Handling Qualities and Performance Aspects of the Simulation of Helicopters Flying Mission Task Elements

Dr. R. Bradley
Dr. D.G. Thomson

Department of Aerospace Engineering
University of Glasgow

Nomenclature

A, B, C	System, control and output matrices
API	Agility Performance Index
AR	Agility Rating
$\mathbf{f_c}$	vector of constraints
n_s, n_c	number of states and controls in API function
p, q, r	components of aircraft angular velocity in body axes
q_i, r_j	weighting constants for API
s	*characteristic dimension of a manoeuvre*
t_m	time taken to complete manoeuvre
$\mathbf{u}$	control vector
$\mathbf{u^*}$	ideal control vector
u, v, w	components of aircraft velocity in body axes
V_f	airspeed
$\mathbf{x}$	state vector
$\mathbf{x_1}$, $\mathbf{x_2}$	vectors of constrained and unconstrained states
$\mathbf{y}$	output vector
ϕ, θ, ψ	aircraft attitude angles
θ_0	*main rotor collective pitch angle*
θ_{1_s}, θ_{1_c}	longitudinal and lateral cyclic pitch angles
$\theta_{0_{tr}}$	tailrotor collective pitch angle

1. Introduction

The need to assess the overall handling qualities of a helicopter by its performance and handling characteristics in a range of typical manoeuvres has been recognised by the authors of the U.S. Handling Qualities for Military Rotorcraft [1]. As part of demonstrating compliance with these requirements, a set of standard manoeuvres, or Mission Task Elements (MTEs) has been defined and criteria for performance and handling have been specified. Of particular interest are the aggressive tasks, and work at Glasgow has been directed towards using inverse simulation [2] to provide an objective evaluation of both the performance and handling qualities aspects of helicopters when flying the aggressive MTEs.

The remainder of this paper continues by defining inverse simulation for the general case before concentrating on the helicopter flight path formulation employed at Glasgow. Next a number of established applications of inverse simulation are reviewed with the aim of identifying those areas which can contribute to an assessment of performance and handling. The later sections pull together this information and discuss those assessments which are achievable by current methods and what further studies are required before useful, objective assessments of performance and handling can be achieved by inverse simulation.

2. Inverse Simulation

2.1 Definition

The simulation exercise of calculating a system's response to a particular sequence of control inputs is well known. It is conveniently expressed as the initial value problem:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}); \quad \mathbf{x}(0) = \mathbf{x}_0, \tag{1}$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}), \tag{2}$$

where **x** is the state vector of the system and **u** is the control vector. Equation (1) is a statement of the mathematical model which describes the time-evolution of the state vector in response to an imposed time history for the control vector **u**. The output equation, (2), is a statement of how the observed output vector **y** is obtained from the state vector.

Inverse simulation is so called because, from a pre-determined output vector **y** it calculates the control time-histories required to produce **y**. Consequently, equations (1) and (2) are used in an implicit manner and, just as conventional simulation attaches importance to careful selection of the input **u**, inverse simulation places emphasis on the careful definition of the required output **y**.

2.2 Application to helicopter.

In the helicopter application discussed here, the state vector is $\mathbf{x} = [u\ v\ w\ p\ q\ r\ \phi\ \theta\ \psi]^T$ and the control vector is $\mathbf{u} = [\theta_0\ \theta_{1_s}\ \theta_{1_c}\ \theta_{0_{tr}}]^T$. The focus of the work at Glasgow is on manoeuvres that are defined in terms of motion relative to an Earth-fixed frame of reference so that the output equation is the transformation of the body-fixed velocity components into Earth axes. For a unique solution to the inverse problem it is necessary to add a further output, a prescribed heading or sideslip profile being the most appropriate choice. The four scalar constraints - three velocity components and one attitude angle - serve to define uniquely the four control axes of the helicopter. The sophistication of the modelling implied by the form of **f** in equation (1) is of central importance since the more complex the basic formulation, the more difficult it is to cast into a useful inverse form. The mathematical model used for this early work was Helistab [3]; Thomson and Bradley [2] have described a method for the unique solution of the inverse problem in this case. Current work at Glasgow University employs an enhanced model, Helicopter Generic Simulation (HGS), [4] which is embedded in the inverse algorithm Helinv. The main features of HGS include a multiblade description of main rotor flapping, dynamic inflow, an engine model, and look-up tables for fuselage aerodynamic forces and moments. The host package, Helinv, incorporates several sets of pre-programmed manoeuvre descriptions which are required as system outputs from the simulation. In fact, the manoeuvres are essentially the input into the simulation and much of the value of Helinv lies in the scope and validity of the library of manoeuvre descriptions which have been accumulated. They include those relating to Nap of the Earth [5], Air-to-air Combat, Mission Task Element [6], and Off-shore Operations. There is also a facility for accessing flight test data. Some examples of these manoeuvres are discussed in the following sections of the paper.

2.3 Linear Formulation

The linear forms of equations (1) and (2) are useful for establishing the general principles of the inverse problem:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}, \tag{3}$$

$$\mathbf{y} = \mathbf{C}\,\mathbf{x} \tag{4}$$

where A, B and C are the system, control and output (or in this context constraint) matrices respectively. It can be shown that the inverse problem for this linear system may be solved uniquely provided:

$$\dim(\mathbf{u}) = \dim(\mathbf{y}) = \text{rank}\begin{bmatrix} CB \\ CAB \\ \dots \\ CA^{n-1}B \end{bmatrix} \tag{5}$$

The more terms that are needed to establish the full rank then the higher the derivatives of **y** that are necessary to calculate the inverse solution. This has implications when defining the flight path; if the time derivatives of position, such as velocity and acceleration, are not continuous then the result may be the calculation of physically unacceptable forms of control response.

The condition expressed in equation (5) is a useful means of determining those constraints which lead to a unique solution for a given reference condition. It also has potential utility in exploring the effectiveness of auxiliary aerodynamic control surfaces or thrust augmentation since these latter features would lead to different forms of the control matrix B.

3. Applications of Inverse Simulation

Prior to the specific consideration of performance and handling qualities, it is valuable to review briefly the relevant areas in which inverse simulation has been applied. The purpose of such an exercise is to set out the current scope of the technique and to identify those properties which may contribute to the use of inverse simulation in quantifying performance and handling qualities.

