configuration when $\mu > 0.40$. This behavior was also observed in Ref. 64. The effect of torsion and dynamic inflow on lead lag regressing mode damping is shown in Fig. 16. Dynamic inflow seems to have a relatively small effect in this case. Furthermore the damping predicted by a flap-lag model is much lower than that predicted by a coupled flap-lag-torsional model, this was also noted in Ref. 64.

The feasibility of simplifying coupled lag-flap-torsional models for blade stability analyses in forward flight was studied in Ref. 61 and it was concluded that the only reliable model under various conditions is the fully coupled model.

Panda and Chopra [62] have also studied flap-lag-torsion stability in forward flight using an offset hinged spring restrained model of a hingeless blade. The effects of pitch-flap and pitch-lag coupling, torsional stiffness and dynamic inflow were considered. The results also confirmed those obtained in Ref. 56, 57 and 64. Subsequently the same authors [63] studied the behavior of hingeless and bearingless rotors in forward flight, including dynamic inflow and using a previously derived finite element method [22]. The results indicated that stiff-in-plane configurations were destabilized by forward flight, while soft-in-plane blade configurations were stabilized by forward flight, which was also found in Refs. 56 and 64.

An experimental and analytical study of the flap-lag stability in forward flight of an isolated, three bladed hingeless model rotor, having a diameter of 1.62 meters was performed in Ref. 65. The rotor was not trimmed and many data points were obtained for high advance ratios and high shaft angles. The purpose of this paper was to determine the adequacy of the linear quasisteady aerodynamic model with dynamic inflow. The results of this correlation study were somewhat inconclusive. Because in some cases the use of dynamic inflow improved the correlation between theory and experiment while in other cases it did not. Fig. 17 shows the lag regressing mode damping at an advance ratio of $\mu = 0.30$ and increasing shaft angles. The lack of correlation for this case was not explained in a convincing fashion and was attributed to stall.

In addition to aeroelastic stability studies in forward flight a smaller number of studies dealt with the aeroelastic response problem due to gusts. Gust response of a coupled rotor/fuselage system in hover and forward flight was studied by Bir and Chopra [66,67]. The blades were represented by a fully coupled flap-lag-torsional model including, moderate deflections. The fuselage had three translational and two rotational (pitch and roll) degrees of freedom. Gusts were represented by a deterministic three dimensional gust field. Some of the more important conclusions of this study were that a complete coupled flap-lag-torsional model of the blade is needed for an accurate response analysis because when the vehicle encounters a gust the blades respond quickly, absorbing the initial impact of the gust. Another important conclusion was that using dynamic inflow can be important for gust response calculations, otherwise the blade response can be overestimated.

The effect of random air turbulence on flap-lag stability in forward flight was considered in Ref. 68, using a random process analysis. It was found that in absence of elastic coupling, turbulence is stabilizing. As indicated by Refs. 56,57 and 61-64 the damping in the flap-lag model is lower (sometimes 300% lower) than in the coupled flap-lag-torsional model. Thus results based on the flap-lag model tend to exhibit excessive sensitivity to gusts. Therefore it appears that the influence of turbulence on blade stability is small.

The rotary-wing aeroelastic problem in forward-flight (after spatial discretization) is governed by nonlinear ordinary differntial equations with periodic coefficients. The numerical treatment of stability and response of such periodic systems is a key ingredient in the solution of these problems. During the last five years a number of reliable efficient numerical schemes for dealing wth such problems have become available and these are described in Ref. 69. Recently the finite element method in the time domain [70] was applied to the solution of periodic systems by Borri [71]. This method is based upon Hamilton's weak principle and consists of the time discretization of the linearized version of this principle. The time discretization utilizes appropriate interpolation functions in time, such as cubic polynomials for example. Application of this method to a periodic system yields a system of linear alegebraic equaitons which have to be solved in an iterative manner to obtain the response of the system. This method can be also used to obtain the transition matrix at the end of one period.

From the discussion presented above it is evident that our analytical understanding of blade behavior in forward flight is improving. However there is considerable need for high quality experimental data on isolated, trimmed hingeless and bearingless rotor blades having simple configurations, i.e., uniform mass stiffness, with zero or linearly varying pretwist and without sweep or droop. Availability of such data, for an advance ratio range of $0 < \mu < 0.45$, could provide a sound basis for verifying and improving forward flight analyses in a systematic manner.

6. Coupled Rotor/Fuselage Aeromechanical Analyses

The aeromechanical instability of a helicopter, on the ground or in flight, is caused by coupling between the rotor and body degrees of freedom. This instability is commonly denoted air resonance when the helicopter is in flight and ground resonance when the helicopter is on the ground. The phyiscal phenomenon associated with this instability is quite complex. The rotor lead-lag regressing mode usually couples with the body pitch or roll to cause an instability. The nature of the coupling which is both aerodynamic and inertial is introduced in the rotor by body or support motion. The importance of developing a mathematically consistent model capable of representing the coupled rotor/fuselage dynamic system has already been discussed in previous reviews [5,6]. A considerable number of such coupled rotor/fuselage analyses which were developed are described below. A number of these models yield good correlation with experimental data.

A relatively comprehensive study by Nagabhushanam and Gaonkar [72] was aimed at determining the influence of various dynamic inflow models and aeroelastic coupling effects on the air resonance problem in forward flight. The model consisted of a number of centrally hinged spring restrained blades having flap and lag degrees of freedom for each blade combined with a fuselage having pitch and roll degrees of freedom. Some of the results obtained were consistent with other results available in the literature. One of the conclusions, namely the deterioration of regressing lag mode damping of soft-in-plane rotors, with increases in advance ratio appears to be somewhat contradictory to other results available.

A much more general coupled rotor/fuselage analysis is one of the many options available in a computer program developed by Johnson [53,73], which had acquired the name CAMRAD (for Comprehensive Analytical Model for Rotorcraft Aerodynamics and Dynamics). This model was used by NASA Langley Research Center to calculate hingeless rotor aeromechanical stability [74]. The model was tested in the Transonic Dynamics Tunnel. The model was a soft-in-plane, four bladed, hingeless rotor with flexures to accommodate flap and lead-lag motion combined with a mechanical feathering hinge to allow blade pitch motion. The support had body pitch and roll motions. The analysis included these degrees of freedom and the dynamic inflow model. The correlation covered the influence of pitch-flap coupling, blade sweep, blade droop, and blade precone as a function of μ, rotor speed and collective pitch. Figure 18 shows the correlation obtained, which was quite good. This code was also used by NASA Ames for hover stability tests of a full scale hingeless rotor [75], and good correlation was obtained.

Johnson also used this code to model the influence of unsteady aerodynamics on hingeless rotor ground resonance [76]. He compared his results with the high quality experimental data obtained by Bousman [77] and obtained the remarkable result that inflow dynamics introduces an additional "inflow mode", which explained previously unresolved questions about the correlation between the theory and the test.

Venkatesan and Friedmann [78,79] developed a mathematical model capable of modeling aeromechanical problems associated with multirotor vehicles, where the two rotors were connected by a flexible supporting structure which also had rigid body degrees of freedom. The blades were modeled as offset hinged spring restrained blades, including geometric nonlinearities. Each blade had flap, lag and torsional degrees of freedom.

A subset of this model, consisting of a three bladed hingeless rotor with flap and lag degrees of freedom for each blade mounted on a gimbal which could pitch and roll, was used in Ref. 80 to simulate the experimental data obtained by Bousman [77]. The results obtained [80], using quasisteady aerodynamics, were in good agreement with the experimental data obtained in Ref. 77, except that the quasisteady model was incapable of predicting the "dynamic inflow mode" found by Johnson [76]. Subsequently both perturbation inflow and dynamic inflow aerodyanmics were incorporated in the coupled rotor/fuselage model [81] and the result obtained with dynamic inflow produced good agreement with the experimental data. Furthermore the "inflow mode" obtained by Johnson was also reproduced. Results illustrating this unsteady aerodynamic effect are shown in Figs. 19 and 20 [81]. Figure 19 shows the variation, of modal frequencies as a function of rotor speed, at zero collective pitch setting, using quasisteady aerodynamics. All frequencies except the one corresponding to 0.7 Hz. are predicted well. When perturbation inflow and dynamic inflow are included the results shown in Fig. 20 indicate, that with dynamic inflow all frequencies are predicted well. Furthermore the "inflow mode", associated with the augmented states introduced but the dynamic inflow model, is also predicted. It is shown in Refs. [48,81] that the identification of this mode is relatively complicated.

Another new program capable of predicting rotorcraft aeromechanical problems, as well as other dynamic problems, is the RDYNE program developed by Sopher and Hallock [82]. This program uses a time-history analysis for rotorcraft dynamics based on dynamical substructures and nonstructural mathematical and aerodynamic components. The program contains both geometrical and aerodynamic nonlinearities and used component mode synthesis to combine various structural elements. The program was applied to ground resonance problems and performed very well.

A modern and modular program, named GRASP, was completed recently [14]. GRASP combines the finite element and multibody approaches and incorporates multiple levels of substructures to provide a powerful tool for the analysis of bearingless rotor aeromechanical problems. GRASP has been designed around the concept of a collection of flexible and rigid bodies connected in an arbitrary manner. The element library of the program contains three elements: (1) an aeroelastic beam element which contains no small angle approximations; (2) an air mass element; and, (3) rigid body mass element. Results for a coupled rotor/body model were obtained, and the eigenvalues of the regressing lag mode damping were compared with results obtained by Ormiston [83]. The correlation between the two sets of results was good. This program was written using modern programming methods, emphasizing clarity and modularity. Despite its many attractive features the program is somewhat limited since it cannot treat blades made of composites, nor can it deal with a variety of problems which lead to equations with periodic coefficients, such as fuselage mass offset from the axis of rotation and blade dissimilarities.

