A Structured Singular Value Technique for the Design of Helicopter Flight Control Systems

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Abstract

In this paper a structured singular value (μ) synthesis has been adopted to design a control law for a blade element, articulated rotor helicopter model. The study was conducted to meet the military rotorcraft handling qualities design specification (ADS-33C) in low-speed flight. In the design a six degree-of-freedom model has been used. The controller minimises the weighted H_∞ norm of the error between the rotorcraft response and the specified reference model while maximising robustness to model uncertainties. Simulations have been carried out using a non-linear model of the helicopter Agusta A-109. These simulations demonstrate that the controller permits to satisfy handling quality requirements and provides good tracking of pilot inputs.

List of Symbols

p, q, r	Angular velocity components along body axes
u, v, w	Translational velocity components along body axes
x, y, z	Components of the position vector in the inertial frame
W_{CONT}, W_{DEL}, W_{IN}, W_P	Weighting functions chosen in the control law design
δ_{col}	Collective stick input
δ_{lat}	Lateral stick input
δ_{long}	Longitudinal stick input
δ_{ped}	Pedal input
Φ, Θ, Ψ	Fuselage attitudes (Euler angles)

I. Introduction

The next generation of helicopters will have to satisfy stringent handling qualities requirements, tailored to specific missions, with severe constraints on the allowable cross-coupling and on the desired frequency responses of the helicopter. Recent experience seems to indicate that, as performance demands increase, the classical single input, single output (*SISO*) design methodologies tend to become more cumbersome to apply and less effective in guaranteeing the desired control requirements. Consequently, many recent researches have explored the application of modern, multivariable (*MIMO*), robust control law design methods to helicopters.

A comprehensive review of such work has been presented in [1], in which the authors identify the hover flight condition as the most challenging for the designers due to the unstable, non-minimum phase characteristics of the helicopter and the rapid change in dynamics in the neighbourhood of hover.

As far as the control design techniques are concerned, the most largely investigated are the eigenstructure assignment [2], [3], [4] and the reference model control [5]. The former can be quite easily used when design requirements can be expressed in terms of desired closed loop modal characteristics. However, in many cases, system design requirements cannot be so simply expressed. Moreover, this technique cannot explicitly take into account control input amplitude and robustness requirements. The model following techniques, on the other hand, have been usually used in the framework of optimal linear quadratic approach [6]. Therefore they have the typical limitations of this approach. In particular, one of its main drawbacks (and of the eigenstructure assignment too) is the destabilising effect of the high frequency rotor dynamics on the rigid body dynamics. In fact, in the presence of high-gain feedback, the frequency range of interest becomes comparable with the one of the coupled rotor-fuselage dynamics. These effects will become even more pronounced if newer technologies, such as bearingless rotors, are introduced into helicopter designs.

To overcome this problem, various authors have suggested the usage of an H_∞ approach [7], [8], which guarantees robustness of the stability with respect to the high frequency dynamics, when coupling with these dynamics is not included in the design process.

This paper proposes a straightforward method of controller synthesis, based on the structured singular value (μ) technique [9], [10], [11], which achieves desired performances objectives and which is robust to unmodeled high frequencies modes. The controller has been designed for a helicopter at hover using the μ toolbox of MATLAB [12]. The design was based on a 9^{th} order model (those describing the rigid-body dynamics) and the unmodeled rotor dynamics were treated as built-in modelling error. The resulting controller was designed to be robust enough to achieve the desired performance despite the unmodeled dynamics. A critical ingredient was the proper selection of the weighting matrices. The final controller design provided closed-loop stability, good decoupling, robustness to unmodeled high frequency dynamics, complying with the quantitative requirements of the military rotorcraft design specifications ADS-33C [13] for a helicopter at hover.

The evaluation of the closed-loop system behaviour has been performed through simulations using a non-linear eighteen-degrees-of-freedom model of the Agusta A-109 helicopter. Actuator dynamics and saturation levels were included.

The paper is organised as follows. The helicopter model used is described in Section II. A set of design specifications based on handling qualities requirements is described in Section III, while the actual design procedure is described in Section IV. Some simulation results are included in Section V. Finally, the conclusions end the paper.

II. Helicopter Model

The linear, time-invariant mathematical model used in this study for the controller design has been obtained from a fully non-linear dynamic model of the Agusta A-109 helicopter, through numerical linearisation about trimmed hover flight condition. The non-linear mathematical model of the helicopter, called HELIDYN, was developed by the authors, on the basis of the work by Howlett [14] and its successive modification [15]. The aerodynamic forces and moments acting on the fuselage and on the other fixed surfaces (horizontal stabilisers, vertical fins, wings) have been calculated using the results presented in [16]. The geometric and physical helicopter parameters have been extracted from the simulation code ARMCOP, validated on the Agusta A-109 [17].

A detailed description of the mathematical model is beyond the scope of this paper, and only a brief outline is presented below. It is a full non-linear model which describes both dynamic and aerodynamic phenomena. The fuselage is modelled as a rigid body. The rotor blades are individually modelled as rigid bodies, and each of them undergoes flap and lag motion. Lead-lag dampers models are also included. A three state Pitt-Peters dynamic inflow model [18] is used to capture the unsteady wake effects at the main rotor while a one state model is used for the wake at the tail rotor. The aerodynamic forces on the rotor are calculated using blade element theory and quasi-steady aerodynamics. Rotor speed degree-of-freedom and engine-governor dynamics are not included in this model. A great effort has been made to put the model in the standard state variable form, so that the model can be directly used for the controller design and for the closed loop simulations by using standard ODE solvers.