3.1 Agility Rating

One of the earliest applications of inverse simulation was an attempt to quantify the agility of a given helicopter configuration through an Agility Performance Index (API) [7]. The difficulty of producing a general definition of the term agility is well known [8] but the API was based on the concept of installed agility, that is, it was dependant on the particular configuration of the helicopter and independent of any pilot model. This independence of a pilot model is a feature of the inverse formulation since it generates a precise piloting task and leaves no scope for other than ideal piloting of the helicopter. The API of a helicopter for a given manoeuvre is determined from the formula:

$$\text{API} = \sum_{i=1}^{n_s} q_i \int_0^{t_m} f(x_i(t))\,dt + \sum_{j=1}^{n_c} r_j \int_0^{t_m} g(u_j(t))\,dt \tag{6}$$

where t_m is the time taken to complete the manoeuvre, q_i and r_j are weighting constants related to state i and control j. The integers n_s and n_c are the number of states and controls to be included in the performance index. The functions $f(x_i(t))$ and $g(u_j(t))$ were selected to penalise large state and control deviations during the manoeuvre: for example,

$$f(x_i(t)) = \left[\frac{x_i(t) - x_{i_{trim}}}{x_{i_{max}} - x_{i_{trim}}}\right]^2$$

where $x_{i_{trim}}$ is the value of state i, in the steady flight condition at the entry to the manoeuvre, and $x_{i_{max}}$ is the maximum value of the state encountered during the manoeuvre. Using this definition low values of API (i.e. small control and state displacements) will imply good agility.

The problem of deciding which single manoeuvre would be used to determine a helicopter's agility was avoided by calculating an Agility Rating (AR) for each class of manoeuvre. Hence each helicopter configuration tested was awarded a series of ratings, one for each manoeuvre class (transient turn, hurdle-hop etc.), and an overall figure for the agility of the vehicle could be derived from these. Within each manoeuvre class the subject helicopter was tested over a series of geometrically similar manoeuvres of varying severity. Manoeuvre severity was varied by altering one of the geometrical flight path parameters, distance to an obstacle of fixed height in the hurdle-hop, for example, (s_{max} to s_{min}) and the constant velocity at which the manoeuvre was flown (V_{fmax} to V_{fmin}). Effectively a surface of API values was calculated for each manoeuvre, and the volume under this surface was taken to be the overall Agility Rating for the manoeuvre. The function used to derive an Agility Rating was defined as

$$AR = \int_{s_{min}}^{s_{max}} \int_{Vf_{min}}^{Vf_{max}} API \; ds \; dVf$$

As low values of API imply good agility, low values of AR also imply good agility. An example of the AR calculation is shown in Figure 1.

The obvious difficulty with such an approach is the appropriate choice of the weights q_i and r_j and, in practice, zero or unity were commonly employed in comparative studies of different helicopter configurations on the basis of whether it was felt that those quantities were significant or not in a particular manoeuvre. Nevertheless, despite this simplified approach, the work established the principle whereby different helicopters could be comparatively assessed for their agility over a range of standard manoeuvres by a reproducible simulation study.

3.2 Design Evaluation

Although ADS 33C [1], is directed towards handling qualities, it is unavoidable that the Mission Task Elements that form part of the aggressive task requirements contain a significant element of performance related criteria which refer to the particular configuration being flown. For example, it is hard to deny a performance element in the Rapid Slalom description: "Manoeuvre rapidly to displace the aircraft 15.2 m laterally from the centreline and immediately reverse direction to displace the aircraft 15.2 m on the opposite side of the centreline. Return to the centreline as quickly as possible. Maintain altitude within ±3 m; maintain airspeed at or above 60 knots; maximum bank angles should be at least 50 degrees". Therefore, the ability to confirm that an existing or projected design can satisfy the criteria, in a performance sense, over the full range of MTEs is of some significance.

In order to apply inverse simulation to the MTEs the first exercise is to convert their description into a flight-path trajectory description. Once this is done then inverse simulation can be used to compare the performance of different helicopter configurations in the execution of the tasks. Such work was reported in Reference 9 where a significant component of the study was the conversion of the task descriptions in ADS 33C into the full set of information required by the inverse method. Some, such as the acceleration / deceleration task, convert in a quite straightforward manner via a specified acceleration profile, but others only define the flight path implicitly. An example of the latter is the Pull-up/push-over which is described only in terms of the load factor profile and which needs the imposition of additional criteria to complete the flight-path definition. When the definition is complete, the availability of an inverse simulation enables a range of performance criteria of candidate helicopters to be investigated against configuration parameters - such as control limits, rotor stiffness and installed power. While it is recognised that these criteria may not be the primary considerations which drive the design of the helicopter, inverse simulation can quickly establish the performance limitations of a given design over the full range of MTEs.

Therefore, in this application, for a given set of flight-path definitions, the validity of the control responses is limited only by the authenticity of the helicopter model, and as will be reiterated in section 5, the performance information produced by inverse simulation gives a direct evaluation of the subject configuration

3.3 Control Strategy

A direct application of inverse simulation can be found in establishing the control strategy to be adopted in order to fly a particular manoeuvre [10]. An example is shown in Figure 2 which shows an assumed velocity profile for a side-step by a typical battlefield/utility type of helicopter. Also shown, in Figure 3, are the control movements, in terms of blade pitch, required to fly this manoeuvre. They show distinct features: the initial broad pulse of lateral cyclic to roll the helicopter into the manoeuvre with the collective increasing at the same time to provide the lateral acceleration. Mid way through the manoeuvre there is a larger pulse of lateral cyclic which acts to turn the rotor so that it can now decelerate the helicopter. A transient drop in collective is also noted at this point which is explained by the requirement to remain at constant height as the helicopter rotates through the vertical position. Such qualitative discussions are easy to construct from the results of the simulation but they have proved to be difficult to establish a priori. In the case of the side-step described here, flight-path data is available from test flights to drive a simulation and one can observe the similarity between the real and modelled velocity profiles and the basic control strategy as described above. A study of this type was reported in Reference 6 and it is clear that for this application inverse simulation can provide useful information once valid flight-path models are available.