The majority of the studies cited above dealt with a rotor/body system where the blades were identical. The interesting effect of blade-to-blade dissimilarities on rotor/body lead-lag dynamics was studied by MuNulty [84]. The most noticeable effect of these dissimilarities was the appearance of additional peaks in the frequency spectrum.

The influence on nonlinear damping on helicopter ground resonance was studied by Tang and Dowell [85]. The analytical model included a three bladed articulated rotor, with each blade having only lead-lag motion, combined with a fuselage which could pitch and roll. The formulation contains both a nonlinear blade damper and a nonlinear landing gear damping. The analytical results were compared with experiments conducted on a model and good agreement was obtained.

7. Active Controls and Their Application to Vibration Alleviation and Blade Stability Augmentation

The use of active controls whereby the pitch of the rotor blade is modified by an automatic control system so as to alleviate dynamic effects represents a typical aeroservoelastic problem. The level and scope of the activity in this area was very substantial. The use of active controls to provide reduction of vibratory loads at the hub, reduction of vibratory loads in the fuselage, gust load alleviation, alleviation of effects due to dynamic stall, stability augmentation in the lead-lag degree of freedom and suppression of coupled rotor/fuselage instabilities were only some of the potential applications considered. A complete review of this subject would require a separate review article. The main objective of this section is to present a concise review of some of the more interesting recent developments.

Two basic approaches for the active control of rotor dynamic problems were considered and implemented. In the first approach the time dependent pitch control is introduced in the fixed system through a conventional swash plate. The majority of studies, which are described in the first part of this section, are based upon this approach. In the second approach the time dependent pitch control, of a particular blade, is introduced in the rotating reference frame and is denoted individual blade control (IBC).

The most important topic, from a practical point of view, is vibration reduction in forward flight using higher harmonic controls (HHC). This approach produces reduced vibration levels in the fuselage, or at the hub, by tailoring the vibratory aerodynamic loads on the blades. Thus vibratory forces and loads are modified, at their source, before they reach the airframe. This is in contrast with conventional means of vibration control [7,8] which deal with vibratory loads after they have been generated. A particularly successful approach to this problem was an adaptive control system which combines recursive parameter estimation with linear optimal control theory. A class of such algorithms has been discussed in detail by Johnson [86] and was also analytically investigated by Molusis, Hammond and Cline [87]. Subsequently wind tunnel tests [88], flight tests [89] and digital simulations [90] have shown that an algorithm, denoted as the "cautious controller" provided good performance. A brief discussion of such an adaptive control system is given below.

The need for an adaptive control system, in which parameters describing the helicopter model are identified on line, follows from the inability of current analytical tools to predict vibration characteristics with sufficient accuracy. Furthermore the sensitivity of vibration characteristics to changes in aircraft configuration and flight condition implies that a constant gain control system might be ineffective. The HHC input in its most general form consists of a harmonic variation of collective and cyclic pitch components

$$\theta_{HHn} = \theta_{HHO} \sin n\psi + \left[\theta_{HHC} \sin(n\psi + \phi_c)\right] \cos\psi +$$

$$\left[\theta_{HHS}(\sin n\psi + \phi_s)\right] \sin\psi \qquad (5)$$

For a four bladed rotor n = 4; this input in the non-rotating system results in 3,4 and 5/rev oscillations in the rotating system.

It is assumed that the helicopter can be represented by a linear, quasistatic frequency domain model relating the output vector Z, consisting of harmonics of vibration, to the input vector θ, consisting of harmonics of blade pitch control at time $\psi_i = i\Delta\psi$, where ψ_i is the sampling time, thus

$$Z_i = Z_o + T\theta_i \qquad (6)$$

where T is the nxm transfer matrix relating output vibration response to input higher harmonic control angles. The sampling interval $\Delta\psi$, should be sufficiently large for the transient to die out and the harmonics to be measured, usually it is taken as once per revolution. The uncontrolled vibration level Z_o and the transfer matrix T are not known, because analytical methods for their prediction are not sufficiently accurate. These quantities are therefore estimated using a Kalman filter, the details of this estimation are presented in Refs. 86, 87 and 90.

The objective function, to be minimized, is the expected value of the performance index

$$J = E(Z_i^T W_Z Z_i) + \theta_i^T W_\theta \theta_i + \Delta\theta_i^T W_{\Delta\theta} \Delta\theta_i \qquad (7)$$

usually the weighting matrices W_Z, W_θ and $W_{\Delta\theta}$ are diagonal and the control law is found by setting $\partial J/\partial\theta_i = 0$. Solution of this relation produces a cautious controller. The control law for such a formulation can be found in Refs. 86, 87 and 90.

When Eq. (6) is based on the uncontrolled vibration level Z_o, sometimes denoted as the open-loop or global model, caution introduces an effective limit on control amplitudes. An alternative approach where Eq. (6) is replaced by

$$Z_i = Z_{i-1} + T(\theta_i - \theta_{i-1}) \qquad (8)$$

known as a closed-loop or local model produces an alternative control law where caution introduces limits on the rate. In either case the cautious controller introduces control limits which compensate for the uncertainty in the parameter estimates. A schematic diagram showing the implementation of such a control system for a digital simulation of control laws [90] is shown in Fig. 21.

Hammond [88] conducted extensive wind tunnel tests, on an aeroelastically scaled, four bladed, articulated helicopter rotor model. A number of alternative algorithms were tested, and it was found that the cautious controller gave very good performance. A typical result [88] showing the variation of the vibratory vertical force with advance ratio is presented in Fig. 22. Reduction between 70-90%, for this vibratory component, were obtained over the range of advance ratios tested. The results also indicated that HHC inputs produce increased edgewise bending moments, torsional moments, and control loads. The increased loads experienced during the tests, were within the design loads. This wind tunnel test was intended to support a subsequent full scale flight demonstration test of OH-6A aircraft equipped with a higher harmonic control system. The results of the full scale tests, which took place in the summer of 1983, were presented in a landmark paper by Wood, Powers, Cline and Hammond [89]. The aircraft was flown from zero airspeed to 100 knots, with the HHC system operated both open loop (manually) and closed loop (computer controlled). Flight test results exhibited significant reduction in helicopter vibrations without undue penalties in blade loads and aircraft performance. Six months later, in 1984, the flights resumed with an improved Kalman filter implementation combined with some hardware improvements. These modifications resulted in improved system performance [89]. Figure 23 shows the closed loop HHC-4P vertical acceleration, at the pilot seat, as a function of airspeed. Comparing Figs. 22 and 23, and recognizing that vertical vibrations at the pilot seat are different from vertical forces measured in the wind tunnel, shows that the full scale tests produced vibration reductions similar to those obtained in the wind tunnel tests.

A comprehensive digital simulation of such an HHC system was conducted by Davis [90]. This study, was a continuation of an earlier study [91], and it was aimed at a comparative study of three basic control algorithms: (1) deterministic, (2) cautious, and (3) dual. A diagram of this system is shown in Fig. 21. Reduction of vibration levels between 75-95% were achieved with HHC angles of less than one degree. The effect of nonlinearity and interharmonic coupling were also considered. This is an important problem because Eqs. (6) or (8) imply a linear or linearized model. A detailed study of the role of nonlinearities (both geometric and aerodynamic) in HHC systems for helicopter vibration was first presented by Molusis [92]. Both Refs. 90 and 92 utilized the G400 [93] helicopter aeroelastic simulation program, to generate their results. Molusis concluded that under certain conditions, nonlinearities could be sufficiently important so as to require modifications in the control algorithms. The simulations conducted at UTRC provided good guidelines for the implementation of a full scale flight demonstration of a HHC-system on the S-76 helicopter [94]. These tests, conducted in the open loop mode only, were the first demonstration of HHC on a 10,000 lb helicopter at speeds up to 150 knots. These successful tests, conducted in early 1985, will eventually lead to flight demonstration of the closed loop system.

Flight tests of an experimental HHC system on a SA349 Gazelle were also conducted in France in 1985 [95]. A detailed description of both the simulations and the flight tests are presented in Ref. 95. The control algorithms were similar to those used in Ref. 87 and 89 and a reduction of 80% in cabin vibrations at an airspeed of 250 km/h was demonstrated.

The higher harmonic control model and the wind tunnel and flight tests discussed above do not imply that these are the only viable approaches for dealing with vibration reduction, gust alleviation and potential performance enhancement. Many other studies have considered alternative approaches and fundamental problems associated with HHC. Shaw et al. [96] have demonstrated a closed-loop HHC system on a dynamically scaled model of a three bladed CH-47D rotor in the Boeing Vertol wind tunnel. Very effective multicomponent vibration suppression was demonstrated up to flight speeds of 188 knots. This vibration suppression was demonstrated with a fixed-gain feedback control which was much simpler and faster than the adaptive control laws used in the studies cited previously. A different approach, based on mathematical programming techniques to determine the HHC angles was presented by Jacob and Lehmann [97]. Wind tunnel test results performed on a model of a BO-105 four bladed hingeless rotor, using a relatively simple cost function, were presented by Lehmann [98]. The special HHC testing facility, discussed in Ref. 98, has a number of unique capabilities. Another different approach to vibration reduction in the fuselage using state-feedback vibration control was proposed and evaluated by DuVal, Gregory and Gupta [99]. While this approach appears to be promising and it needs further study.