The mathematical model is based on a total of 18 degrees of freedom: 3 rigid body translations and 3 rigid body rotations for the fuselage; flap and lag angular motion for each of the four rigid blades; 3 main rotor dynamic inflow and 1 tail rotor dynamic inflow. The total number of states is 32:

- twelve associated with the motion of the fuselage: the six velocities in the body-fixed reference frame, the three Euler angles Φ, Θ, Ψ and the fuselage position in the inertial frame;
- two for each blade to describe the lag motion and two for the flap motion. The blade equations are derived enforcing the momentum equilibrium at the hinge. The moments are generated by the inertial forces, aerodynamic forces and the lag damper forces;
- three to describe the airmass dynamics at the main rotor. The equations used can be found in [19];
- one to describe the inflow at the tail rotor.

The trimming conditions were obtained by using the algorithm proposed in [20]. It is based on a periodic shooting approach with Newton-Raphson iterations and it exploits the symmetry of the rotor to minimise computational workload. The technique has a considerably fast convergence: in our simulations at most 8 iterations were necessary to produce a periodic trim condition wherever in the flight envelope.

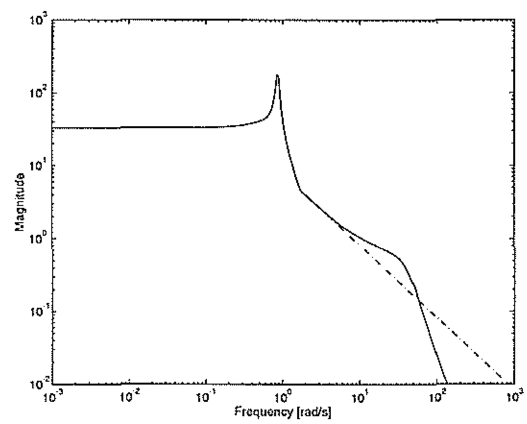

Figure 1 . Full order and reduced order (dashdot) maximum singular values

A linear time-invariant model was obtained from HELIDYN using a procedure similar to the one proposed in [15]. This model was then reduced to eight order,

Table 1 . Eigenstructure of the reduced helicopter model

Eigenvalues	−3.1097	0.0370∓0.8710j	0.3081±0.5735j	−0.9446	−0.3095±0.0272j
Eigenvectors	*roll*	*side velocity*	*forward velocity*	*pitch*	*heave-yaw*
u	−0.0022	−0.0402±0.2025j	−0.2371∓0.4496j	−0.4126	−0.0035∓0.0533j
v	0.0668	0.3430∓0.0191j	−0.1177±0.2271j	0.0108	−0.0031±0.0236j
w	0.0058	0.0087±0.0085j	−0.0245∓0.0183j	−0.0196	−0.0734∓0.4675j
p	0.9436	−0.4722±0.1159j	0.2137∓0.0071j	0.0176	0.0301±0.0343j
q	0.0208	−0.0380±0.3253j	0.2017∓0.3331j	0.6252	0.0321±0.0472j
r	0.0053	0.0500∓0.2467j	−0.1892±0.0330j	0.0114	−0.6056∓0.6315j
Φ	−0.3054	0.0928±0.5427j	0.1528±0.2708j	−0.0173	0.0142±0.0039j
Θ	−0.0068	0.3549±0.0556j	0.5865±0.0179j	−0.6615	0.0007∓0.0399j

retaining the state associated with the rigid body and discarding all the others, using the method presented in [21] (see Figure 1). This model exhibits some unfavourable open-loop handling qualities characteristics that are typical of a conventional helicopter in hover. These include: unstable rigid body modes, non-minimum phase zeros and high level of cross-coupling (see Table 1).

III. Handling Qualities Requirements

An unaugmented (open-loop except for the pilot) helicopter exhibits unacceptable responses in hover. The response to the collective, longitudinal and lateral cyclic, and pedals are highly coupled and unstable. Pilot workload is high and precise control is difficult without augmentation.

In this paper we will consider a design for a Hover Hold controller for precision hover that meets level 1 handling qualities with poor usable Cue Environment (UCE=3) [13].

The pilot has good control over the angular acceleration of the helicopter, but the direct control of translation is poor. The control sensitivity (the pitch and roll rate commanded by cyclic) is high in hover because of low damping. The combination of high sensitivity and only indirect control of translational velocity is conducive to pilot induced oscillations, and increases the difficulty of the control task. In hover and at low speed, direct control over the forces would be more desirable, in order to obtain direct command of the helicopter velocity and displacement.

More specifically, for this task it is desired to have pilot longitudinal stick commands correspond to forward velocity, lateral stick to side velocity, collective to heave velocity and pedal to yaw. This is a velocity command system and it is intended to provide good handling qualities during precise hover manoeuvres. It is also desired that the control laws provide gust attenuation, acceptable robustness in the presence of variations in parameter and meet handling qualities requirements.

For this mission task, the handling qualities [13] foresee the following type of responses: Translational Rate Command and Position Hold (*TRC+PH*), Rate Command Direction Hold (*RCDH*), Rate Command Height Hold (*RCHH*).