The sequencing and co-ordination of control movements made during the execution of a manoeuvre are potentially a significant contributory factor in the assessment of handling qualities therefore one may expect the type of investigation described in this section to be recalled in the discussion concerning handling qualities in section 6 below.

4. Inverse Simulation - Rationale

There appears to be some reluctance to afford to inverse simulation the same credibility that is attached to conventional simulation techniques. Often this reluctance is expressed as doubting the realism of the manoeuvre descriptions. Their description in terms of smooth profiles of velocity components, angles of climb and load factors is seen as an unjustifiable over simplification - yet what evidence there is [6] suggests that the assumed profiles are realistic. Moreover, the detail of the profile tends to be of secondary importance once the gross features of the manoeuvre are captured.

The essential difference between inverse and conventional simulation can be found in descriptions of the corresponding flight tasks. Consider a helicopter initially in trimmed flight at a safe distance from the ground. The injection by the pilot of a standard test input, such as a doublet, on one of the channels - without regard to the subsequent behaviour of the helicopter - relates to the open loop situation of conventional simulation, as described by equations (1) and (2). Now consider the alternative situation where the pilot is flying a specific tracking manoeuvre at a constant altitude and close to the ground. The pilot works hard to follow the track precisely and thus closes the loop between the flight path and the helicopter's controls. The tight coupling between the flight path and the controls provides a feedback which is the essence of inverse simulation. It formulates the limiting case of an ideal pilot. The linear description of equations (1) and (2) expressed in equations (3) and (4) illustrates this property since the condition of equation (5) allows the control **u** to be expressed in the form:

$$u = D\,x + E\,z \qquad (7)$$

where D and E depend on C, B and A and z can be obtained from y and its derivatives. The substitution of this form into equation 1 gives:

$$\dot{x} = (A + BD)x + (BE)\,z \qquad (8)$$

Appendix 1 shows how the specific constrained dynamics described in [7] are equivalent to (7) and hence (8). The different system matrices of equations (3) and (8) indicate the possibility of distinguishing between open loop and constrained dynamics. Table 1 shows how the eigenvalues of the two systems differ for a typical helicopter of the battlefield/utility type trimmed at 80 knots level flight. In particular, the introduction of four zero eigenvalues corresponding to the four constraints imposed on the dynamics should be noted in the case of the flight-path constrained system used at Glasgow. Other authors [e.g. 11] have used inverse simulation with constraints applied to the components p, q and r, of the angular velocity and the normal velocity component w in order to drive an inverse simulation with flight data or as part of a controller. In this kinematically constrained system there are only two degrees of freedom remaining - effectively those of the u and v velocity components. An example of the eigenvalues for such a system is also shown in Table 1.

Open Loop System
-10.5442
-0.6539 ± 2.2548 i
-3.1993
0.1333 ± 0.3759 i
-0.0259
-0.4092

Flight-path Constrained System
0.0027 ± 8.6651 i
-0.2399 ± 5.4186 i
0.0
0.0
0.0
0.0

Kinematically Constrained System
-0.1070
-0.0234
0.0
0.0
0.0
0.0

Table 1. Eigenvalues of Battlefield/Utility Helicopter at 80 kts

The fact that this distinctive behaviour of the constrained system has been observed in flight data from Near-Earth trials [12] gives strong credence to the validity of the inverse approach in this context. On the other hand experience has shown that the use of flight data from test inputs applied during Up-and-away flight is difficult to relate to the results of flight-path constrained simulation.

The conclusion may be drawn that the inverse method is currently particularly suited to those applications for which it is designed - that is for well-defined manoeuvres close to the ground where the pilot can use the full range of cues to achieve a precise flight path. It is particularly valuable in such applications because, as has been noted elsewhere [13], the flight path constraint ensures that the helicopter does not drift into unrepresentative flight regimes during the manoeuvre - as can be the case with conventional simulations being driven by measured control inputs.

5. Performance

The authors of ADS-33C [1] in their introduction note the potential for assessing handling qualities by simulation. It is now possible to examine the conclusions above and assess the potential for inverse simulation being a useful tool for evaluating the performance of a given helicopter configuration. The necessary ingredients for an inverse simulation are a completely defined manoeuvre trajectory and an appropriate helicopter model. Once these are available the algorithm produces a full set of the control, kinematic dynamic and performance information. One may conclude that since the helicopter data is likely to be the primary data that is available - as it is the helicopter itself which is the candidate for assessment - then the remaining information is that relating to trajectory definition. While these can be quite straightforward to represent mathematically and an extensive library of manoeuvre descriptions is now available, the need for comparisons with flight tests grows as the range and scope of manoeuvres widens. Therefore, given a menu of suitable flight path trajectories, the application of inverse simulation should be a valuable routine exercise: an example of which is described in the following section.

5.1 Example of the Use of Inverse Simulation in Performance Studies

In this example we shall examine the effect of increasing mass on a helicopter's performance whilst flying a mission task element. Two configurations of the same aircraft, of the battlefield/utility type, have been chosen. The baseline configuration has a mass of 3500kg and it is assumed that its centre of gravity is directly below the rotor hub. This is compared with the same aircraft carrying a load of 500kg internally which is assumed to have shifted the centre of gravity a distance of 15cm aft of the rotor hub. All other parameters in the HGS model are identical for both cases.

Possibly the MTE where mass is most critical is the transient turn, and it is the performance of the example helicopter flying this manoeuvre which will be studied. The description of this manoeuvre given in ADS-33C specifies a 180 degree heading change initiated from a velocity of 120 knots, altitude is to be maintained within tight limits, and the manoeuvre is to be completed within 10 seconds. This information on its own is insufficient to create the mathematical representation of the manoeuvre which is required by Helinv, hence a few extra assumptions about how the manoeuvre will be flown have to be made. It is assumed that, from a rectilinear flight trajectory, the pilot will roll the aircraft to an appropriate bank angle, then hold this angle until the 180 degree heading change is approached, at which point the aircraft will be rolled in the opposite direction to achieve straight line flight. If it is further assumed that constant altitude is desirable, and that to perform the task as quickly as possible, the entry speed of 120 knots is maintained, then the turn rate profile shown in Figure 4 is sufficient to obtain the required mathematical representation. A full description of how the flight path can be obtained from the turn rate profile and airspeed is given by Thomson and Bradley, [5], but the basic principle involves varying the maximum turn rate until the manoeuvre is completed within 10 seconds. This is achieved with a turn radius of 155m and the resulting maximum normal load factor is 2.75. Note that the fraction of the manoeuvre spent in the transients must also be specified and in this case a value of 15% was chosen after examination of flight test data from similar manoeuvres [6].