All the studies cited above were aimed at vibration reduction in forward flight. Active control systems also offer the potential for cost effective solutions to other dynamics problems. A natural extension of adaptive control system approach is to apply it to gust alleviation. The feasibility of such a system was studied analytically by Saito [100], using a four bladed aritculated blade model. Response to step and sinusoidal gusts was considered. Gust response alleviation, between 50-100% was obtained.

Two fundamental studies on the use of active controls to augment rotor/fuselage stability were recently completed. Straub and Warmbrodt [102] performed an analytical study of ground resonance with airloads. The control system was modeled using state variable feedback with appropriate gain and phase. The analytical model of the coupled rotor/fuselage system was represented by an offset hinged spring restrained, three bladed hingeless rotor with flap and lag degrees of freedom. The rigid fuselage had pitch, roll, lateral and longitudinal translations. The model was based on quasisteady aerodynamics and contained geometrical nonlinearities. The configuration analyzed is shown in Fig. 24. The feedback was applied through a conventional swash plate, active pitch input to the k^{th} blade was

$$\theta_{Ak} = \theta_{AC(\psi)}\cos\psi_k + \theta_{AS(\psi)}\sin\psi_k \tag{9}$$

To control the linearized constant coefficient system written multiblade coordinates, three "active" actuators in the fixed system were used. Using state variable feedback this control problem was explored in detail to obtain good physical understanding of the problem. The study assumed that all states are known. It was found that a 1% augmentation in critical damping of the regressing lag mode could be obtained with a 0.3 degree of blade cyclic lead-lag feedback.

In a second paper, Straub [102] used the same mathematical model to study the linear optimal control problem (LQG) of a four bladed articulated rotor in ground resonance. The solution for the control law was the deterministic optimal controller with linear feedback of all the state variables. The optimal gain was obtained from the solution of the algebraic Ricatti equation. Analytical results were generated in order to simulate the behavior of an articulated four bladed, H-34 rotor, with 4% hinge offset mounted on the Rotor Test Apparatus (RTA) in the 40 x 80 wind tunnel. Figure 25 illustrates, that a simple reduced state controller, using only control involving position and velocity of the hub $u = u(x,y,\dot{x},\dot{y})$, and without gain scheduling, yields a stable system at all speeds. This result should be compared with u = o (3200 Nms) which corresponds to nominal lag dampers, whereas all other result in this plot are based on lag damping reduced by a factor of ten (C_ζ = 320 Nms). The influence of this suboptimal controller, with four feedback loops $u = u(x,y,\dot{x},\dot{y})$, on the modal frequencies is shown in Fig. 26. It is seen that only small changes occur at the coalesce rotor speeds and that frequencies are not changed at other rotor speeds. This clearly indicates that improvements in system stability, as a result of active control, are strictly due to increased regressing lead-lag mode damping.

An alternative to control through the conventional swash plate is the individual blade control (IBC) approach in which each blade is individually controlled in the rotating frame over a wide range of frequencies. This control concept was pioneered by Kretz [103] however a considerable amount of the more recent work in this area was done by Ham and his associates [104]. Reference 104 contains a detailed review of this work. Using a simple wind tunnel model combined with the concept of modal control, a number of important applications of this method were considered. These applications were: (1) gust alleviation, attitude stabilization and vibration alleviation [105]; (2) lag damping augmentation [106]; (3) stall flutter suppression [107]; and (4) flapping stabilization in forward flight [108].

An important contribution in this area was the recognition that by multiblade coordinate transformation, individual blade control laws could be implemented through a conventional swashplate [105]. The practical applications of this control concept are currently being evaluated by industry, Ref. 109 is representative of such a feasibility study conducted at Bell Helicopters.

8. Application of Structural Optimization to Vibration Reduction

The higher harmonic control, for vibration reduction, discussed above, modifies the unsteady aerodynamic loads acting on the blade and thus reduces the aeroelastic response of the blade. A somewhat similar goal can be accomplished by "aeroelastic tunning" of the blade, using changes in blade twist, sweep and mass or stiffness distribution [110]. Both methods are similar because they reduce the vibrations at the source, namely the rotor. Instead of the conventional design approach, used in Ref. 110, one can use modern structural optimization to reduce vibrations in rotorcraft [111]. In Ref. 111 the various techniques available for vibration reduction in rotorcraft using structural optimization were explored and reviewed with considerable detail. The only prudent approach for reduction of blade vibrations in forward flight, when changing mass and stiffness distribution, combined with changes in blade tip sweep is one in which aeroelastic constraints on blade stability are enforced. This requirement complicates the problem because a fully coupled aeroelastic stability and response analysis in forward flight has to be coupled with a structural optimization program. Therefore only a few studies having this capability are available [111-114]. In Ref. 112-114 modern structural optimization was used to reduce vibration levels in forward flight. The objective function minimized was the oscillatory vertical hub shear at $\mu = 0.30$. Behavior constraints are the frequency placements of the blade in flap, lag and torsion, combined with the requirement that aeroelastic stability margins in hover remain unaffected by the optimization process. Stiff-in-plane, hingeless, blade optimization is discussed in Refs. 112 and 113, a detailed treatment of the soft-in-plane configuration is given in Ref. 112 and 114. Figure 27 shows the influence of structural optimization on the vertical hub shears after two stages of optimization, D_0 refers to the intial design and D_{II} indicates the final design. Other results obtained indicated hub shear reductions between 15-40%, and blade weight reductions between 9-20%.

Structural optimization was also used in a systematic study by Davis and Weller [115] in which a hierarchy of dynamic problems were considered. These were: (1) maximizing a bearingless rotor inplane structural damping, due to shearing of the elastomer; (2) frequency placement of blade natural frequencies; (3) minimizing hub vibratory shear using a simplified model for rotor aerodynamics (40% reduction in hub shears was obtained); and (4) minimizing rotor vibration indices. Various optimization algorithms, problem formulations and solution strategies were also considered in this comprehensive study. This excellent study could be extended to include aeroelastic constraints, without too much difficulty.

9. Aeroelastic Analysis and Testing of Special Configurations

The purpose of this section is twofold. First the aeroelastic analysis of some special configurations is described. Often those configurations are unique and thus it is inconvenient to include them in any of the previous topics described in this paper. The second part of this section describes recent dynamic tests which have been performed to validate various analyses and designs.

The aeroelastic stability of two somewhat unusual rotor configurations, a constant lift rotor (CLR) and a free tip rotor (FTR), were analyzed by Chopra [116] for the case of hover. The CLR-configuration employs a pitch control input to rotate several independent airfoil sections which are free to pivot around a continuous spar, allowing them to change their pitch so as to obtain the desired lift. For this blade rigid body flap and lag motion was assumed at the root hinge, and each strip was assumed to undergo independent torsional motion. Stability boundaries are obtained in a conventional manner using a linearized stability analysis in hover [5]. The influence of several parameters on blade dynamics was examined. The free-tip rotor blade consists of two sections: an inboard section similar to that of a conventional blade with a pitch control system and a small outboard section (about 5-10% of radius) freely pitching on its spar with control input of pitch motion. Thus a free tip rotor is similar to a constant lift rotor with two sections. Under appropriate conditions both configurations were found to be free of aeroelastic instabilities.

The analysis of circulation control rotors was also considered by Chopra [117,118]. In Ref. 117 the aeroelastic stability of a hingeless circulation control rotor blade, in hover, was analyzed using finite elements [22]. The airfoil characteristics associated by the blowing, which produces circulation control, was taken from experimental data. With the exception of the effects of blowing this analysis was similar to Ref. [22]. The importance of including the second lag mode in the analysis was noted and it was found that blowing can have significant effects on blade stability. In Ref. 118 a much more comprehensive study of bearingless circulation control rotors was conducted, using tabular aerodynamics and dynamic inflow. The influence of dynamic inflow was found to be very small on this type of rotor. It was concluded that the expected levels of internal structural damping appear adequate to stabilize the mildly damped fundamental and second lag mode.

The aeroelastic stability of a two bladed rotor on flexible unsymmetrical support was analyzed by Chen [119]. Due to the complicated interaction between the aeroelastic and parametric excitation problems, present in this configuration, it was difficult to obtain conclusive results.

Using the mathematical model developed in Ref. 54 Venkatesan and Friedmann [120] have analyzed the aeromechanical stability of a hybrid heavy lift multirotor vehicle in hover. This model was intended to simulate the dynamic behavior of the hybrid heavy lift helicopter built by the Piasecki Aircraft Co., which crashed during flight tests, in the early summer of 1986. This dynamic model consisted of two rotors, connected by a flexible supporting structure and combined an envelope providing buoyant lift. Each four bladed articulated rotor was modeled using fully coupled, nonlinear, flap-lag-torsional dynamics, the vehicle had six rigid body degrees of freedom and the supporting structure had two bending and one torsional degree of freedom. The aeromechanical stability model had a total of 31 degrees of freedom, and thus it represents one of the more complicated aeromechanical problems considered. The results obtained indicated a potential for air and ground resonance type instabilities.

An interesting case study of a coupled pitch-flap-lag instability involving the coupling of higher chord and flap bending modes combining with a reactionless torsion mode, observed in the tests of an experimental rotor in hover, was presented by Neff [121].