In the *TRC+PH*, constant control input must result in constant translational rate and the rotorcraft must hold position if the force on the cockpit controller is zero. *TRC+PH* systems are preferred in the nap of the earth manoeuvres in fair to poor usable cue environments. In fair usable cue environments considerable concentration is required for the pilot to perceive pitch or roll attitude and lateral, longitudinal or vertical translational rates. The translational rate response to step cockpit cyclic control position or force should have a qualitative first order appearance with an equivalent rise time (time to 0.632 of steady state) between 2.5 and 5 s. Moreover the pitch and roll attitude should not exhibit objectionable overshoots for level 1. In *PH* the rotorcraft shall be capable of automatically holding its position with respect to a ground fixed or shipboard hover reference maintaining its position within 3 m diameter circle during a 360° turn completed between 10 and 45 s (depending whether the manoeuvre is aggressive or not) in the presence of steady state wind of up to 18 m/s.

In the *RCHH*, constant collective input must result in constant vertical rate and the rotorcraft must hold the altitude if the force on the cockpit controller is zero. The vertical rate response should have a qualitative first order appearance with a time constant less than 5 s (level 1) following a step collective input. Moreover for level 1 it shall be possible to produce the vertical rate of 0.81 m/s after 1.5 s.

In the *RCDH*, the response to pedal force or position input must result in constant yaw rate and the rotorcraft must hold its heading if the input is null. The bandwidth is defined in such a way to capture the greatest speed of response that can be obtained without any tendency towards pilot induced oscillations. In the simplest case the bandwidth is the frequency at which the phase lag between the pilot's input and the heading is -135°. This bandwidth should exceed 3.5 rad/s and the minimum allowable equivalent damping should be 0.35 for level 1. The heading angle should return to within 10 percent of peak in less than 10 s and remain within the specified 10 percent for at least 30 s in the presence of a pulse input in the pedal control actuator.

Eventually, in the cases above, control inputs necessary to achieve a response in one axis shall not

Table 2 . Reference model

	δ_{LONG}	δ_{LAT}	δ_{COL}	δ_{PED}
$\dot{x}$ (ft/s)	$\frac{-30}{4s+1}$	0	0	0
$\dot{y}$ (ft/s)	0	$\frac{-10}{4s+1}$	0	0
$\dot{z}$ (ft/s)	0	0	$\frac{-20}{4s+1}$	0
Ψ (rad)	0	0	0	$\frac{1}{s(0.25s+1)}$

result in objectionable responses in one or more of the other axes.

From the point of view of developing a control law, the handling qualities requirements are criteria which can be used to evaluate the design performance. Indeed the criteria often describe tests for system evaluation. To meet the handling qualities requirement the aircraft must respond to the time varying input signals coming from the pilot tracking a desired output response.

This fact with the need to provide the required stability and robustness to unmodeled dynamics and parameter uncertainties led us to the use of a model following technique.

The reference model used is synthesised in Table 2. Time constants have been chosen to comply with the requirements previously stated, while the gains have been chosen taking into account both the maximum velocity for a helicopter at hover (5 knots) and input command ranges.

While good handling and tracking of inputs are characteristics which are evident, the properties of robustness to unmodeled dynamics can be erroneously neglected. In fact if the mathematical model of the system used to develop the control law does not include flap and lag dynamics (see Figure 1 and Figure 5), these dynamics may destabilise the closed-loop system or at least compromise the performance assured for the reduced model.

Thus robustness, and especially *robust performance*, is another important design consideration. For this reason the controller has been designed using the structured singular value technique.

IV. μ Synthesis Controller Design

The general framework of μ-synthesis, shown in Figure 2, is based on describing the system as a linear fractional transformation (*LFT*). Any linear interconnection of inputs, outputs and commands along with perturbations and a controller can be viewed in this context and rearranged to match this diagram, as shown afterwards. The term S represents the system interconnection structure, Δ the uncertainties and K the control law. The vector $\boldsymbol{y}$ is composed of measurement signals provided to the controller, while the vector $\boldsymbol{e}$ consists of the error signals such as performance index signals, actuators signals levels, and so on. The first input $\boldsymbol{u}$ is a vector consisting of manipulable control signals from the feedback compensator K and the second set of inputs $\boldsymbol{d}$ is composed of the external signals, such as disturbances and measurement noise. The third set of inputs and outputs w and z are just fictitious signals used to characterise the model uncertainties.

This framework is the typical H_∞ framework and the stabilising controller can be found using the well-known state-space methods [12], [22]. The H_∞ controller guarantees *nominal performance* and *robust stability*:

- *nominal performance*: the closed-loop system achieves nominal performance if the performance objective is satisfied for the nominal plant model;
- *robust stability*: the closed-loop system achieves robust stability if the closed-loop system is stable for all possible plants as defined by the uncertainty description Δ.

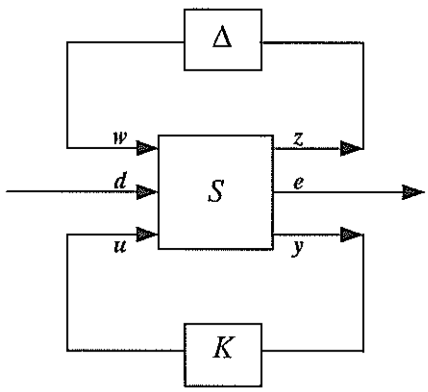

Figure 2 . General μ-synthesis framework

What the H_∞ controller does not guarantee is the *robust performance* requirement. Robust performance is achieved if the closed-loop system is stable for all possible plants and if, in addition to that, the performance objective too is satisfied for all possible plants as defined by the uncertainty description Δ. The Δ block is normalised to 1 for the synthesis procedure.