Having defined the helicopter configurations and specified the manoeuvre, it is possible to perform inverse simulations of these configurations flying it. The control time histories generated are shown in Figure 5, from which the overall control strategy can be deduced. The manoeuvre is initiated by a pulse in lateral cyclic to roll the aircraft, note that there is little difference in the amount required between the two configurations. As the aircraft rolls, as shown in Figure 6, collective (and hence thrust) must be added to maintain altitude. There is also a forward motion of the longitudinal stick (denoted by negative longitudinal cyclic) to maintain constant forward speed. The manoeuvre is performed without sideslip and tail rotor collective is used to ensure this condition is met. The initial pulse in lateral cyclic is opposed by a similar pulse in tailrotor collective which then increases beyond its level flight trim position to offset the extra torque produced by increased main rotor collective. The main differences between the time histories of the two aircraft lie in the collective and longitudinal plots. The baseline configuration requires less collective firstly because it is lighter, but one must also consider the effect of shifting the centre of gravity aft of the rotor hub. This produces a nose up pitching moment which must be countered by forward stick if velocity is to be maintained, which explains the 4.5 degrees of extra forward longitudinal cyclic required by the loaded configuration. The longitudinal tilt of the thrust vector is in addition to the lateral tilt required for rolling, and hence is a contributory factory in the 2.5 degrees of extra collective required by the heavy configuration. Examination of Figure 6 shows that the roll angle history which was suggested by the manoeuvre definition is obtained, and the maximum bank angle reached was approximately 70 degrees, with roll rates of approximately 70 degrees/second encountered in the transients.

The advantage of this method becomes apparent when it is realised that the collective limit of this configuration is 20 degrees. Consequently on examination of the collective time history in Figure 5 it is clear that the loaded configuration is close to the limiting case for this manoeuvre. It then follows that the limiting case for various aircraft masses and centre of gravity positions could be obtained by repeated inverse simulation of the manoeuvre thereby allowing the aircraft configuration envelope for this MTE to be derived. This of course could be extended to include the whole series of MTEs.

5.2 Future Needs

For performance studies the issue of future needs is clear cut as far as inverse simulation is concerned. One may assume that a helicopter model of appropriate validity is available from the initial design activity. For a performance investigation, the facilities offered by flight mechanics models such as HGS have so far proved to be adequate. The sole remaining unknown in a performance study is the set of manoeuvres over which to evaluate the design. Although several of the descriptions in Reference 6 have been validated against flight data there are many manoeuvres which need flight tests conducted and corresponding formal descriptions derived from the results.

6. Handling Qualities

The application to handling qualities is not so apparent. Certain steps in this direction have been made. Whalley [8] uses a simplified helicopter model and piloted flight simulation to make limited headway. McKillip [14] also used an inverse model in controller design to minimise pilot control activity. A major difficulty is that handling qualities have traditionally been quantified by consolidating a number of subjective evaluations. The aim in any simulation activity is to remove the subjective nature of the assessment and this suggests the formulation of a Handling Qualities Index (HQI). The problem of representative flight path definition discussed in section 5 remains but it should be clear that the information produced when assessing performance is strongly relevant to handling qualities. An obvious example is the set control movement profiles. They can be processed to give rates of change, auto and cross correlations etc. all of which might be expected to have a bearing on handling qualities. In addition, the attitude angles can significantly modify the cue environment of the pilot, so one may conclude that the data gathered during inverse simulation at least contains much of the data relating to handling qualities. An index of the same form as used for agility, (6) above, can be formed but there would probably be more terms and merely taking arbitrary values for the weights would undermine its credibility until there had been extensive validation.

Some technical problems concerning the use of inverse simulation in this context remain. First there is the problem of hitting control limits during a manoeuvre - currently the method merely notes the exceedances. There are possible options for selecting a subsequent control strategy which depend on minimising the distance from the required trajectory, but the method currently being considered would remove the limited control from those which can be varied and uses the criterion from equation (5) to determine which constraints may be retained.

Another factor is related to the ideal piloting required by the inverse method. The modification of the inherent dynamics of the helicopter by constrained manoeuvring flight is of considerable importance to the handling as perceived by the pilot. If the helicopter becomes significantly unstable in one of its modes one would expect that this would become a factor in the handling qualities assessment. But would, in practice, the pilot actually induce such instability by determinedly adhering to the defined trajectory ? It seems more likely that he would reduce his demands on the system until the problem became manageable even if this meant some deviation from the desired flight path. Therefore the pilot would become less than ideal and the inverse formulation of the feedback (5) would no longer apply. This relaxation of the constraint can be illustrated by including a variable lag between the requested and actual control vector and write the linear system as:

$$\dot{\mathbf{x}} = A\,\mathbf{x} + B\,\mathbf{u} \tag{9}$$

$$\mathbf{u}^* = D\,\mathbf{x} + E\,\mathbf{f} \tag{10}$$

$$\dot{\mathbf{u}} = k\,(\mathbf{u}^* - \mathbf{u}) \tag{11}$$

where k is a scalar gain which is assumed the same for all axes. The vector $\mathbf{u}^*$ is the ideal control which is lagged by (11) into the actual control $\mathbf{u}$. The extremes of open loop and inverse formulation are equivalent to $k = 0$ and $k = \infty$ respectively. Figure 7 shows how the eigenvalues migrate as k varies between its limits. Development of such an approach offers the possibility of investigating realistic 'nearly' inverse simulations.

7. Conclusions

The paper has discussed the scope of current work in inverse simulation and considered its potential contribution to performance and handling qualities evaluation. Specific conclusions can be made:

(a) Current mathematical models, such as HGS, are adequate for basing inverse flight mechanics studies on.

(b) Flight-path constrained inverse simulation is particularly suited to tasks where the pilot is cued by the ground, such as in NOE manoeuvres.

(c) Flight tests should be made to validate the flight-path models currently being developed.