A thorough design oriented treatment of the aeromechanical aspects of hingeless/bearingless rotor system was presented in Ref. 122. The importance of blade parameters such as droop, control system flexibility and blade stiffnesses are discussed. It is noted that lead-lag damping levels in bearingless main rotors is lower than that for hingeless rotors. This property of bearingless rotors is illustrated by Fig. 28 where comparisons of measured and predicted lead-lag damping (in the rotating system) are shown. The data was obtained in whirl tower tests of actual full scale rotors. A somewhat similar study on scaled bearingless main rotors was also done by Weller and Peterson [123].

A very comprehensive experimental investigation of bearingless model rotor stability was undertaken by Dawson [124]. The emphasis was on isolated blade stability. Five different configurations were tested and a significant body of high quality data was obtained. The experimental results were compared with analytical results obtained by the FLAIR analysis [125], in general the agreement between experiment and theory was acceptable except in cases when blade flexibility played a strong role. Most of the differences between predicted and experimental damping data occurred at high pitch angles. Figure 29 compares the lead-lag damping of Configurations 4 and 5 [124] as pitch angle is varied. For Configuration 4 the blade is preconed 2.5 deg. with respect to the flexbeam, while for Configuration 5 the flexbeam is preconed by 2.5 deg. with respect to the hub. Measurements of lead-lag damping at low pitch could not be obtained due to flutter. The theoretical model underpredicts damping at the higher pitch angles.

Additional test results obtained on similar two and three bladed bearingless rotors are described by Bousman and Dawson [125]. These results are particularly interesting because a pitch-flap type of flutter, attributed to unsteady aerodynamic effects, was observed.

A design oriented parameteric study of the aeromechanical stability, in air resonance, of bearingless rotors in hover was conducted by Hooper [127] using the FLAIR program [125]. In this study it was found that precone angle of the blade relative to the flexbeam and vertical offset of the snubber attachment point can produce beneficial blade dynamic behavior. This theoretical, design oreinted study, also complements the previously mentioned combined experimental/analytical studies [122-124].

The behavior of bearingless rotors, for tail rotor applications, was also studied. Thus, Ref. 122 also contains useful information on the dynamic behavior of bearingless tail rotors. The aeroelastic characteristic of the AH-64 bearingless tail rotor were presented by Banerjee [128]. The elastomeric shear attachment of the flexbeam to the hub introduces beneficial damping and modal characteristics which yields an aeroelastically stable rotor. This rotor was extensively tested in wind tunnel covering most operating conditions.

Finally, it should be mentioned that another potential tool for aeroelastic research on rotors are dynamically scaled wind tunnel models. The development and testing of a 27% dynamically scaled model of AH-64 main rotor is described in Ref. 129. This scaled down version, of an existing full scale rotor offers the potential for studying in detail the aeromechanical behavior of the rotor and can be also used to simulate other aeroelastic problems.

10. Concluding Remarks

It is evident from this survey that the level of activity in rotary-wing aeroelasticity during the last few years was very substantial. Comparing this activity to its fixed-wing counterpart gives one the impression that the center of gravity in the field of aeroelasticity is shifting from fixed-wing configurations to rotary-wing configurations. This is not surprising in view of the historical fact that helicopters are thirty years behind fixed-wing aircraft in development. Judging by the recent research, discussed in this paper, it appears that this thirty year gap might be narrowing. However, the level of complexity present in rotary-wing systems, which imposes stringent demands on the level of sophistication required in the analysis, implies that rotary-wing aeroelasticity is far from being a mature field of research. Much additional research is needed before rotary-wing aeroelasticity will achieve the level of maturity which currently exists in the fixed-wing field.

A number of topics where additional research has the potential for significant payoffs in terms of improved helicopter designs are:

1. Generation of an experimental data base for the validation of both isolated blade and coupled rotor/fuselage in forward flight. This test data should be obtained for hingeless, bearingless and articulated rotor configurations with simple geometries and properties (i.e., no sweep and droop, constant mass and stiffness distribution, and zero or linear pretwist).

2. Correlation studies based on this data to validate forward flight analyses.

3. Improved unsteady aerodynamics, in the time domain, for compressible and transonic regimes which are suitable for incorporation in aeroelastic analyses in hover and forward flight.

4. Development of improved methods for dynamic load predictions, on blades, which could lead to multidisciplinary optimization of rotor systems with simultaneous aeroelastic performance and acoustic constraints.

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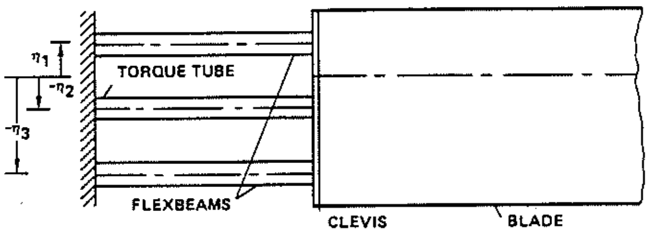
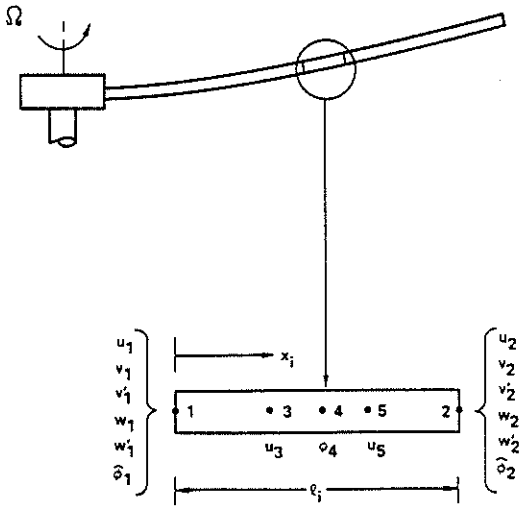


Figure 1. Analytical model of a bearingless blade Ref. (22)

Figure 2. Finite element model showing nodal degrees of freedom (Ref. 22)



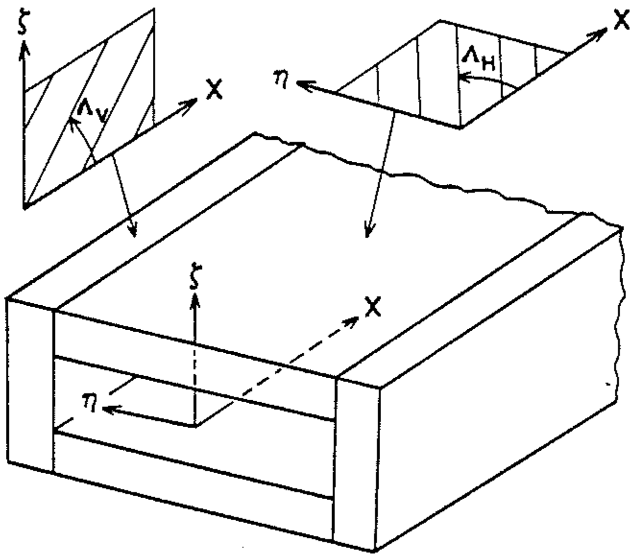


Figure 3. Composite box beam representing blade spar. Ply angle orientation with respect to reference coordinate: Λ_V - vertical laminates, Λ_H for horizontal laminates (Ref. 25)

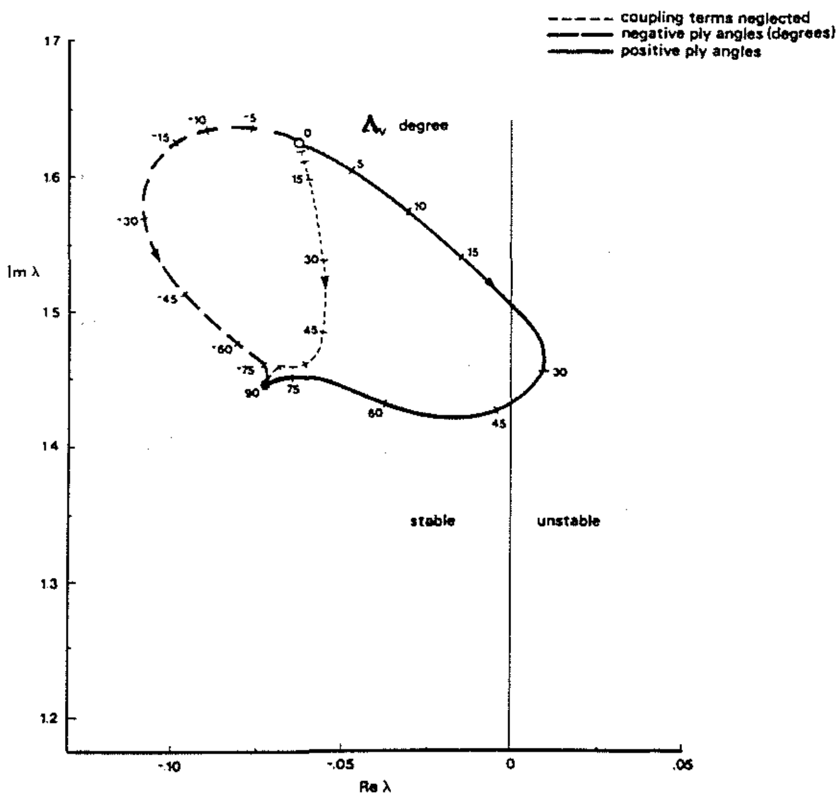


Figure 4. Root locus (eigenvalue) plot for lag mode of a composite blade with symmetric laminates, C_T/σ = 0.10 (Ref. 25)