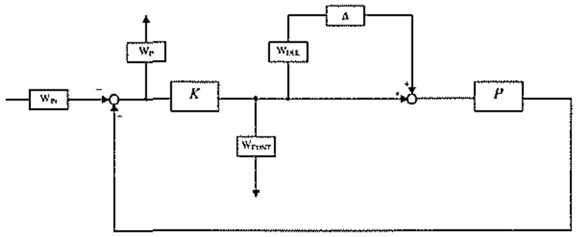

Figure 3 . Feedback design structure

The robust performance requirement can be satisfied only if a μ synthesis technique is adopted [9], [10]. The *D-K* iteration, which is an approximation to the μ synthesis, is a practical approach to design control systems with the goal of robust performance [9]. This technique integrates H_∞ optimisation methods for the controller synthesis and the structured singular value for analysis. An upper bound on μ is found by scaling the closed-loop system by a matrix D obtained from solving the μ analysis problem. The *D-K* iteration is carried out until a satisfactory controller is constructed.

The block diagram adopted to synthesise the μ output feedback controller is shown in Figure 3, and rearranged as depicted in Figure 4 to build the interconnection structure that exactly fits the general framework of μ-synthesis previously described.

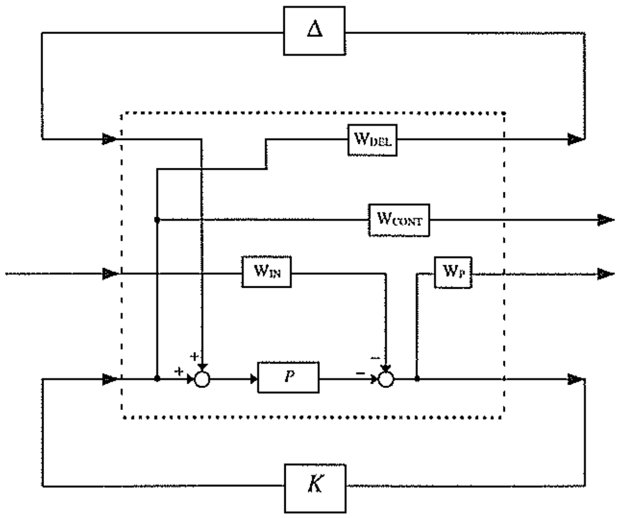

Figure 4 . Interconnection design structure

All input-output relationships in Figure 3 and Figure 4 are frequency dependent. In particular:

- P is the reduced order model of the helicopter; this model has eight states, the eight states associated with rigid-body dynamics (see Table 1) and four inputs, the four commands available on a one-rotor helicopter;
- W_{DEL} is a weight which models the uncertainty in the nominal model;
- W_{CONT} is a weight on the control input which penalises the amplitudes of the commands especially at high frequencies;
- W_{IN} is a filter used to characterise the signals to be tracked by the helicopter model;
- W_P is the performance weight on the controlled variables. These variables are shown in Table 3.

Table 3 . Controlled variables

u, v, w	translational velocities in body axes
Φ, Θ, Ψ	Euler angles
r	yaw rate in body axes

Weighting functions selection - The cost function W_{DEL} has been chosen basing on the error made approximating the full-order model with the reduced model (see Figure 5). It was chosen diagonal and equal for each of the four helicopter inputs:

$$W_{DEL_i} = 10\frac{s+0.2}{s+200}.$$

With such a choice not only stability but also performance are assured for the complete model.

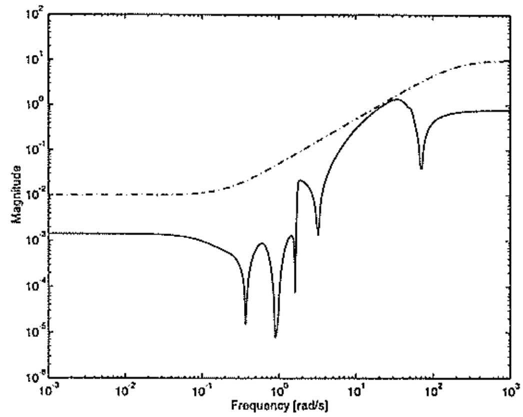

Figure 5 . Uncertainty weighting function (dashdot) and relative error between the full order and the reduced model

The function W_{IN} has been determined basing on the reference model shown in Table 2. So it was of the first order for u, v and w and of the second order for ψ. It was chosen diagonal to minimise cross-coupling.

The role of the weighting function W_{CONT} was to limit the control action at high frequencies and to maximise the system robustness by limiting the controller gain in the frequency range in which the plant uncertainty is high. For the helicopter case it was found that a diagonal matrix with differentiator type functions rendered acceptable designs, i.e.:

$$W_{CONT_i} = K_i\left(1+s\tau_i\right).$$

Because the cost function W_{CONT} is not proper, it could not be implemented directly in the state-space solution. This problem was overcome adding a high frequency pole at the function.

Eventually, the performance weight W_P was chosen to assure an integral action on the controlled variables. The time constants and the gains were determined basing on the desired performance of the closed loop system.

The controller found after some D-K iterations was then reduced using a balanced realisation model reduction method [21]. The controller was reduced to the smallest order satisfying the μ test (see Figure 6). This procedure led to a 21^{st} order controller.

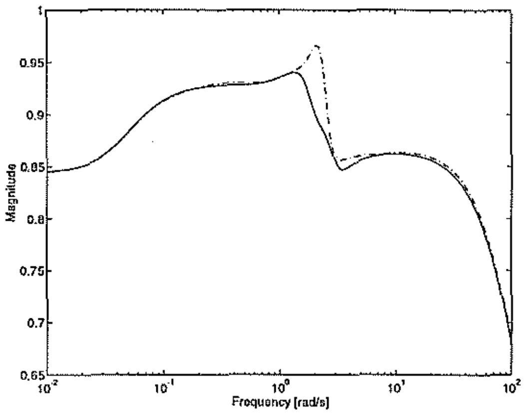

Figure 6 . μ plot of the closed loop system with the full order controller and the reduced controller

V. Simulation results

Simulations have been carried out using the scheme sketched in Figure 7.