(d) Handling qualities assessment by inverse simulation presents a number of technical challenges to the flight dynamicist and modeller.

(e) Performance-assessment by inverse simulationpresents only the problem of valid flight path modelling.

(f) The rewards of being able to allocated a handling qualities rating to a configuration simply through a simulation study are high. Such a rating has the potential of being able to capture a significant element of the information encompassed in the ratings derived from piloted tests.

Acknowledgements

The author wishes to thank The Royal Society for their continued support of this work. The co-operation Dr Gareth Padfield of the Defence Research Agency, RAE Bedford, is also appreciated, with particular reference to the use of look-up tables and helicopter configurational data.

References

1. Anon, Aeronautical Design Standard, Handling Qualities Requirements for Military Rotorcraft. ADS-33C, August 1989.

2. Thomson, D.G., Bradley, R., Development and Verification of an Algorithm for Helicopter Inverse Simulation, Vertica, Vol. 14, No. 2, May 1990.

3. Padfield, G.D., A Theoretical Model for Helicopter Flight Mechanics for Application to Piloted Simulation, Royal Aircraft Establishment, TR 81048, April 1981.

4. Thomson, D.G., Development of a Generic Helicopter Mathematical Model for Application to Inverse Simulation, University of Glasgow, Dept. of Aerospace Engineering Internal Report No. 9216, June 1992.

5. Thomson, D.G., Bradley, R., Modelling and Classification of Helicopter Combat Manoeuvres, Paper No. 5.9.1, Proceedings of the 17th ICAS Conference, Sept. 1990.

6. Thomson, D.G., Bradley, R., Mathematical Definition of Helicopter Manoeuvres, University of Glasgow, Dept. of Aerospace Engineering Internal Report No. 9225, June 1992.

7. Thomson, D.G., An Analytical Method of Quantifying Helicopter Agility, Paper 45, Proceedings of the 12th European Rotorcraft Forum, Garmisch-Partenkirchen, Federal Republic of Germany, September 1986.

8. Whalley, M.S., Development and Evaluation of an Inverse Solution Technique for Studying Helicopter Maneuverability and Agility, NASA TM 102889, July 1991.

9. Thomson, D.G., Bradley, R., The Use of Inverse Simulation for Conceptual Design, 16th European Rotorcraft Forum, Glasgow, September 1990.

10. Thomson, D.G., Bradley, R., An Investigation of Pilot Strategy in Helicopter Nap-of-the-Earth Manoeuvres by Comparison of Flight Data and Inverse Simulations, Proceedings of the Royal Aeronautical Society Conference: Helicopter Handling Qualities and Control, London, November 1988.

11. Von Grunhagen, W., Inverse Simulation - A Tool for the Validation of Simulation Programs - First results. DLR Technical Note (To be published)

12. Thomson, D.G., Bradley, R., Prediction of the Dynamic Characteristics of Helicopters in Constrained Flight, The Aeronautical Journal, Vol. 94, No. 940, Dec. 1990.

13. Bradley, R., Padfield, G.D., Murray-Smith, D.J. Validation of Helicopter Mathematical Models. IMC Symposium on Model Validation, London 1989.

14. McKillip, R.M., Perri, T.A., Helicopter Flight Control System Design and Evaluation for NOE Operations Using Controller Inversion Techniques, 45th Annual Forum of the American Helicopter Society, Boston, May 1989.

Appendix : Linear Representation of the Inverse Problem

Reference [12] considers the linear helicopter system

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

where $\mathbf{x}$ = the state vector = $[u\ v\ w\ p\ q\ r\ \phi\ \theta]^T$,
$\mathbf{u}$ = the control vector = $[\theta_0\ \theta_{1_s}\ \theta_{1_c}\ \theta_{0_{tr}}]^T$,

and A and B are the system and control matrices. To recast this linear system in the inverse form the state vector is partitioned into two sub-vectors, $\mathbf{x}_1$ containing those variables which are strongly influenced by the manoeuvre constraints and $\mathbf{x}_2$, containing those which are not. For the flight path constraints described earlier, the strongly influenced states will be the three translational velocities u, v, w (through the requirement to follow a defined trajectory) and yaw velocity, r (through the constraint on heading). This gives the partitioned system

$$\left[\begin{array}{c}\dot{\mathbf{x}}_1 \\ \hline \dot{\mathbf{x}}_2\end{array}\right] = \left[\begin{array}{c|c}A_{11} & A_{12} \\ \hline A_{21} & A_{22}\end{array}\right]\left[\begin{array}{c}\mathbf{x}_1 \\ \hline \mathbf{x}_2\end{array}\right] + \left[\begin{array}{c}B_1 \\ \hline B_2\end{array}\right][\mathbf{u}]$$

where $\mathbf{x}_1 = [u\ v\ w\ r]^T$, and $\mathbf{x}_2 = [p\ q\ \phi\ \theta]^T$. Expanding this into two linear simultaneous equations, then, from the first of them, the control vector is given by

$$\mathbf{u} = B_1^{-1}\,[\dot{\mathbf{x}}_1 - A_{11}\,\mathbf{x}_1 - A_{12}\,\mathbf{x}_2] \qquad \text{(A1)}$$

and, by substitution into the second, the unconstrained states can be expressed in terms of the constraint influenced states

$$\dot{\mathbf{x}}_2 = [A_{22} - (B_2B_1^{-1})\,A_{12}]\,\mathbf{x}_2 + [\{A_{22} - (B_2B_1^{-1})\,A_{12}\}\,\mathbf{x}_1 + (B_2B_1^{-1})\,\dot{\mathbf{x}}_1] \qquad \text{(A2)}$$