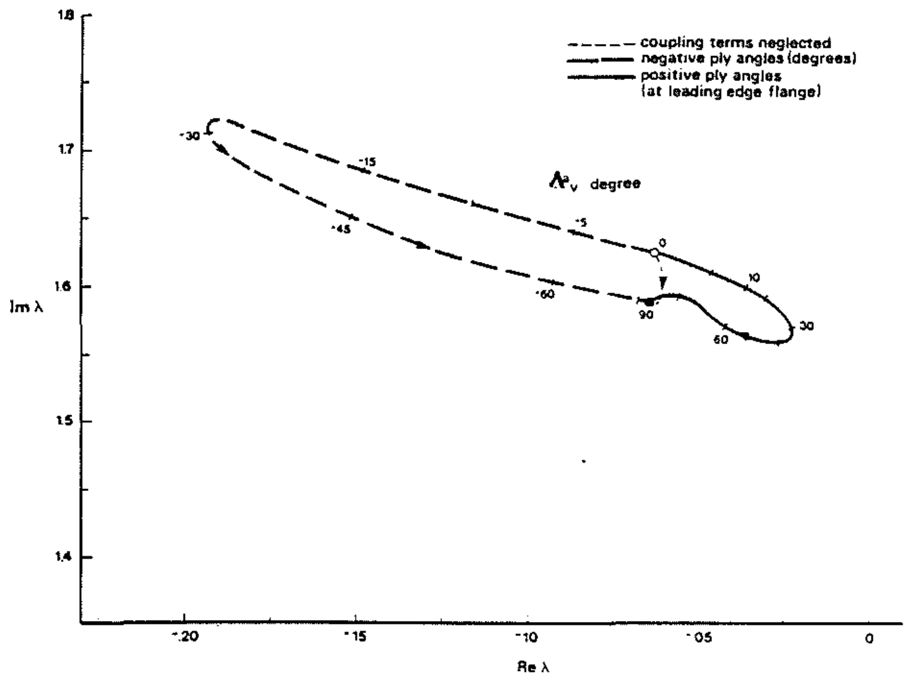
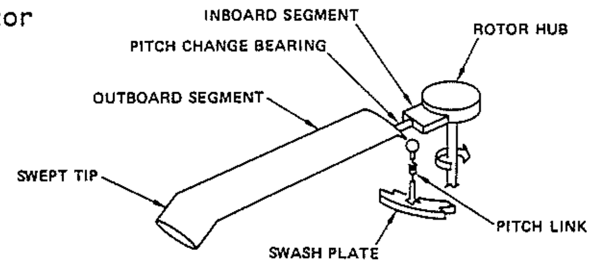


Figure 5. Root locus (eigenvalue) plot for lag mode of a composite blade with antisymmetric laminates, C_T/σ = 0.10 (Ref. 25)

Figure 6. Swept-tip hingelesss rotor blade model (Ref. 28)



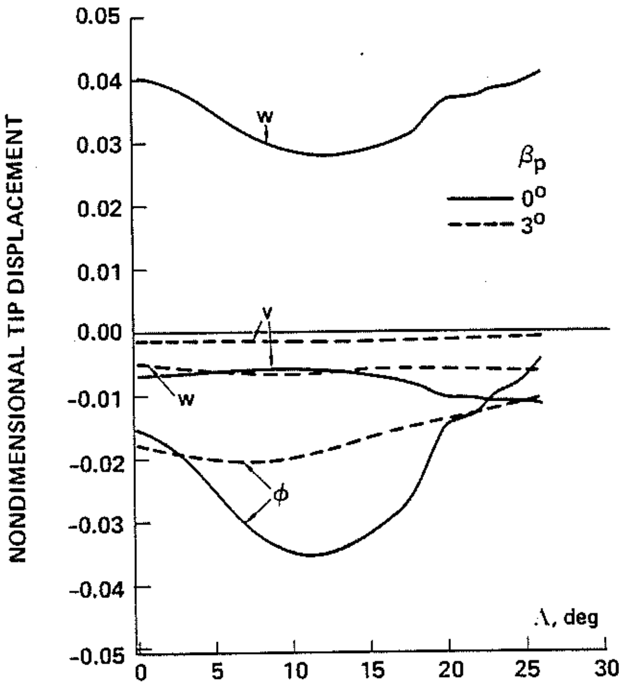


Figure 7. Effect of sweep and precone on hover equilibrium position, from Ref. 28 (ω_{L1} = 1.147; ω_{F1} = 1.125; ω_{T1} = 3.176 γ = 5.5, ℓ_T = 0.1R; θ_o = 0.1432; σ = 0.07)

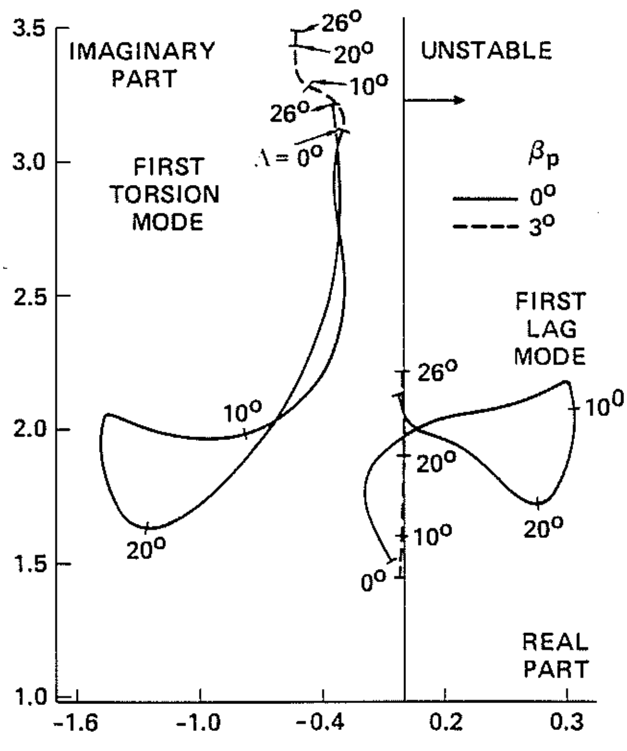


Figure 8. Effect of sweep and precone on aeroelastic stability in hover, (or real and imaginary part of eigenvalue) from Ref. 28, with same data as Fig. 7

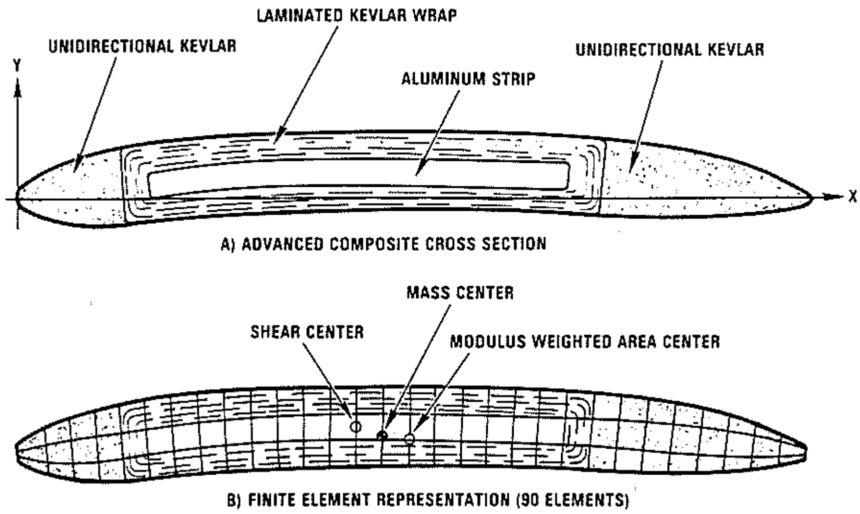


Figure 9. Analysis of an advanced composite airfoil cross section (Ref. 30)

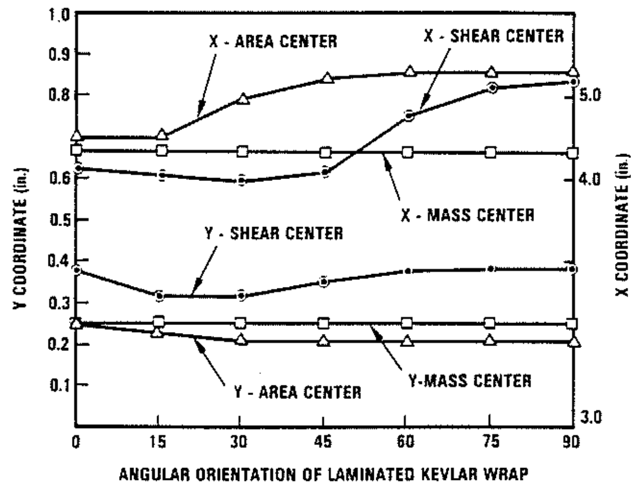


Figure 10. Location of mass, area and shear centers on an advanced composite cross section with different angles of wrap (Ref. 30)