In Figure 8 the transient response characteristics due to a longitudinal step input is shown.

In Figure 9 the transient response characteristics due to a lateral step input is shown.

In Figure 10 the transient response characteristics due to a collective step input is shown.

In Figure 11 the transient response characteristics due to a pedal step input is shown.

Examining the responses, it can be seen that excellent decoupling between axes is achieved. The u, v, w responses have a qualitative first order appearance with an equivalent rise time of about 4 seconds. Moreover the delay present in the height response is less than 0.2 seconds. This has been evaluated comparing the height response to a first order response with delay of 0.2 seconds.

The achievable vertical rate in 1.5 s is about 0.9 m/s assuring level 1 flying qualities. In Figure it is possible to notice the periodic behaviour of angular rates once that the steady state condition has been reached.

In Figure 11 it can be noticed that a 360° turn is accomplished in about 20 s, and the rotorcraft maintains its position. The equivalent damping for the mid-term response is greater than 0.35.

Finally in Figure 12 the heading hold performance in the presence of a step in the pedal control actuator is depicted.

The heading angle shall return to within 10 percent of peak in less than 10 s and remain within the specified 10 percent for at least 30 s (level 1).

The controlled variables are supposed to be measured by an Inertial Navigation Unit.

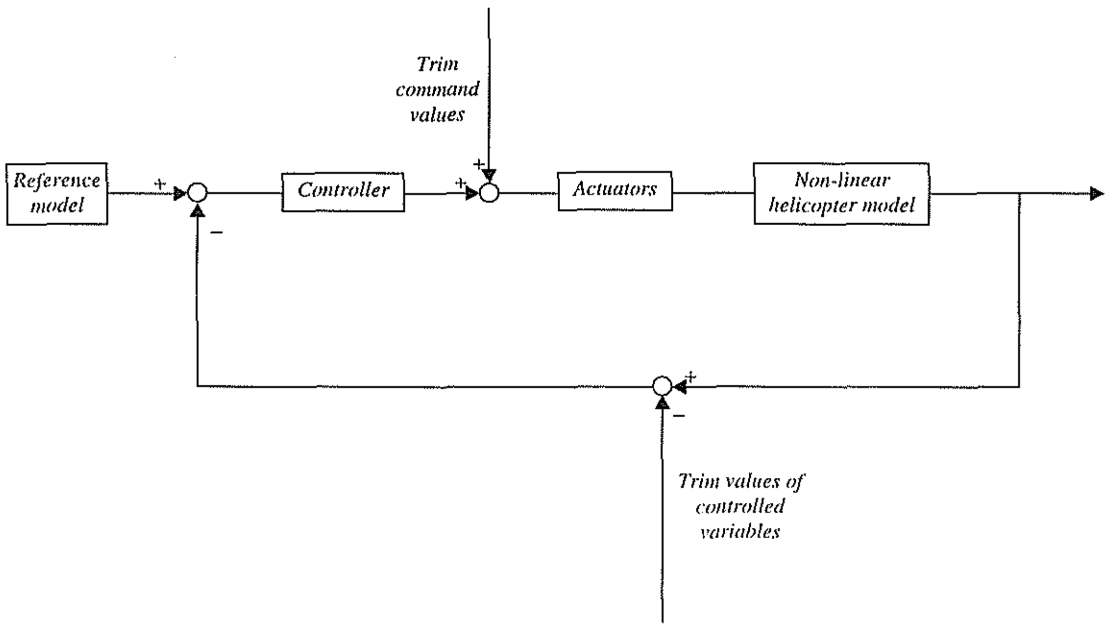

Figure 7 . Simulation scheme

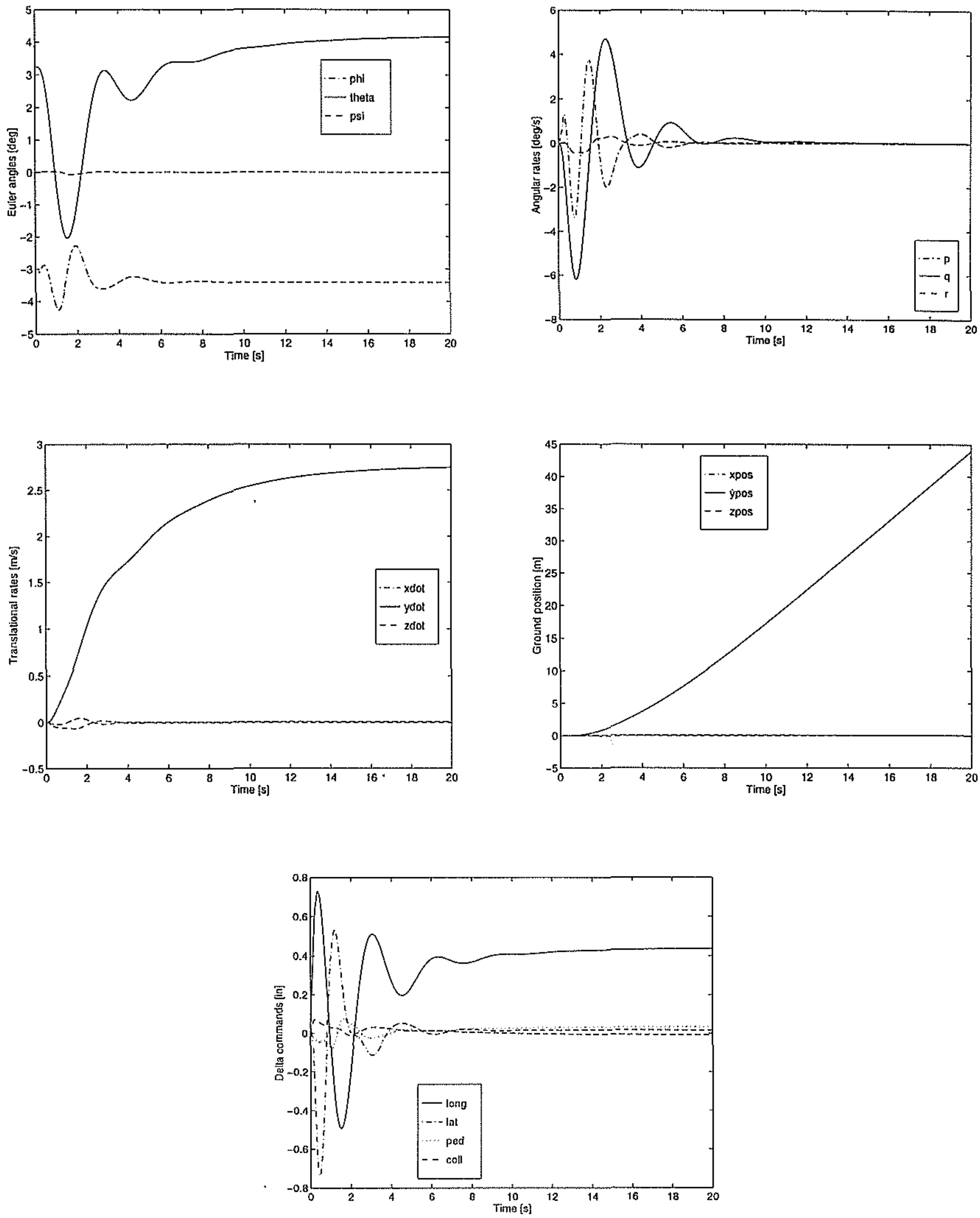

Figure 8 . Response to a longitudinal step input

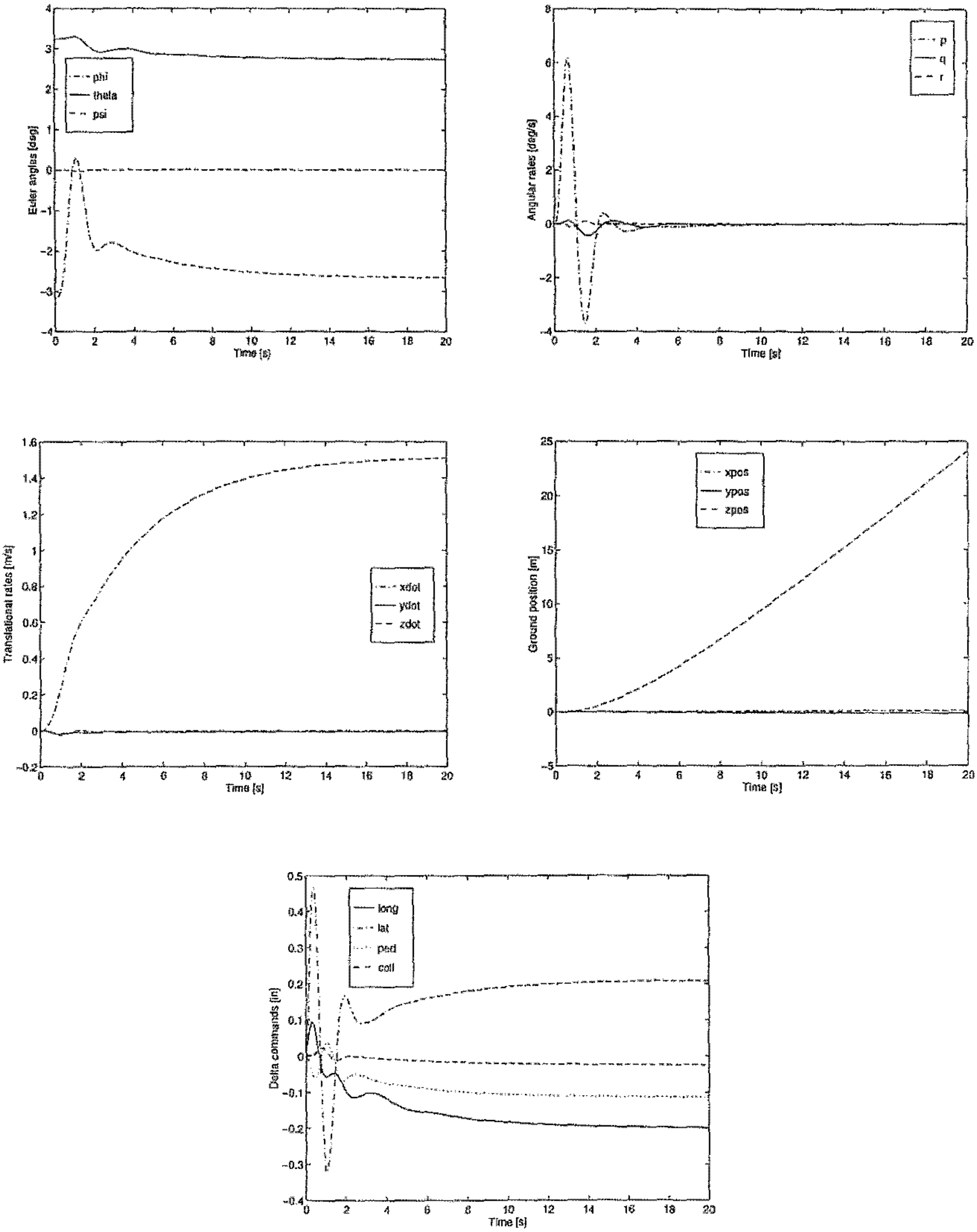

Figure 9 . Response to a lateral step input

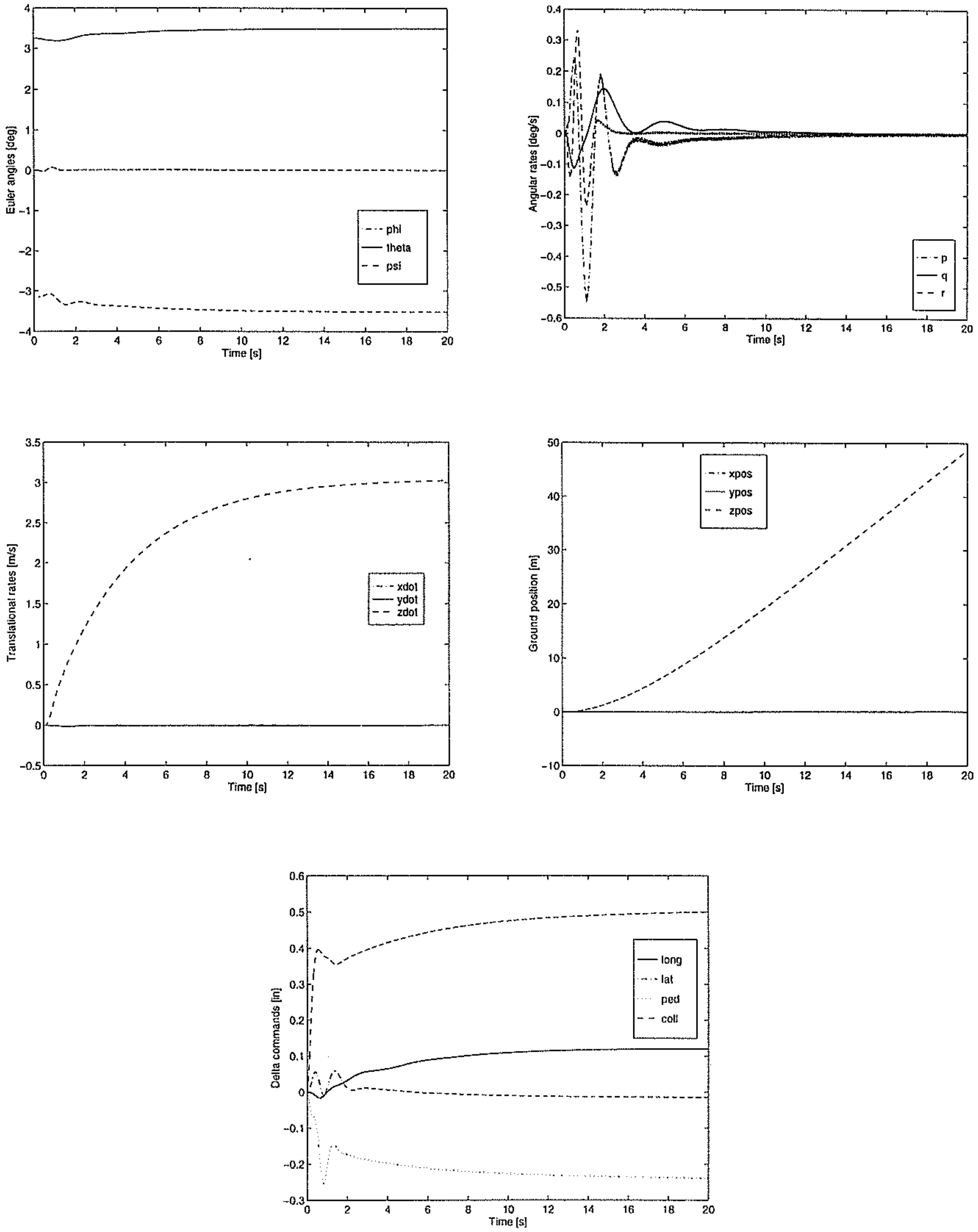

Figure 10 . Response to a collective step input

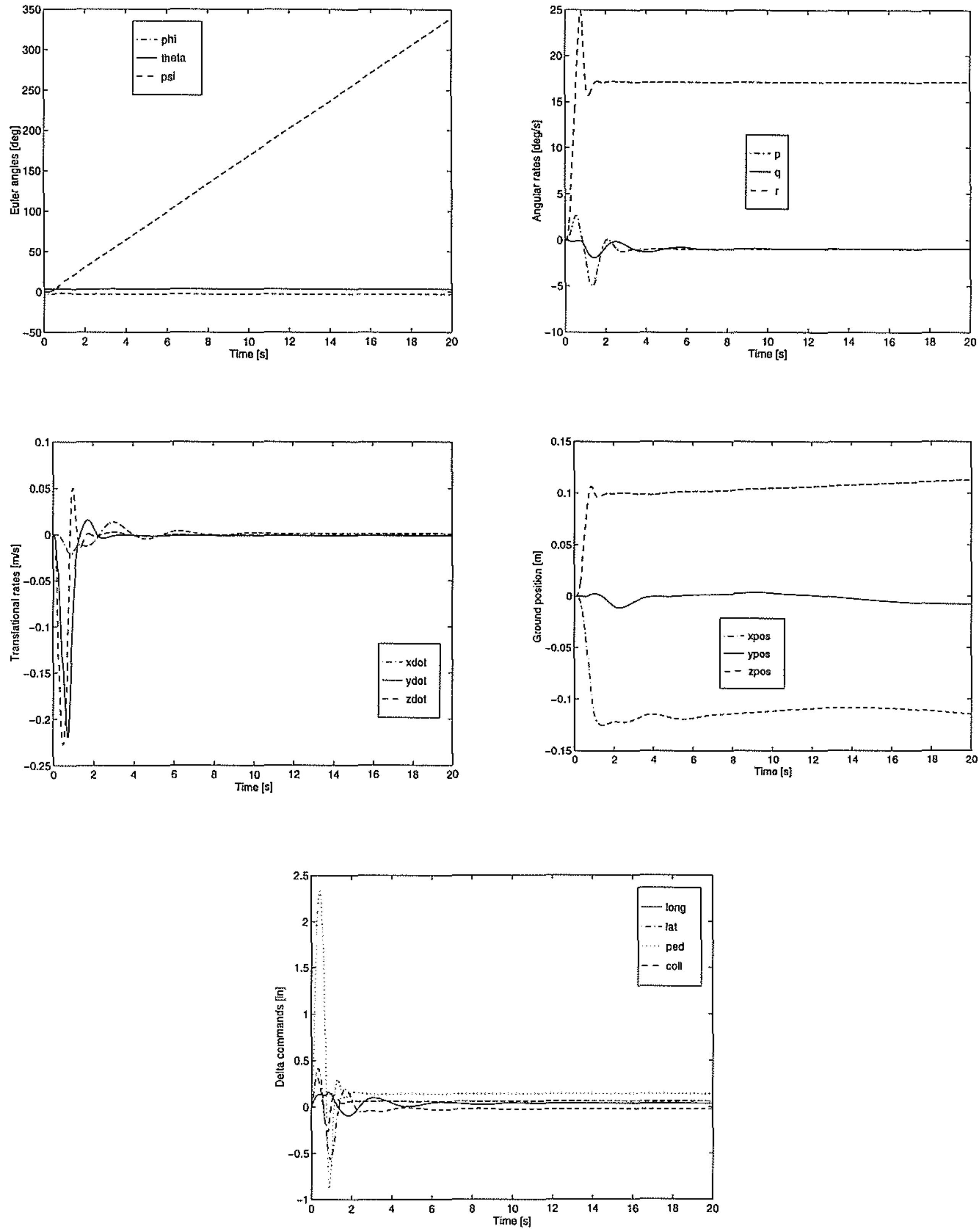

Figure 11 . Response to a pedal step input

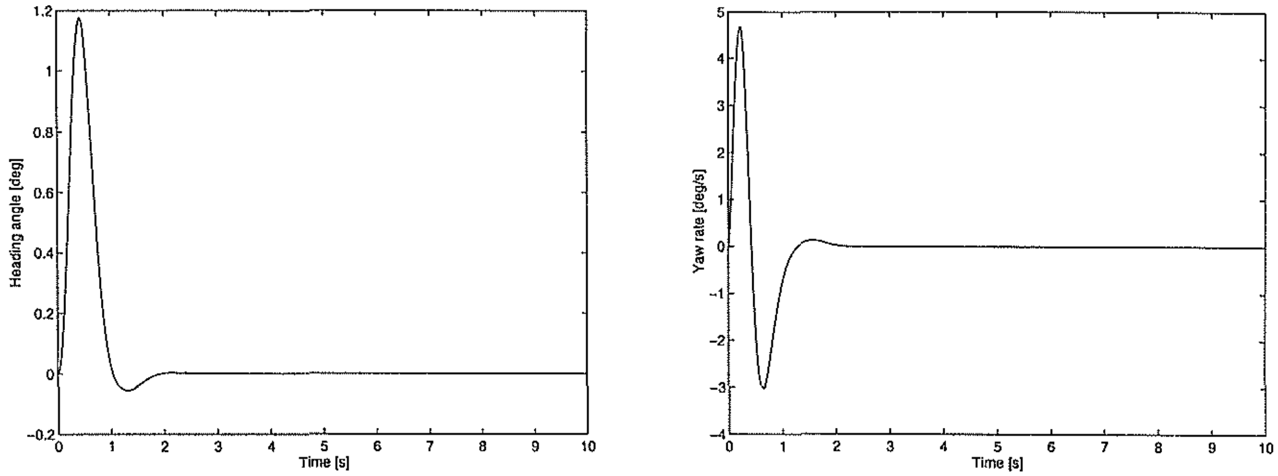

Figure 12 . Response to a disturbance

Conclusions

In this paper a structured singular value (μ) synthesis has been adopted to design a hover hold controller for the Agusta A-109 helicopter. The controller has been designed using the modelling following technique, minimising the weighted H_∞ norm of the error between the reference model and the augmented rotorcraft and maximising robustness to high frequency dynamics not included in the reduced order design model.

Simulations have been carried out using a non linear blade element model of the helicopter. These simulations demonstrate that the augmented helicopter meets the military rotorcraft handling qualities design specification (ADS-33C) and guarantees high level of decoupling between axes.

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