In Reference12 the constraint vector is written in the form

$$\mathbf{f}_c = [x_e\ y_e\ z_e\ \psi]^T$$

then it can be shown [12] that

$$\dot{\mathbf{x}}_1 = T_1\,\bar{\mathbf{f}}_c + T_2\,\dot{\mathbf{x}}_2 + T_3\,\mathbf{x}_2 + T_4\,\mathbf{x}_1 \qquad \text{(A3)}$$

where T_1 - T_4 are matrices composed of functions of the trim values of the states. Substitution of equation (A2) into equation (A3) to eliminate $\dot{\mathbf{x}}_2$ yields

$$\dot{\mathbf{x}}_1 = S_3\,\bar{\mathbf{f}}_c + S_2\,\mathbf{x}_2 + S_1\,\mathbf{x}_1$$

where

$$S_1 = [I - T_2(B_2B_1^{-1})]^{-1}\,[T_4 + T_2(A_{21} - (B_2B_1^{-1})A_{11}],$$

$$S_2 = [I - T_2(B_2B_1^{-1})]^{-1}\,[T_3 + T_2(A_{22} - (B_2B_1^{-1})A_{12}],$$

and

$$S_3 = [I - T_2(B_2B_1^{-1})]^{-1}\,[T_1].$$

Finally, substitution in (A1) gives:

$$\mathbf{u} = B_1^{-1}\,[(S_1 - A_{11})\,\mathbf{x}_1 - (S_2 - A_{12})\,\mathbf{x}_2 + S_3\,\bar{\mathbf{f}}_c]$$

or, equivalently,

$$\mathbf{u} = [\, B_1^{-1}\ (S_1 - A_{11}) : B_1^{-1}\ (S_2 - A_{12})]\ \mathbf{x} + S_3\ \bar{\boldsymbol{\xi}}_c$$

which is of the required form:

$$\mathbf{u} = D\ \mathbf{x} + E\ \mathbf{z}$$

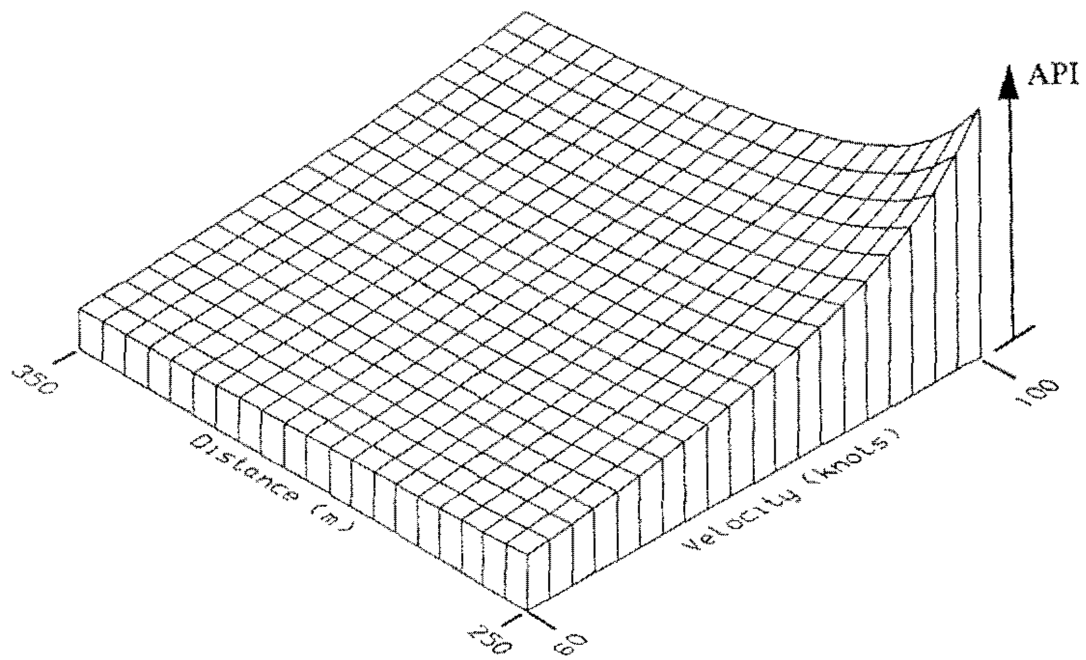

Figure 1 : Typical Agility Surface for the Calculation of an Agility Rating

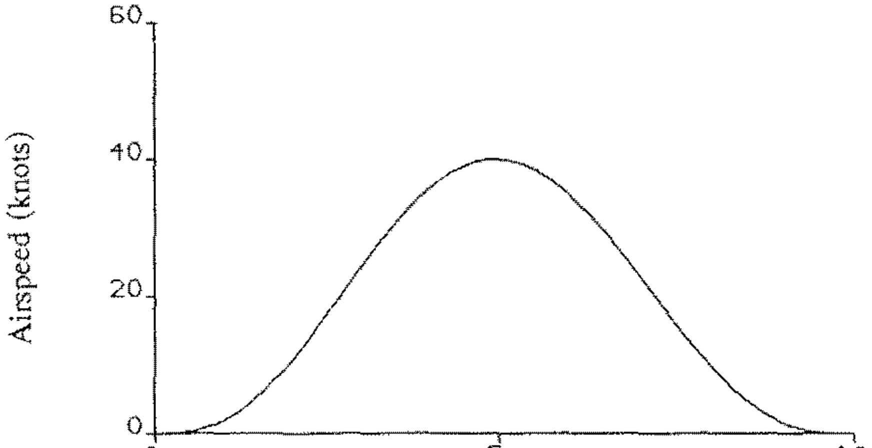

Figure 2 : Modelled Velocity Profile for a Side-step Manoeuvre

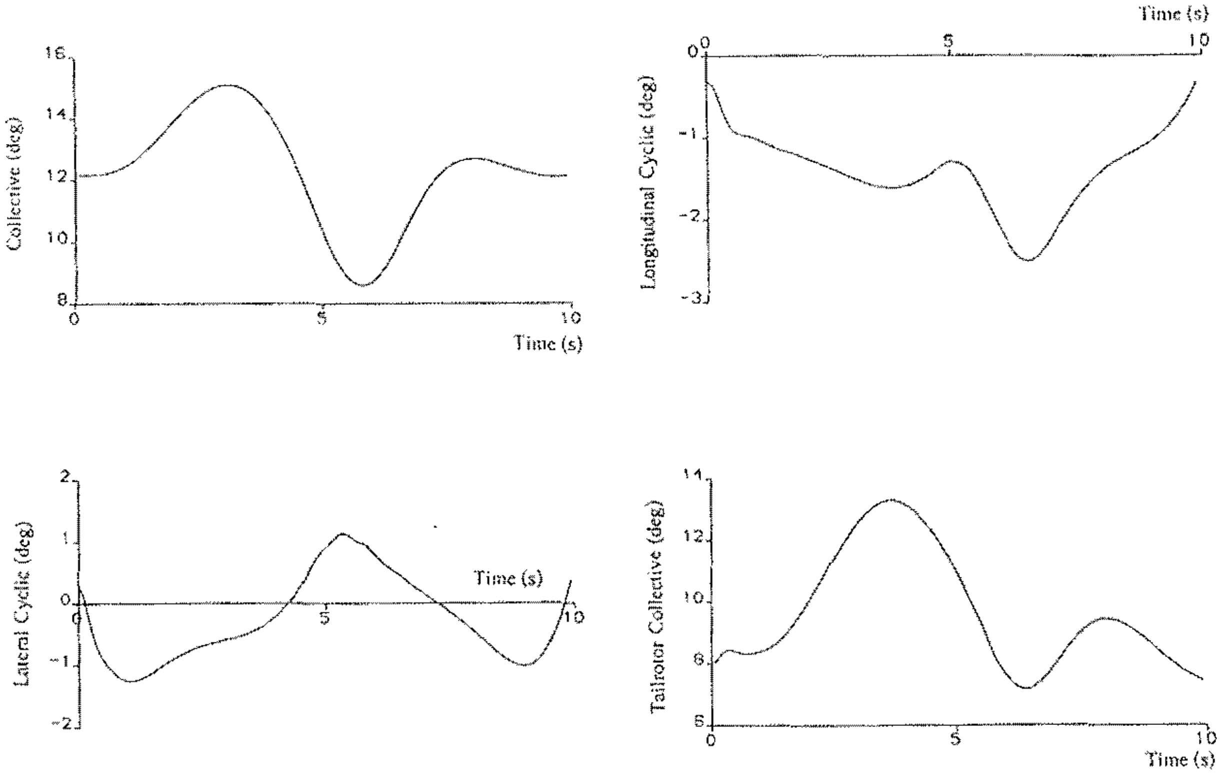

Figure 3 : Simulated Control Time Histories for Battlefield/Utility Helicopter Flying a Side-step Manoeuvre