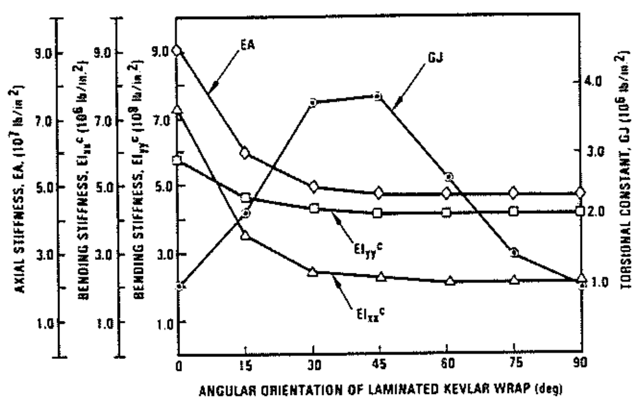


Figure 11. Structural constants of an advanced composite cross section with different angles of wrap (Ref. 30)

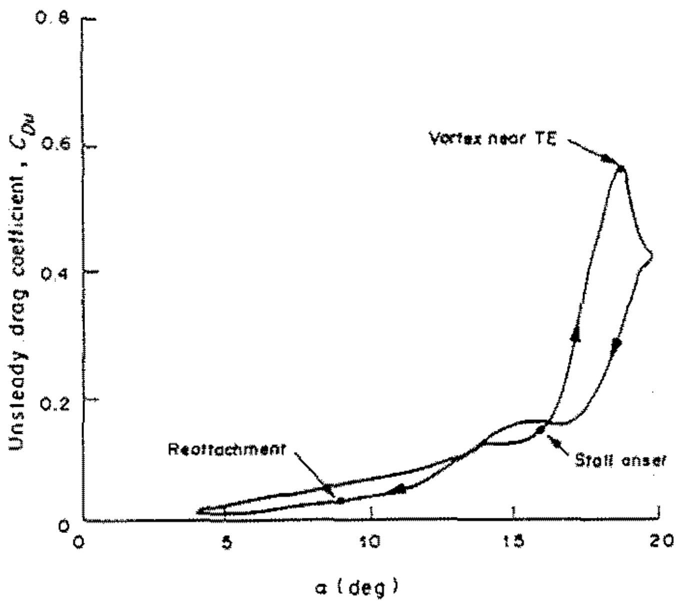
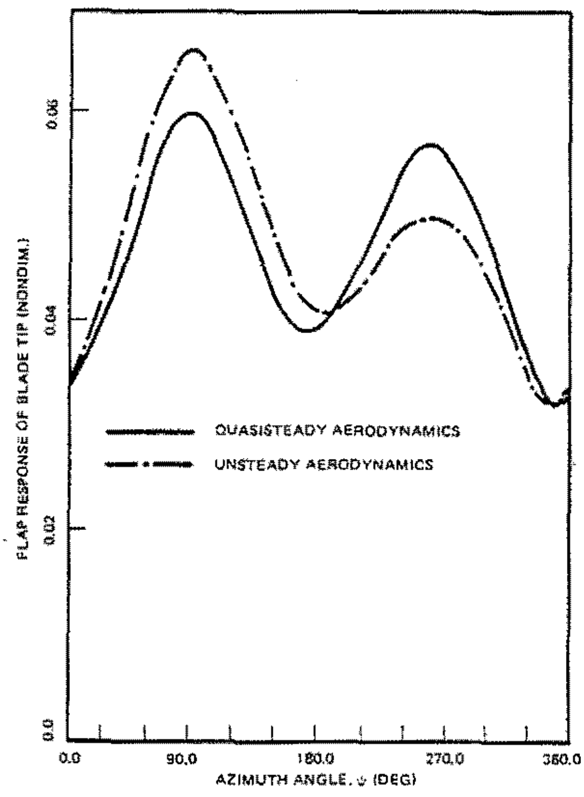


Figure 12. Typical unsteady drag coefficient loop data, SC1095 airfoil, α_o = 12 deg; $\overline{\alpha}$ = 8 deg; k = 0.10 deg (Ref. 35)



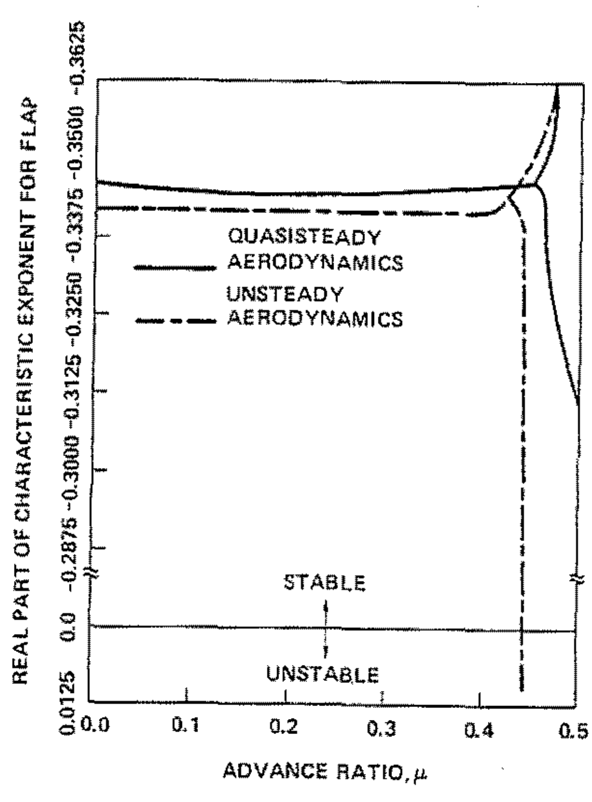


Figure 13. Comparison of flap response calculated using quasisteady and unsteady aerodynamics for μ = 0.40, from Ref. 43 (C_W = 0.005; γ = 5.5; σ = 0.07; $\overline{\omega}_{F1}$ = 1.125; $\overline{\omega}_{L1}$ = 0.732; R_c = 0.0)

Figure 14. Real part of characteristic exponent for flap calculated using quasisteady and unsteady aerodynamics, same data at Fig. 13, (Ref. 43)

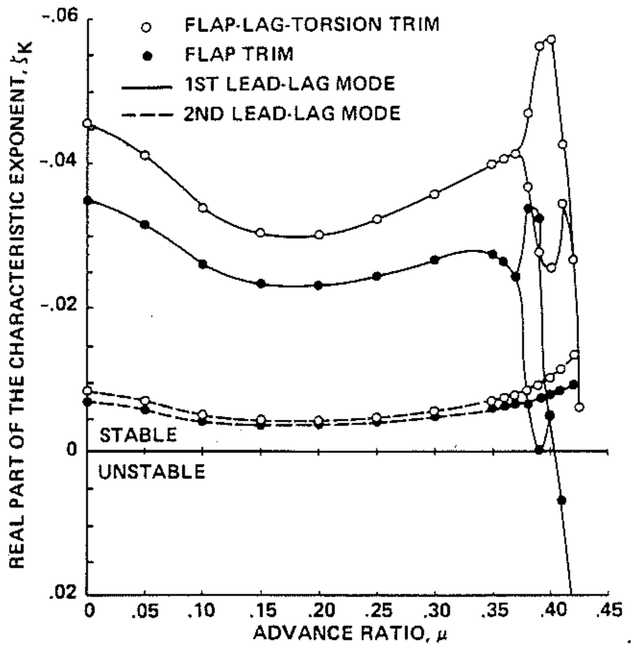


Figure 15. The effect of number of degrees of freedom used in trim analysis on lead-lag damping versus advance ratio, stiff-in-plane blade, from Ref. 56 ($\overline{\omega}_{L1}$ = 1.40; $\overline{\omega}_{F1}$ = 1.15; $\overline{\omega}_{T1}$ = 3.0; σ = 0.10; R_C = 1.0; propulsive trim)

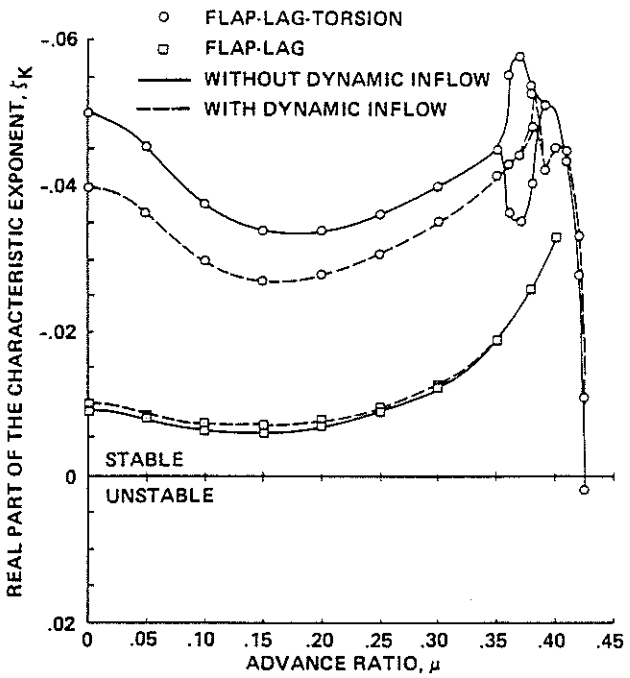


Figure 16. The effect of torsion and dynamic inflow on lead-lag regressing mode damping versus advance ratio, stiff-in-plane blade, from Ref. 56 (same data as Fig. 15)