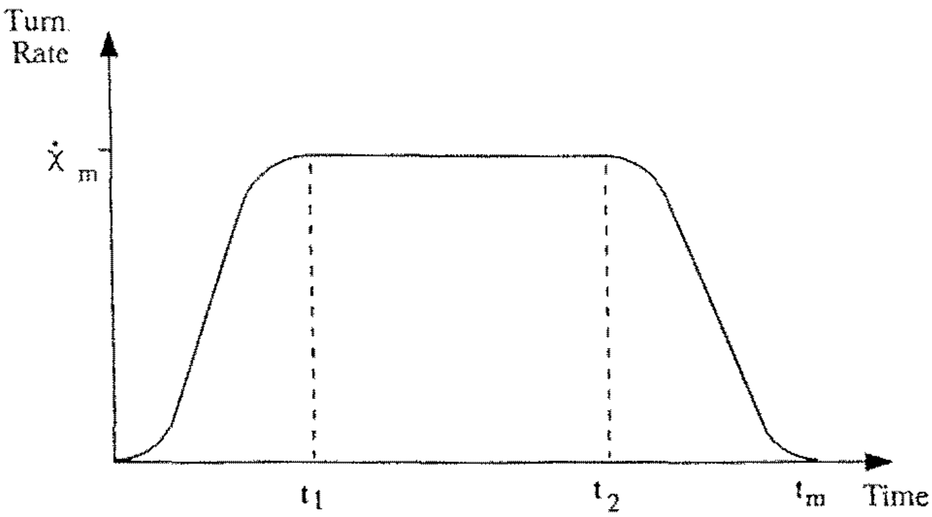

Figure 4 : Assumed Turn Rate Function for a Transient Turn Mission Task Element

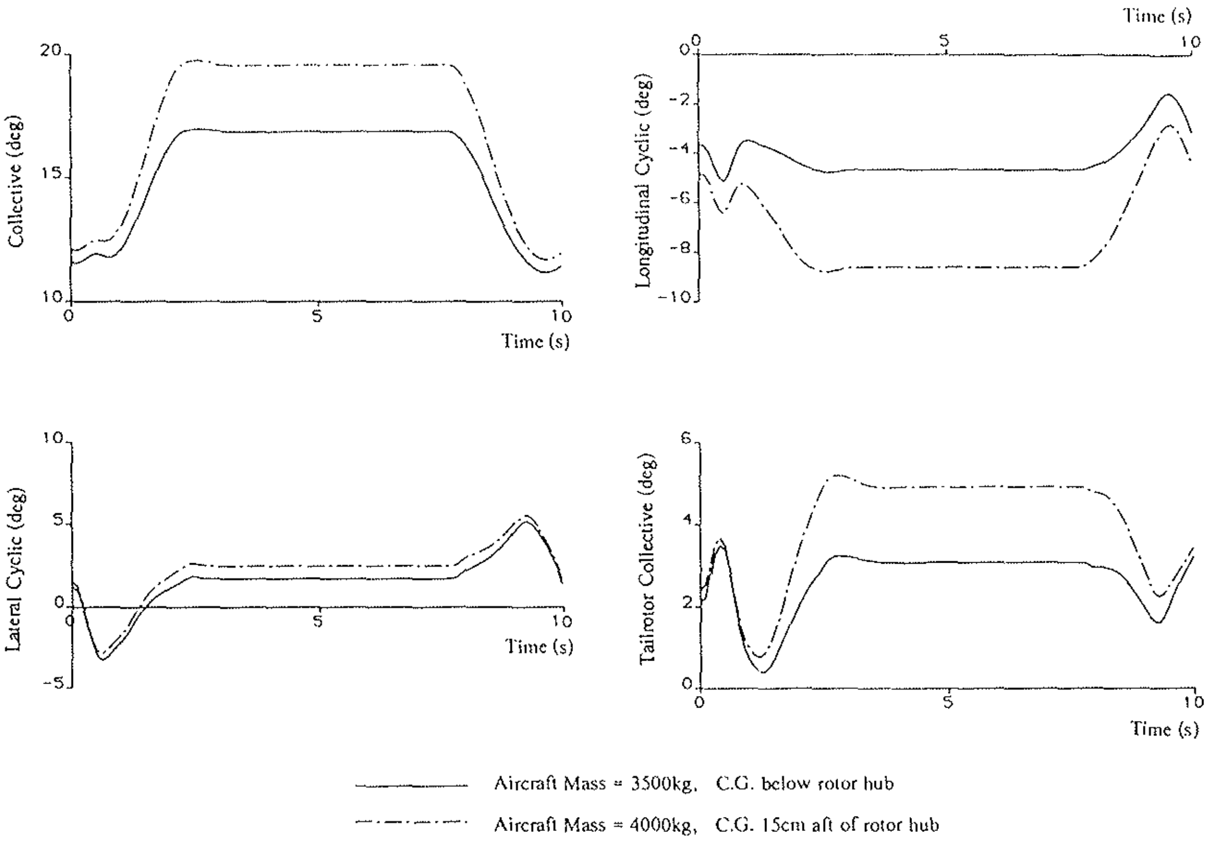

Figure 5 : Simulated Control Time Histories for a Battlefield/Utility Helicopter Flying a Transient Turn Mission Task Element

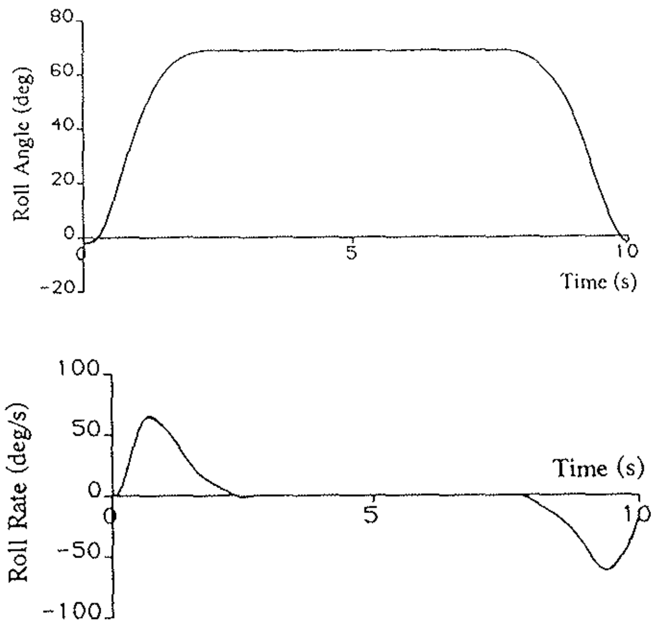

Figure 6 : Simulated Roll Angle and Roll Rate for a Battlefield/Utility Helicopter Flying a Transient Turn Mission Task Element

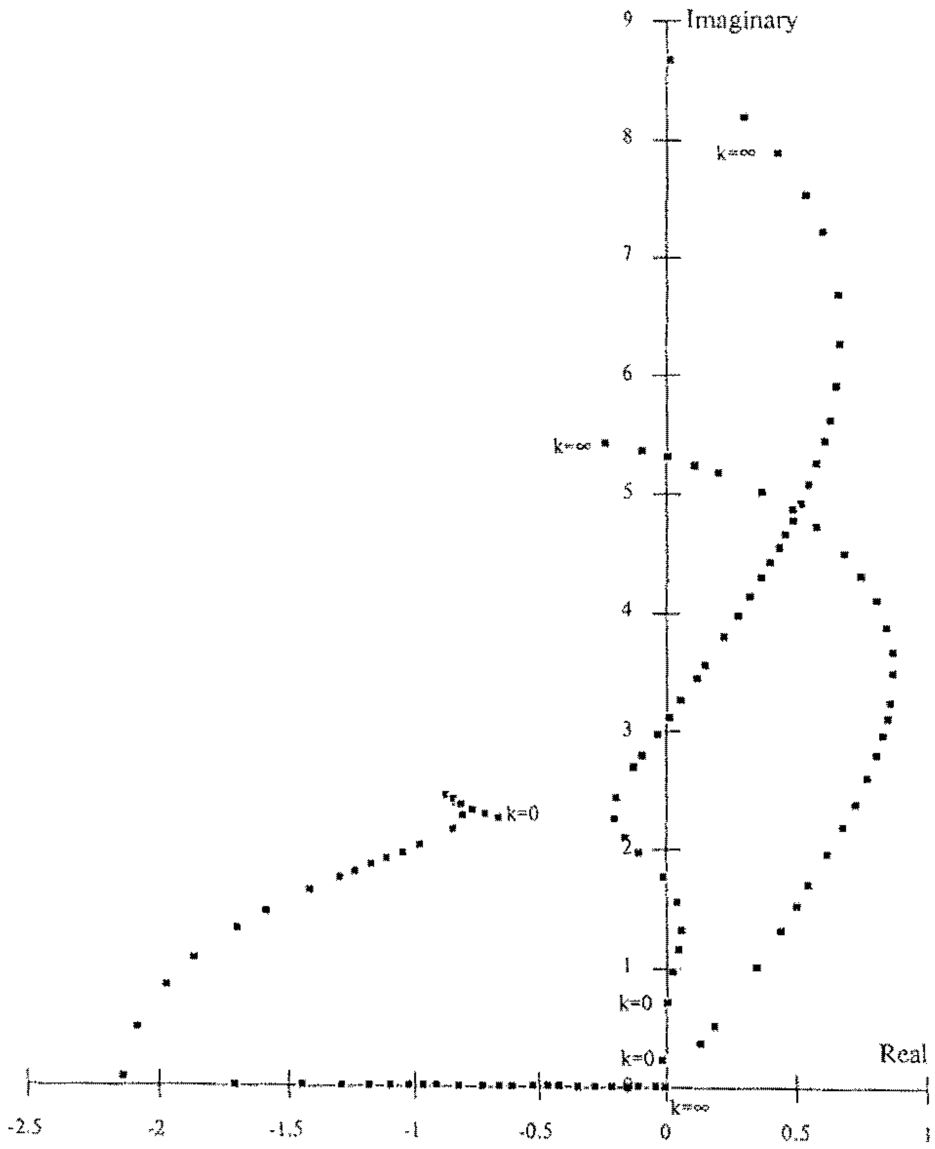

Figure 7 : Migration of Eigenvalues with Gain