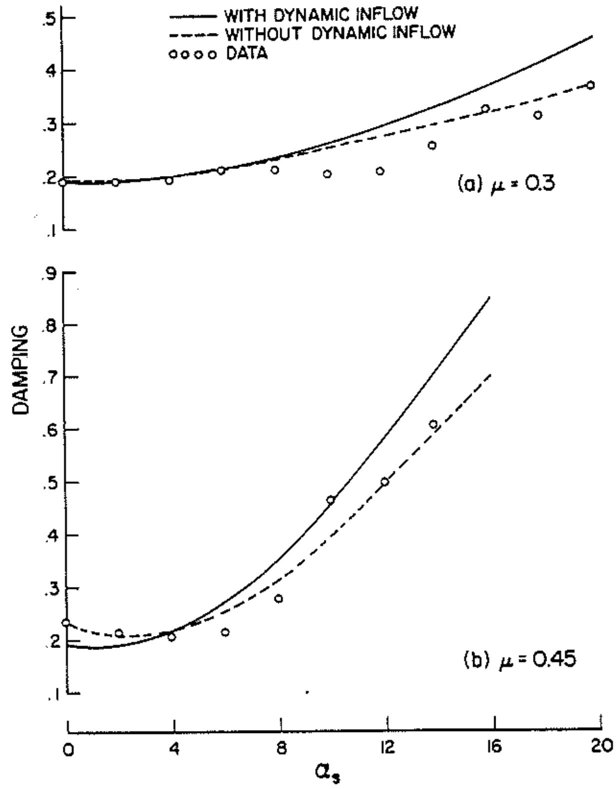


Figure 17. Lag regressing mode damping correlations in substall and stall, from Ref. 65 (Ω = 1000 RPM; R_c = 0; θ_0 = 0 deg.)

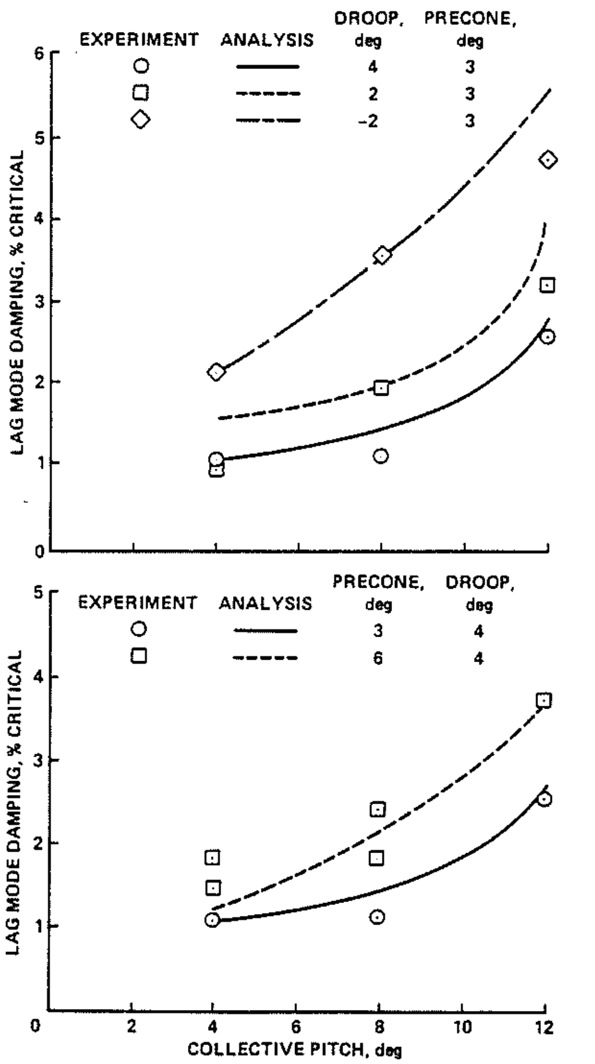


Figure 18. Blade regressing lag mode damping ratio as a function of collective pitch at an advance ratio of μ = 0.30; from Ref. 74, (δ_3 = 42.5 degrees; R = 1.38 m; ΩR = 90 m/s in Freon 12; σ = 0.10)

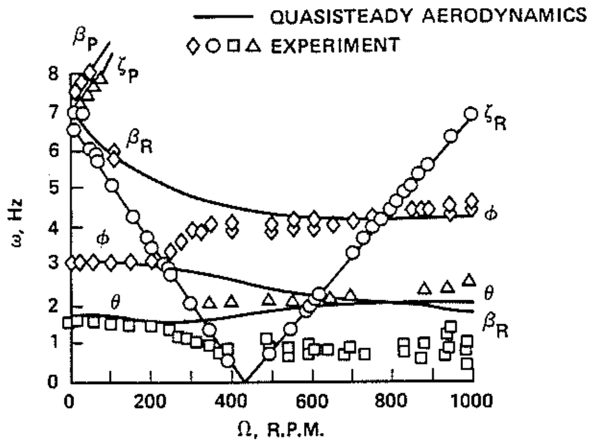


Figure 19. Variation of modal frequencies with Ω; $\theta_c = 0$, Configuration 4, (Ref. 81)

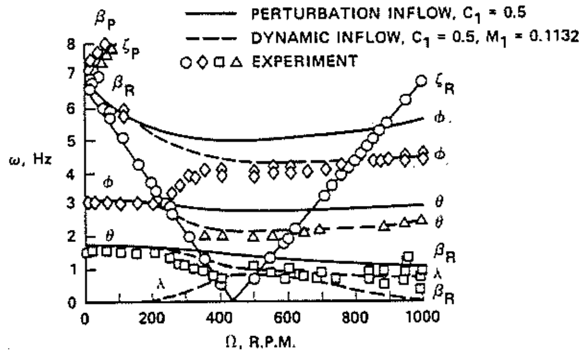


Figure 20. Variation of modal frequencies with Ω; $\theta_c = 0$, Configuration 4, (Ref. 81)

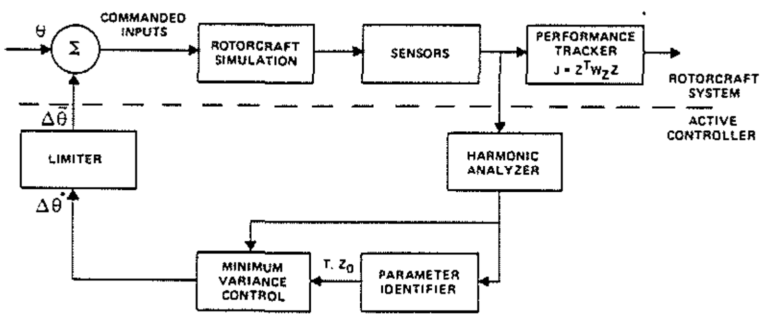


Figure 21. Schematic block diagram of closed-loop adaptive HHC system, used in simulations (Ref. 90)

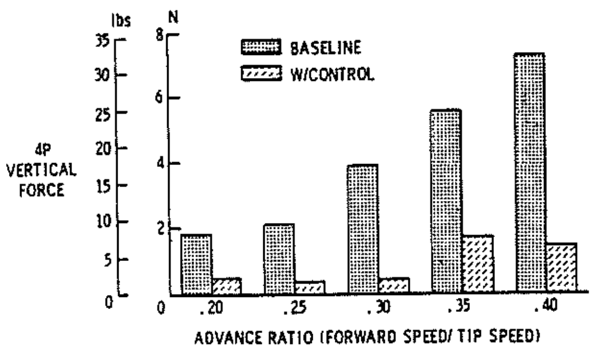


Figure 22. Variation of vibratory vertical force with advance ratio, using adaptive HHC in wind tunnel test (Ref. 88)

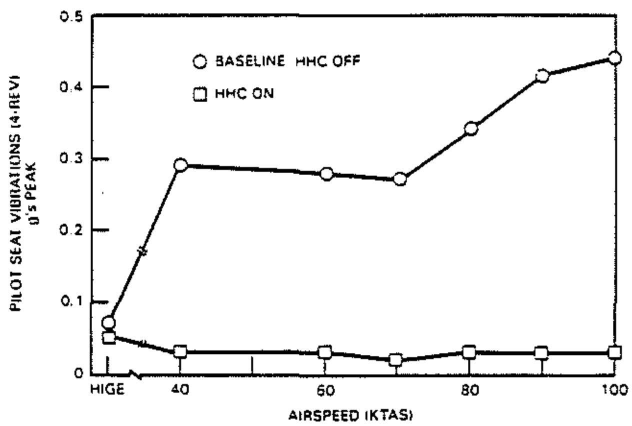


Figure 23. Vertical vibration reduction at pilot seat, 1984 Software (Ref. 89)

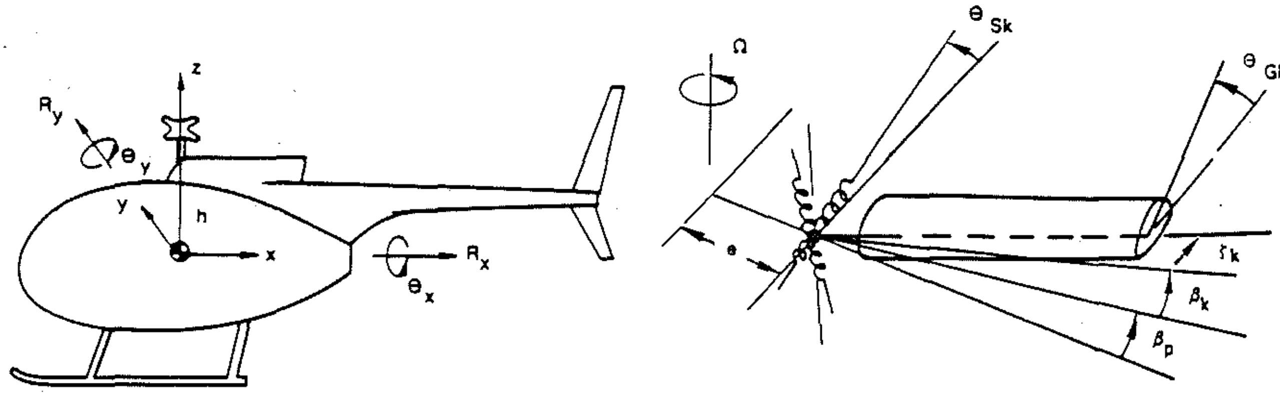


Figure 24. Fuselage and rotor model used in coupled rotor/body simulation for active controls (Ref. 101)

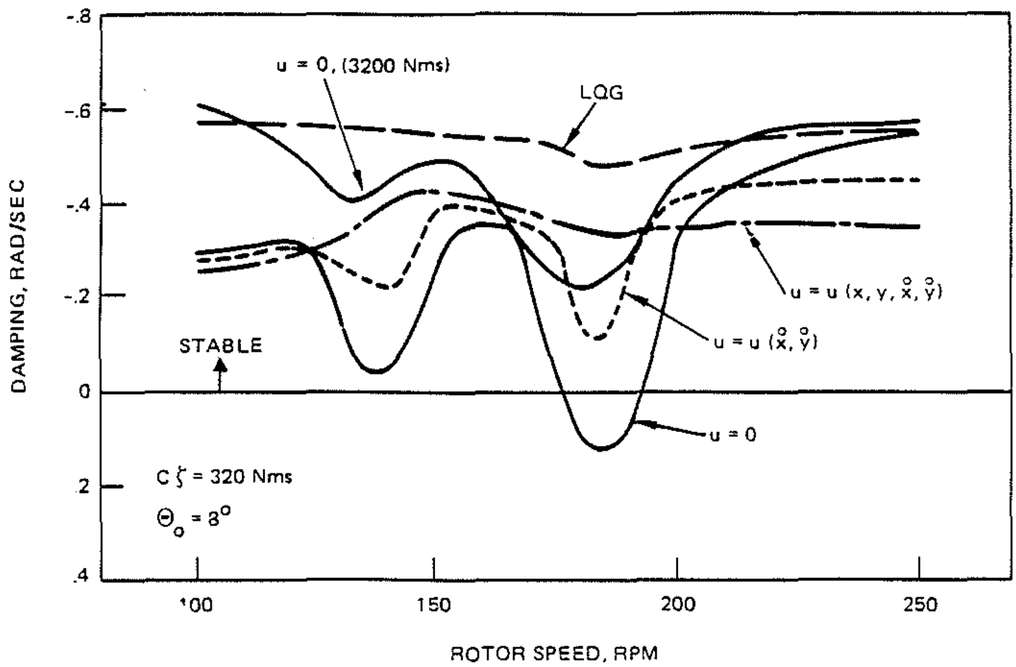


Figure 25. Aeromechanical stability with optimal controller and three reduced state, constant gain feedback systems for H-34/RTA (Ref. 102)

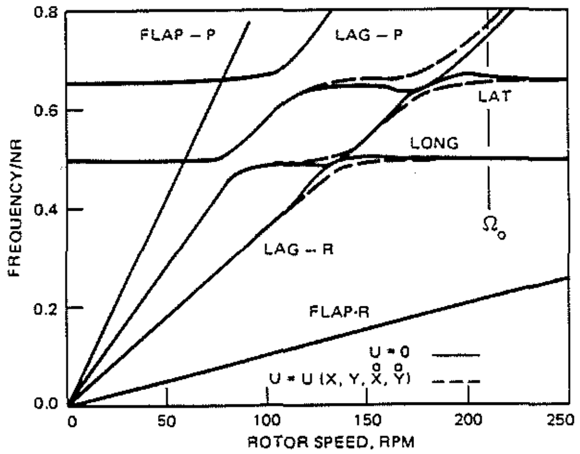


Figure 26. Effect of feedback control on H-34/RTA modal frequencies (Ref. 102)

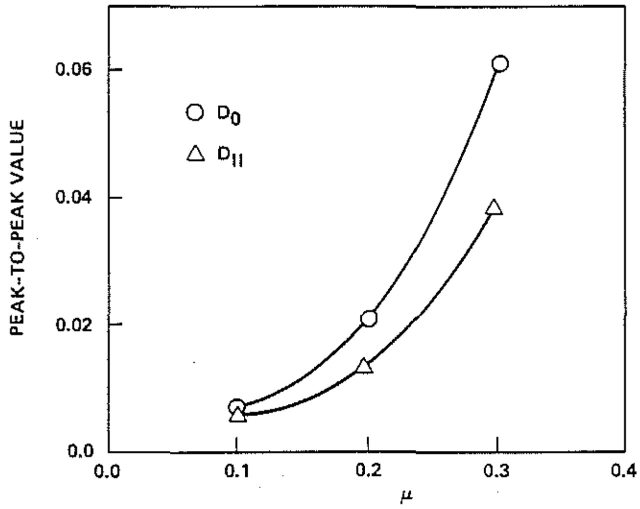


Figure 27. Vertical hub shears, nonlinear, peak-to-peak values, (nondimensionalized), comparison of initial and final designs after two stages of optimization, soft-in-plane blade (Ref. 114)

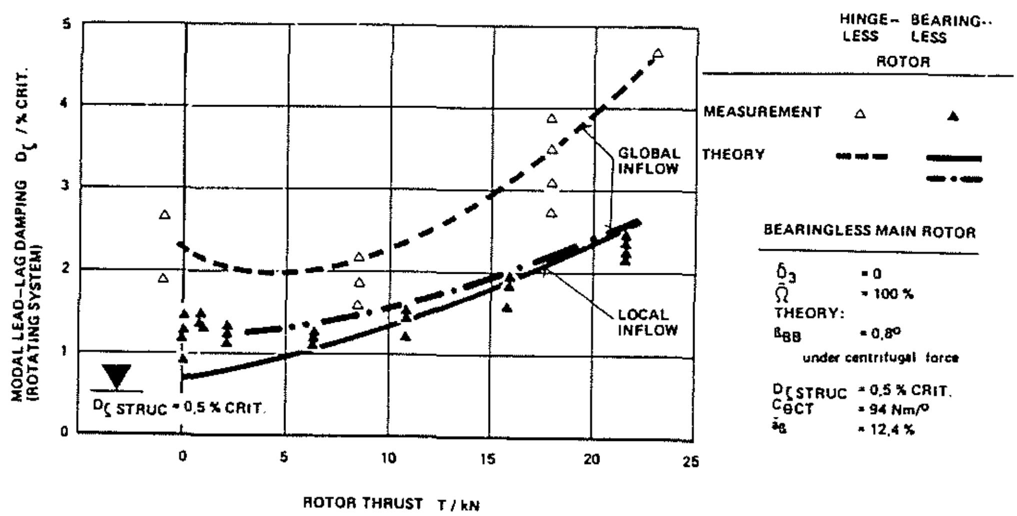


Figure 28. Comparison of lead-lag damping of experimental bearingless main rotor and B0105 hingeless rotor, obtained on whirl tower (Ref. 122)

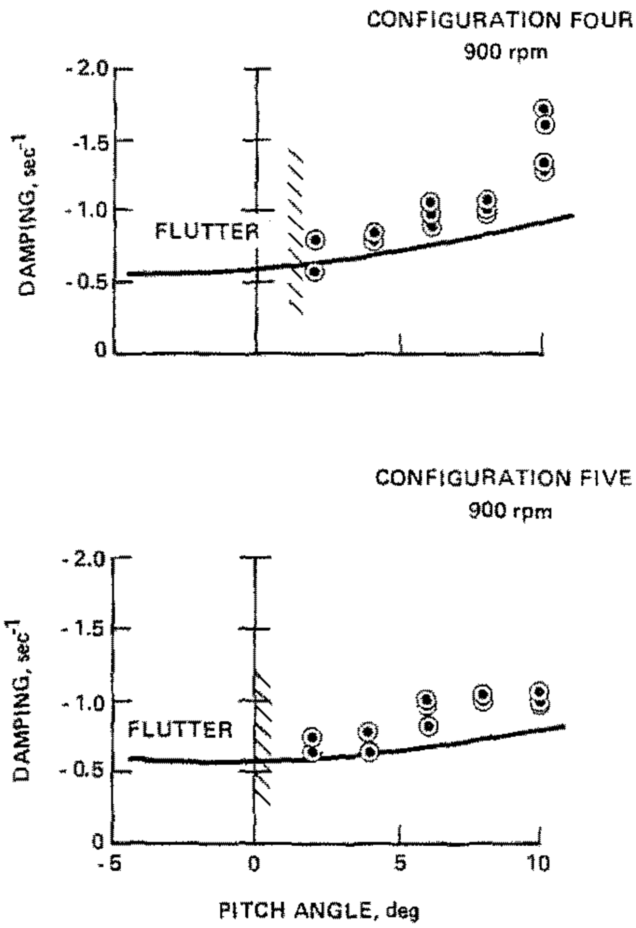


Figure 29. Lead-lag damping versus blade pitch angle at Ω = 900 RPM for configurations 4 and 5 (Ref. 124)